Dual Condensates

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Phys. Rev. **D75** (2007) 114003
Phys. Rev. **D77** (2008) 114018
and work in progress

St.Goar, 31th August 2009

Center transformations and spectral sums: lattice

String tension from low lying eigenvalues

Dual condensates

Spectral sums for continuum theory

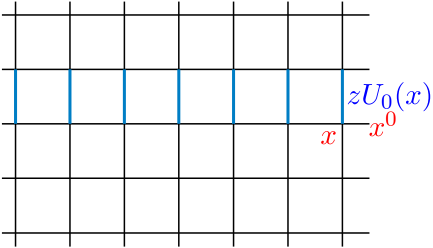
Conclusions

Center transformations and spectral sums

finite temperature: periodic Euclidean time

center transformation with $z \in \mathcal{Z}$

$$\{U_\mu(x)\} \longrightarrow \{{}^zU_\mu(x)\}, \quad U_0(x^0,\boldsymbol{x}) \to zU_0(x^0,\boldsymbol{x}), \quad x^0 \text{ fixed}$$



loop $\mathcal{C}$ winds n-times:

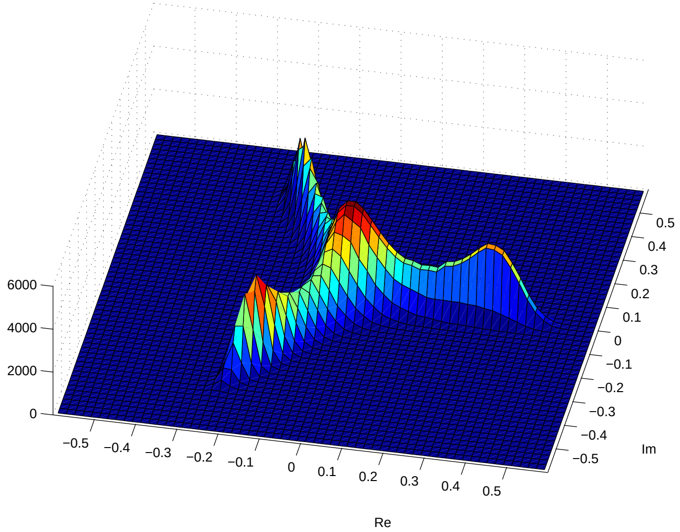
$$\mathcal{W}_\mathcal{C} \longrightarrow z^n \mathcal{W}_\mathcal{C}$$

Polyakov loop $\mathcal{P}(\boldsymbol{x})$:

$$\mathcal{P}(\boldsymbol{x}) \longrightarrow z\mathcal{P}(\boldsymbol{x})$$

action and measure of gluodynamics invariant

Polyakov loop $P(\boldsymbol{x}) = \mathrm{tr}\,\mathcal{P}(\boldsymbol{x})$ order parameter



histogram of $P = \mathrm{tr}\mathcal{P}$ for quenched $SU(3)$

- Dirac operator $\mathcal{D}$ with fixed boundary conditions:

$$\text{spectral data:} \qquad \{\lambda_p, \psi_p\}$$

- chiral condensate

$$\langle\bar\psi(x)\psi(x)\rangle = \frac{1}{Z}\int \mathcal{D}U\, e^{-S[U]} \det\mathcal{D}\cdot\langle x|\mathrm{tr}\,\mathcal{D}^{-1}|x\rangle$$

- spectral resolution $\rightarrow$ spectral sum

$$\langle x|\mathrm{tr}\mathcal{D}^{-1}|x\rangle = \sum_p \frac{1}{\lambda_p}\,\bar\psi_p(x)\psi_p(x)$$

- beyond chiral condensate (fermion density, UV-filters)

$$\langle x|\,\mathrm{tr}\,\mathcal{C}f(\mathcal{D})|x\rangle = \sum_p f(\lambda_p)\,\bar\psi_p(x)\,\mathcal{C}\psi_p(x)$$

- center transformations (ct): $\mathcal{D}, \lambda_p, \psi_p \longrightarrow \mathcal{D}_z, {}^z\lambda_p, {}^z\psi_p$

$$\langle x|\,\mathrm{tr}\mathcal{C}f(\mathcal{D})|x\rangle \rightarrow \langle x|\,\mathrm{tr}\mathcal{C}f(\mathcal{D}_z)|x\rangle$$

- dual spectral sums (Fourier transform)

$$\mathcal{S}_{\mathcal{C},f}(x) = \sum_k z_k^* \langle x|\mathrm{tr}\,\mathcal{C} f\left(\mathcal{D}_{z_k}\right)|x\rangle$$

- order parameters for center symmetry

$$\mathcal{S}_{\mathcal{C},f}(x) \xrightarrow{\mathrm{ct}} z\,\mathcal{S}_{\mathcal{C},f}(x) \quad \text{(since} \quad \mathcal{D}_{z_k} \xrightarrow{\mathrm{ct}} \mathcal{D}_{z z_k})$$

- space-time averages: $\mathcal{S}_{\mathcal{C},f}$
- Gattringers original choice:
  $\mathcal{C} = \mathbb{1},\ f(\mathcal{D}) = \mathcal{D}^{N_t}$ $\rightarrow$ exact relation to thin Polyakov loop

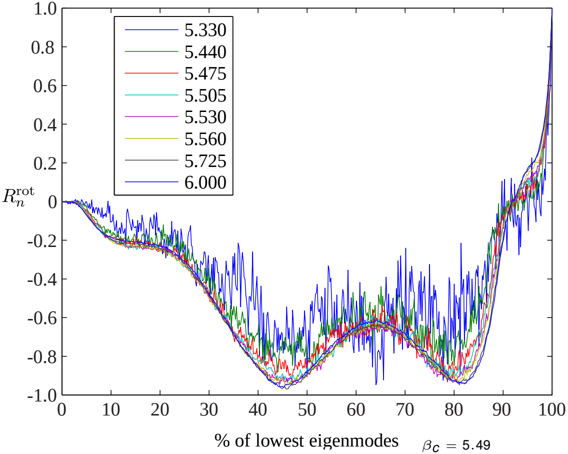
$$\Sigma^{(N_t)}(x) \equiv \sum_{k=1}^{n_D} z_k^* \langle x|\mathrm{tr}\left(\mathcal{D}_{z_k}\right)^{N_t}|x\rangle = \kappa'\, P(\boldsymbol{x})$$

$\mathcal{D}$ with NN, hopping parameter expansion of $\langle x|\mathrm{tr}\mathcal{D}^{N_t}|x\rangle$.

analytical/numerical results:

(Gattringer; Bilgici et al.; Synatschke, Wozar, AW)

- small eigenvalues more affected as large ones by ct
- above T_c more affected as below T_c
- dependence on $\arg(\mathcal{P})$
- but: $\Sigma^{(N_t)}$ dominated by large λ_p: $R = \Sigma^{(N_t)}/(\kappa' P)$



$\beta_c = 5.49$

- continuum limit: need UV-improved spectral sums

$$f(\lambda) \stackrel{\lambda\to\infty}{\longrightarrow} 0$$

- prominent: propagator sums (Synatschke, Wozar, AW)

$$\Sigma^{(-1)}(x) = \sum_k z_k^* \,\langle x|\mathrm{tr}\,\frac{1}{\mathcal{D}_{z_k}}|x\rangle$$

$$\Rightarrow \langle\Sigma^{(-1)}\rangle \qquad \text{dual condensate}$$

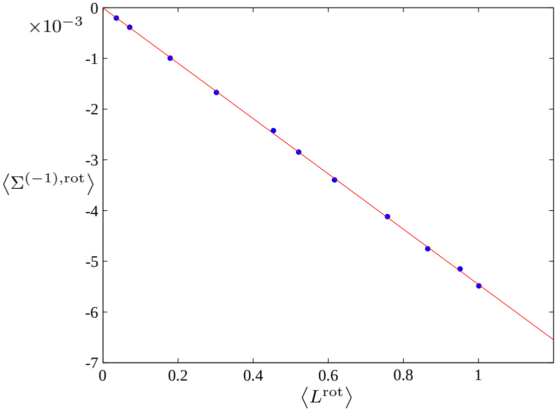
- hopping parameter expansion and simulations $\rightarrow$

$$\Sigma^{(-1)}(x) \propto P(\boldsymbol{x}) \quad , \quad \Sigma^{(-1)} \propto L = \frac{1}{V_s}\sum_{\boldsymbol{x}} P(\boldsymbol{x})$$

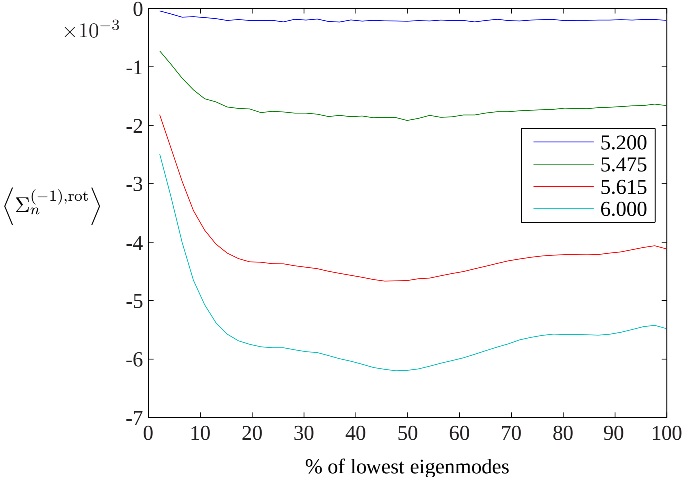
- SU(N): Fourier transform of chiral condensate

dual condensate for SU(3) from MC-simulations

$$f(\mathcal{D}) = \frac{1}{\mathcal{D}} \Longrightarrow \text{propagator sum } \Sigma^{(-1)} \Longrightarrow \langle \Sigma^{(-1)} \rangle$$



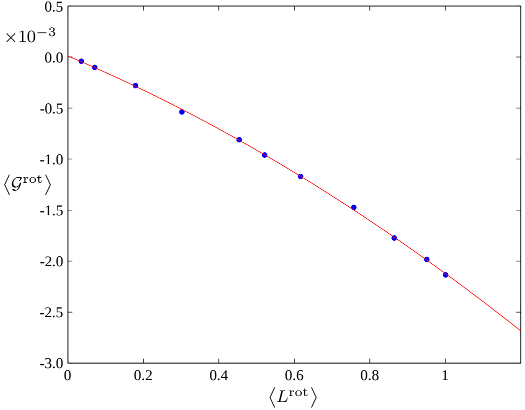
$\langle \Sigma^{-1,\mathrm{rot}} \rangle$ as function of $\langle L^{\mathrm{rot}} \rangle$, small $4^3 \times 3$ lattice, quenched dual condensate $\langle\Sigma^{(-1)}\rangle$ increases with T



$T < T < T < T$ (Synatschke, Wozar, AW)

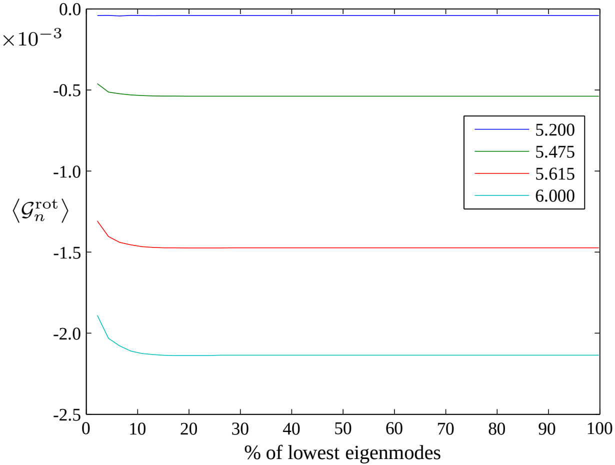
Gaussian spectral sums from MC-Simulations

$$f(\mathcal{D}) = e^{-\mathcal{D}^2} \Longrightarrow \text{heat kernel sum } \mathcal{G} \Longrightarrow \langle \mathcal{G} \rangle$$



$\langle \mathcal{G}^{\rm rot} \rangle$ as function of $\langle L^{\rm rot} \rangle$, small $4^3 \times 3$ lattice, quenched

partial Gaussian sums $\mathcal{G}_n$ converge quickly, lowest 3%



IR-dominance, small $4^3 \times 3$ lattice, $n_D = 2304$, quenched

String tension from low lying eigenvalues

- static quark potential

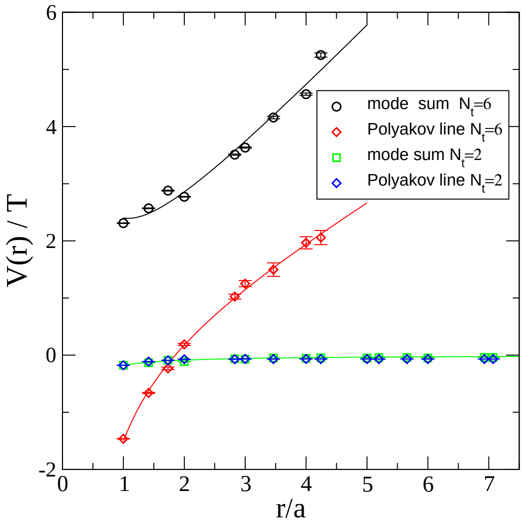
$$V(r) = -T\log\langle P(\boldsymbol{x})P(\boldsymbol{x}+r\boldsymbol{e}_3)\rangle$$

$P \to \kappa'\Sigma^{(N_t)} \Rightarrow$ all eigenfunctions/values needed

- use partial Gaussian spectral sum $\mathcal{G}_n$, for SU(2)

$$\mathcal{G}_n(x) := \frac{1}{8}\sum_{p=1}^{n}\left(|\psi_p(x)|^2 e^{-\lambda_p^2/\mu^2} - |{}^z\psi(x)|^2 e^{-{}^z\lambda_p^2/\mu^2}\right)$$

- $\mathcal{G}_n(\boldsymbol{x})$ instead of $P(\boldsymbol{x})$: $V(r) \to V_n^{\mathcal{G}}(r)$
- staggered fermions, improved action
- simulation parameters: $\beta = 1.35$, $\sigma a^2 = 0.1244(7)$
- lattices: $12^3 \times 6 \Longrightarrow T = 0.7\,T_c$
  $12^3 \times 2 \Longrightarrow T = 2.1\,T_c$
- 12 000 configurations each, $T_c \leftrightarrow N_t = 4$



potential from P and $\mathcal{G}_n$, $N_t = 6$ confined, $N_t = 2$ deconfined
lowest 50 eigenvalues of $\approx 50\,000$ eigenvalues

Dual condensates

- twist with arbitrary $z = e^{-i\phi}$, $\phi \in [0, 2\pi]$

$$
\begin{aligned}
U_0(x^0, \boldsymbol{x}) &\longrightarrow e^{-i\phi} U_0(x^0, \boldsymbol{x}), \quad x^0 \text{ fixed} \\
\mathcal{P}(\boldsymbol{x}) &\longrightarrow e^{-i\phi} \mathcal{P}(\boldsymbol{x})
\end{aligned}
$$

- imaginary chemical potential $\mu \propto \phi$
- non-periodic $U(N)$ gauge transformation $g(x) = e^{i\phi x^0/N_t}$

$$
\begin{aligned}
U_\mu(x), \psi(x) &\Longleftrightarrow {}^\phi U_\mu(x), {}^\phi\psi(x) \\
\text{twisted } U_\mu(x) &\Longleftrightarrow \text{twisted } {}^\phi\psi(x)
\end{aligned}
$$

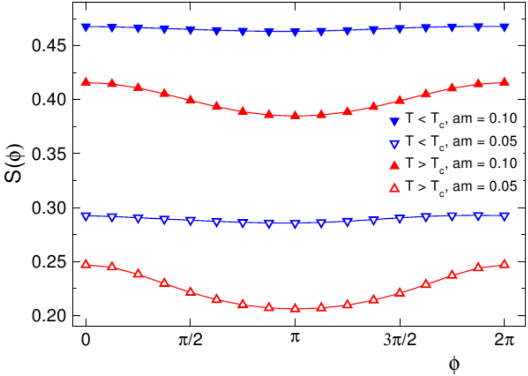
- spectral problem with boundary angle ϕ:

$$
\mathcal{D}\, {}^\phi\psi_p = {}^\phi\lambda_p\, {}^\phi\psi_p, \quad {}^\phi\psi_p(x^0 + N_t) = -e^{i\phi}\, {}^\phi\psi_p(x^0)
$$

- condensates

$$\bar\psi(x)\psi(x)\big|_\phi = \langle x|\mathrm{tr}\,\mathcal{D}^{-1}|x\rangle_\phi$$
$$S(\phi) = \frac{1}{V}\int dx\,\langle\bar\psi(x)\psi(x)\rangle_\phi$$

- below T_c: no ϕ-dependence, above T_c: ϕ-dependence



$T = 255$ MeV
$a = 0.129$ fm

$T = 337$ MeV
$a = 0.098$ fm

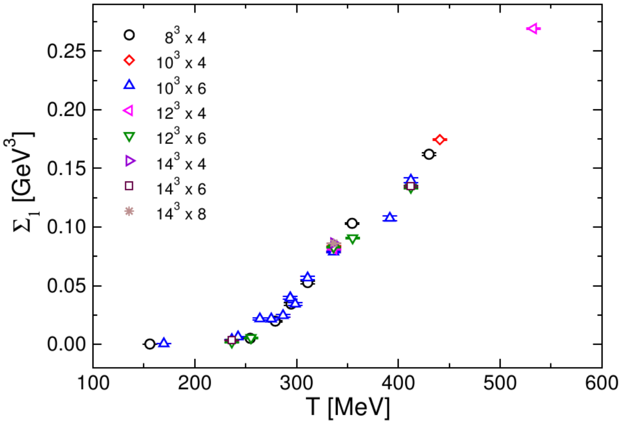
$12^3 \times 6$
quenched

Lüscher-Weiss action

from Bilgici et al. (2008)

- dual condensate

$$\langle \Sigma^{(-1)} \rangle = \int_0^{2\pi} \frac{d\phi}{2\pi} \, S(\phi)\, e^{-i\phi} \quad \text{vanishes below } T_c$$



$12^3 \times 6$, quenched, Bilgici et al. (2008)

With dynamical fermions

- Is condensate $2\pi/3$ or only 2π-periodic?
- 'gauge transformation' $g(x^0+N_t,\boldsymbol{x})=e^{i\phi}\,g(x^0,\boldsymbol{x})$
  $\phi=2\pi k/3 \Rightarrow$ can choose $g(x)\in SU(3)$

$$\begin{array}{rcl} U_\mu(x),\,\psi(x) & \Longleftrightarrow & {}^\phi U_\mu(x),\,{}^\phi\psi(x) \\ \psi \text{ antiperiodic} & \Longleftrightarrow & {}^\phi\psi \text{ boundary angle } 2\pi k/3 \end{array}$$

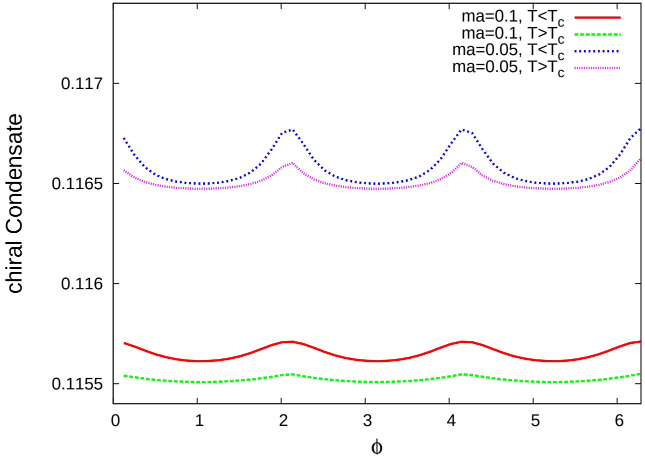
- twist in fermionic determinant and spectral sum

$$S(\phi)=\frac{1}{Z_\phi}\int \mathcal{D}U\, e^{-S[U]}\det(\mathcal{D}_\phi)\,\langle x|\mathrm{tr}\,D_\phi^{-1}|x\rangle$$

$2\pi/3$-periodic (Roberge-Weiss)
(cp. Braun, Haas, Marhauser, Pawlowski: ERGE-results)

- dual condensate vanishes for all T; instead

$$\langle\tilde{\Sigma}^{(-1)}\rangle=\int\frac{d\phi}{2\pi}\,S(\phi)\,e^{-3i\phi}$$



flat connections, $\langle P\rangle_\phi$ from simulations, 80 ϕ-values
$2\pi/3$-periodic condensates (Synatschke, Wozar, AW)

- quenched approximation, dynamical simulations, Schwinger-Dyson (Bilgici et al. 2009, Fischer 2009)

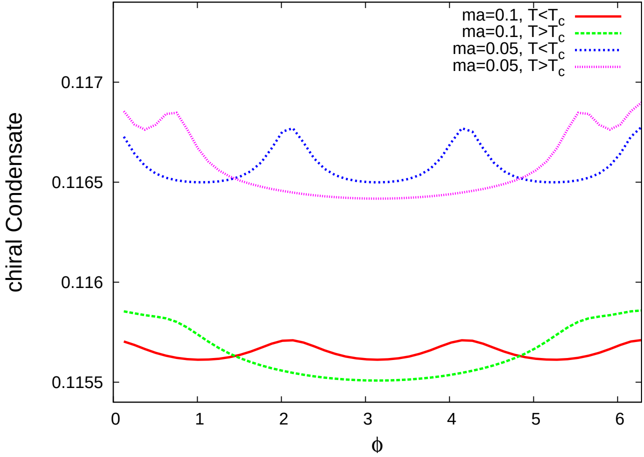
$$S'(\phi) = \frac{1}{Z}\int \mathcal{D}U\, e^{-S[U]}\,\det(\mathcal{D})\,\langle x|\mathrm{tr}\,D_\phi^{-1}|x\rangle$$

- ensemble ϕ-independent $\Rightarrow$ fixed $\langle P(\boldsymbol{x})\rangle \neq 0$
- not compatible with

$$\langle P(\boldsymbol{x})\rangle_{\phi=2\pi k/3} = e^{2\pi ik/3}\,\langle P(\boldsymbol{x})\rangle_{\phi=0}$$

- Roberge-Weiss symmetry violated for $\langle P\rangle \neq 0$
- now ϕ not imaginary chemical potential
- expansion in dressed Polyakov loops $\Rightarrow \Sigma^{(-1)} \propto P_1$

$$\langle x|\mathrm{tr}\,\mathcal{D}_\phi^{-1}|x\rangle = \sum_{n\in\mathbb{Z}} e^{in\phi}P_n, \quad P_n = \sum_{\mathrm{wind}(\mathcal{C}_x)=n} \alpha_{\mathcal{C}_x} W(\mathcal{C}_x)$$



flat connections, $\langle P \rangle$ from simulations, 80 ϕ-values
2π-periodicity, $\Sigma^{(-1)} \propto P$ (Synatschke, Wozar, AW)

Spectral sums for continuum theory

- finite temperature: A_μ periodic, ψ periodic
- $\mathcal{D} = i\gamma^\mu D_\mu + im \Rightarrow \{\lambda_p, \psi_p\}$
- center transformation = 'gauge transformation'

$$^gA_\mu = g(A_\mu + i\partial_\mu)g^{-1}, \quad g(x_0+\beta, \boldsymbol{x}) = z\, g(x_0, \boldsymbol{x}),$$

^gA still periodic, but

$$^g\psi(x_0 + \beta, \boldsymbol{x}) = -z\, {}^g\psi(x_0, \boldsymbol{x})$$

- formally $\mathcal{D}_{^gA} = g\mathcal{D}_A g^{-1}$
- change of gauge invariant quantity depends on z only

write: $\mathcal{D}_A \to \mathcal{D}_z,\ \lambda_p \to {}^z\lambda_p,\ \psi_p(x) \to {}^z\psi_p(x)$

- spectral sums as above

$$\mathcal{S}_f(x) = \sum_k z_k^* \langle x|\mathrm{tr}\, f(\mathcal{D}_{z_k})|x\rangle$$

- order parameters for center symmetry
- Schwinger-model q-instantons
  $\tau = \beta/L$, $\eta = \pi q \tau$, $\mu_p =$ eigenvalues of $\mathcal{D}^2$:

$$\mathcal{S}_f(\boldsymbol{x}) = -\frac{q}{L} P(\boldsymbol{x}) \sum_{p=0}^{\infty} f(\mu_p) \left\{ L_p(\eta) + L_{p-1}(\eta) \right\} e^{-\eta/2}$$

- Gaussian sum, $f(\mathcal{D}) = \exp(t\mathcal{D}^2)$:

$$\mathcal{G}_t(\boldsymbol{x}) = -\frac{q}{L} P(\boldsymbol{x}) \coth(tB)\, \exp\left(-\frac{\eta}{2}\coth(tB)\right)$$

- similar result for t'Hooft T^4-instantons

- for arbitrary A_μ: no counterterms needed
  (see Langfeld, Synatschke, AW)
- simple mock-model:

$$p^2 \quad \text{on} \quad \psi(x+L) = e^{2\pi i\phi}\,\psi(x) \Longrightarrow \mu_n \propto (n+\phi)^2$$

- difference of heat kernels for periodic/antiperiodic bc

$$K_{\rm per}(t) - K_{\rm aper}(t) = \frac{L}{\sqrt{\pi t}} \sum_{n\,\rm odd} e^{-(nL)^2/4t} = o(t^\infty)$$

- difference of zeta-functions

$$\begin{aligned}\zeta_{\rm per}(s) - \zeta_{\rm aper}(s) &= \frac{L^{2s}}{(2\pi)^{2s}} {\sum}' \left(\frac{1}{n^{2s}} - \frac{1}{(n+\frac{1}{2})^{2s}}\right) \\ &= \frac{L^{2s}}{\pi^{2s}}\, 2\big(2^{1-2s}-1\big)\zeta_R(2s)\end{aligned}$$

$\Rightarrow$ no poles on whole complex s-plane, same for $S_\zeta(x)$

Conclusions, remarks

- spectral sums: order parameters for center symmetry
- continuum: spectral sums exist for almost all $f(\mathcal{D})$
- explicit results for t'Hooft instantons on T^4 (with μ)
- Polyakov loop locally from lowest $\lambda_p, \psi_p(x)$
- chiral condensate: Fourier transform of spectral sum
- direct relation (circumvent Roberge-Weiss!)

  dual condensate $\Longleftrightarrow$ $\langle$dressed Polyakov loops$\rangle$
- relation

  CSB $\Longleftrightarrow$ confinement
- very recent results:

  dynamical fermions — Bilgici, Bruckmann, Gattringer, Hagen

  Schwinger-Dyson approach — Fischer

  renormalization group — Braun, Haas, Marhauser, Pawlowski