QCD transition temperature: approaching the continuum

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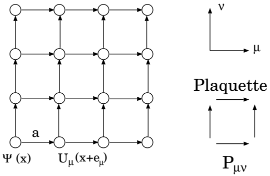
new ('09) results of the Wuppertal-Budapest group
(about scaling and lattice artefacts)

EMMI Workshop, September 1 2009, St. Goar

Outline

Lattice QCD introduction



Fundamental fields

Gauge fields:

$U_\mu(x) \in SU(3)$ live on the links (μ index)

Quark fields:

$\Psi(x)$, $\bar{\Psi}(x)$ anti-commuting Grassmann variables live on the sites

Wilson fermions: computationally expensive

Staggered fermions: faster, BUT taste symmetry violation
(only one pseudogoldstone pion instead of three)
fermion doubling is avoided by rooting: "good, bad or ugly?"

Lattice formulation

$$Z = \int dU d\Psi d\bar{\Psi} e^{-S_E} \tag{1}$$

S_E is the Euclidean action

Parameters:
gauge coupling g
quark masses m_i $(i = 1..N_f)$
(Chemical potentials μ_i)
Volume (V) and temperature (T)

Finite $T \leftrightarrow$ finite temporal lattice extension

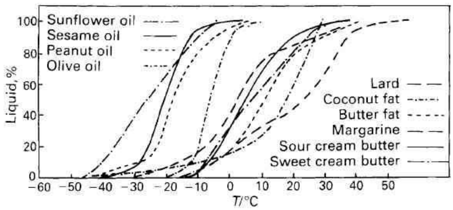
$$T = \frac{1}{N_t a} \tag{2}$$

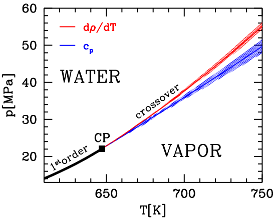
Continuum limit: $a \to 0 \Longleftrightarrow N_t \to \infty$

The nature of the QCD transition

Y.Aoki, G.Endrodi, Z.Fodor, S.D.Katz, K.K.Szabo, Nature, 443 (2006) 675

analytic transition (cross-over) $\Rightarrow$ it has no unique T_c:
examples: melting of butter (not ice) & water-steam transition

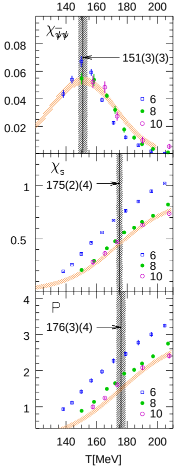




above the critical point c_p and $d\rho/dT$ give different T_cs.
QCD: chiral & quark number susceptibilities or Polyakov loop
they result in different T_c values $\Rightarrow$ physical difference

The transition temperature: results and scaling

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46



Chiral susceptibility

T_c=151(3)(3) MeV

ΔT_c=28(5)(1) MeV

Quark number susceptibility

T_c=175(2)(4) MeV

ΔT_c=42(4)(1) MeV

Polyakov loop

T_c=176(2)(4) MeV

ΔT_c=38(5)(1) MeV

Literature: discrepancies between T_c

Bielefeld-Brookhaven-Riken-Columbia Coll. (+MILC='hotQCD'):

M. Cheng et.al, Phys. Rev. D74 (2006) 054507

T_c from $\chi_{\bar{\psi}\psi}$ and Polyakov loop, from both quantities:

T_c=192(7)(4) MeV

Wuppertal-Budapest group (WB):

Y. Aoki, Z. Fodor, S.D. Katz, K.K. Szabo, Phys. Lett. B. 643 (2006) 46

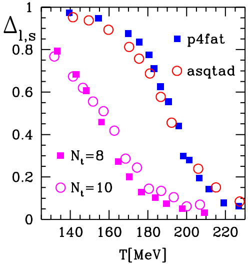
chiral susceptibility: T_c=151(3)(3) MeV
Polyakov and strange susceptibility: T_c=175(2)(4) MeV

'chiral T_c': ≈40 MeV; 'confinement T_c': ≈15 MeV difference

both groups give continuum extrapolated results with physical m_π

Literature: discrepancies between T dependences

Reason: shoulders, inflection points are difficult to define?
Answer: no, the whole temperature dependence is shifted



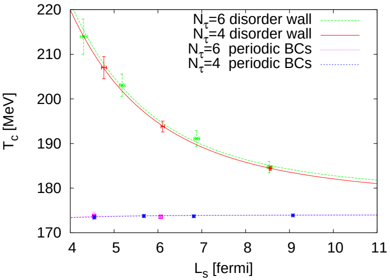
for chiral quantities ≈35 MeV; for confinement ≈15 MeV
this discrepancy would appear in all quantities (eos, fluctuations)

150 MeV transition temperature: isn't it a bit too small?
lattice works in V$\rightarrow\infty$, which gives much smaller T_c

T_c strongly depends on the geometry

nanotube-water doesn't freeze, even at hundred degrees below 0^oC

exploratory study: A. Bazavov and B. Berg, Phys.Rev. D76 (2007) 014502
use 'confined' spatial boundary conditions: more like experiments



large deviation (upto 30 MeV) from the infinite volume limit
if $V\rightarrow\infty$ is 150 MeV a 100 fm^3 system might have 170 MeV

Possible reasons for the discrepancy

"Non-lattice artefact/formulation" related reasons

a. bug in the codes

b. systematic errors are largely underestimated

"Lattice artefact/formulation" related reasons

a. the pion mass is not small enough:
'hotQCD' 230MeV $\Rightarrow$ shift of 5 MeV, WB: 135 MeV pseudogoldstone

b. not small enough lattice spacings: new 'hotQCD'/WB upto N_t=8/12

c. actually it is not QCD, what we are studying
(most large scale thermodynamics studies use staggered fermions)

Discretization errors in the transition region

we always have discretization errors: nothing wrong with it as long as

a. result: close enough to the continuum value (error subdominant)
b. we are in the scaling regime (a^2 in staggered)

various types of discretization errors $\Rightarrow$ we improve on them (costs)

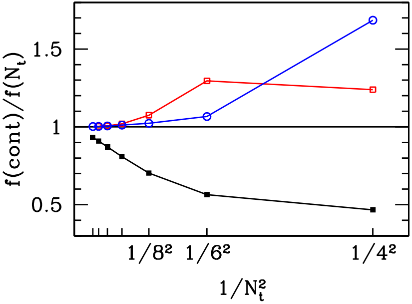
we are speaking about the transition temperature region
interplay between hadronic and quark-gluon plasma physics
smooth cross-over: one of them takes over the other around T_c

both regimes (low T and high T) are equally important
improving for one: T$\gg T_c$, doesn't mean improving for the other: T$<T_c$

example: 'expansion' around a Stefan-Boltzman gas (van der Waals)
for water: it is a fairly good description for T$\gtrsim$300^o
claculate the boiling point: more accuracy needed for the liquid phase

Examples for improvements, consequences

how fast can we reach the continuum pressure at T=∞?



p4 action is essentially designed for this quantity T$\gg T_c$

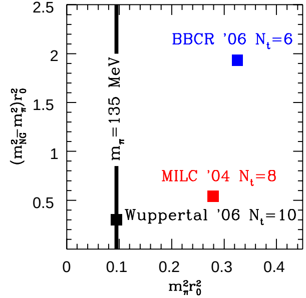
asqtad designed mostly for T=0 physics (but good at high T, too)

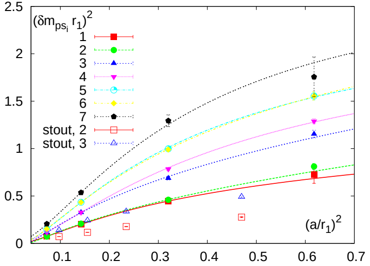
stout-smeared one-link converges slower but in the a^2 scaling regime (e.g. extrapolation from N_t=8,10 provides a result within about 1%)

Chiral symmetry breaking and pions

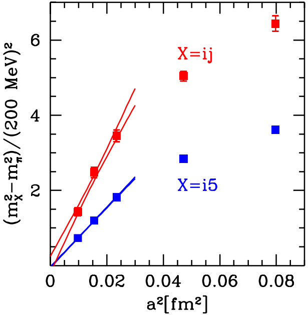
transition temperature for remnant of the chiral transition:
balance between the chirally broken and chirally symmetric sectors
chiral symmetry breaking: 3 pions are the pseudo-Goldstone bosons

staggered QCD: 1 pseudo-Goldstone instead of 3 (taste violation)
staggered lattice artefact ⇒ disappears in the continuum limit
WB: stout-smeared improvement is designed to reduce this artefact





Scaling for the pion splitting



scaling regime is reached if a^2 scaling is observed
asymptotic scaling starts only for N_t>8 (a$\lesssim$0.15 fm): two messages

a. N_t=8,10 extrapolation gives 'p' on the $\approx$1% level: good balance

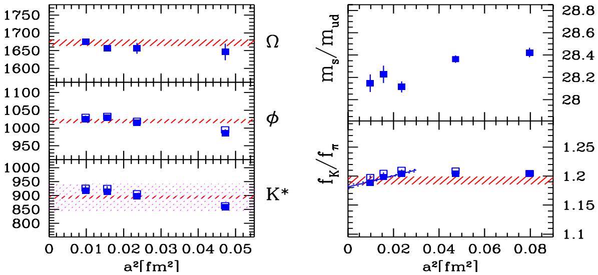
b. stout-smeared improvement is designed to reduce this artefact
most other actions need even smaller 'a' to reach scaling

Setting the scale in lattice QCD

in meteorology, aircraft industry etc. grid spacing is set by hand

in lattice QCD we use g,m_{ud} and m_s in the Lagrangian ('a' not)
measure e.g. the vacuum mass of a hadron in lattice units: $M_\Omega a$
since we know that M_Ω=1672 MeV we obtain 'a' and T=1/N_ta

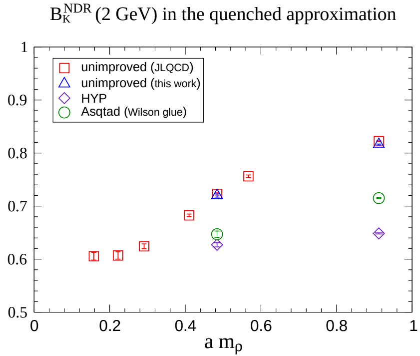
Y.Aoki et al. [Wuppertal-Budapest Collaboration] arXiv:0903.4155



independently which quantity is taken (we used physical masses)
⇒ one obtains the same 'a' and T, result is safe

Scaling of B_K in quenched simulations

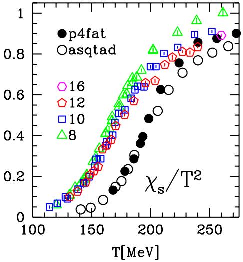
HPQCD and UKQCD Collaborations, Phys. Rev. D73 (2006) 114502



unimproved action has large scaling violations
asqtad action is somewhat better
HYP smeared improvement ⇒ almost perfect scaling

T>0 results: strange susceptibility

Y.Aoki et al. [Wuppertal-Budapest Collaboration] arXiv:0903.4155

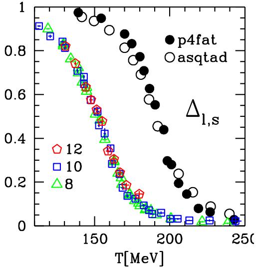


- 'hotQCD' results are on N_t=8, WB results are on N_t=8,10,12,(16)
- 'hotQCD': results with two different actions are almost the same
- WB: for large T one extrapolates according to the known a^2 behaviour
- WB: no change in the lattice results compared to our 2006 paper
- note, that the experimental value of f_K decreased by 3% since 2006

about 20 MeV difference between the results

T>0 results: chiral condensate

Y.Aoki et al. [Budapest-Wuppertal Collaboration] arXiv:0903.4155



'hotQCD' results are on N_t=8, WB results are on N_t=8,10,12
'hotQCD': results with two different actions are almost the same
WB: no lattice spacing dependence observed for N_t=8,10,12
WB: no change in the lattice results compared to our 2006 paper

about 35 MeV difference between the results

transition temperatures for various observables

	$\chi_{\bar\psi\psi}/T^4$	$\chi_{\bar\psi\psi}/T^2$	$\chi_{\bar\psi\psi}$	$\Delta_{l,s}$	L	χ_s
WB'09	146(2)(3)	152(3)(3)	157(3)(3)	155(2)(3)	170(4)(3)	169(3)(3)
WB'06	151(3)(3)	-	-	-	176(3)(4)	175(2)(4)
BBCR	-	192(4)(7)	-	-	192(4)(7)	-

renormalized chiral susceptibility, renormalized chiral condensate
Polyakov loop and strange quark number susceptibility

no change compare to our 2006 data (errors are reduced)
note, that the experimental value of f_K decreased by 3% since 2006
Particle Data Group now gives f_K=155.5(2)(8)(2) MeV (error 0.5%)

r_0 is not directly measurable:

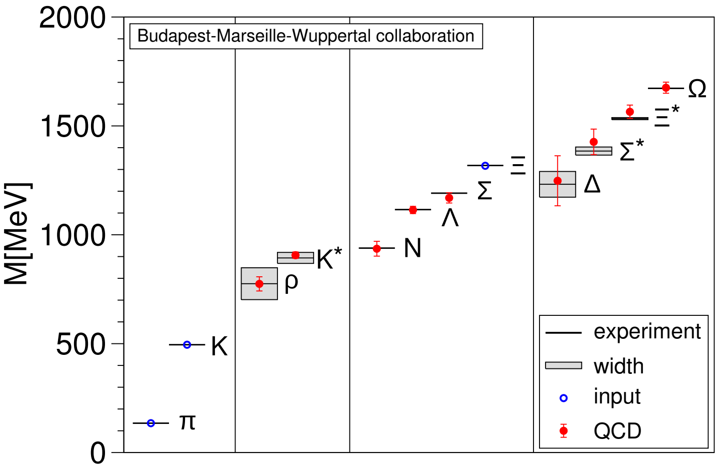
ETM:0.444(4) fm, QCDSF:0.467(6) fm,

HPQCD&UKQCD:0.469(7) fm, PACS-CS:0.492(6)(+7) fm

Summary

- new (2009) results for the transition temperature
- three major improvements since 2006
  a. at T=0 all simulations are done with physical quark masses
  b. to verify that the results are independent of the scale setting
  we use 5 experimentally well-known quantites: $f_K, f_\pi, m_{K^*}, m_\Omega, m_\Phi$
  c. even smaller lattice spacings: N_t=12 (in one case N_t=16)
- all findings are in complete agreement with our 2006 results
- Particle Data Group reduced the experimental value of f_K: 3%
- discrepancy between Wuppertal-Budapest & 'hotQCD' results
  a. for the remnant of the deconfinement transition: about 20 MeV
  b. for the remnant of the chiral transition: about 35 MeV
  $\Rightarrow$ finding the reason: task for the future
- Wilson fermions: theoretically cleaner option

Final result for the hadron spectrum



CP violation, K^0–$\bar{K}^0$ mixing and B_K

$K_L \to e_R^+ \nu_L \pi^-$: 20.2% and $K_L \to e_R^- \bar{\nu}_L \pi^+$: 0%
the reason is the maximal C-violation

do we know the absolute definition of left and right?
exchange also left and right $K_L \to e_L^- \bar{\nu}_R \pi^+$: 20.2%

do it more precisely:
K_L slightly prefers to decay into $e^+ \nu \pi^-$ than $e^- \bar{\nu} \pi^+$

$$\frac{\Gamma(K_L \to e^+ \nu \pi^-)}{\Gamma(K_L \to e^- \bar{\nu} \pi^+)} = 1.007 = 1 + f(\bar{\eta}, \bar{\rho} ...)\ \langle \bar{K}^0 | \mathcal{H} | K^0 \rangle = 1 + f(\bar{\eta}, \bar{\rho} ...) \frac{8}{3} m_K^2 f_K^2 B_K$$

CKMfitter Group, UTfit Collab. still use quenched B_K from 1997

Importance sampling

$$Z = \int \prod_{n,\mu} [dU_\mu(n)] e^{-S_g} \det(M[U])$$

we do not take into account all possible gauge configuration

each of them is generated with a probability $\propto$ its weight

importance sampling, Metropolis algorithm:
(all other algorithms are based on importance sampling)

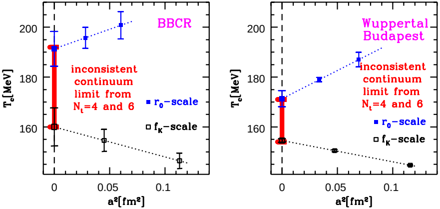
$$P(U \rightarrow U') = \min\left[1, \exp(-\Delta S_g) \det(M[U'])/\det(M[U])\right]$$

gauge part: trace of 3$\times$3 matrices (easy, without M: quenched)
fermionic part: determinant of $10^6 \times 10^6$ sparse matrices (hard)

more efficient ways than direct evaluation (Mx=a), but still hard

Consequences of the non-scaling behaviour

for large 'a' no proper a^2 scaling (e.g. due to large m_π splitting)
how do we monitor it, how to be sure being in the scaling regime?
dimensionless combinations in the a→0 limit:
$T_c r_0$ or T_c/f_K for the remnant of the chiral transition



N_t=4,6: inconsistent continuum limit
N_t=6,8,10: consistent continuum limit (stout-link improvement)

independently which quantity is taken one obtains the same T_c
signal: extrapolation is safe, we are in the a^2 scaling regime

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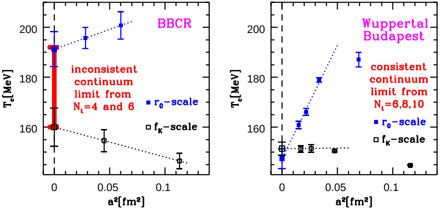

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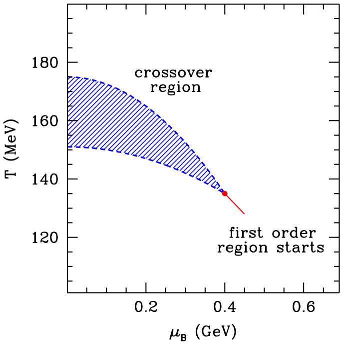
Scenarios for $\mu > 0$

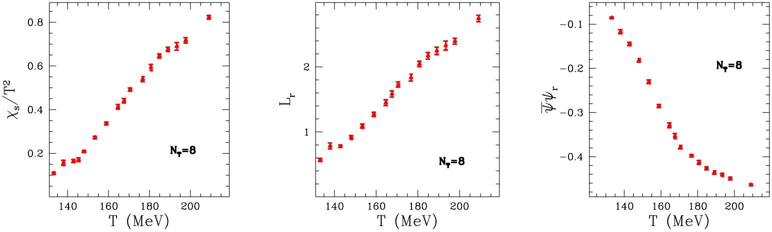


Does the crossover region shrink or expand?

The curvature can affect the existence of the critical endpoint

Estimate: if $\mu_{crit} = 360$ MeV $\rightarrow \Delta\kappa \approx 0.02$

Observables

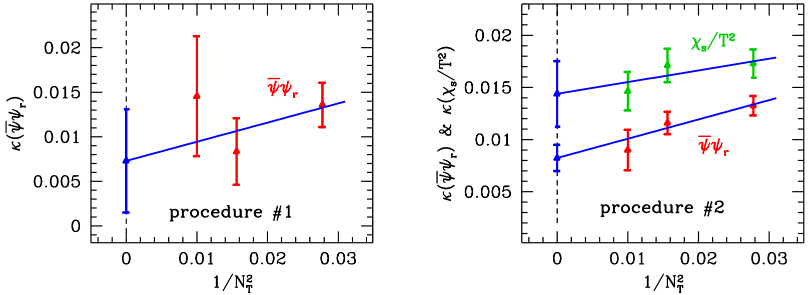


T_c is defined as inflection point or given value ($\frac{\mathcal{O}(T{=}0)+\mathcal{O}(T{=}\infty)}{2}$)
μ dependence by Taylor expansion $\rightarrow$ curvature

Two procedures:

1. determine inflection point as a function of μ
2. average shifts for different T-s

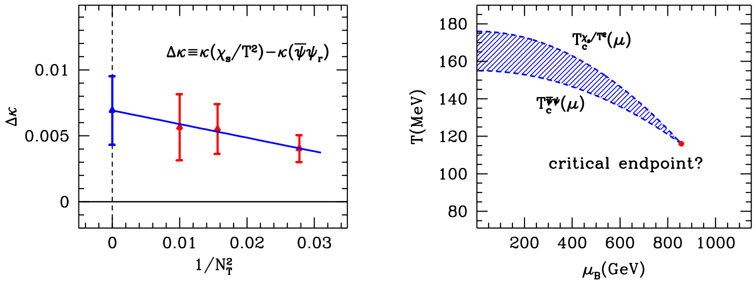
Preliminary results



Continuum extrapolated results from $N_t = 6, 8$ and 10; both procedures

Polyakov loop result consistent with χ_s/T^2

Preliminary results



Difference $\Delta\kappa \equiv \kappa(\chi_s/T^2) - \kappa(\bar\psi\psi_r)$ not consistent with zero

Necessary condition for the critical point

But not sufficient

Strength of the transition from individual quantities →

more statistics needed

more statistics in the Taylor method

determining the inflection point needs 10-times more statistics
to that end the whole T dependence should be determined
this gives more than just the inflection point
a clear signal for broadening or shrinking will be seen

$a \to 0$ can be done with present resources

the Taylor procedure gives only the leading order term(s) in μ
N_t=4 unimproved staggered experience [Fodor-Katz'01, Fodor-Katz'04]
the leading order terms are insensitive to the critical point $\Rightarrow$
evaluation of the whole determinant, we need all the terms in μ

our action (smeared improved): better at T=0 & evaluation possible
for p4, asqtad or hisq no such eigenvalue structure (det) is known

(it gives certainly more information than just the leading order terms)

memory/CPU requirements for full determinants

N_t=4 & N_s=8,10,12 needed 1 GB memory & 25 CPU years (in '04)
memory requirements grow as N_t^6, CPU requirements as N_t^9

accumulate the same statistics (shown by the first CPU row)
to reach the same μa: exponentially more configs are needed
'05 observation: applicability range $\propto V^{-0.35}$ and $\mu a \propto V^{-0.25}$
$\Rightarrow$ additional increase of the statistics (second CPU$^+$ row)

N_t	4	6	8	10
memory [GB]	1	11	64	244
'04 CPU [kyears]	0.025	1	13	95
'04 CPU$^+$ [kyears]	0.025	1	18	150
machine [year]	cluster	cluster	2 BG/P	15 BG/P

$\Rightarrow$ N_t=6,8,10: our present resources are not enough for that