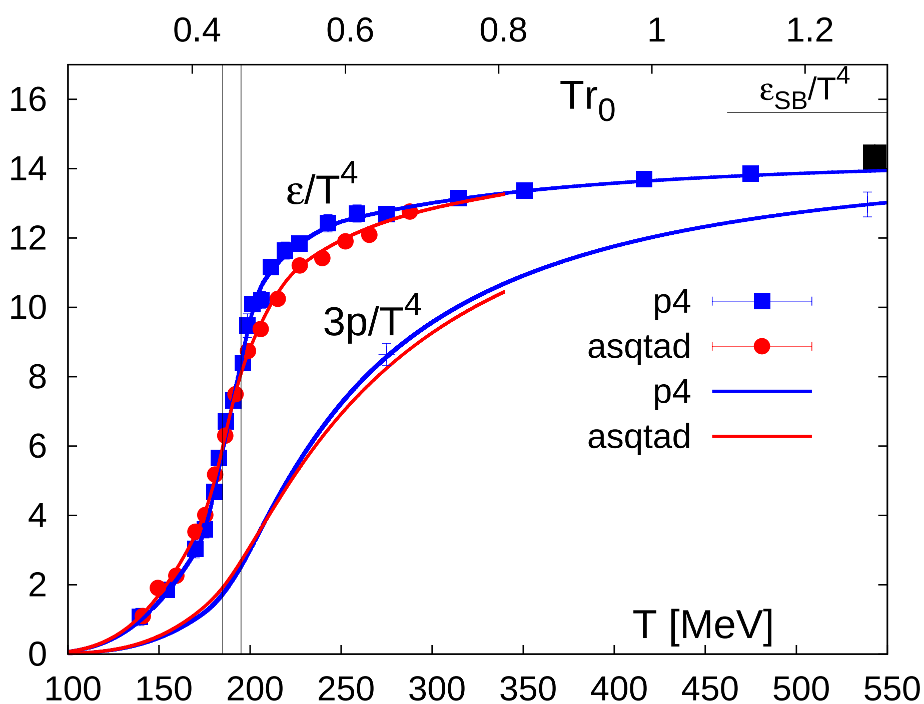
(Magnetic) Equation of State of (2+1)-flavor QCD

Frithjof Karsch, BNL& Bielefeld University

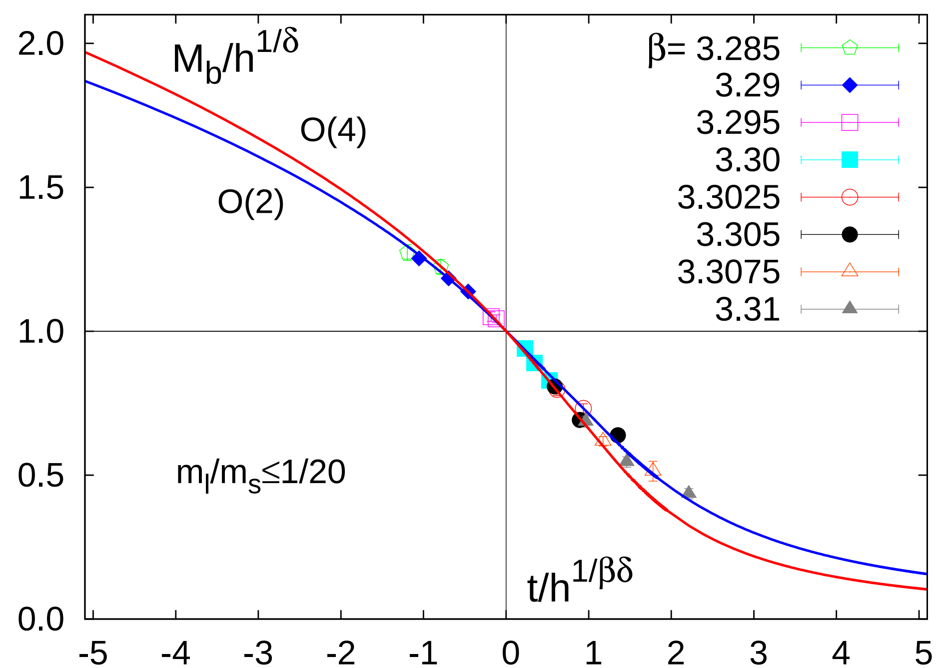
QCD equationS of state

$$\frac{\epsilon}{T^4} = \frac{1}{VT^2}\frac{\partial \ln Z(T,V,m_l)}{\partial T} \quad , \quad \frac{\langle\bar\psi\psi\rangle_l}{T^3} = \frac{1}{VT^3}\frac{\partial \ln Z(T,V,m_l)}{\partial m_l}$$

equation of state



magnetic equation of state



Magnetic Equation of State of (2+1)-flavor QCD†

Frithjof Karsch, BNL& Bielefeld University

- QCD and O(N) models, critical behavior and

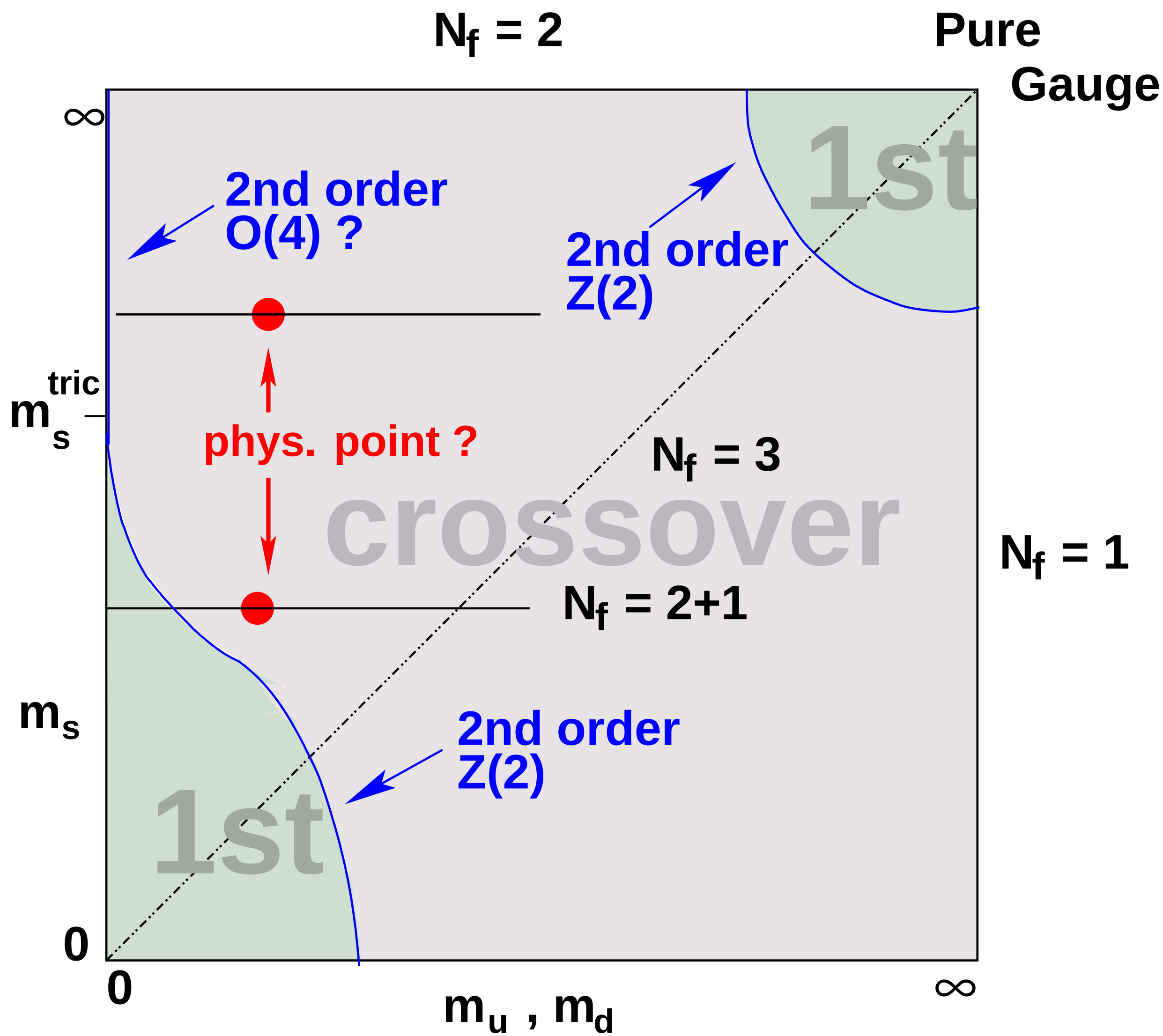
  scaling functions: magnetic EoS
- (2+1)-flavor QCD

  scaling of the chiral order parameter
- scaling of susceptibilities?

  status
- Conclusions

† This talk is based on preliminary numerical results obtained by the RBC-Bielefeld-GSI collaboration

Why (2+1)-flavor QCD?



Chiral limit of finite-T QCD on the lattice

attempts to verify the universality conjecture for the chiral phase transition through lattice calculations so far are ambiguous; even the possibility of a 1^{st} order transition is discussed

- while Wilson fermions recover chiral symmetry only in the continuum limit, staggered fermions have a $O(2)$ rather than the full $O(4)$ symmetry for finite lattice cut-off
- some features of $O(N)$ scaling have been observed in lattice calculations with staggered fermions, e.g. 'correct scaling' of the critical coupling as function of the quark mass
- Wilson fermions seem to yield the 'correct' magnetic equation of state
- lattice calculations so far fail to show the 'correct' $O(N)$ scaling of the chiral susceptibility

2 (+1)-flavor QCD and O(N) spin models

physics of QCD at low energies as well as close to the chiral phase transition is described by effective, O(N) symmetric spin models

- $T = 0$: chiral symmetry breaking at $T = 0,\ m_q = 0$ as well as leading temperature and quark mass dependent corrections are related to universal properties of 4-dimensional, $O(4)$ symmetric spin models
- $T \simeq T_c$: chiral symmetry restoration at $T = T_c,\ m_q = 0$ as well as leading temperature and quark mass dependent corrections are related to universal properties of 3-dimensional, $O(4)$ symmetric spin models

R. Pisarski and F. Wilczek, PRD29 (1984) 338
K. Rajagopal and F. Wilczek, hep-ph/0011333
A. Pelissetto and E. Vicari, Phys. Rept 368 (2002) 549

Spontaneous Symmetry Breaking

$O(N)$ spin models in d-dimensions

- non-vanishing expectation value, M, of the scalar field, $\Phi_{||}$, parallel to the symmetry breaking field H
- $(N-1)$ transverse (Goldstone) modes give corrections for non-zero H (spin waves); controlled by M and the decay constant F for Goldstone modes

$$M_H = M_0 \left(1 - \frac{N-1}{32\pi^2} \frac{M_0 H}{F_0^4} \ln\left(\frac{M_0 H}{F_0^2 \Lambda_M}\right) + \mathcal{O}(H^2)\right) \quad , \quad d = 4$$

$$M_H = M_0 \left(1 + \frac{N-1}{8\pi} \frac{(M_0 H)^{1/2}}{F_0^3} + \mathcal{O}(H)\right) \quad , \quad d = 3$$

P. Hasenfratz and H, Leutwyler, NPB343, 241 (1990)

D.J. Wallace and R.K.P. Zia, PRB12, 5340 (1975)

Spontaneous Symmetry Breaking (cont.)

- (chiral) susceptibilities diverge below T_c for $H \to 0$

$$\chi_H = \frac{\mathrm{d}M_H}{\mathrm{d}H} \sim \langle \Phi_{||}^2 \rangle - \langle \Phi_{||} \rangle^2 \sim \begin{cases} H^{-1/2} & , \; d = 3 \\ -\ln H & , \; d = 4 \end{cases}$$

- divergence in the zero-field (chiral) limit

$$\chi_{H=0}(T) = \begin{cases} \infty & , \; T \le T_c \\ A(T - T_c)^{-\gamma} & , \; T > T_c \end{cases}$$

- divergence at T_c

$$\chi_H(T = T_c) = H^{1/\delta - 1} \quad , \quad T = T_c$$

crit. exp. O(2) [O(4)]: $\gamma = 1.32\ [1.45]$, $1 - 1/\delta = 0.79\ [0.79]$

O(N) spin models in 3-dimensions

influence of Goldstone modes on spontaneous symmetry breaking below T_c and the consistency with critical behavior at T_c has been established in numerical simulations J. Engels and T. Mendes, NP B572 (2000) 289

- $T < T_c$:
  $M(t,h) = c_0(T) + c_1(T) h^{1/2} \;\Rightarrow\; \chi_M = \partial M/\partial h \sim h^{-1/2}$
- $T \simeq T_c$: $t = 0:\; M(0,h) \sim h^{1/\delta}$, $h = 0:\; M(t,0) \sim t^{\beta}$
- magnetic equation of state
  scaling functions control order parameter and its susceptibility, e.g.

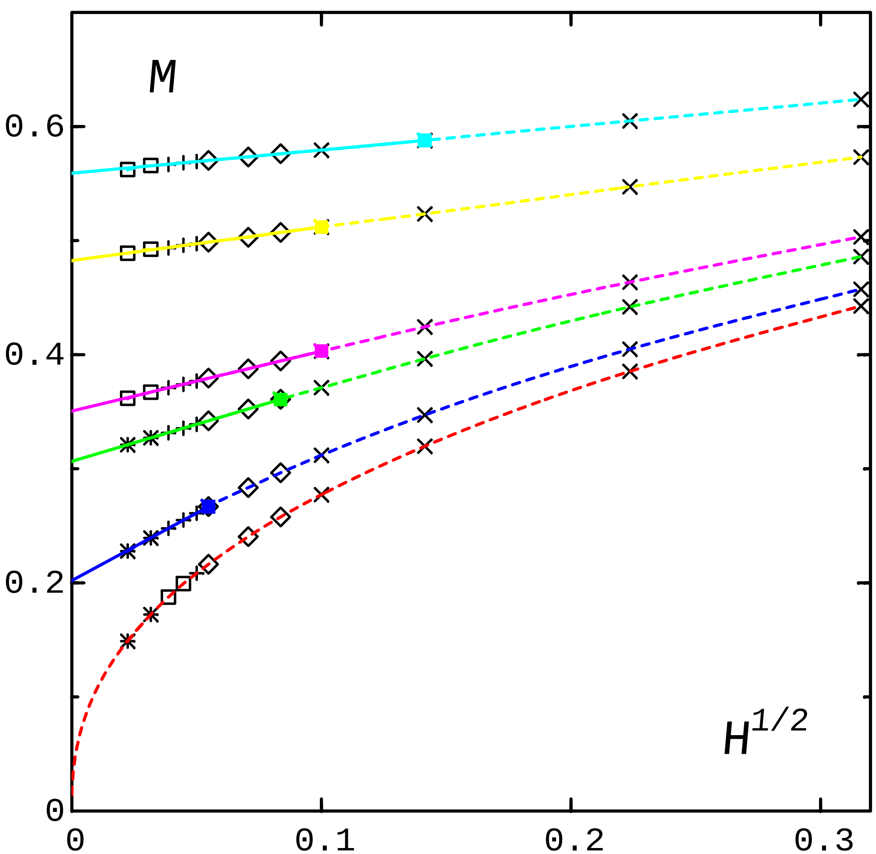
$$M(t,h) = h^{1/\delta} f_G(t/h^{1/\beta\delta}) \quad , \quad \chi_M = h^{1/\delta - 1} f_\chi(t/h^{1/\beta\delta})$$

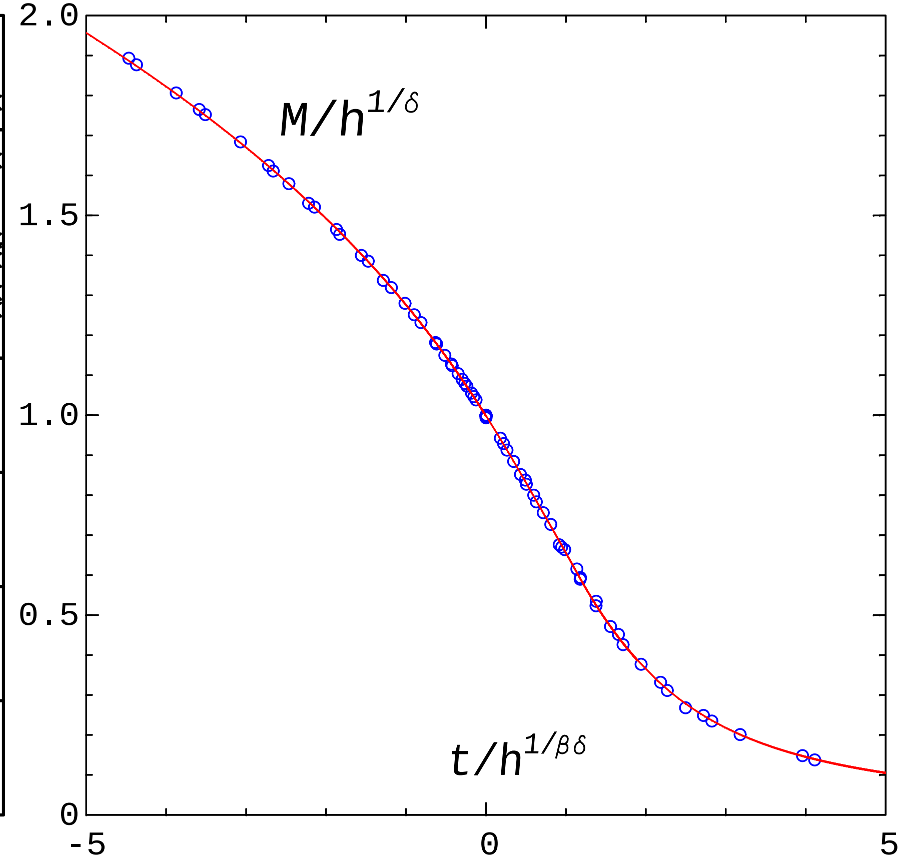
with $f_G(0) = 1$ and $\lim_{z\rightarrow -\infty} f_G(z) \sim (-z)^{\beta}$

$\Leftrightarrow$ scaling fields t and h need to be normalized

$$t = \frac{1}{t_0} \frac{T - T_c}{T_c} \quad , \quad h = \frac{H}{h_0}$$

3-d, O(4) models close to T_c

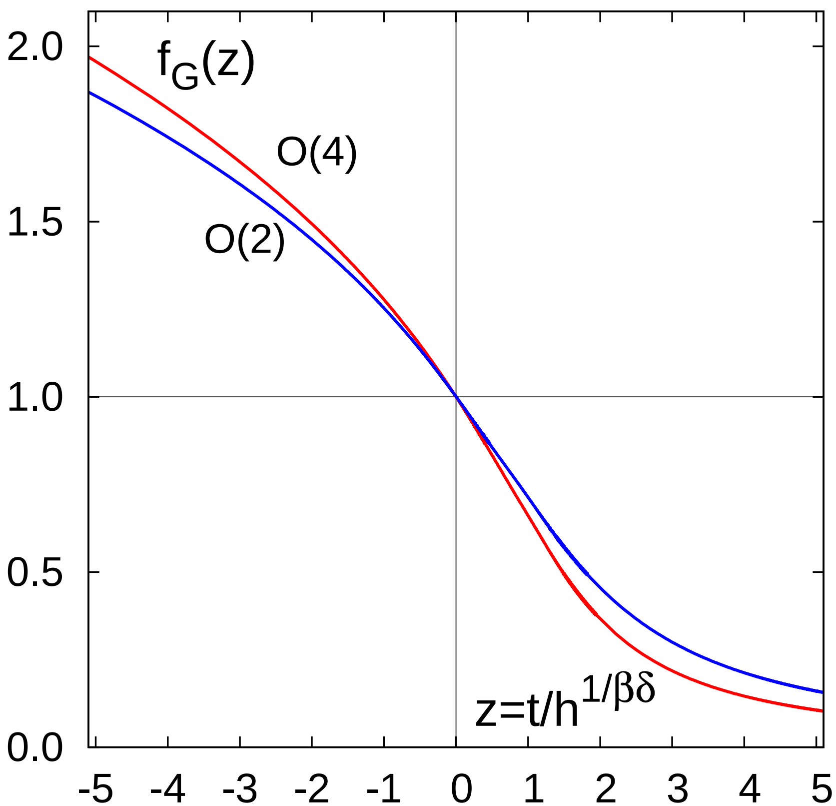


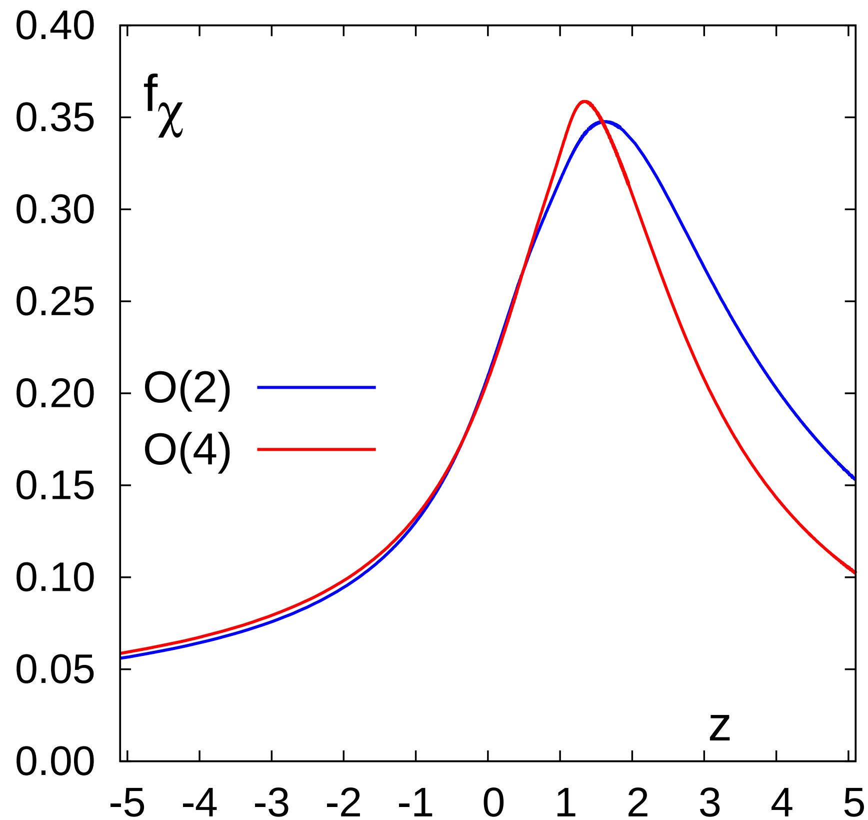


J. Engels and T. Mendes 2000

- condensate shows $\sqrt{H}$ dependence and $O(4)$ scaling
- magnetic equation of state reflects $O(4)$ scaling including Goldstone modes

3-d, O(N) scaling functions





$$f_\chi(z) = \frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta} f'_G(z)\right)$$

O(2): J. Engels et al., 2001

O(4): J. Engels et al., 2003

- $\chi_M = \partial M/\partial h \sim h^{-1/2} = h^{1/\delta - 1} f_\chi(t/h^{1/\beta\delta})$

Goldstone mode & magnetic EoS

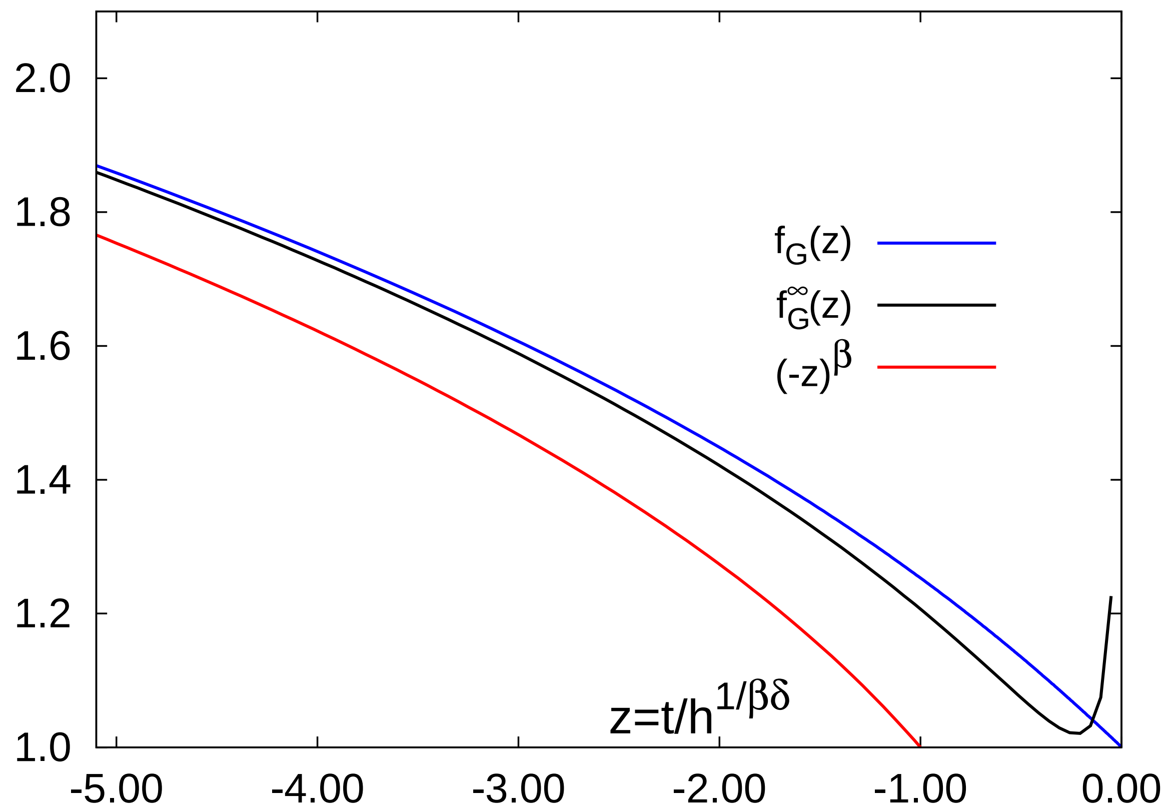
$z \longrightarrow -\infty$

$$f_G(z) \sim (-z)^{\beta}(1 + \tilde{c_2}\beta(-z)^{-\beta\delta/2})$$

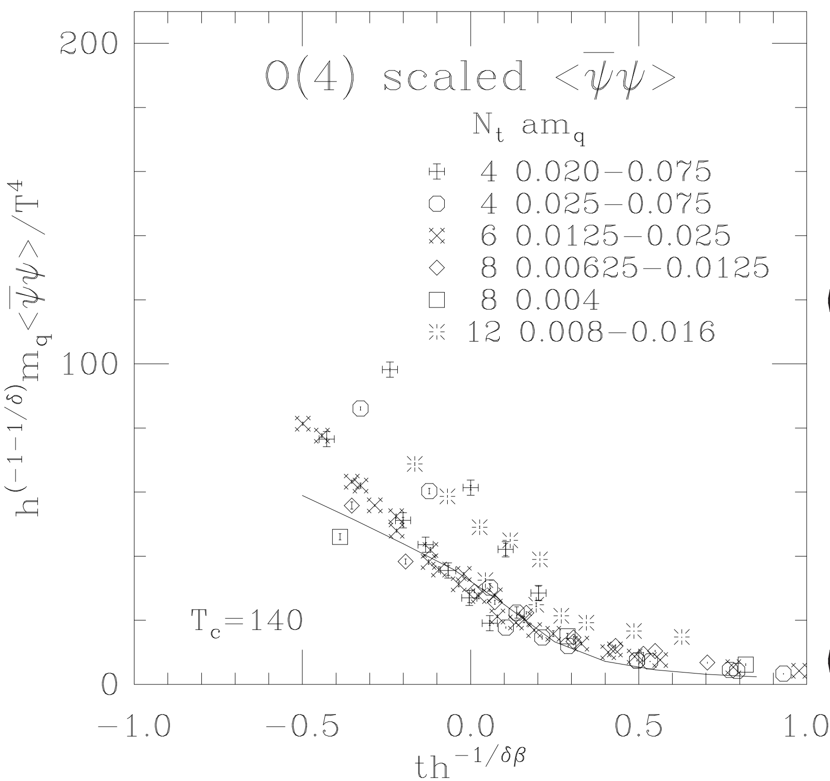
$$\Rightarrow \quad M \sim (-t)^{\beta} + c(t)h^{1/2}$$

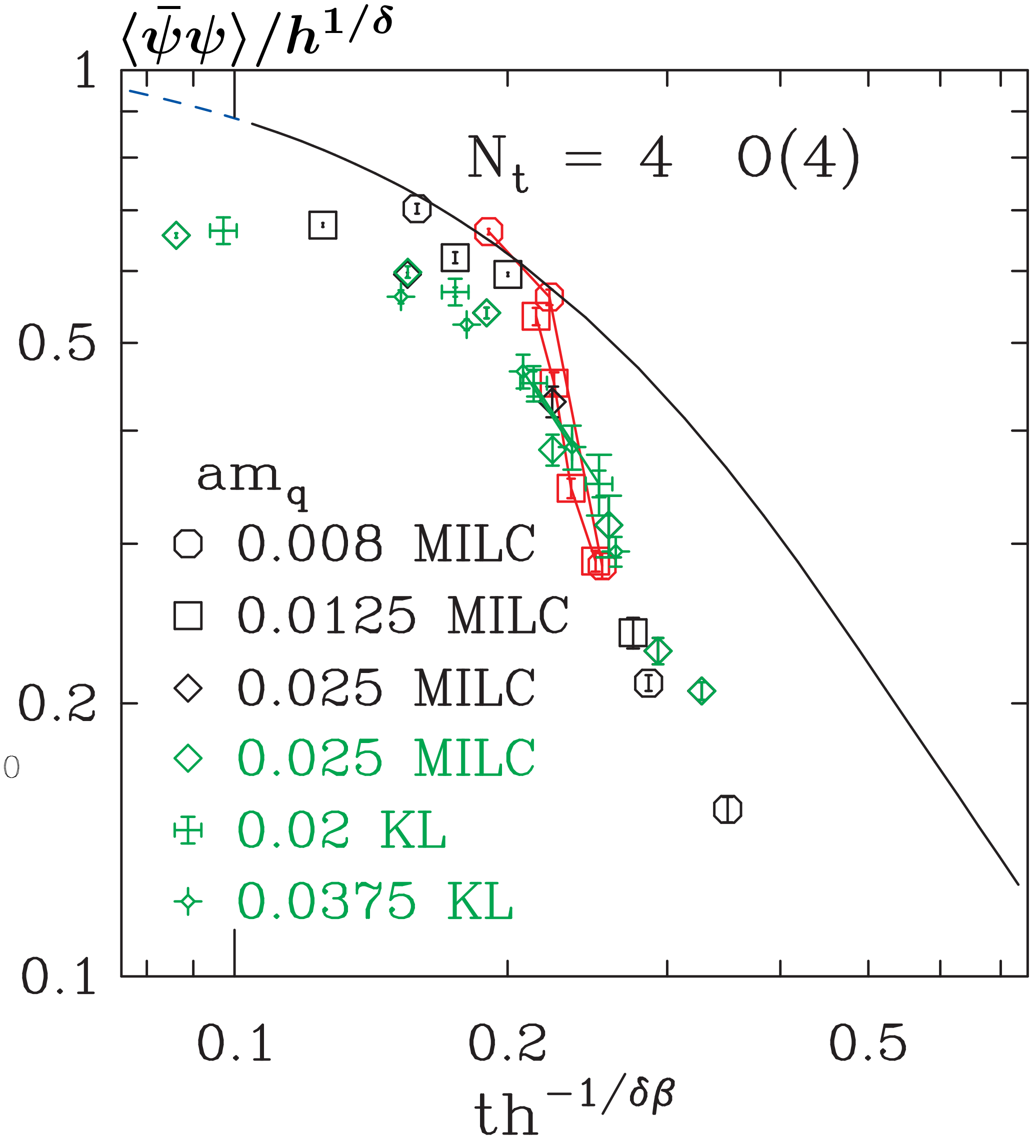
$$f_\chi(z) \sim (-z)^{\beta(1-\delta/2)}$$

Goldstone modes give dominant h-dependent contribution to magnetic EoS



O(N) scaling in QCD?

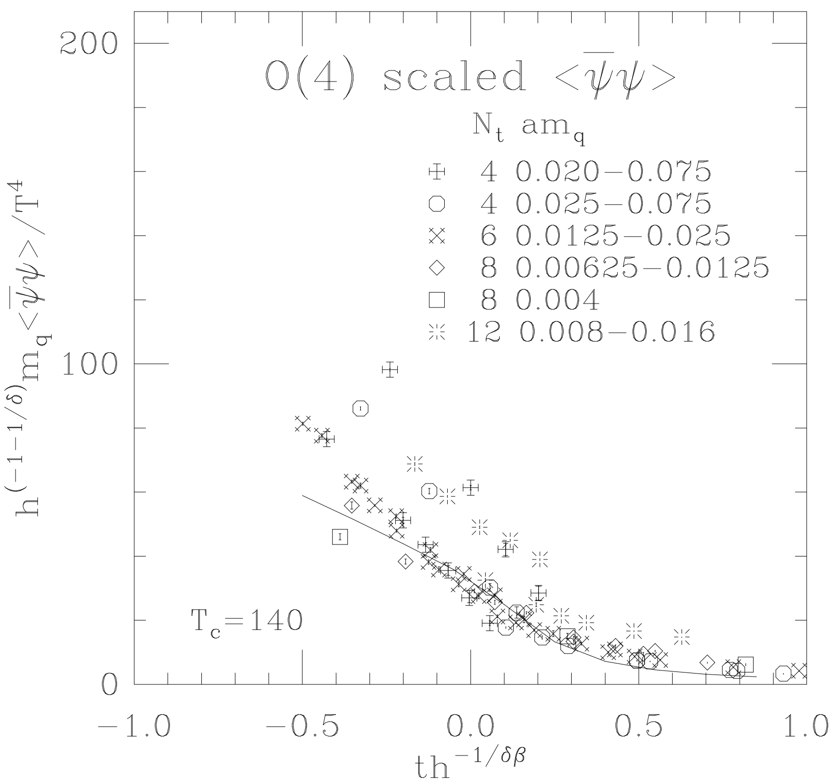


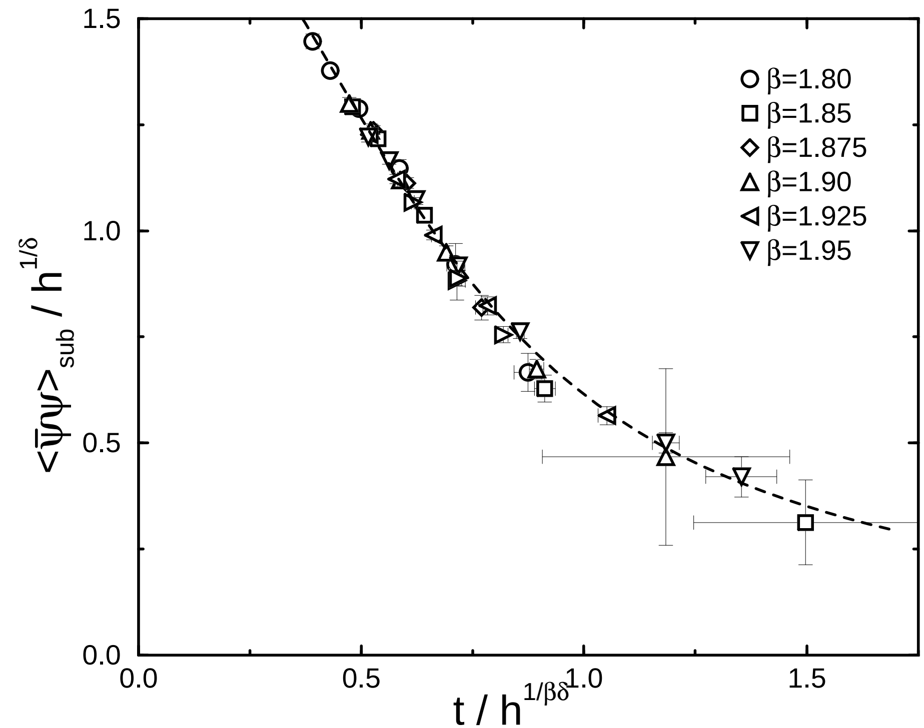


MILC, 2000, 2001

- 2-flavor QCD, staggered fermions; no evidence for O(N) scaling

O(N) scaling in QCD?





MILC, 2001

- 2-flavor QCD, staggered fermions; no evidence for scaling

CP-PACS, 2001

- 2-flavor QCD, Wilson fermions; only $z > 0$, no evidence for Goldstone modes

Scaling analysis in (2+1)-flavor QCD

(RBC-Bielefeld and hotQCD Collaborations)

QCD with 2 light and a 'physical' strange quark mass;

staggered fermions, p4 and asqtad actions, RHMC simulations

- calculations have been performed on $N_\sigma^3 N_\tau$ lattices for $N_\tau = 4,\ 6$ and 8
- check volume dependence: $4 \le N_\sigma / N_\tau \le 8$
- at present, most detailed analysis with p4-action for $N_\tau = 4$:

$$1/80 \le m_l/m_s \le 2/5$$

physical value: $m_l/m_s \simeq 0.05 \Rightarrow 75\ \mathrm{MeV} \le m_\pi \le 320\ \mathrm{MeV}$

$\Rightarrow$ find evidence for Goldstone modes in the broken phase

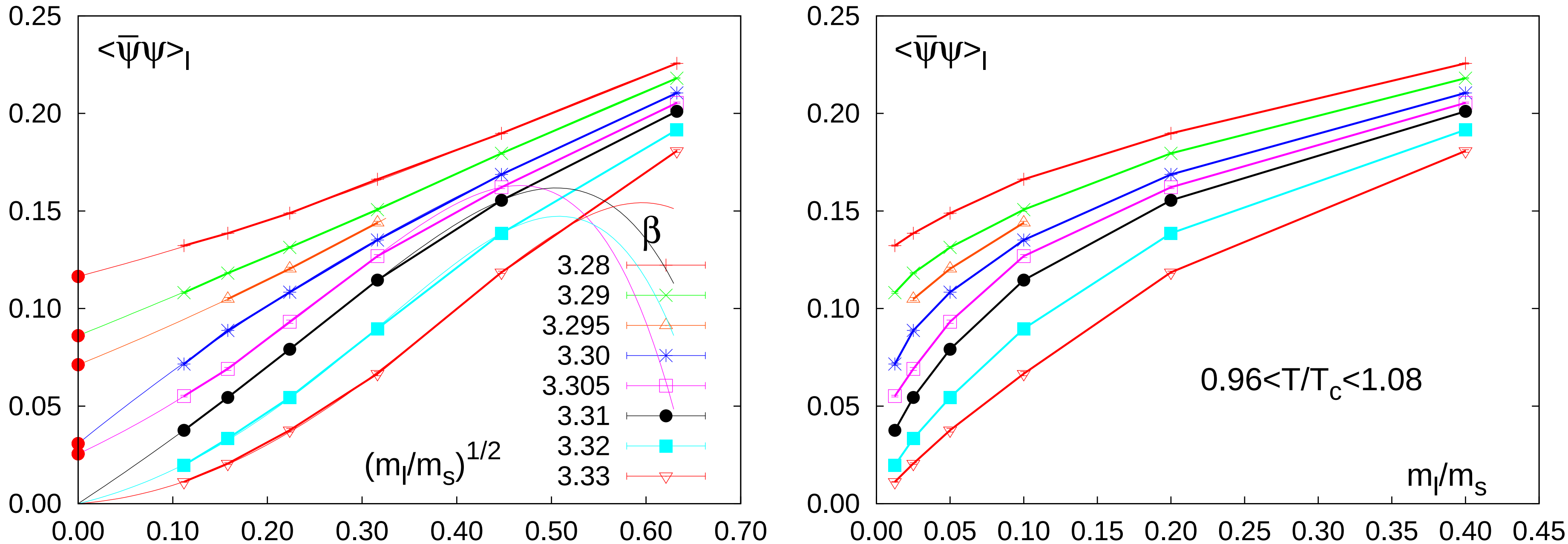
$\Rightarrow$ find evidence for $O(N)$ scaling

$\Rightarrow$ determine normalization of scaling fields

Chiral condensate: $N_\tau = 4$:

(RBC-Bielefeld collaboration, in preparation)

$$\langle\bar\psi\psi\rangle_l = \frac{1}{N_\sigma^3 N_\tau}\frac{1}{4}\frac{\partial \ln Z}{\partial m_l a}$$



- evidence for $\sqrt{m_l}$ term in $\langle\bar\psi\psi\rangle$

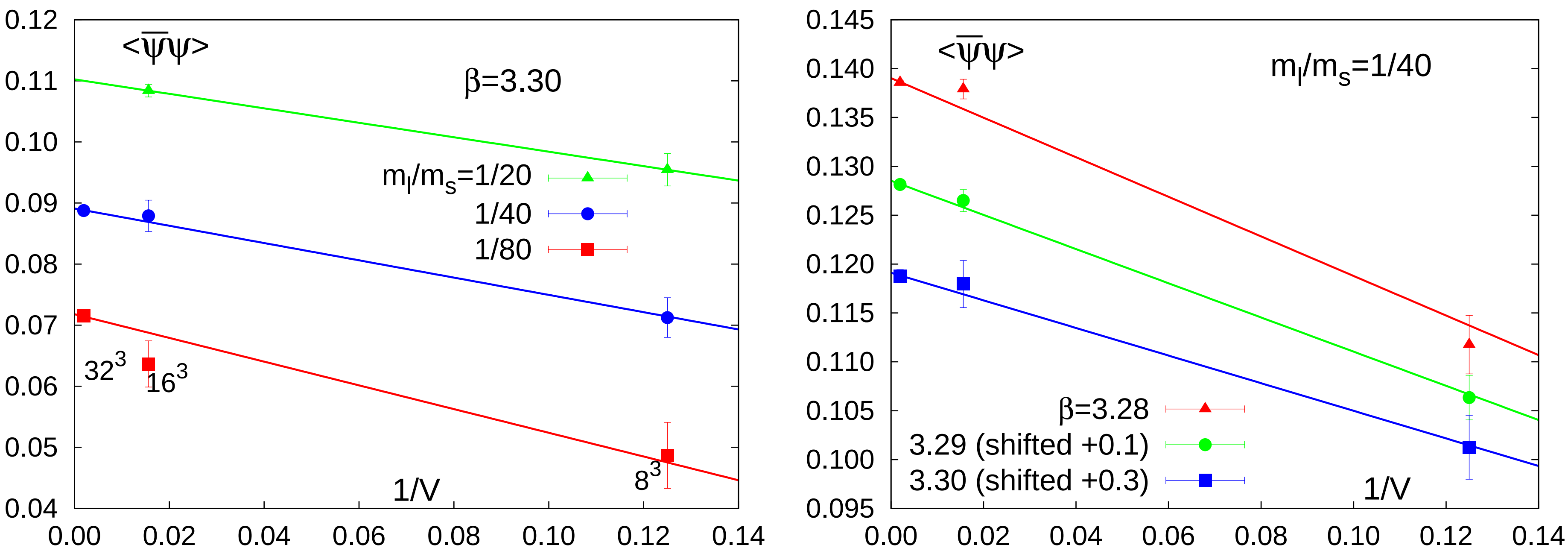
  for orientation: $\beta = 3.28$ $T \simeq 188$ MeV,
  $\beta = 3.30$ $T \simeq 196$ MeV

- Statistics:

  20.000-40.000 trajectories per $(\beta,\ m_q)$

Volume dependence of the chiral condensate

$$\langle\bar\psi\psi\rangle(T,m_l,V) \quad = \quad \langle\bar\psi\psi\rangle + A(T,m_l)/V$$



- no strong temperature dependence of $1/V$ corrections ($\beta_c \simeq 3.30$)

  for orientation: $\beta = 3.28$ $T \simeq 188$ MeV, $\beta = 3.30$ $T \simeq 196$ MeV
- $1/V$ corrections are under control

Renormalized chiral order parameter

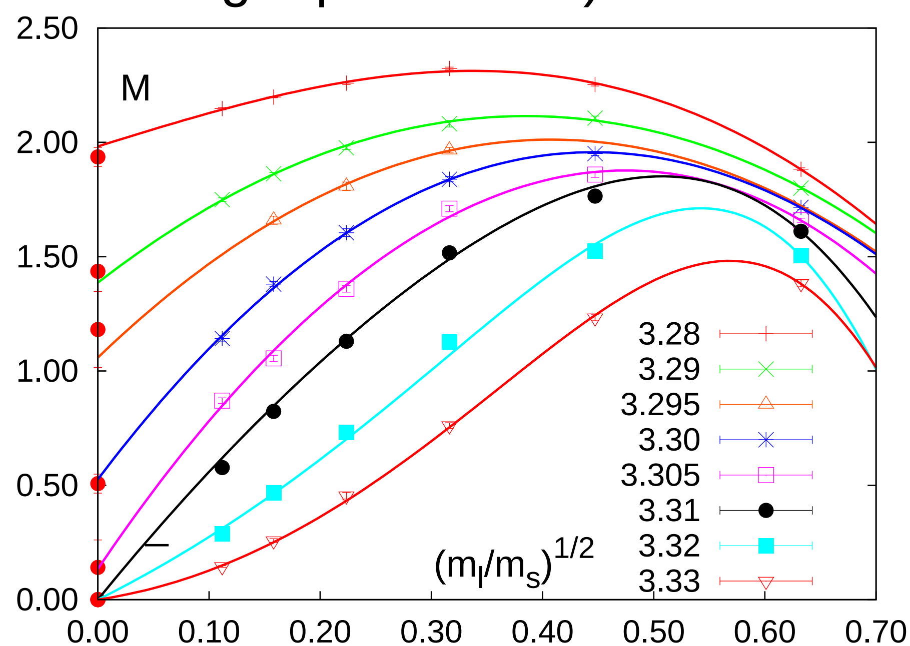
$$M \equiv \frac{1}{T^4} m_s \left(\langle \bar{\psi}\psi \rangle_l - \frac{m_l}{m_s} \langle \bar{\psi}\psi \rangle_s \right)$$

- eliminates additive divergence $\sim m_q/a^2$
- multiplicative renormalization takes care of anomalous dimension
- reduces to chiral condensate (times strange quark mass) in the chiral limit

$\Rightarrow$ well-defined continuum limit
improved (universal) scaling properties?

- magnetic EoS for M coincides with that for

$$M_0 = \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4}$$



• contribution linear in m_l eliminated

O(N) scaling analysis; $m_l/m_s \le 1/20$

$$M \equiv h^{1/\delta} f_G(z) \qquad ; \qquad z = t/h^{1/\beta\delta}$$

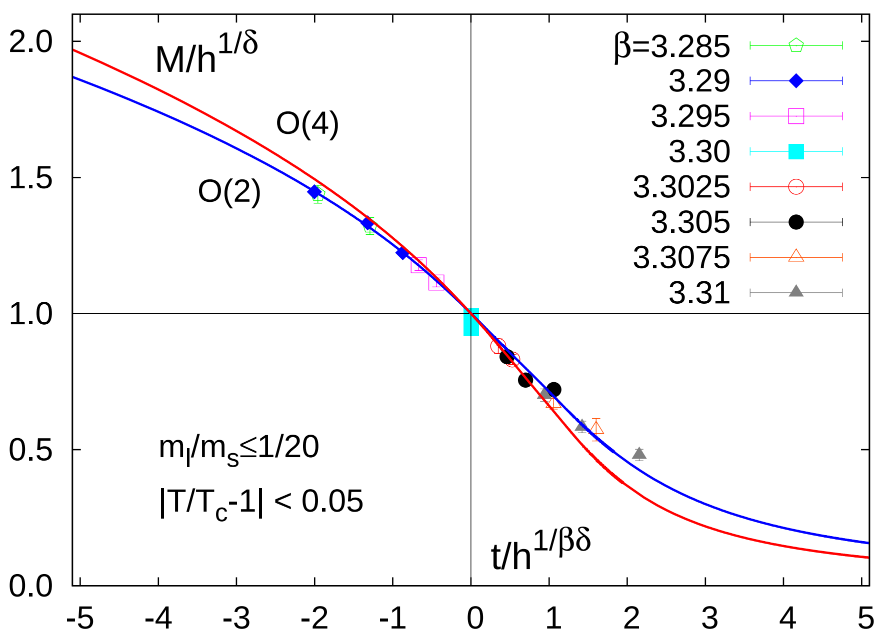
- 3 parameter fit: t_0, h_0, T_c $\qquad t = \frac{1}{t_0}\frac{T - T_c}{T_c}\ ,\ h = \frac{1}{h_0}\frac{m_l}{m_s}$
- use only data for $m_l/m_s \le 1/20$, $\beta \in [3.285, 3.31]$

O(N) scaling analysis; $m_l/m_s \le 1/20$

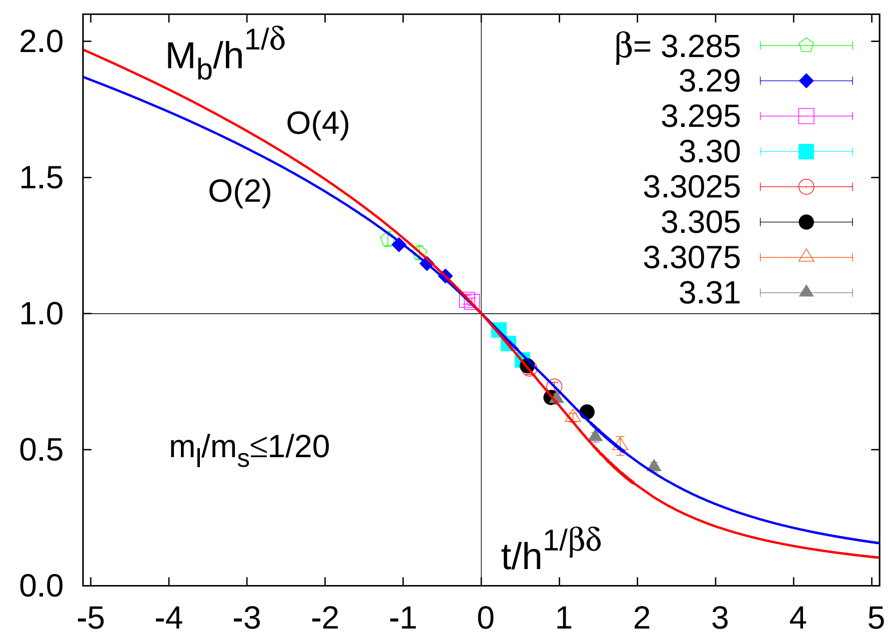
$$M \equiv h^{1/\delta} f_G(z) \qquad ; \qquad z = t/h^{1/\beta\delta}$$

- 3 parameter fit: t_0, h_0, T_c $\qquad t = \frac{1}{t_0}\frac{T - T_c}{T_c}\ , \ h = \frac{1}{h_0}\frac{m_l}{m_s}$
- use only data for $m_l/m_s \le 1/20$, $\beta \in [3.285, 3.31]$

renormalized order parameter



un-renormalized order parameter



$\Rightarrow t_0, h_0, T_c$ should agree for $h \to 0$, $t \to 0$

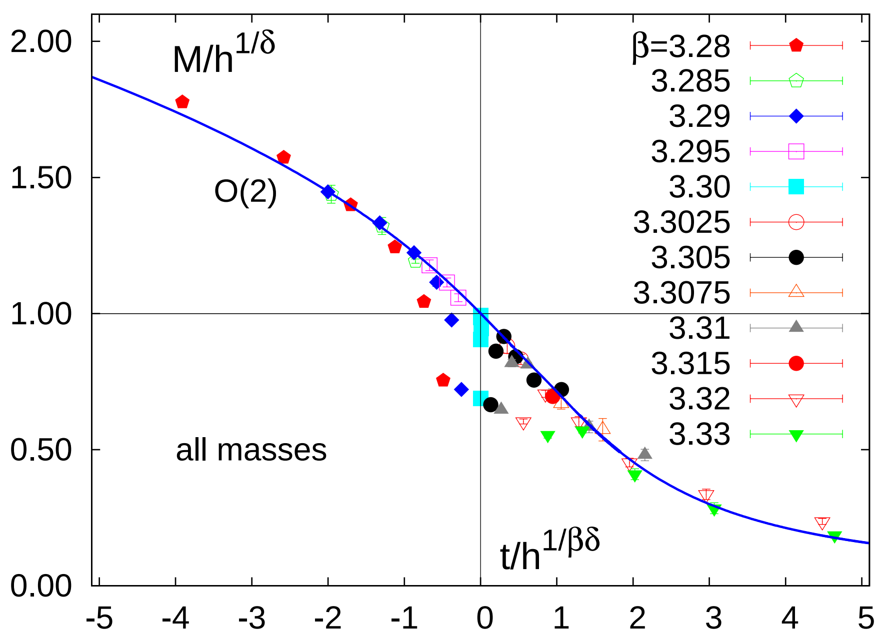
O(N) scaling analysis; all masses

$$M \equiv h^{1/\delta} f_G(z) \qquad ; \qquad z = t/h^{1/\beta\delta}$$

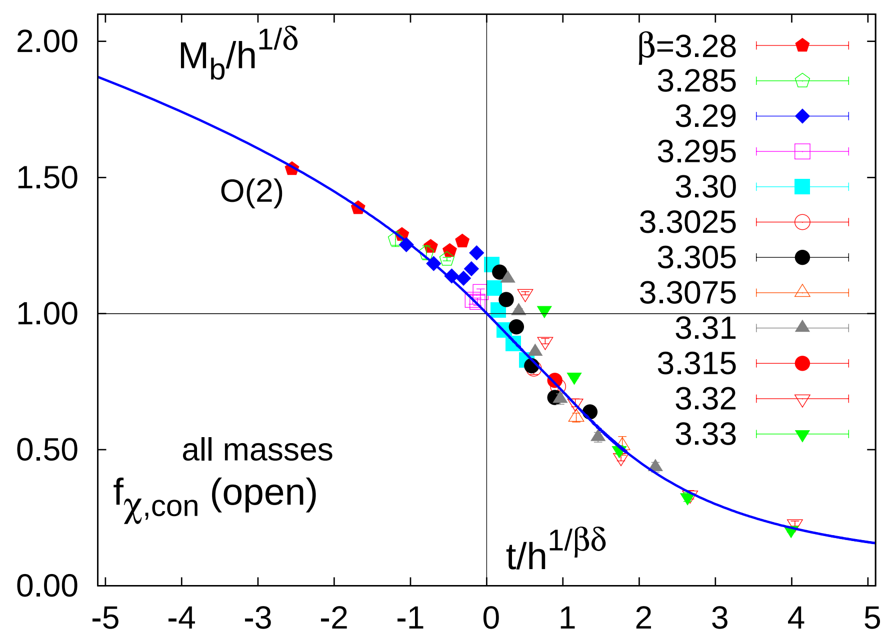
$$t = \frac{1}{t_0}\frac{T - T_c}{T_c} \; , \; h = \frac{1}{h_0}\frac{m_l}{m_s}$$

- no fit: use t_0, h_0, T_c from low mass fit

renormalized order parameter



un-renormalized order parameter

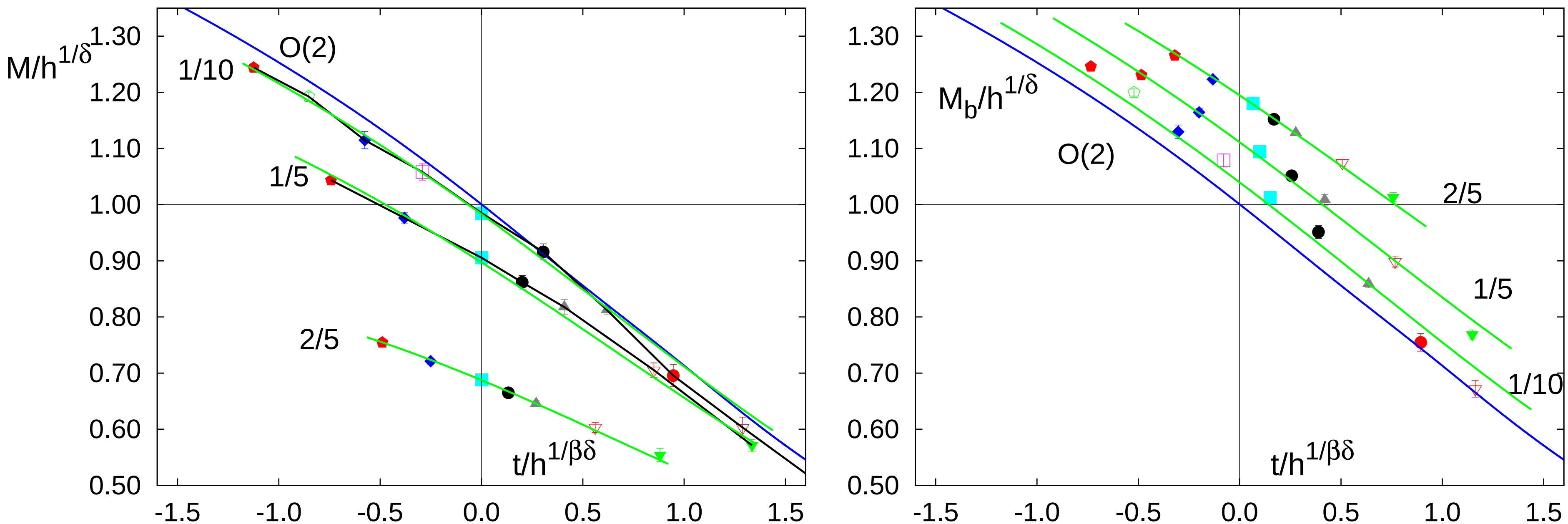


$\Rightarrow$ different scaling violations for $t \neq 0$, $h > 0$

Scaling violations for $m_l/m_s \gtrsim 1/10$

$$M(t,h) = h^{1/\delta} f_G(t/h^{1/\beta\delta}) + a_t th + b_1 h + b_3 h^3 + b_5 h^5$$

$b_1 \equiv 0$ $\qquad$ $b_1 = 0.0013(3)$



- sizeable contributions from regular part for $m_l/m_s > 1/10$

The QCD scaling fields

$$M \equiv h^{1/\delta} f_G(z) \quad ; \quad z = t/h^{1/\beta\delta}$$

$$t = \frac{1}{t_0}\frac{T - T_c}{T_c} \; , \; h = \frac{1}{h_0}\frac{m_l}{m_s}$$

- fits to low mass data with different cuts on m_l/m_s

$(m_l/m_s)_{\max}$	M	h_0	t_0	T [MeV]
1/20	renorm.	0.0049(5)	0.0048(2)	195.6(4)
1/20	un-renorm.	0.0022(5)	0.0038(2)	194.6(4)
1/40	renorm.	0.0043(5)	0.0047(5)	195.4(4)
1/40	un-renorm.	0.0026(5)	0.0039(5)	194.8(4)

T_c from $r_0 = 0.469$ fm;

$N_\tau = 4$: no continuum limit

The QCD scaling fields

$$M \equiv h^{1/\delta} f_G(z) \quad ; \quad z = t/h^{1/\beta\delta}$$

$$t = \frac{1}{t_0} \frac{T - T_c}{T_c} \; , \; h = \frac{1}{h_0} \frac{m_l}{m_s}$$

- a unique combination:

$$\begin{aligned} z &= (6.9 - 8.6) \frac{T - T_c}{T_c} \left(\frac{m_l}{m_s} \right)^{-1/\beta\delta} \\ &\simeq (4.7 - 5.9) \frac{T - T_c}{T_c} \left(\frac{m_\pi}{m_K} \right)^{-2/\beta\delta} \end{aligned}$$

using $m_\pi / m_K = 0.435(2)$ for $m_l / m_s = 0.1$

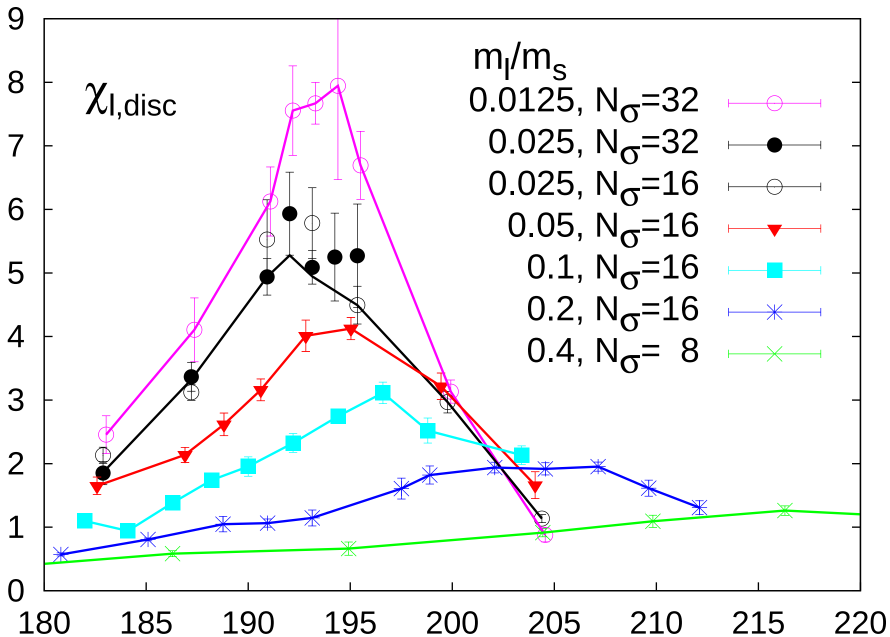
preliminary; continuum extrapolation is still missing

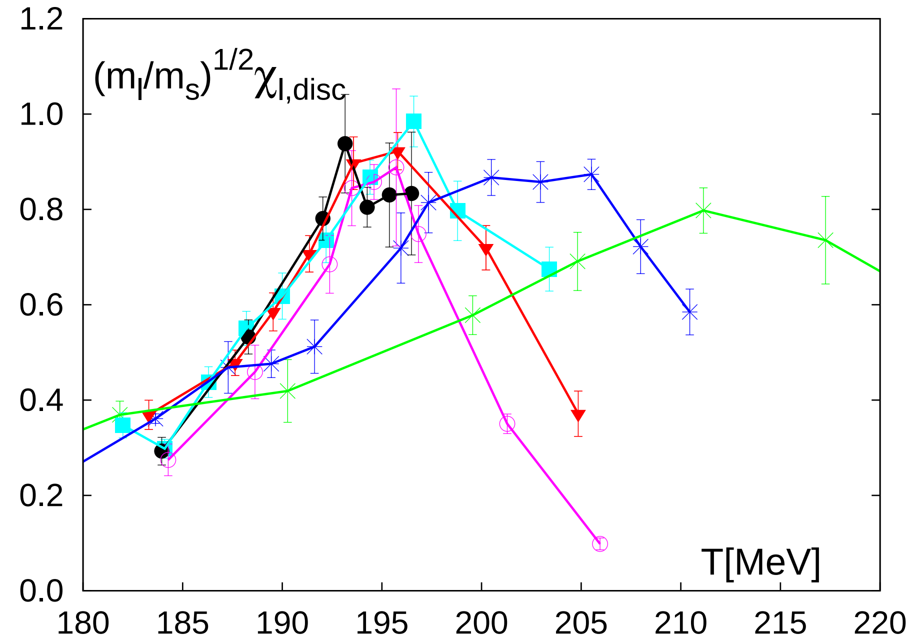
Chiral susceptibility

$N_\tau = 4$: $1/80 \le m_l/m_s \le 2/5$

$$\chi_m^l = \frac{\partial \langle \bar\psi\psi\rangle_l}{\partial m_l a} \equiv \chi_{disc} + \chi_{con}$$

$$\chi_{disc} = \frac{1}{N_\sigma^3 N_\tau}\left(\frac{n_f}{4}\right)^2 \left(\left\langle (\mathrm{Tr} D_l^{-1})^2 \right\rangle - \left\langle \mathrm{Tr} D_l^{-1} \right\rangle^2\right)$$





- evidence for $1/\sqrt{m_l}$ singularity in χ_{disc}

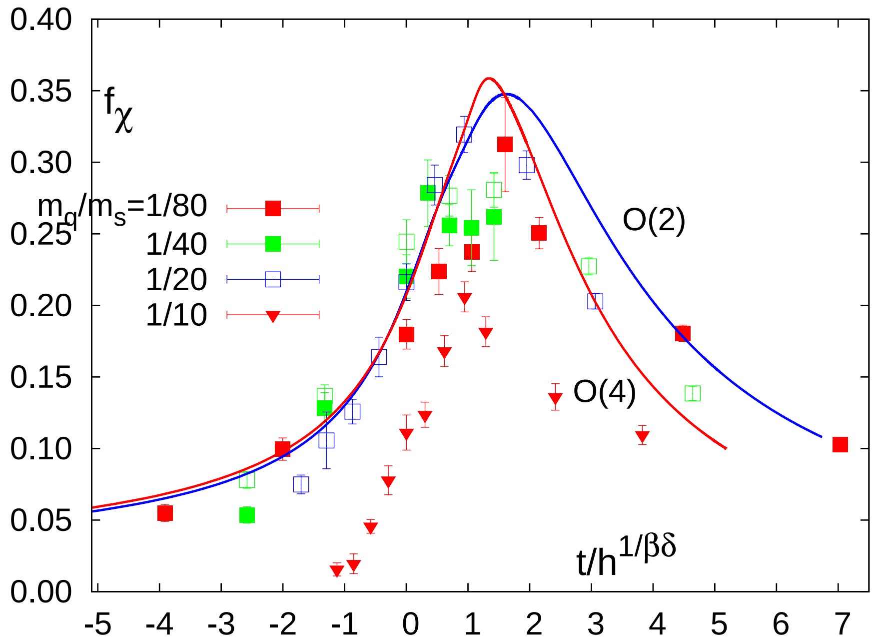
shown is only the disconnected contribution

Order parameter susceptibility

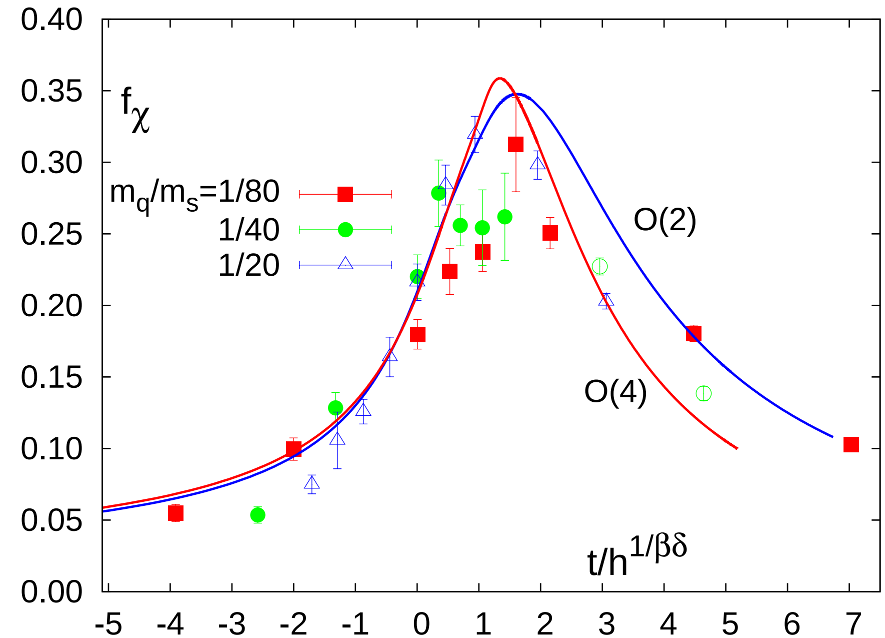
$$\chi_M \equiv \left(\frac{\partial M}{\partial h}\right)_t = h^{1/\delta-1} f_\chi(t/h^{1/\beta\delta})$$

$$= h_0 N_\tau^2 m_s^2 \left(\chi_m^l - \frac{N_\tau^2}{m_s}\langle\bar{\psi}\psi\rangle_s - \frac{m_l}{m_s}\chi_m^{ls}\right)$$

all β and m_l/m_s



$|\beta - \beta_c| \le 0.015$, $m_l/m_s \le 1/20$



- fluctuations more sensitive to cut-off and/or quark mass effects?

Conclusions

- Goldstone modes control properties of the chiral condensate in the confined phase already for 'physical' quark mass values

  3-d, O(N) scaling close to T_c

- (2+1)-flavor QCD

  the magnetic equation of state is consistent with 3-d, O(N) scaling; no indications for a 'nearby' first order phase transition