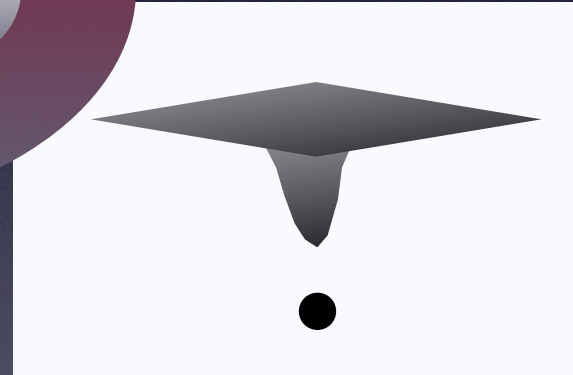
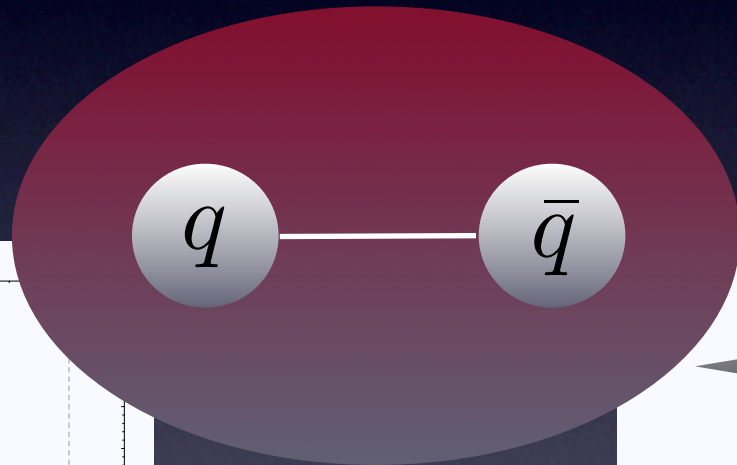
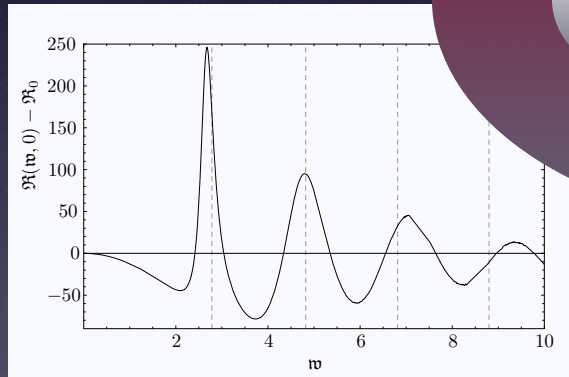


Useful results from the AdS/CFT correspondence?

*-holographic quarks, mesons
and the Super-Yang-Mills phase diagram*



EMMI workshop 'Quarks, Hadrons and the Phase Diagram of QCD',
St Goar, 31st August 2009

Matthias Kaminski
IFT-UAM/CSIC Madrid

Outline

I. Invitation: AdS/QGP Correspondence

II. Quarks

III. Mesons

IV. SYM phase diagrams

V. Holographic hydrodynamics

VI. Summary



I. Invitation: AdS/QGP Correspondence

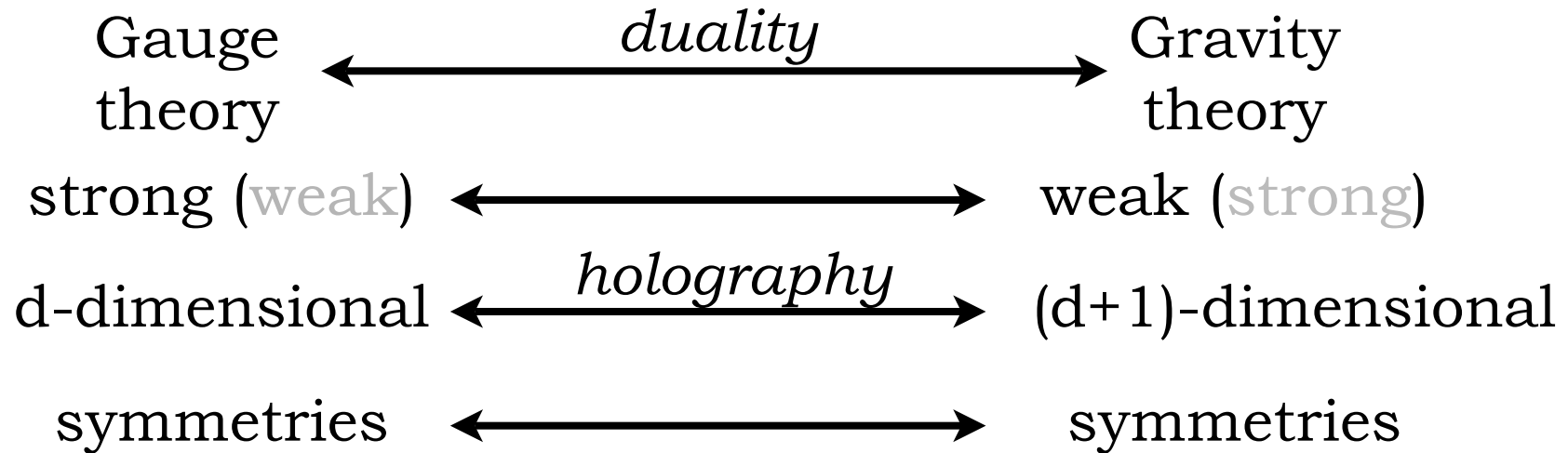
-What can we do with gauge/gravity?

- Compute observables in strongly coupled QFTs
- Meson spectra/melting
Review: [Erdmenger,Evans,Kirsch,Threlfall 0711.4467]
- Quark energy loss, Jets
- Thermodynamics/Phase diagrams
- Holographic hydrodynamics (beyond Muller-Israel-Stewart)
- Transport coefficients (e.g. ‘universal’ viscosity bound)
- Model QCD equation of state ($v_s, \xi/s$ match lattice-QCD)
- Deconfinement & Break: Chiral, Conformal, SUSY
- Condensed matter applications (strongly corr. electrons)
- [AdS/QCD (bottom-up approach)]



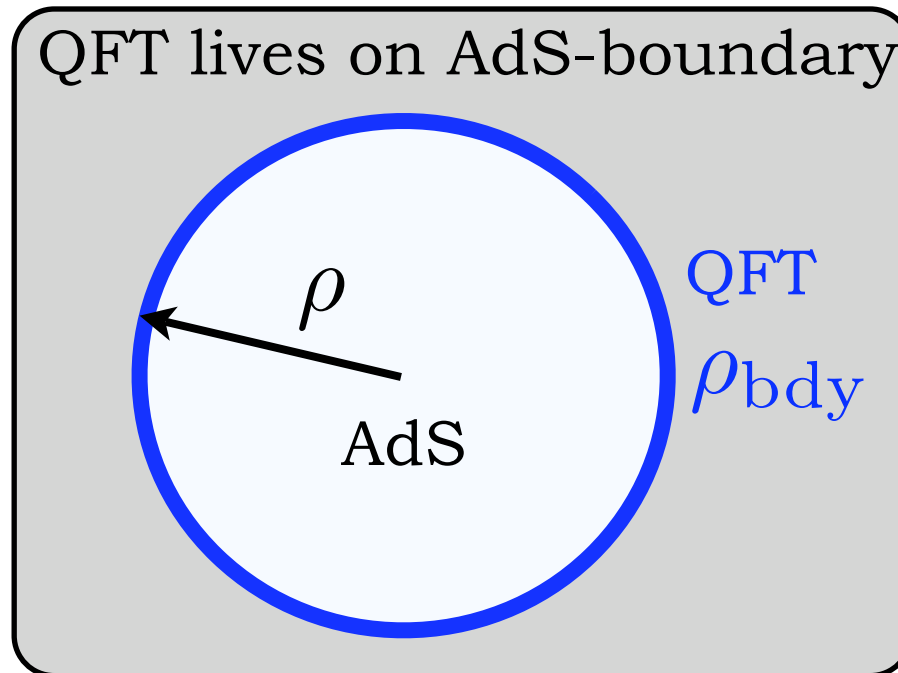
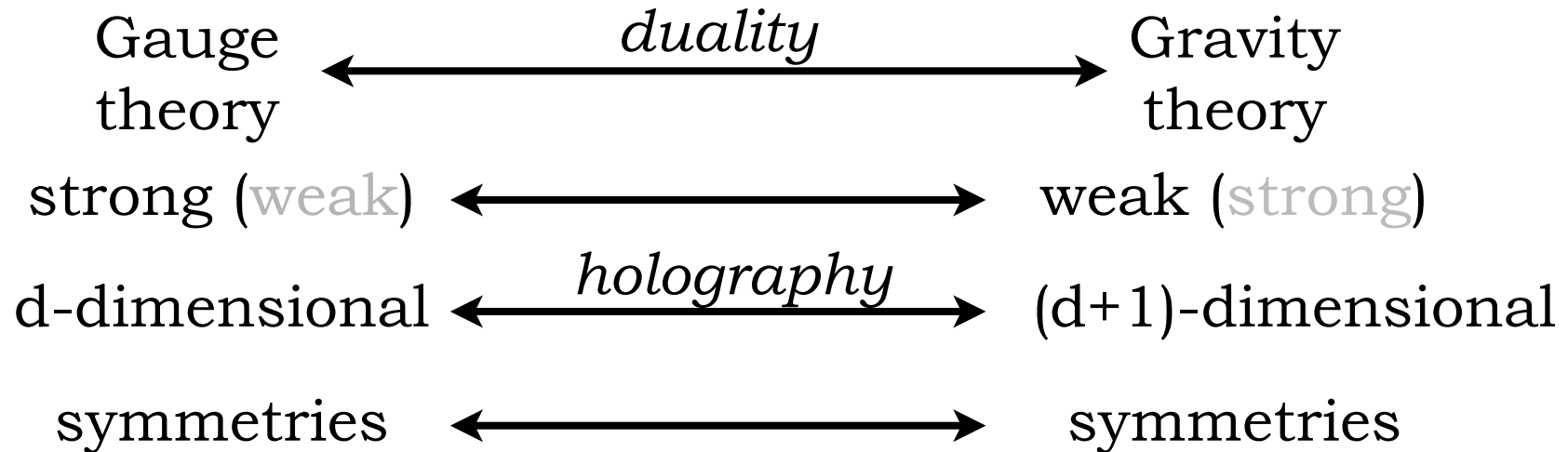
I. Invitation: AdS/QGP Correspondence

-General features of gauge/gravity $\text{Large } N, \text{ large } \lambda = g_{\text{YM}}^2 N$



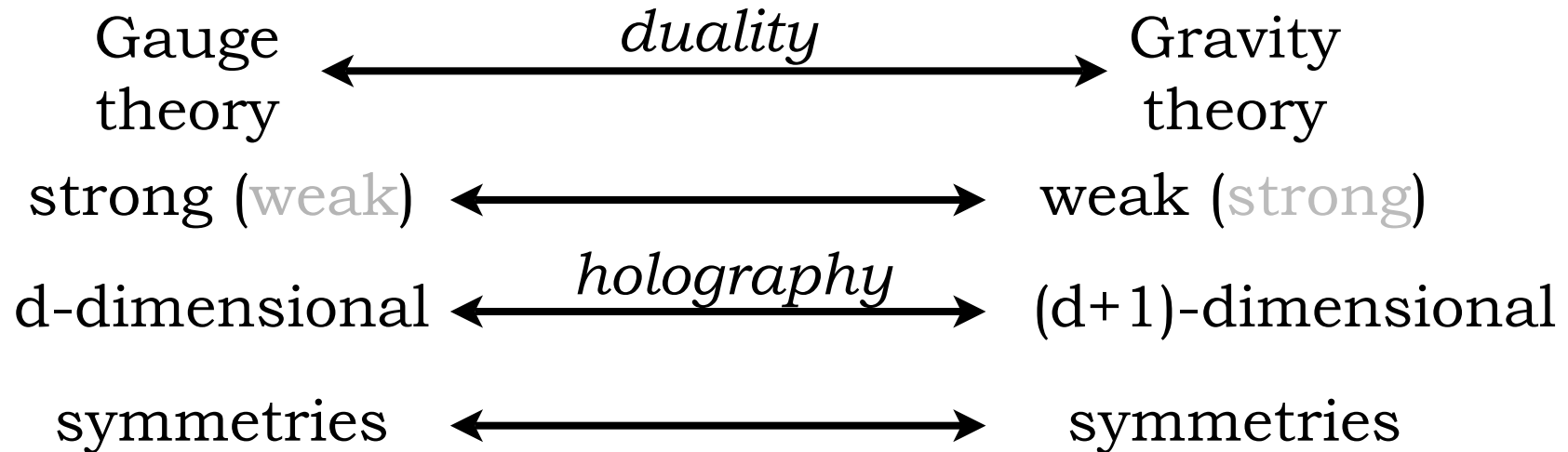
I. Invitation: AdS/QGP Correspondence

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I. Invitation: AdS/QGP Correspondence


-General features of gauge/gravity Large N , large $\lambda = g_{\text{YM}}^2 N$



➔

Dictionary:

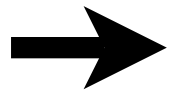
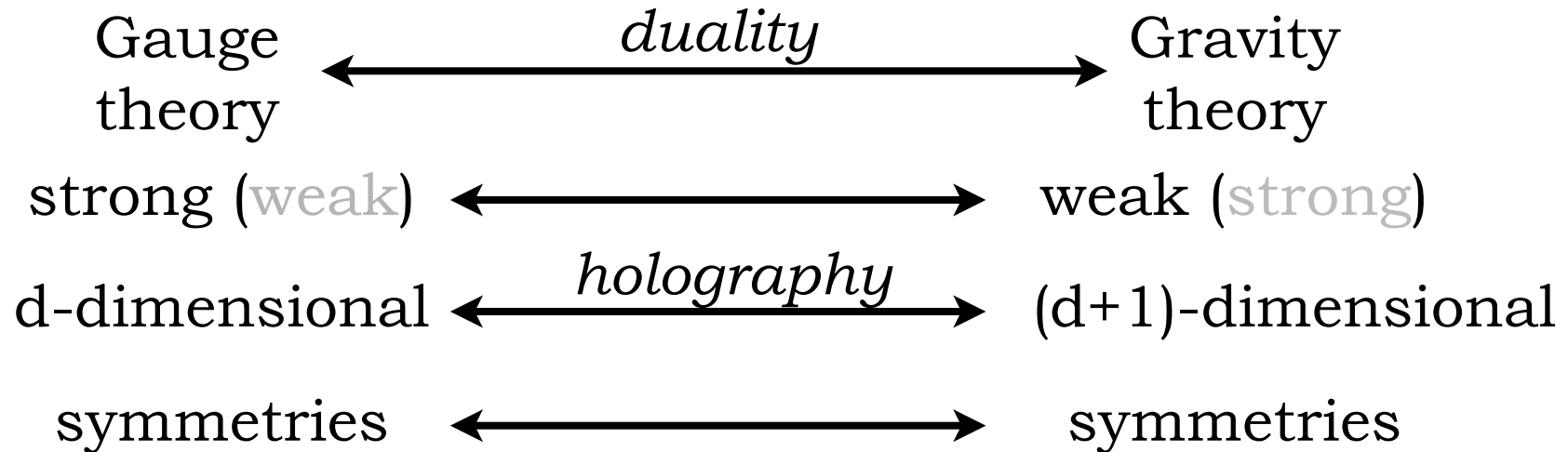
operator J_μ	\longleftrightarrow	field A_μ
correlator G^{ret}	\longleftrightarrow	$\frac{\delta^2}{\delta A_{\text{bdy}} \delta A_{\text{bdy}}} S_{\text{Sugra}}$
QFT FEATURE (energy scale) (phase transition)	\longleftrightarrow	GEOMETRY (radial coord.) (geom. transition)





I. Invitation: AdS/QGP Correspondence

-General features of gauge/gravity Large N , large $\lambda = g_{\text{YM}}^2 N$



Boundary asymptotics:

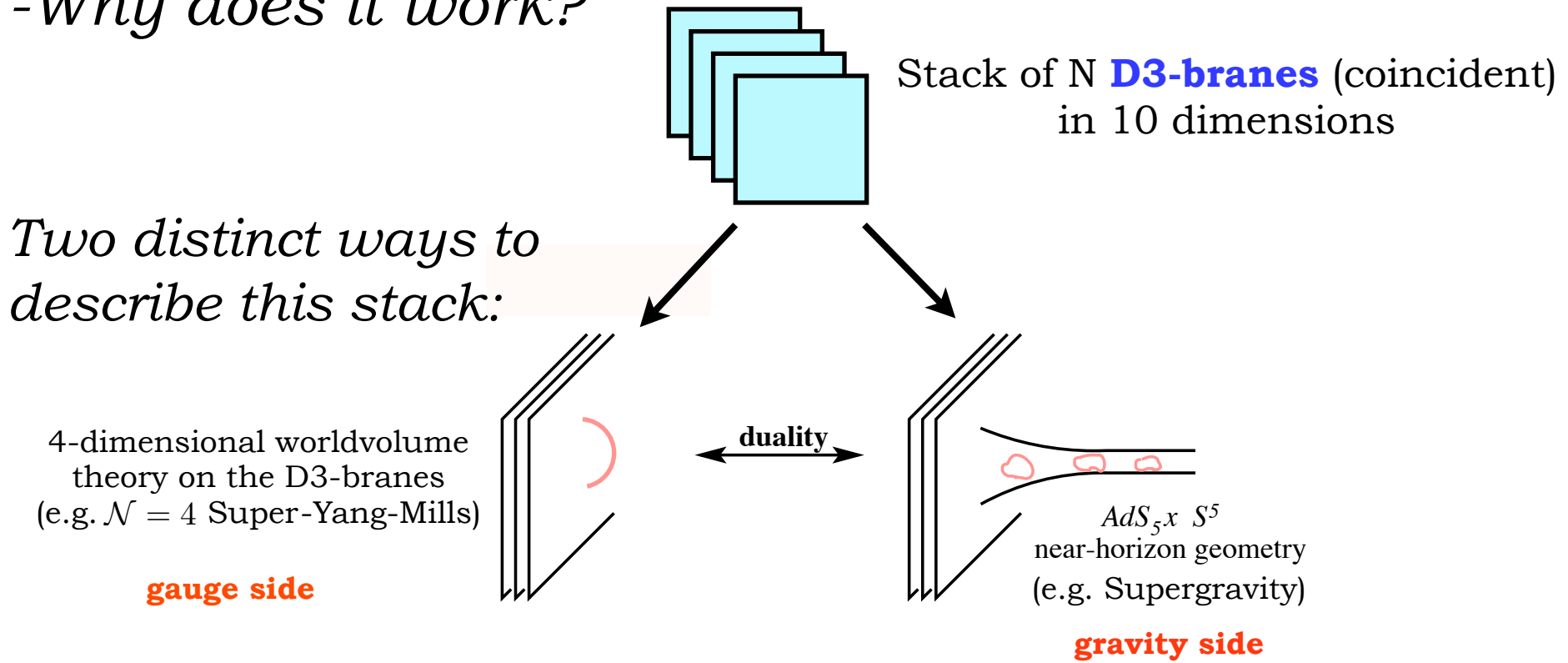
$$A = \underbrace{A^{(0)}}_{\text{non-normalizable (source)}} + \frac{\underbrace{A^{(2)}}_{\text{normalizable (vev)}}}{\rho^2} + \dots$$

$A^{(2)} = \langle J \rangle$

$(\rho \rightarrow \infty)$

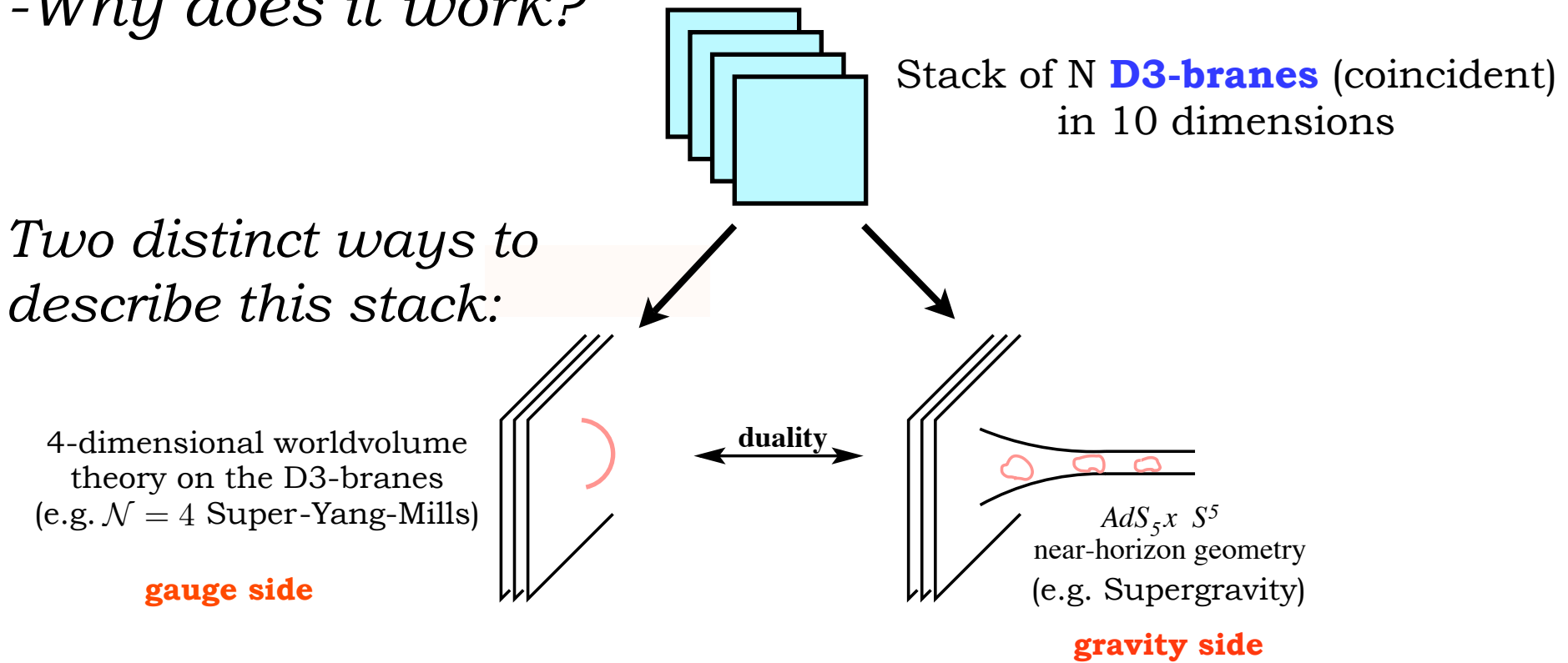
I. Invitation: AdS/QGP Correspondence

-Why does it work?



I. Invitation: AdS/QGP Correspondence

-Why does it work?



-How does it work?

Add/change geometric objects on 'gravity side':

Geometric setup:
Strings/Branes



Find solution
configuration



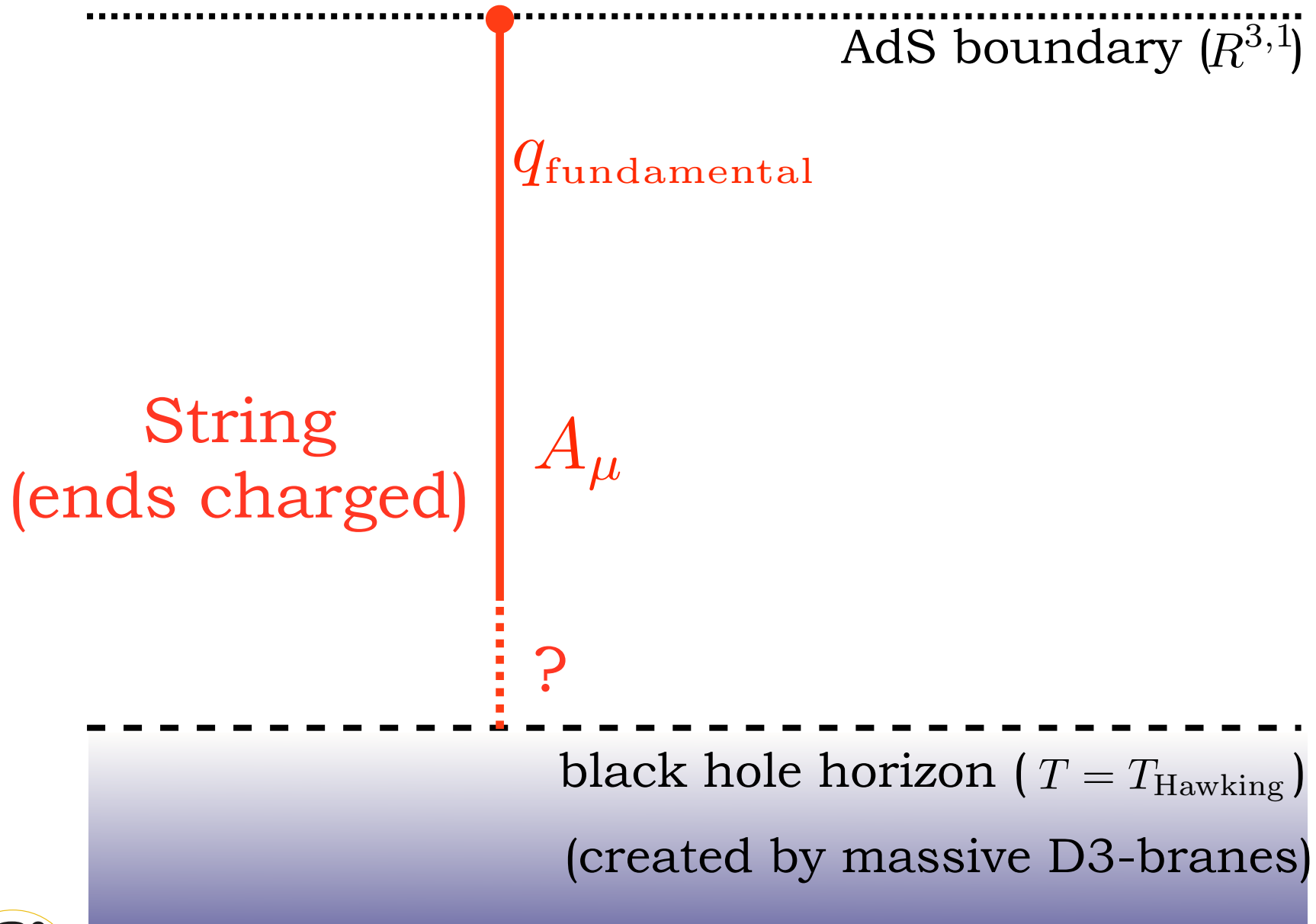
Field Theory
result

Example: Schwarzschild radius corresponds to temperature

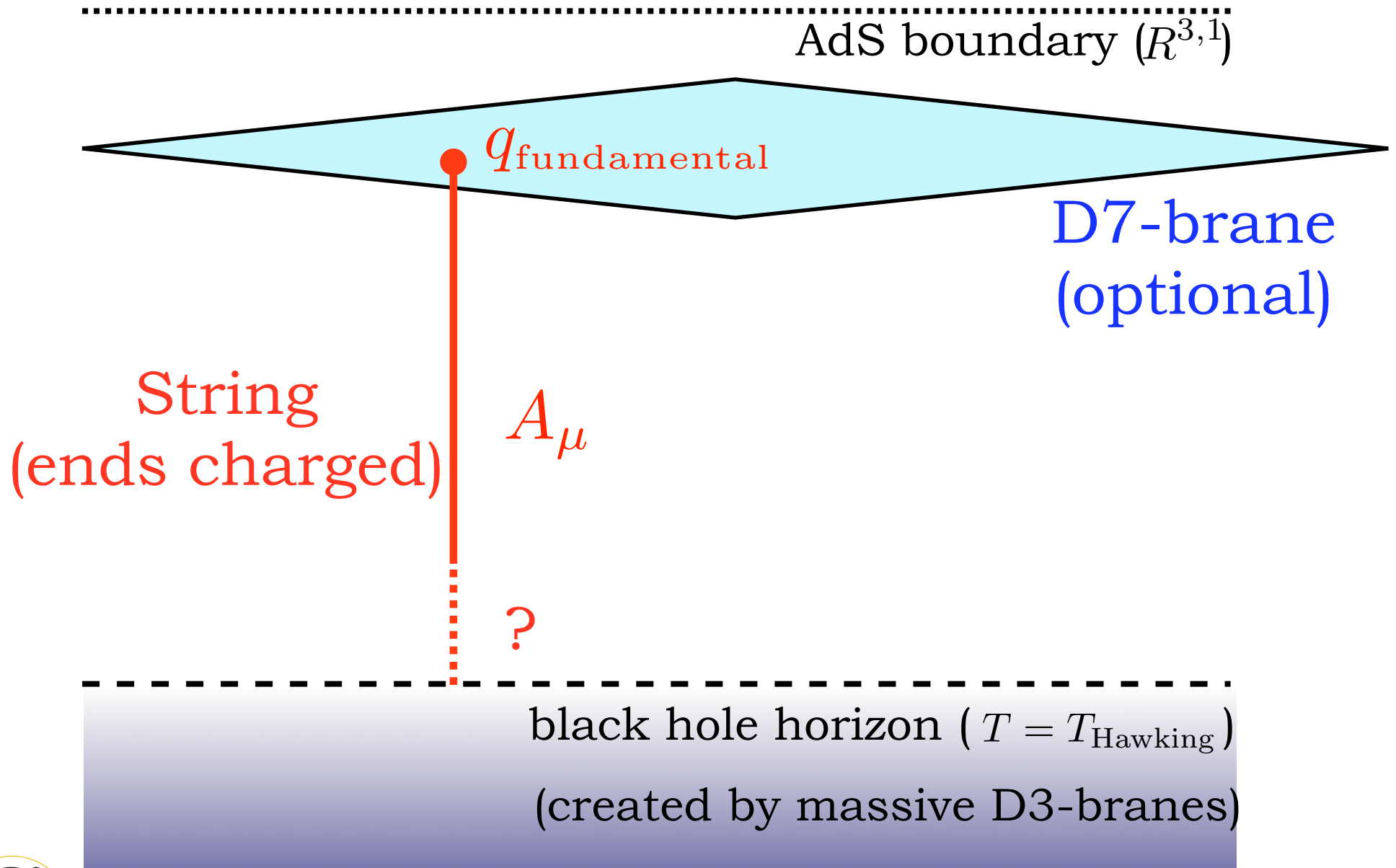
II. Quarks

- Geometric setup*
- Gravity solution*
- Results*
- Discussion*

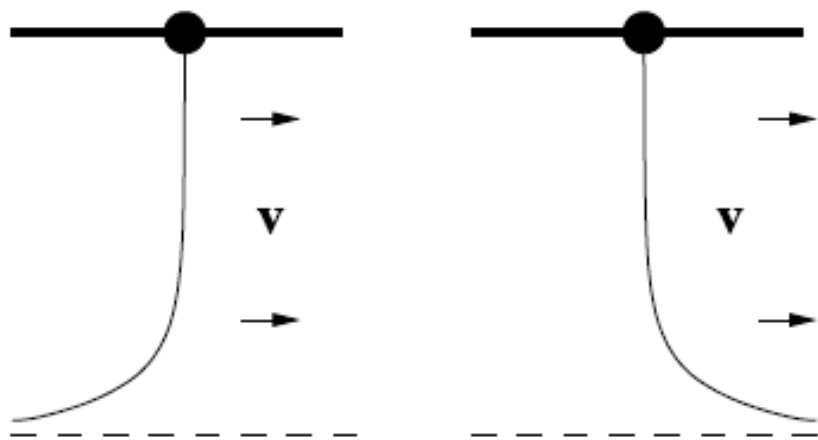
II. Quarks - Geometric setup (general)



II. Quarks - Geometric setup (general)



II. Quarks - Gravity solutions & results (drag)



[Herzog, Karch, Kovtun, Kozcaz, Yaffe
hep-th/0605158]

[Casalderrey-Solana, Teaney
hep-th/0605199]

[Gubser hep-th/0605182]

- left: trailing string solution
- right: unphysical analytic solution

- quark E,p-loss rate:

$$\frac{dp}{dt} = \frac{1}{v} \frac{dE}{dt} = -\frac{\pi}{2} \sqrt{\lambda} T^2 \frac{v}{\sqrt{1-v^2}}$$

- equilibration times:

$$\frac{dp}{dt} = -\frac{p}{\tau_q}, \quad \tau_q = \frac{2m_q}{\pi T^2 \sqrt{\lambda}}$$

$$\tau_{\text{charm}} \approx 2 \text{ fm}$$

$$\tau_{\text{bottom}} \approx 6 \text{ fm}$$

II. Quarks -Discussion

- Drag on heavy quarks in thermal SYM
 - Viscous drag has upper bound: $\mu \leq 2\pi T$ $\left(\frac{dp}{dt} = -\mu p\right)$
 - Mechanism: Energy & Momentum flow along string
 - not scattering (string fluctuations)
 - not glueball emission (closed string 'emission')
 - rather like a wake of a boat [Gubser,Pufu,Yarom 0706.0213]
[Chesler,Yaffe 0706.0368]
-

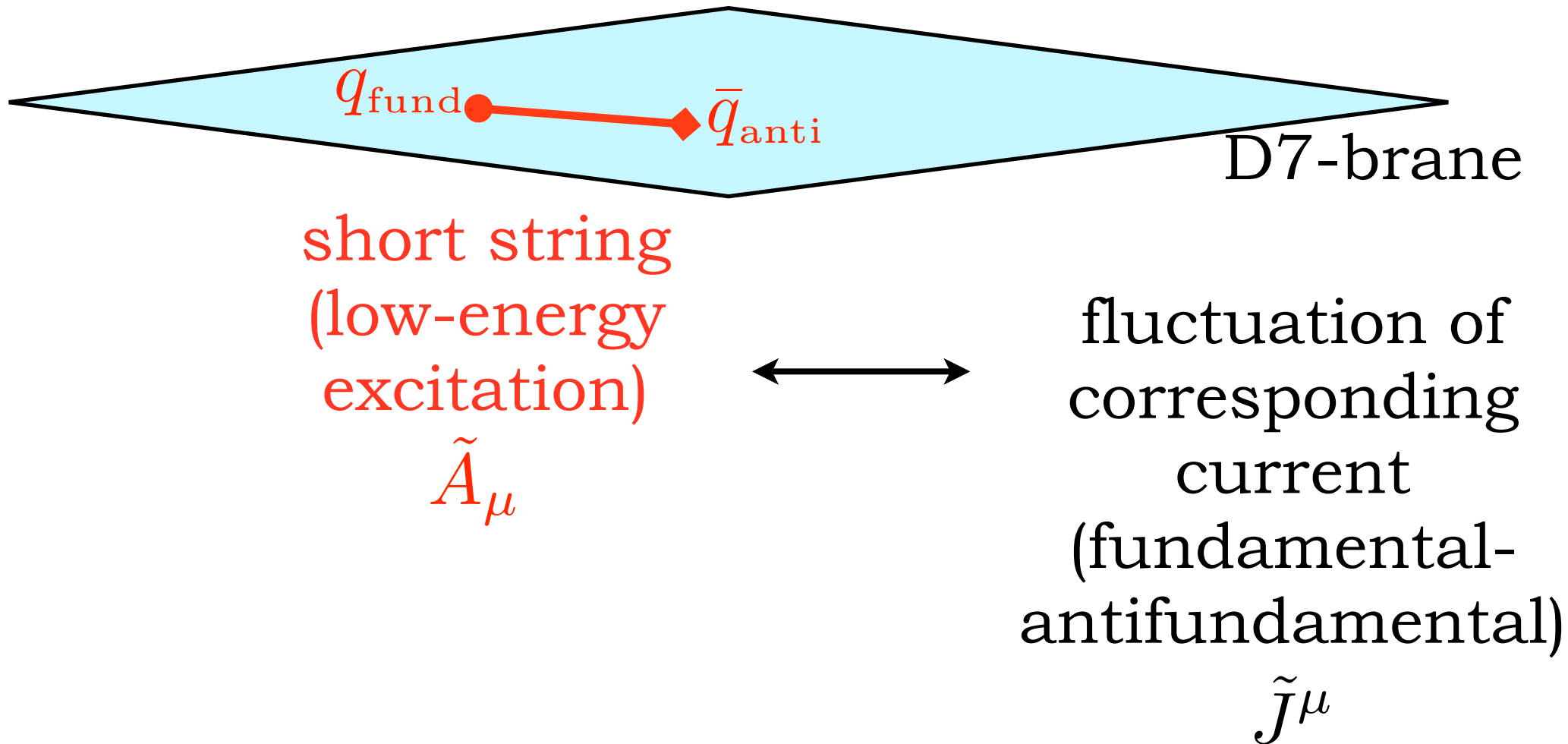
- Jets:*
- complete energy density and flux (heavy quark at v)
 - supersonic: Mach cone
 - near cone: most of energy flux flows orthogonal to front
 - diffusion wake behind quark
 - complete computation vs hydro: hydrodynamics valid!
[Chesler]

III. Mesons

- Geometric setup*
- Gravity solution*
- Results*
- Discussion*

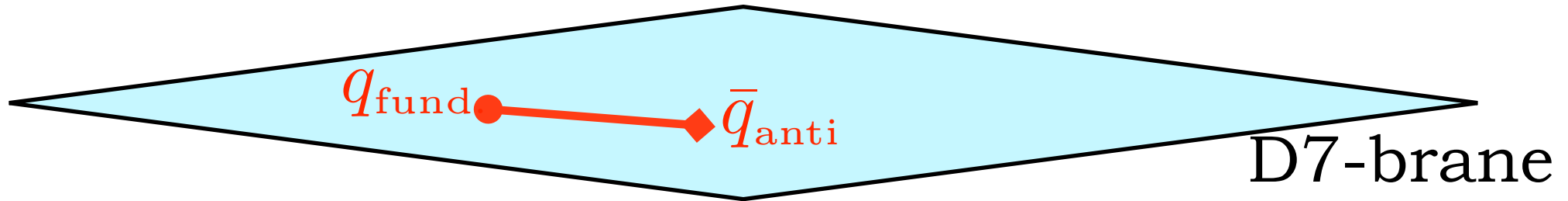
III. Mesons - *Geometric setup*

- Stationary gravity 'background' gives equilibrium thermodynamics
- Gravity 'fluctuations' give field theory dynamics



III. Mesons - Geometric setup

- Stationary gravity 'background' gives equilibrium thermodynamics
- Gravity 'fluctuations' give field theory dynamics



short string
(low-energy
excitation)

$$\tilde{A}_\mu$$



fluctuation of
corresponding
current
(fundamental-
antifundamental)

$$\tilde{j}^\mu$$

Chemical potential:

$$\hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu$$

[Nakamura et al., hep-th/0611021]

[Myers et al., hep-th/0611099]

III. Mesons -Gravity solution

[Erdmenger, M.K., Rust 0710.0334]

Effective action:
$$S_{D7} = \int d^8x \sqrt{|\det\{[g + F] + \tilde{F}\}|}, \quad F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$$



III. Mesons -Gravity solution

[Erdmenger, M.K., Rust 0710.0334]

Effective action:
$$S_{D7} = \int d^8x \sqrt{|\det\{\underbrace{[g + F]}_G + \tilde{F}\}|}, \quad F_{\mu\nu} = \partial_{[\mu}A_{\nu]}$$

Equation of motion:
$$0 = \tilde{A}'' + \frac{\partial_\rho[\sqrt{|\det G|}G^{22}G^{44}]}{\sqrt{|\det G|}G^{22}G^{44}}\tilde{A}' - \frac{G^{00}}{G^{44}}\rho_H^2\omega^2\tilde{A}$$

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[Erdmenger, M.K., Rust 0710.0334]

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Equation of motion:

‘Curved’ Maxwell equations:

$$\partial_\mu F^{\mu\nu} = 0$$

$$\partial_\mu \left(\sqrt{-G} G^{\mu\nu} G^{\rho\sigma} F_{\nu\sigma} \right) = 0$$

$$\partial_\mu \left(\sqrt{-G} G^{\mu\nu} G^{\rho\sigma} \partial_{[\nu} \tilde{A}_{\sigma]} \right) = 0$$

III. Mesons -Gravity solution

[Erdmenger, M.K., Rust 0710.0334]

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Boundary conditions:
$$\tilde{A} = (\varrho - \varrho_H)^{-i\omega} \left[1 + \frac{i\omega}{2} (\varrho - \varrho_H) + \dots \right]$$

III. Mesons -Gravity solution

[Erdmenger, M.K., Rust 0710.0334]

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Translation to gauge theory by duality:
$$A_\mu \xleftrightarrow{\text{AdS/CFT}} J^\mu$$

(source)

III. Mesons - Gravity solution

[Erdmenger, M.K., Rust 0710.0334]

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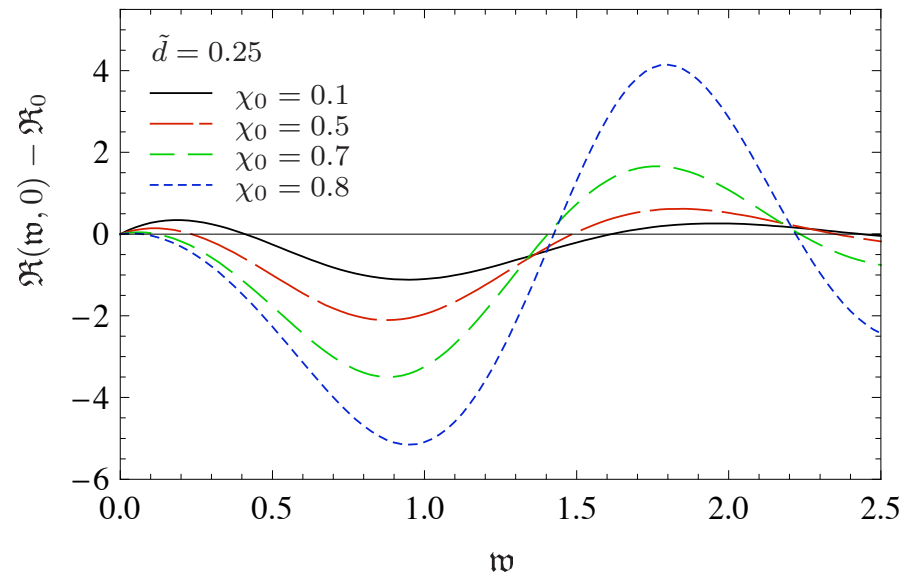
Gauge correlator:
[Son et al. '02]

$$G^{\text{ret}} = \frac{N_f N_c T^2}{8} \lim_{\rho \rightarrow \rho_{\text{bdy}}} \left(\rho^3 \frac{\partial_\rho \tilde{A}(\rho)}{\tilde{A}(\rho)} \right)$$

III. Mesons -Results

[Erdmenger, M.K., Rust 0710.0334]

Finite baryon density:



Thermal spectral function:

$$\mathfrak{R}(\omega, \mathbf{q}) = -2 \text{Im} G^{\text{ret}}(\omega, \mathbf{q})$$

$$L(\varrho) = \varrho \chi(\varrho)$$

$$\chi_0 = \chi(\rho) \Big|_{\rho \rightarrow \rho_H} \sim \frac{m_{\text{quark}}}{T}$$

III. Mesons -Results

[Erdmenger, M.K., Rust 0710.0334]

Finite baryon density:

Lower temperature

Thermal spectral function:

$$\Re(\omega, \mathbf{q}) = -2 \text{Im} G^{\text{ret}}(\omega, \mathbf{q})$$

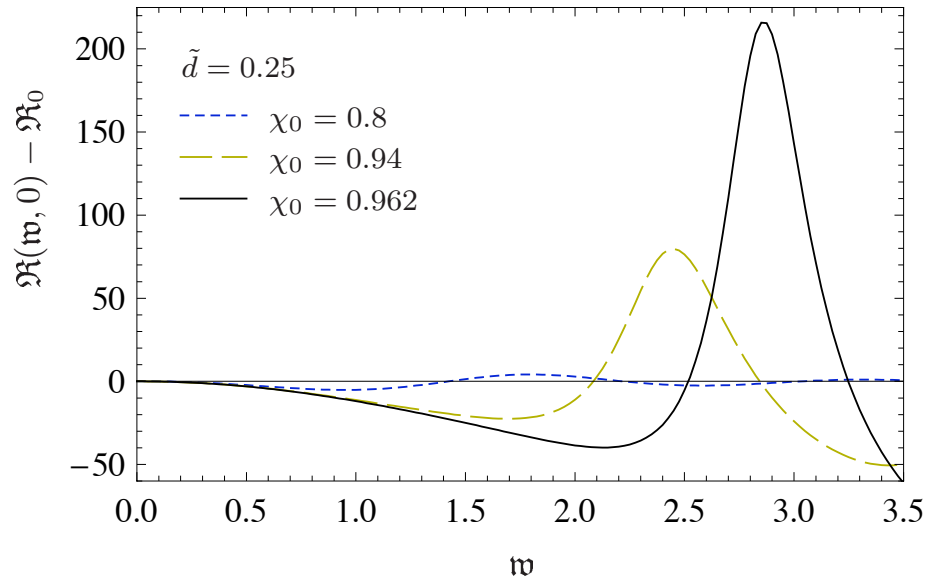
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III. Mesons -Results

[Erdmenger, M.K., Rust 0710.0334]

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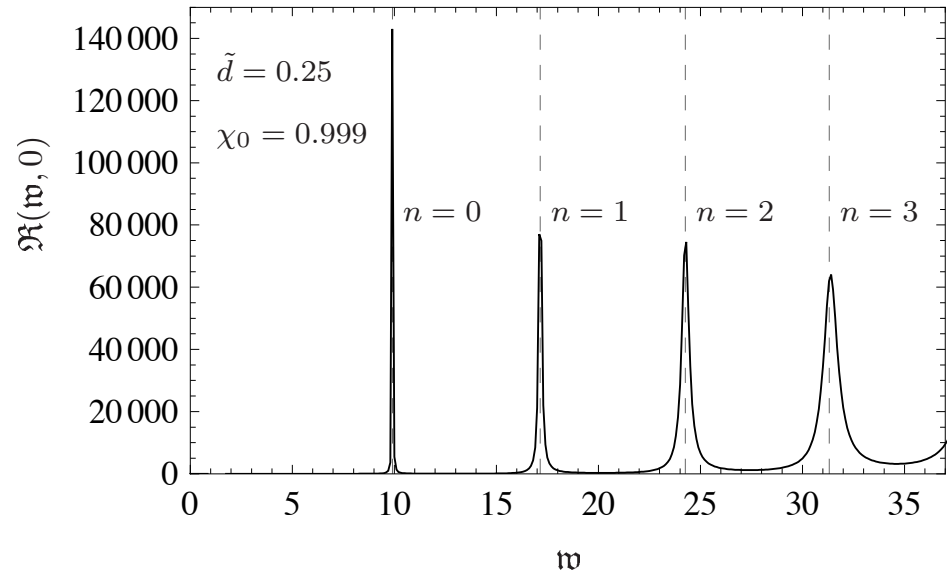
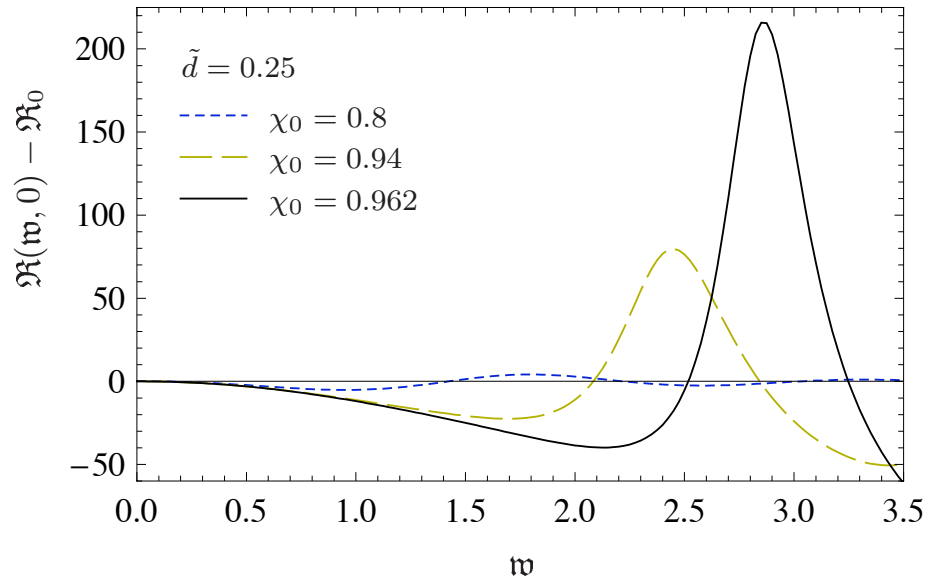
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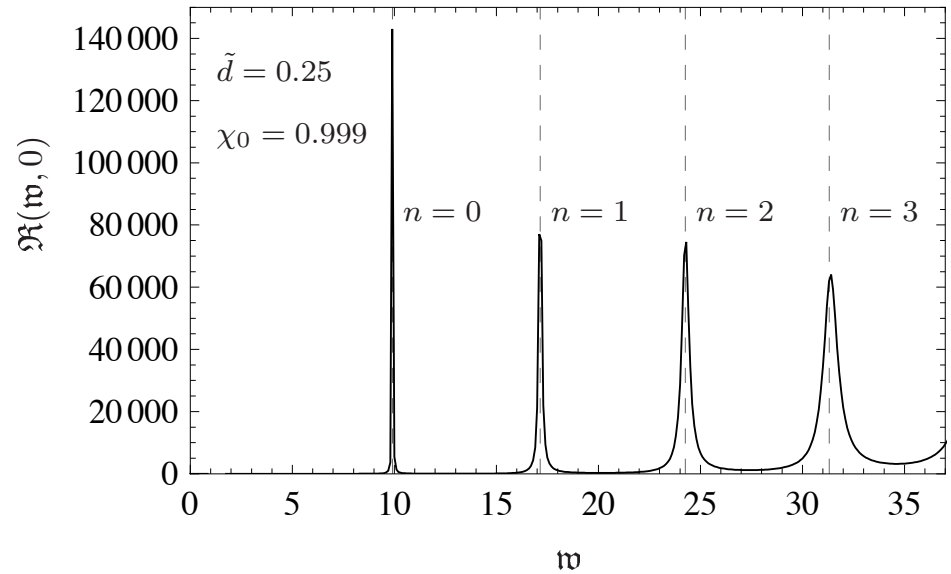
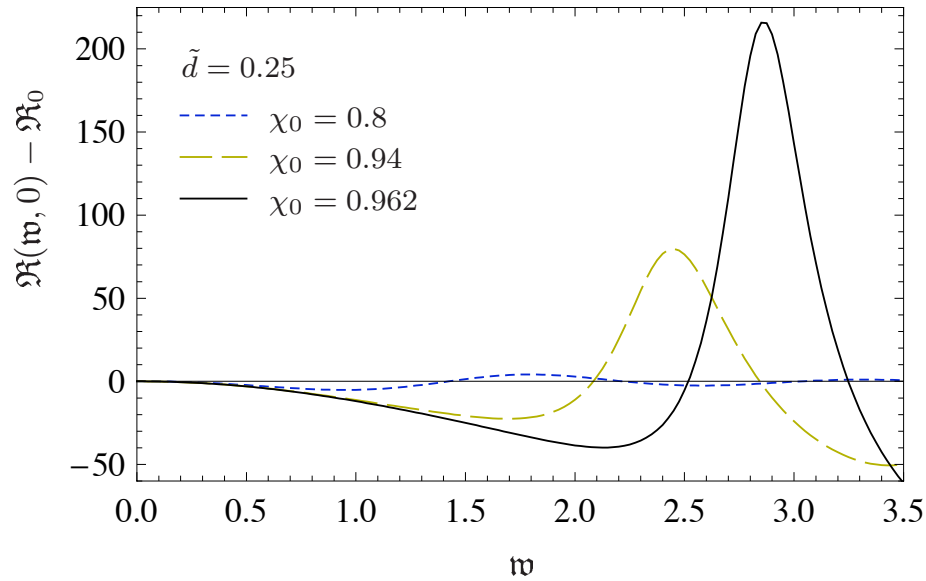
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III. Mesons -Results

[Erdmenger, M.K., Rust 0710.0334]

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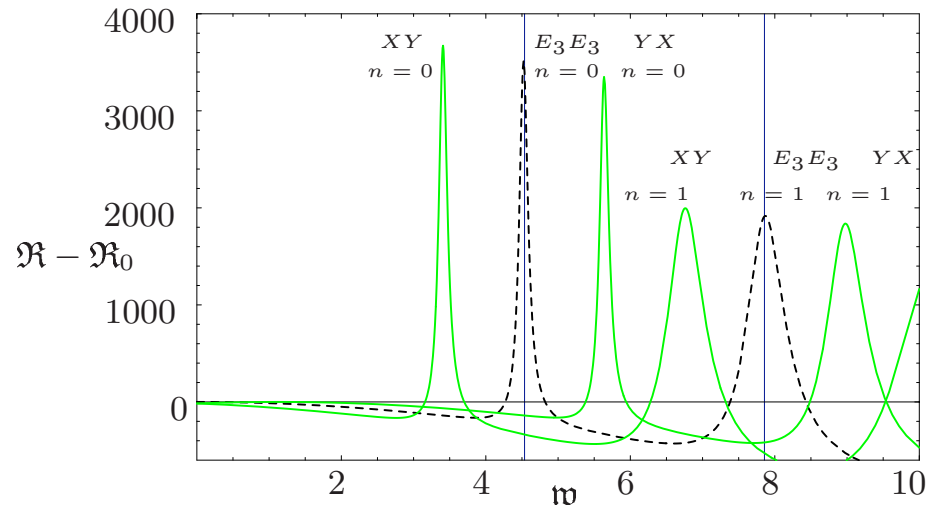
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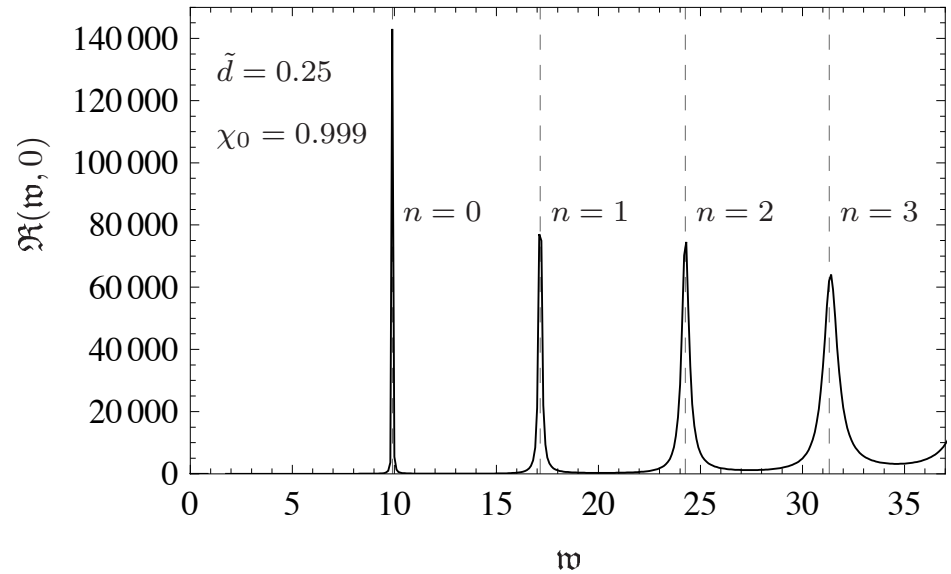
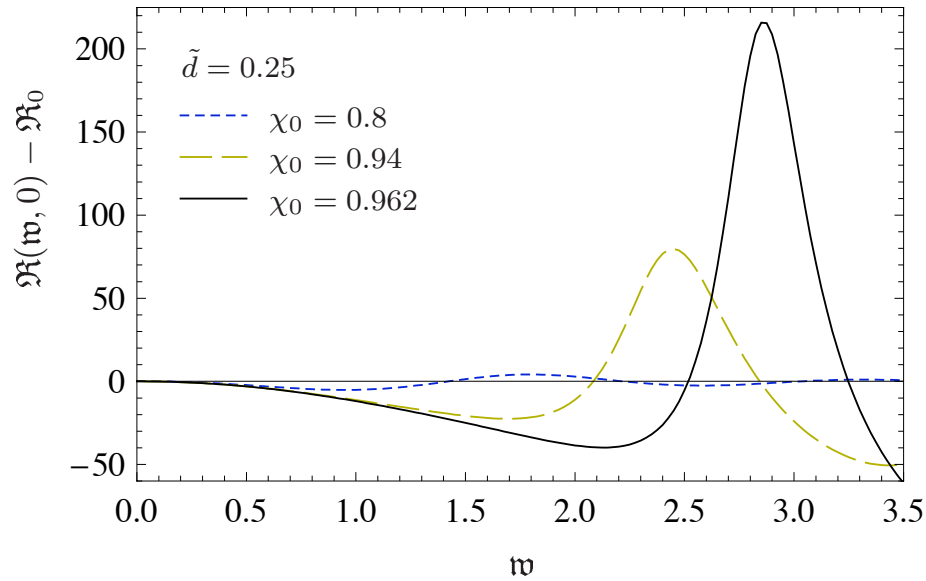
Finite isospin density:



III. Mesons -Results

[Erdmenger, M.K., Rust 0710.0334]

Finite baryon density:



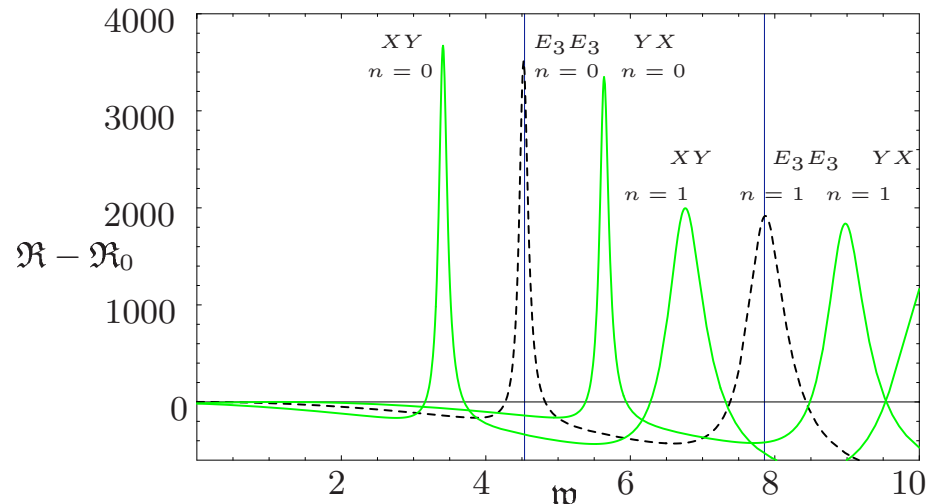
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$$\chi_0 = \chi(\rho) \Big|_{\rho \rightarrow \rho_H} \sim \frac{m_{\text{quark}}}{T}$$

Finite isospin density:



Analytically: [PhD thesis '08]



III. Mesons -Discussion

- Quite stable quark bound states survive deconfinement
- Resonances: vector mesons (like QCD's Rho-meson)
- Correlators encode transport coefficients (Kubo formulae)
- Poles of correlators in complex frequency plane are QNMs

Other hadron results:

Review: [Erdmenger,Evans,Kirsch,Threlfall 0711.4467]

- Charmonium diffusion suppressed at strong coupling:

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D p_i \quad [Dusling,Erdmenger, M.K., Rust,Teaney,Young 0808.0957]$$

$$\tau_{\text{relax}}^{\text{strong}} \approx 4\tau_{\text{relax}}^{\text{weak}}$$

- Baryons modeled by classical solutions *[Witten, hep-th/9805112]*
- Problem: N quarks needed, N large *[Sfetsos,Siampos 0807.0236]*
- Even worse: baryons with less than N quarks allowed

IV. Super-Yang-Mills Phase Diagrams



IV. Super-Yang-Mills phase diagram

-Gravity setup

- D_3 -branes

[Karch, Katz; hep-th/0205236]



IV. Super-Yang-Mills phase diagram

-Gravity setup



D₇-branes

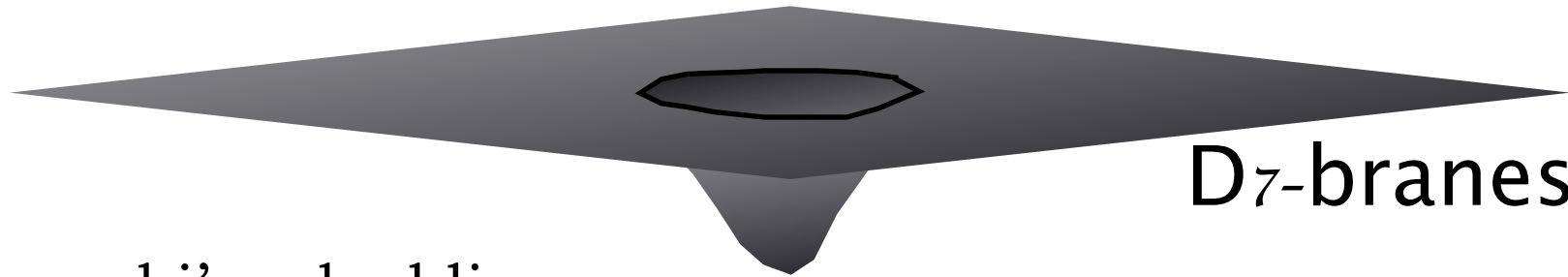
'Minkowski' embedding

• D₃-branes

[Karch, Katz; hep-th/0205236]

IV. Super-Yang-Mills phase diagram

-Gravity setup



D₇-branes

'Minkowski' embedding

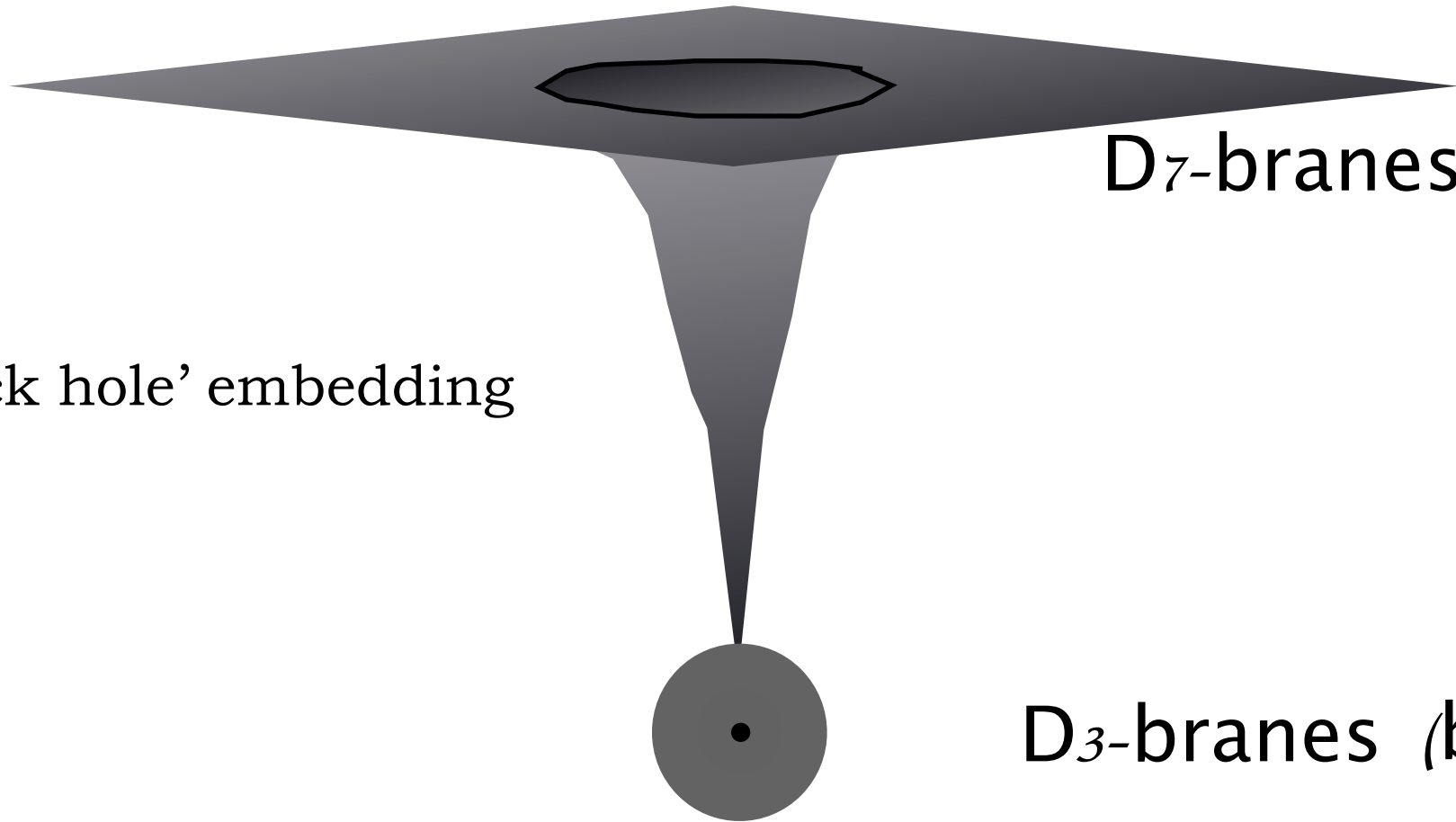


D₃-branes (black)

[Karch, Katz; hep-th/0205236]

IV. Super-Yang-Mills phase diagram

-Gravity setup



D₇-branes

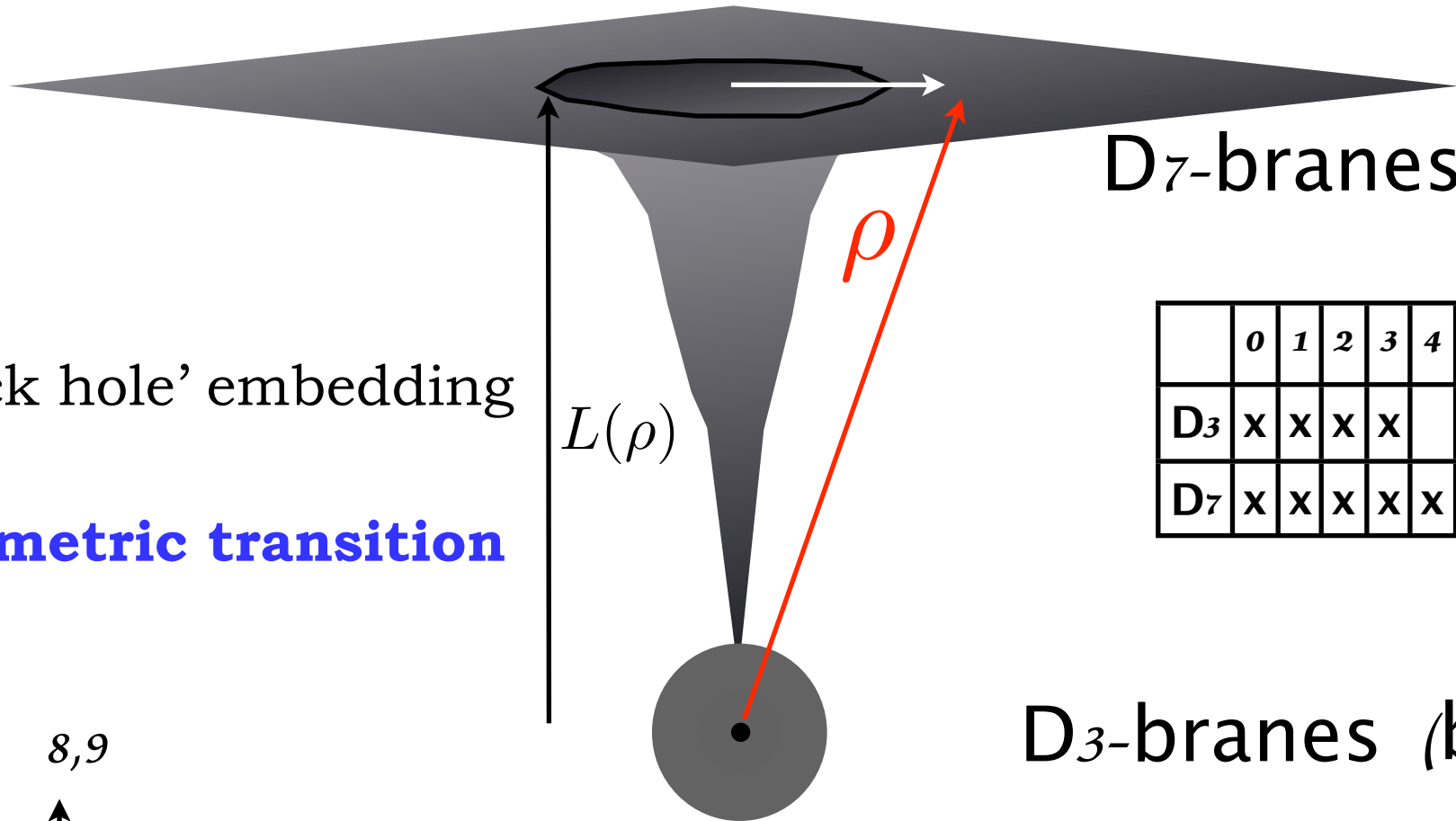
'Black hole' embedding

D₃-branes (black)

[Karch, Katz; hep-th/0205236]

IV. Super-Yang-Mills phase diagram

-Gravity setup



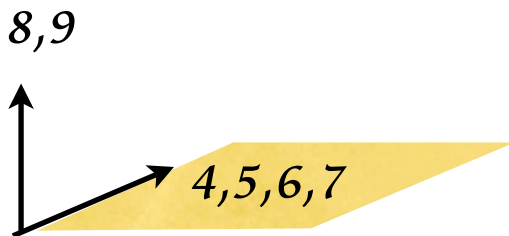
D₇-branes

	0	1	2	3	4	5	6	7	8	9
D ₃	x	x	x	x						
D ₇	x	x	x	x	x	x	x	x		

'Black hole' embedding

Geometric transition

D₃-branes (black)

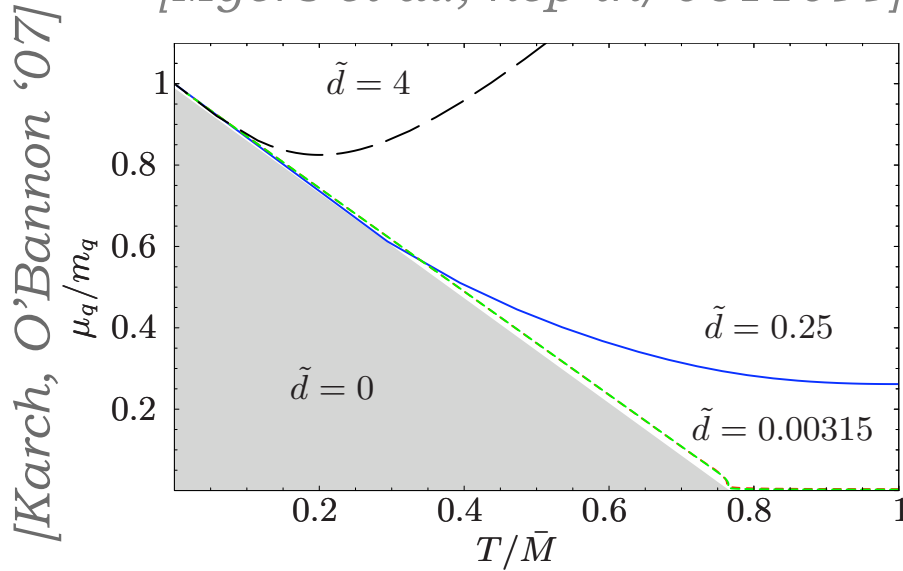


[Karch, Katz; hep-th/0205236]

IV. Super-Yang-Mills phase diagram -Results

Baryonic phase diagram:

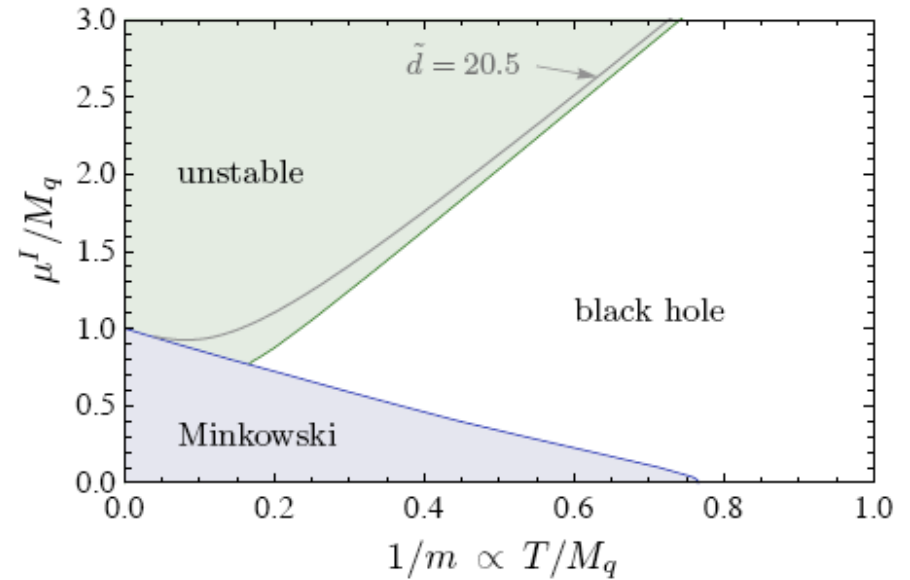
[Erdmenger, M.K., Rust 0710.0334]
 [Myers et al., hep-th/0611099]



- meson melting transition
- continuous range
- first order range
- no confinement

Isospin phase diagram:

[Erdmenger, M.K., Kerner, Rust 0807.2663]

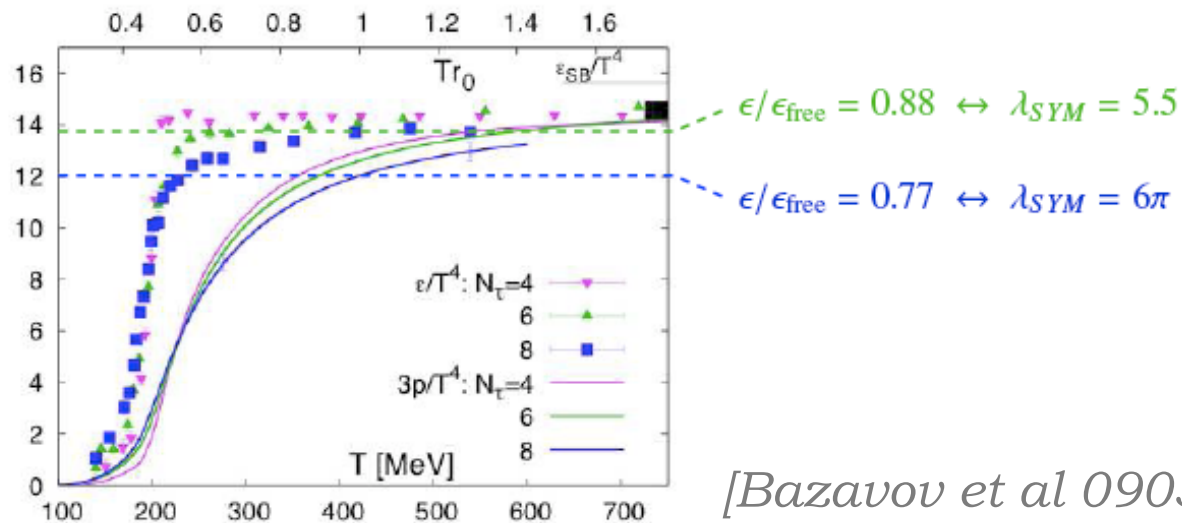


- meson melting
 - flavor fluct. instability
 - superconducting phase
- [Ammon, Erdmenger, M.K.,
 Kerner 0903.1864]

IV. Super-Yang-Mills phase diagram

-How useful are these SYM results?

- SYM-coupling not running
- Finite T: SUSY broken, non-conformal, BUT: large N
- Energy densities (free theories): $\epsilon_{\text{SYM}} = 39T^4 \gg \epsilon_{\text{QCD}} = 16T^4$
- SYM vs QCD equation of state:



- QCD-equation of state modeled with gravity potential; speed of sound and bulk viscosity similar to lattice-QCD

IV. Super-Yang-Mills phase diagram

-Other phases at strong coupling

- Sakai-Sugimoto model (D4, D8 and anti-D8-branes)
 - chiral symmetry breaking (CSB)
 - deconfinement can be tuned to coincide with CSB

- Short thermalization times: ($\tau_{\text{RHIC, therm}} = 0.6 \text{ fm}/c$)

$$\tau_{\text{therm}} = 0.4 \text{ fm}/c \text{ [Friess et al. hep-th/0611005]}$$

$$\tau_H = 0.3 \text{ fm}/c \text{ [Amado et al. 0710.4458]}$$

- Black hole formation (far-from-eq. isotropization)
[Chesler, Yaffe 0812.2053]

V. Conformal hydrodynamics

-First order hydrodynamics

Conservation equations

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu j^\mu = 0$$

Constitutive equations

$$T^{\mu\nu} = \frac{\epsilon}{3}(4u^\mu u^\nu + g^{\mu\nu}) + \Pi^{\mu\nu}$$

$$j^\mu = nu^\mu - \sigma T(g^{\mu\nu} + u^\mu u^\nu)\partial_\nu \left(\frac{\mu}{T}\right) + \xi \omega^\mu$$

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu\partial_\lambda u_\rho$$

[Erdmenger, Haack, M.K., Yarom 0809.2488]

New vorticity term arises!
(related to triangle anomaly)

$$\partial_\mu j^\mu = -\frac{1}{8}C\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

$$\xi = C \left(\mu^2 - \frac{2}{3} \frac{\mu^3 n}{\epsilon + P} \right)$$

[Son, Surowka 0906.5044]

(see also chiral magnetic effect)

[talk by H.J. Warringa]



V. Conformal hydrodynamics

- New coefficient at first order hydrodynamics (\sim viscosity)
- ξ completely determined by C and equation of state
- 3 ways to compute ξ :
 - E, p conservation & Weyl symmetry (conf.rescaling)
 - positivity of entropy current (anomaly requires new coeff)
 - directly in specific holographic model (microscopic)
- Second order hydro: even more new terms (beyond MIS)
[see also talk by Rischke]

➔ Relativistic hydrodynamics needs to be completed.

VI. Summary

- Perils: large N & 't Hooft coupling (conformality, SUSY)
- Terrifying agreement with lattice & 'QCD'
- Heavy quarks: jets & drag (viscosity bound)
- Vector mesons survive deconfinement
- Baryon/Isospin phase diagrams with meson melting
- Flavor superconducting phase at high isospin density
- Hydrodynamics: neglected terms at the order of viscosity



Answer: YES!

VI. Summary

- Perils: large N & 't Hooft coupling (conformality, SUSY)
- Terrifying agreement with lattice & 'QCD'
- Heavy quarks: jets & drag (viscosity bound)
- Vector mesons survive deconfinement
- Baryon/Isospin phase diagrams with meson melting
- Flavor superconducting phase at high isospin density
- Hydrodynamics: neglected terms at the order of viscosity

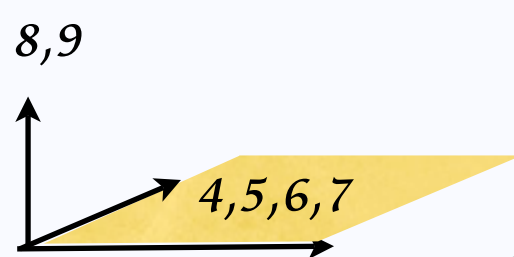
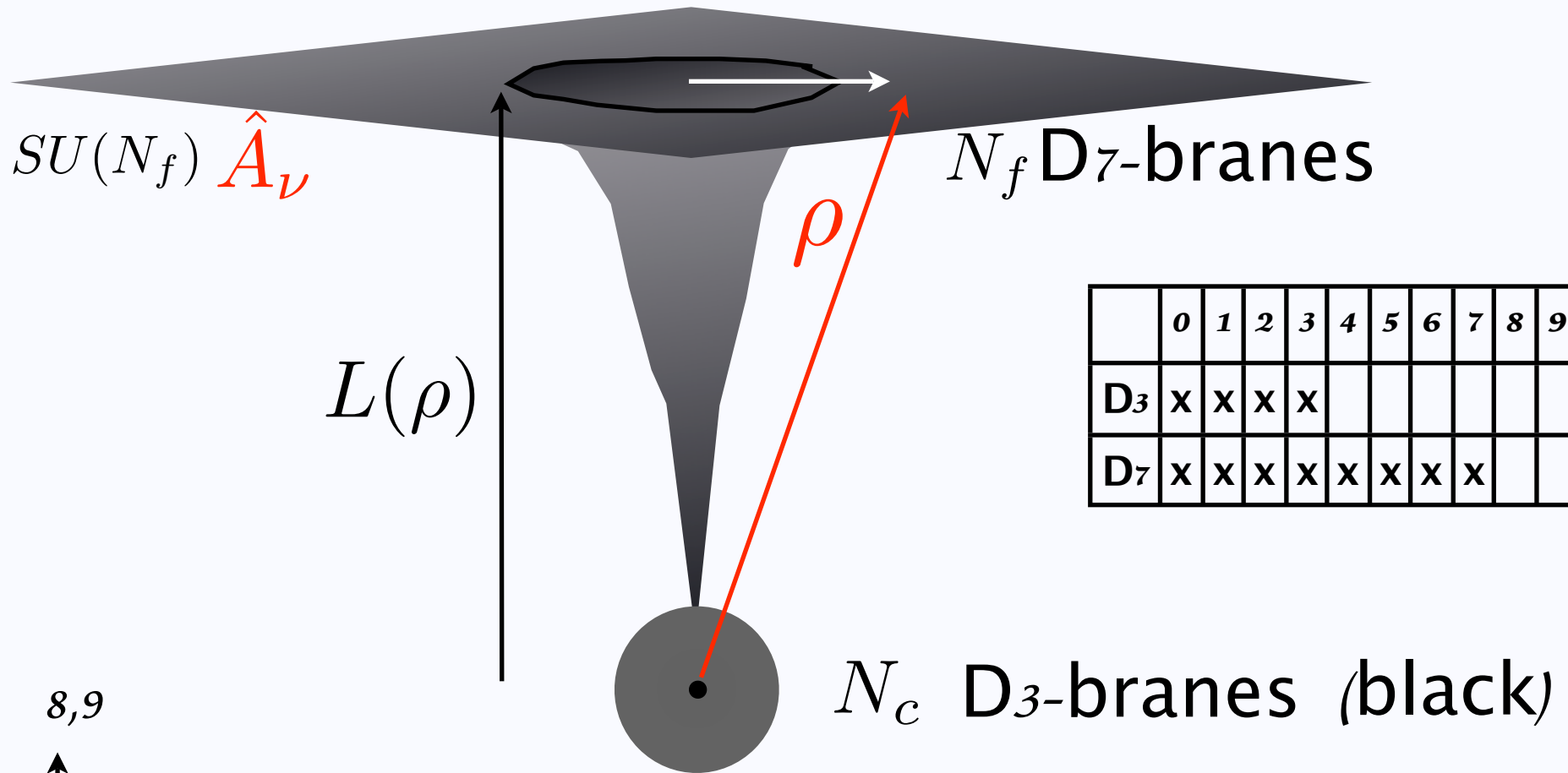
➔ *Useful results from the AdS/CFT correspondence?*

Answer: YES!



APPENDIX: -Geometric setup (detailed)

[Karch, Katz; hep-th/0205236]



Chemical potential: $\hat{A}_\mu = \delta_{\mu 0} A_0 + \tilde{A}_\mu$

[Nakamura et al., hep-th/0611021]

[Myers et al., hep-th/0611099]

APPENDIX: Extension of the correspondence

		<i>Univer- sality</i>	Original AdS/CFT correspondence	AdS Schwarzschild black hole (D3/D7)
Gauge		QCD	$\mathcal{N} = 4$ SuperYangMills	thermal Yang-Mills
Gravity		?	Type II Sugra in AdS	TypeII Sugra in AdS Schwarzschild b.h.
Gauge theory symmetry	non- conf.	✓	⊙	✓
	non- SUSY	✓	⊙	✓
Relations				$T \leftrightarrow \text{horizon}$ $\mu_B, \mu_I \leftrightarrow A_0(\rho)$

$$g_{YM}^2 = g_s$$

$$\frac{R^4}{(\alpha')^2} = 4\pi N_c g_s \equiv \lambda$$

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