# The phase diagram of QCD from imaginary chemical potentials

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"Quarks, Hadrons, and the Phase Diagram of QCD", St. Goar, september 3, 2009

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based on arXiv:0909.0254

# 1 – Introduction

**Exploring the phase diagram of QCD at finite $T$ and finite baryon chemical potential $\mu_B$ by lattice QCD simulations is highly non-trivial because of the sign problem. The fermion determinant appearing in the expression for the full QCD partition function**

$$Z(T) \equiv \int \mathcal{D}U e^{-S_G[U]} \det M[U]$$

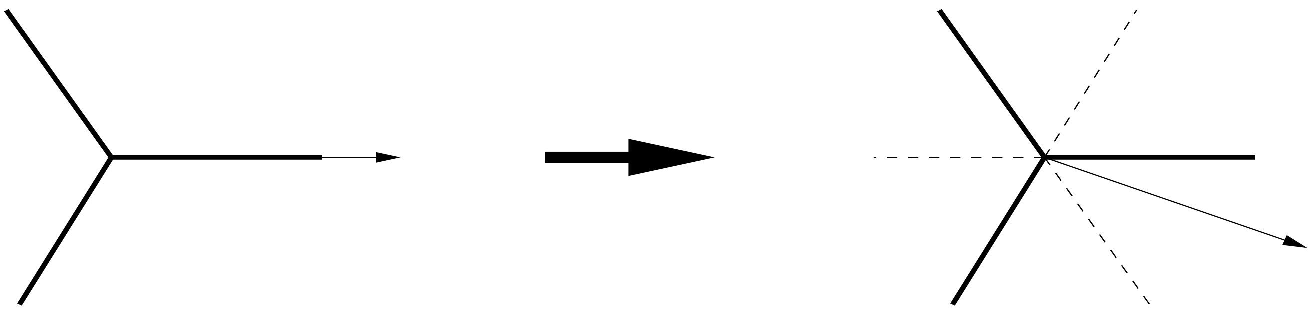
**is a complex quantity if $\mu_B \neq 0$.**

**If instead the chemical potential is purely imaginary, $\mu_B = 3\mu_q = i\mu_I$, $\det M$ is real again and numerical simulations are feasible. The phase diagram in the $T$-$\mu_I$ plane can be mapped out systematically.**

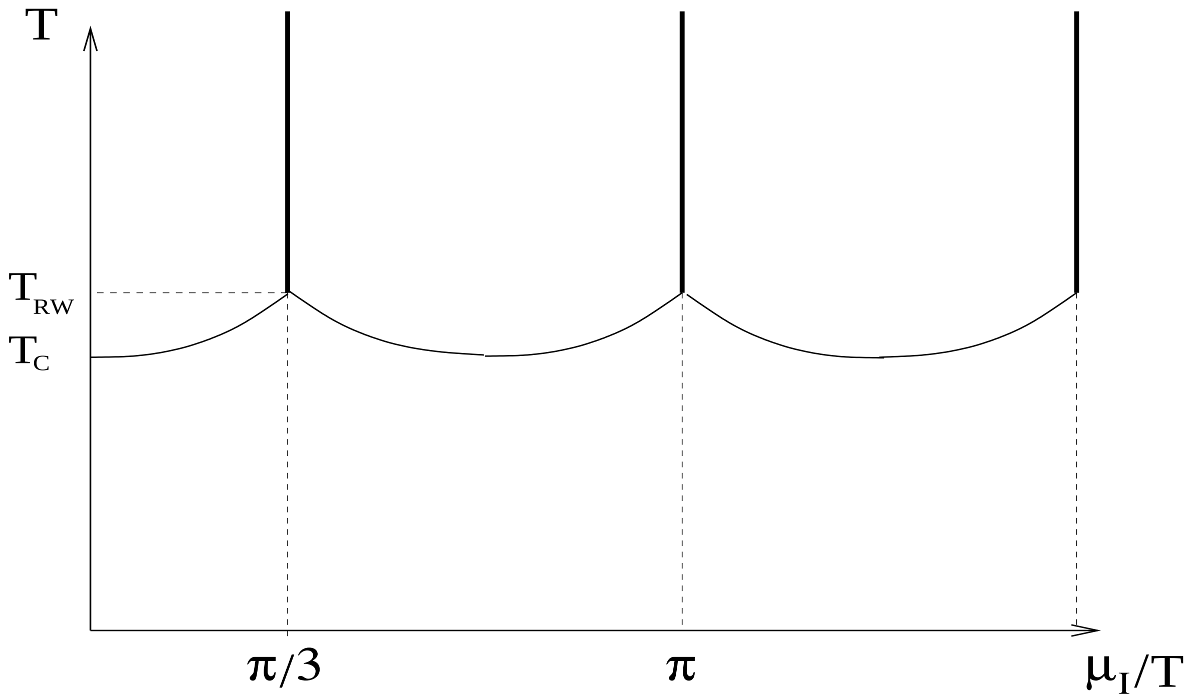
**What do we learn from doing that?**

## Sketch of the $T$-$\mu_I$ phase diagram

- an imaginary chemical potential is equivalent to a rotation of fermion boundary conditions in temporal direction by an angle $\theta_q = \mu_I/T$
- an amount $2\pi k/N_c$ of this rotation, with $k$ integer, can be cancelled by a center transformation. Hence the partition function has periodicity $2\pi/N_c$ in $\theta_q$ (Roberge and Weiss)
- the periodicity is smoothly realized at low $T$. Instead in the high $T$ regime first order phase transitions occur for $\theta_q = (2k+1)\pi/N_c$ at which the Polyakov loop $\langle L \rangle$ suddenly jumps from one center sector to the other (RW lines)



**zero chemical potential** **non zero imaginary chemical potential**




- **The RW line must end at some endpoint $T_{\rm RW}$**

- **The diagram is completed by the analytic continuation of the physical deconfinement/chiral transition line, which repeats periodically over the plane. Numerical results show that such line touches the RW line right on its endpoint.**

**Determining the location of the deconfinement line may be useful to get part of the physical line by analytic continuation**

Ph. de Forcrand and O. Philipsen, Nucl. Phys. B 642, 290 (2002); Nucl. Phys. B 673, 170 (2003)

M. D'Elia and M.P. Lombardo, Phys. Rev. D 67, 014505 (2003); Phys. Rev. D 70, 074509 (2004)

V. Azcoiti, G. Di Carlo, A. Galante and V. Laliena, Nucl. Phys. B 723, 77 (2005).

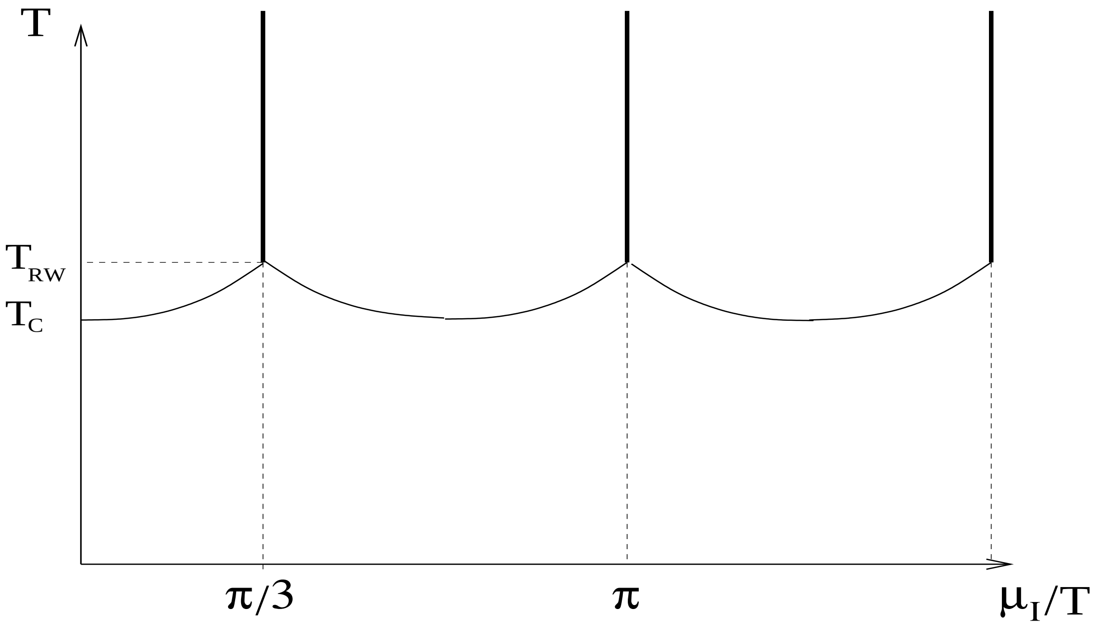
H. S. Chen and X. Q. Luo, Phys. Rev. D 72, 034504 (2005).

P. Giudice and A. Papa, Phys. Rev. D 69, 094509 (2004)

P. Cea, L. Cosmai, M. D'Elia and A. Papa, Phys. Rev. D 77, 051501 (2008); Phys. Rev. D 80, 034501 (2009).

**Here we are interested in a different question:**

## What is the order of the RW endpoint?




**Along the RW lines the theory possesses an exact $Z_2$ symmetry: the system is in perfect equilibrium between two center sectors. The symmetry is spontaneously broken for $T > T_{\rm RW}$, where a phase transition in $T$ takes place.**

**Such symmetry is better appreciated at $\theta_q = \pi$ (periodic b.c.), where it corresponds to charge conjugation. The RW endpoint is equivalent to the finite spatial size transition at which charge symmetry is spontaneously broken which has been studied in other contexts T. DeGrand, R. Hoffmann and J. Najjar, JHEP 0801, 032 (2008); B. Lucini, A. Patella and C. Pica, Phys. Rev. D 75, 121701 (2007); B. Lucini and A. Patella, Phys. Rev. D 79, 125030 (2009)**

# Two possibilities

- **The endpoint is second order.**

  In this case the universality class is Ising 3d by symmetry, the corresponding critical behaviour may in principle influence physics at $\theta_q = 0$ M. D'Elia, F. Di Renzo and M.P. Lombardo, Phys. Rev. D 76, 114509 (2007); H. Kouno, Y. Sakai, K. Kashiwa and M. Yahiro, arXiv:0904.0925 [hep-ph].

- **The endpoint is first order.**

  In this case it is actually a triple point with two further first order lines departing from it. Those are naturally identified with (part of) the analytic continuation of the physical critical line. More interesting consequences follows ...

**The first order hypothesis is surely realized close enough to the quenched limit, $am_q \to \infty$, where $\theta_q$ becomes completely irrelevant and the RW endpoint coincides with the usual quenched deconfining transition, $T_{\rm RW} = T_c$**

## 2 – NUMERICAL RESULTS

**We have investigated QCD with two degenerate flavors, standard plaquette action, standard staggered fermion formulation (square root), RHMC algorithm.**

**Two values of the bare quark mass:** $am_q = 0.075$ **and** $am_q = 0.025$

**Lattices** $L_s^3 \times L_t$ **with** $L_t = 4$ **and** $L_s = 8,\ 12,\ 16,\ 20,\ 32$**.**

**We have worked at fixed** $\theta_q = \pi$ **and the temperature** $T = 1/(L_t a(\beta, m_q))$ **has been changed by tuning the inverse gauge coupling** $\beta$**.**

**Collected statistics are of the order of** $50 - 100$**K trajectories for the** $\beta$ **values closest to the critical point.**

**At $\theta_q = \pi$, the broken symmetry is charge conjugation and the imaginary part of the Polyakov loop is a possible order parameter. We study its susceptibility**

$$\chi \equiv L_s^3 \left( \langle \mathrm{Im}(L)^2 \rangle - \langle \mathrm{Im}(L) \rangle^2 \right) \tag{1}$$

**its expected finite size scaling behaviour is the following**

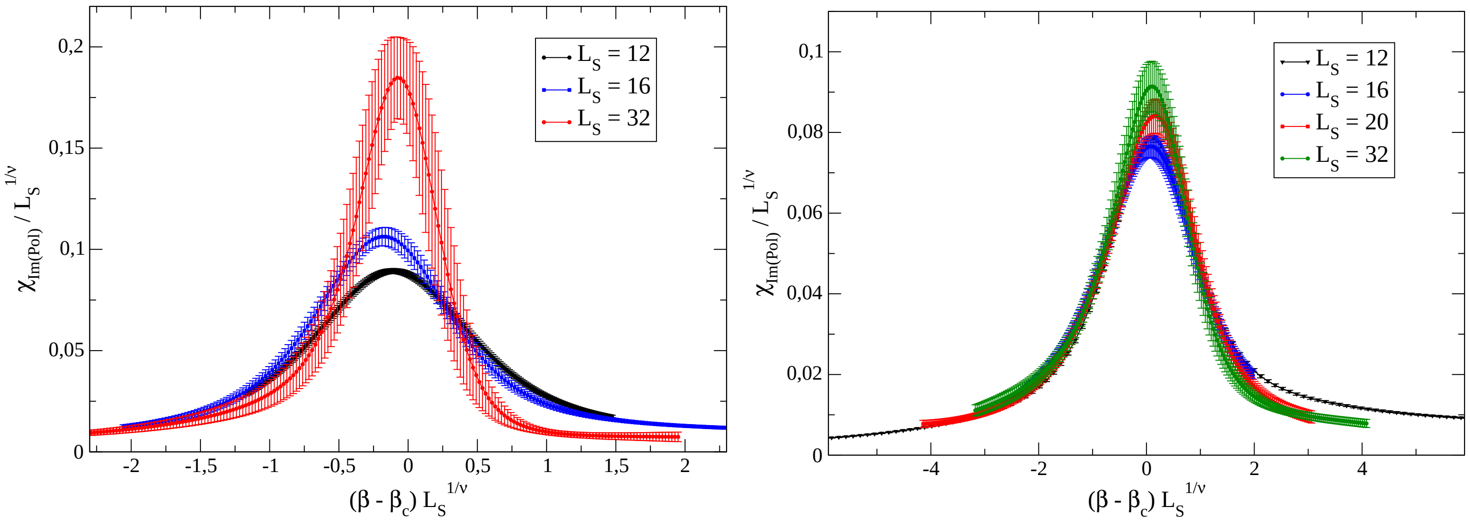
$$\chi = L_s^{\gamma/\nu}\, \phi(\tau L_s^{1/\nu}) \quad \Longrightarrow \qquad \chi / L_s^{\gamma/\nu} = \phi(\tau L_s^{1/\nu}) \tag{2}$$

**where $\tau \equiv (T - T_{\mathrm{RW}})/T_{\mathrm{RW}} \sim (\beta - \beta_{\mathrm{RW}})$,**

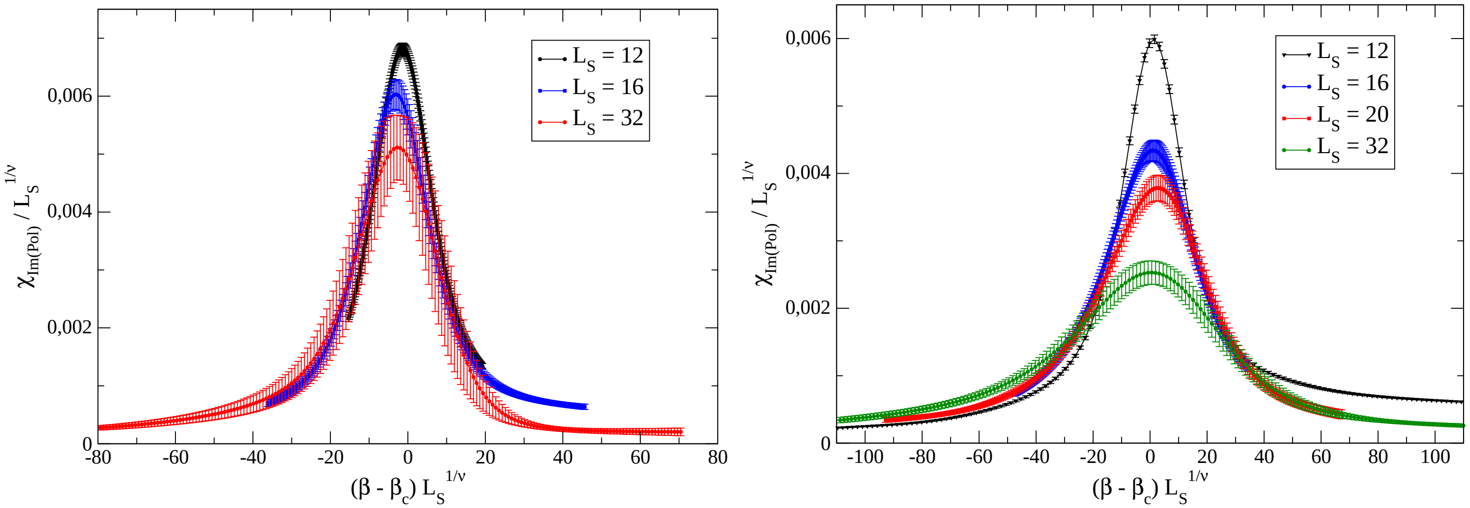
**Table of relevant critical indexes**

| | $\nu$ | $\gamma$ |
|---|---|---|
| **Ising 3d** | **0.63** | **1.24** |
| $1^{st} Order$ | $1/3$ | **1** |

# 2nd order




# 1st order



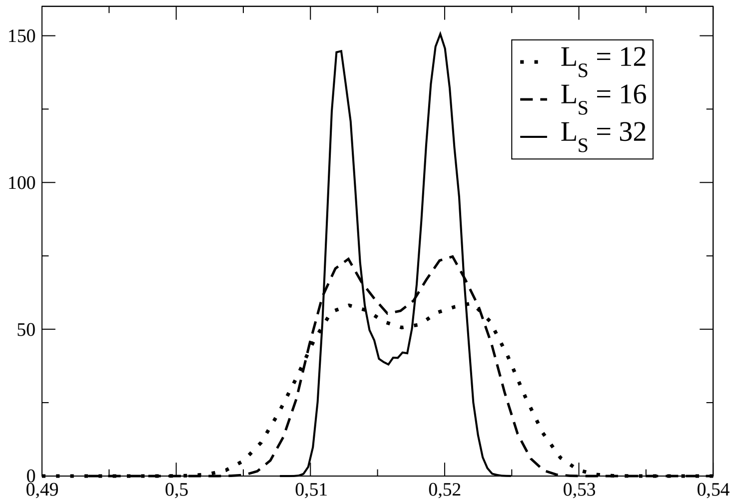

$am_q = 0.025$

$\beta_{\mathrm{RW}} = 5.3388$

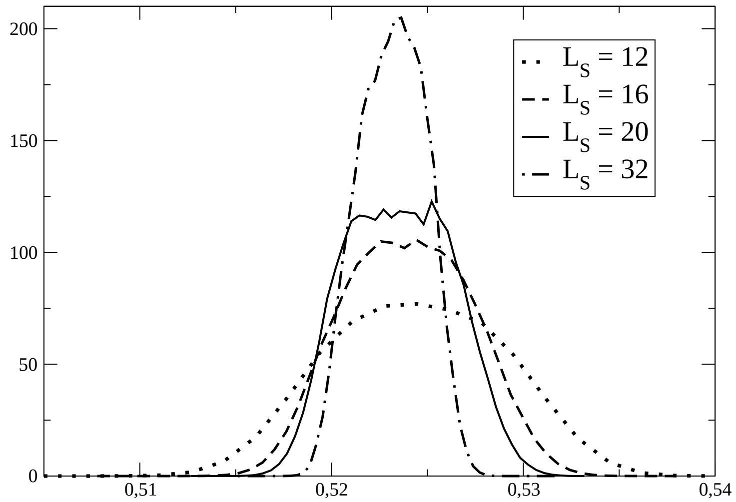
$am_q = 0.075$

$\beta_{\mathrm{RW}} = 5.3969$

# reweighted plaquette distribution at the critical coupling



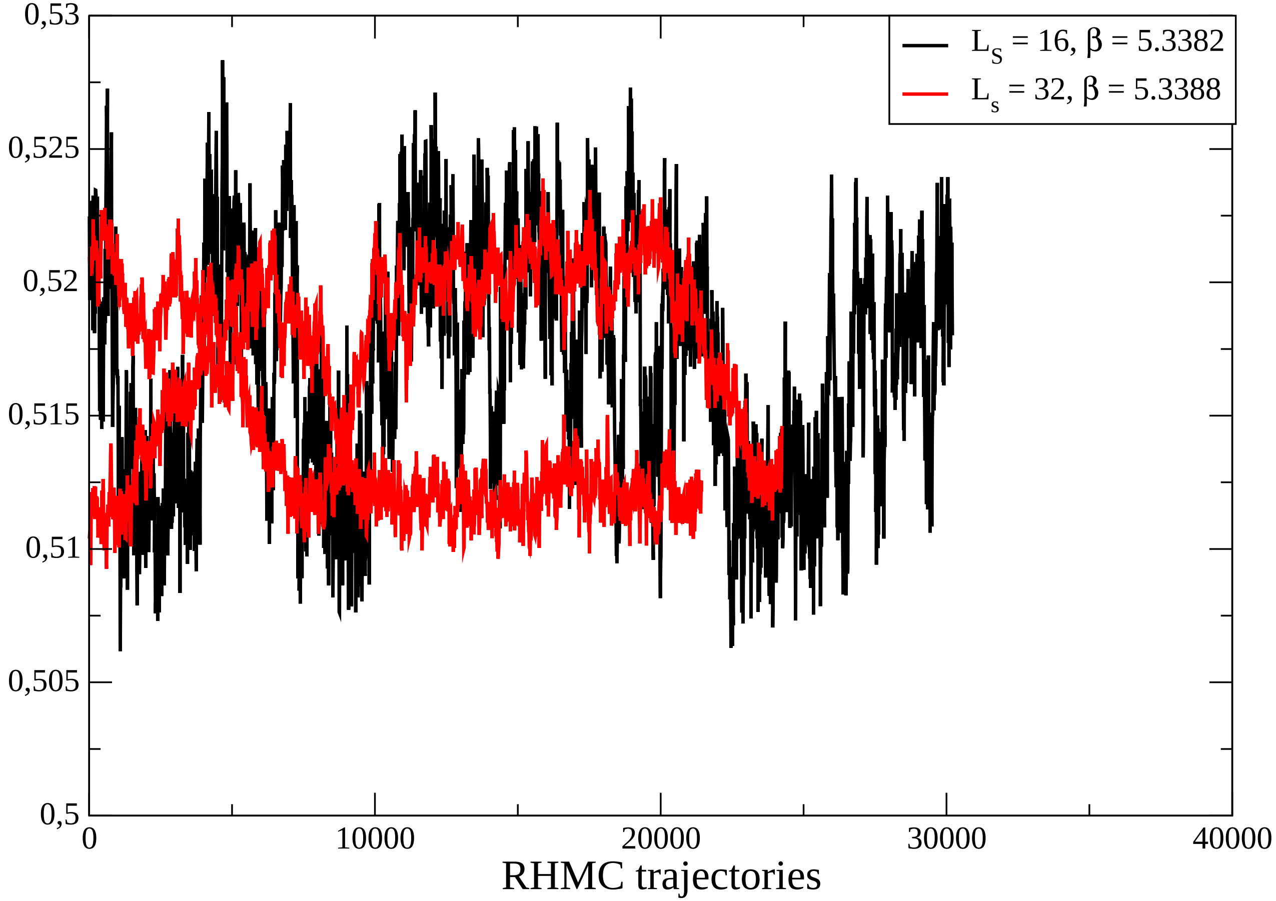

$am_q = 0.025$



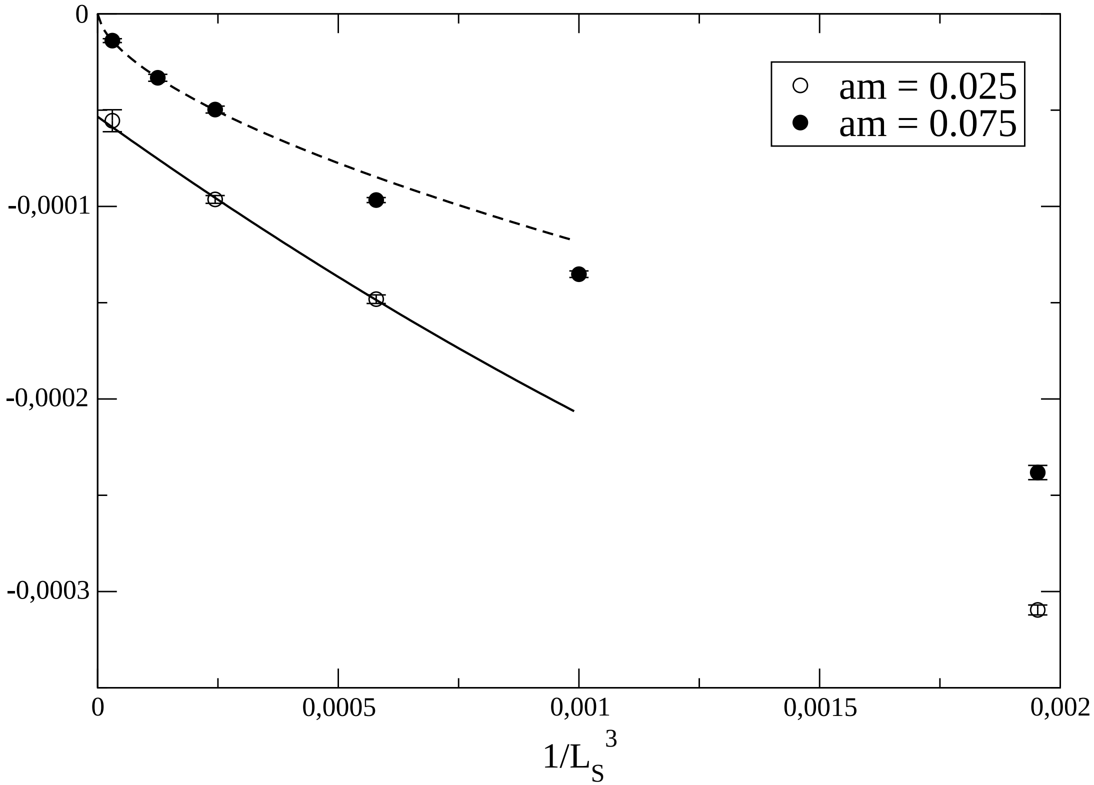

$am_q = 0.075$

**Double peak distributions are visible at the lower quark mass, but not at the higher quark mass**

The development of metastabilities as $L_s \longrightarrow \infty$ for the lower quark mass is also visible from Monte-Carlo histories of the plaquette:



Challa-Landau-Binder cumulant of the spatial plaquette



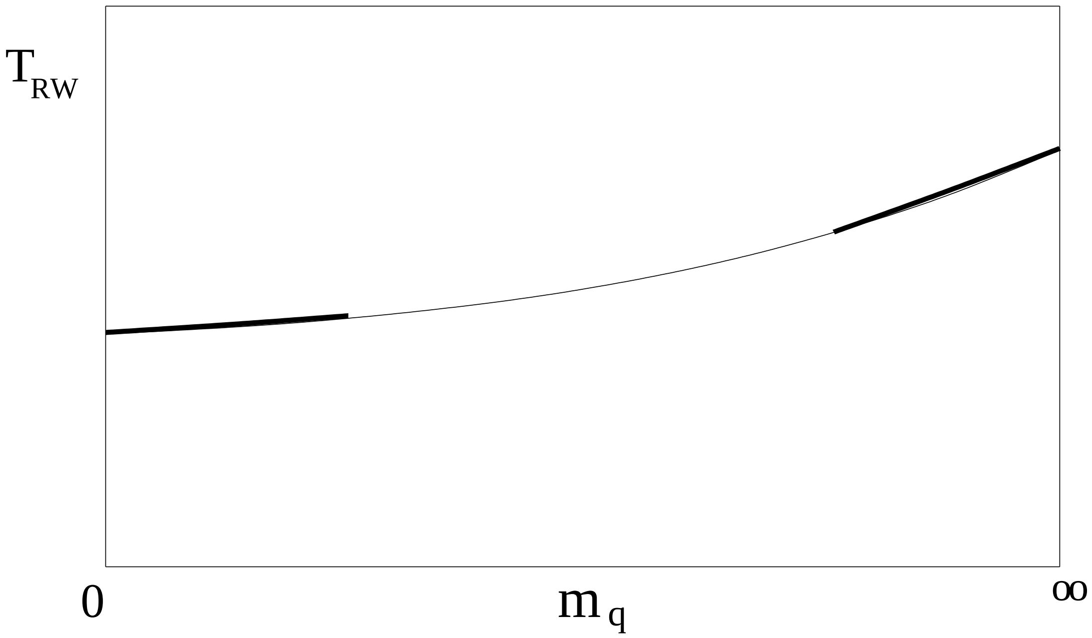

**also the infinite volume limit of the Binder-Challa-Landau cumulant**

$$\frac{1}{3}\left(1-\frac{\langle P^4\rangle}{\langle P^2\rangle^2}\right)$$

**is consistent with a first order at** $am_q = 0.025$ **but not at** $am_q = 0.075$**.**

Both the scaling of the order parameter susceptibility and the search for double peak distribution lead to the following conclusion:

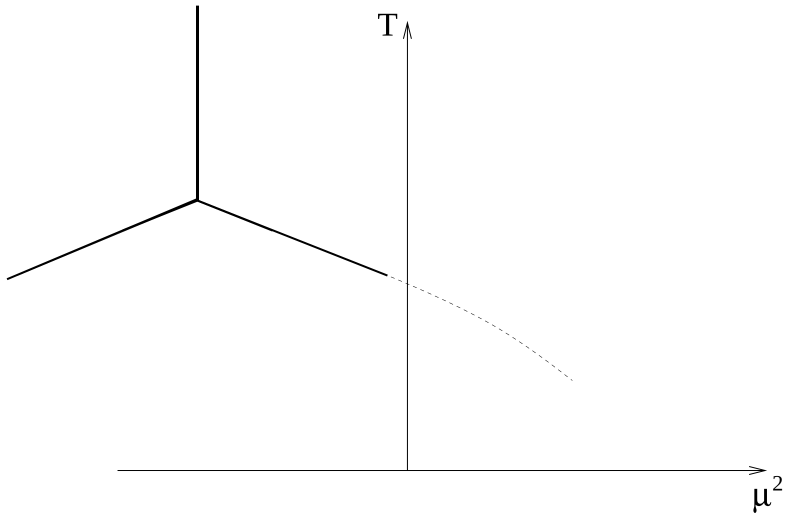
The transition is first order at the lower quark mass $am = 0.025$. It is weaker and likely second order at the higher quark mass $am_q = 0.075$. Since at $am_q = \infty$ the transition must be first order again, the following scenario takes place:

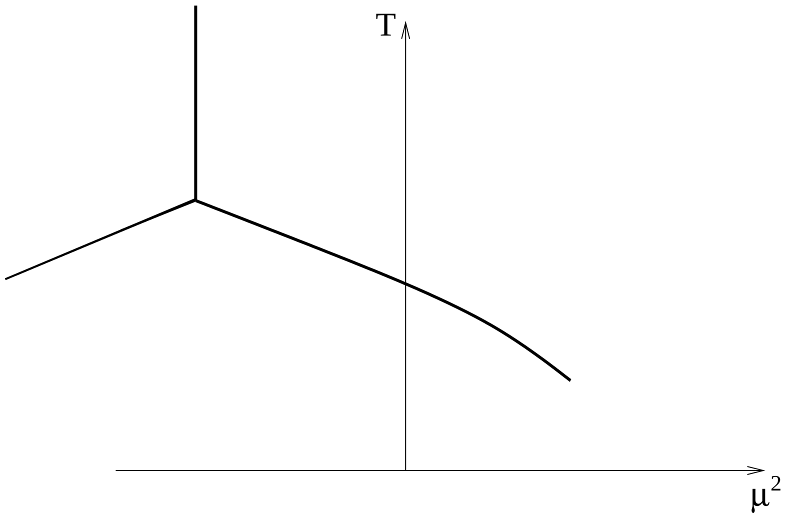



Since for $am_q = 0.025$ the pion is already quite heavy, the first order chiral region includes physical quark masses.

## 3 – Discussion and speculations

When the RW endpoint is first order, what is the fate of the first order line departing from it?



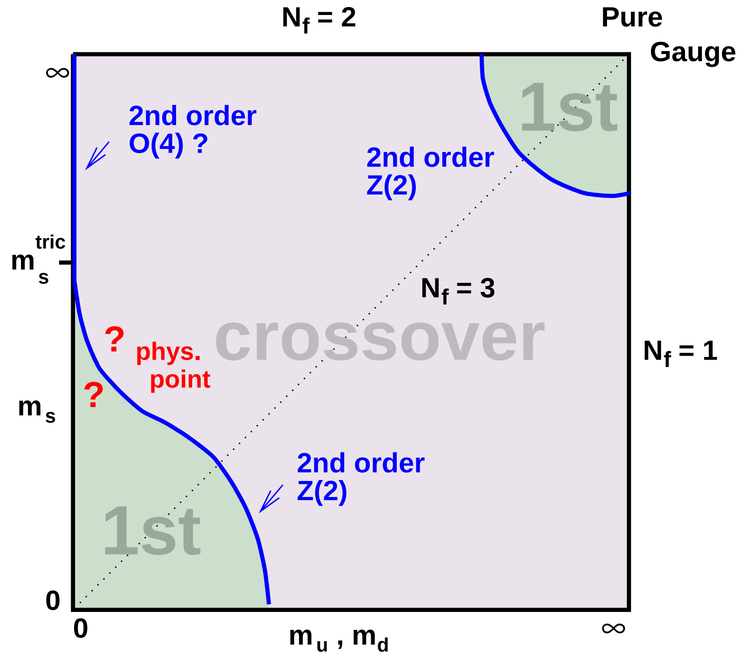

1) It could have a 2nd order endpoint at $\mu^2 < 0$ whose critical behaviour could have strong influence on the $\mu^2 = 0$ physics

2) It could cross the $\mu^2 = 0$ axis

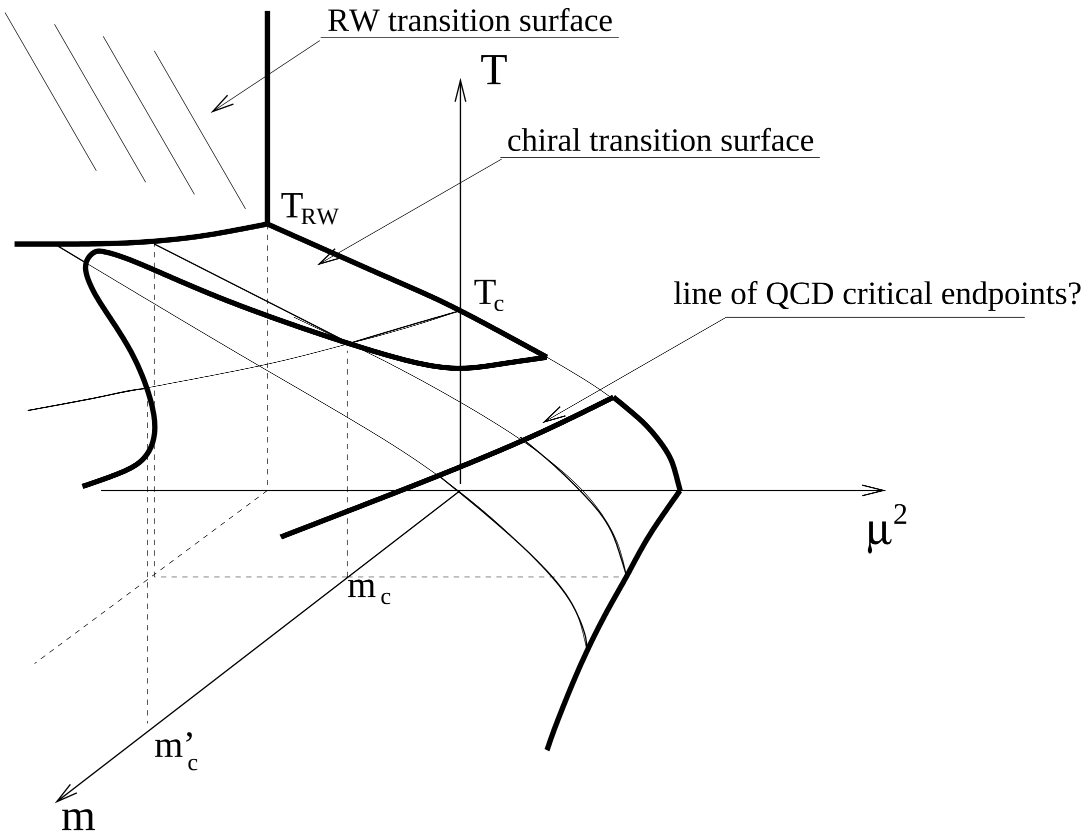
Second possibility is verified in the quenched limit, could be likely again in the chiral limit where the RW endpoint gets stronger.

# a pure speculation ...

Suppose a similar scenario happens for different number of flavors (i.e. RW endpoint weakens and get stronger again when decreasing the quark masses)




What if the first order regions in the Columbia plot are interesections with the first order line (hyper-surface) departing from the RW endpoint? This is true for the quenched corner




**In this purely speculative scenario, the phase diagram for $N_f = 3$ in the $T$-$\mu^2$-$m$ plane would look like above.**

**That is supported by the recent results from Ph. de Forcrand and O. Philipsen:**

**the chiral critical mass is a decreasing function of $\mu^2$**

# 4 – CONCLUSIONS

- **For $N_f = 2$ QCD, standard staggered and plaquette action, $L_t = 4$, the RW end-point is first order for low quark masses, as in the quenched limit.**
  **That should be checked by further studies going closer to the continuum limit.**

- **One can make more speculative conjectures starting from that, which could be checked by systematic studies of the phase diagram in the $T$-$\mu_I$ plane and for different number of flavors.**