# QCD equation of state

at *Nt=8*

Szabolcs Borsanyi
Wuppertal

<u>outline</u>

I. Nf=0
II. Nf=3
III. Nf=2+1
IV. On the Tc controversy

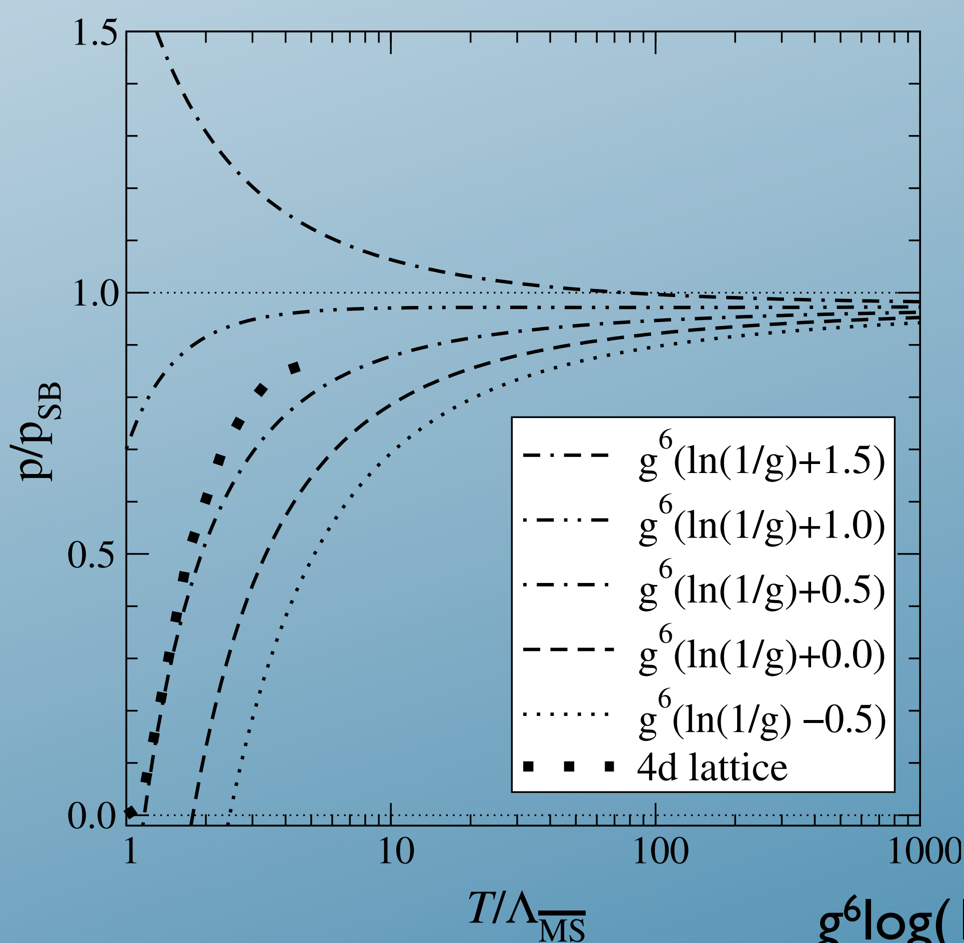
<u>based on mostly unpublished work by</u>

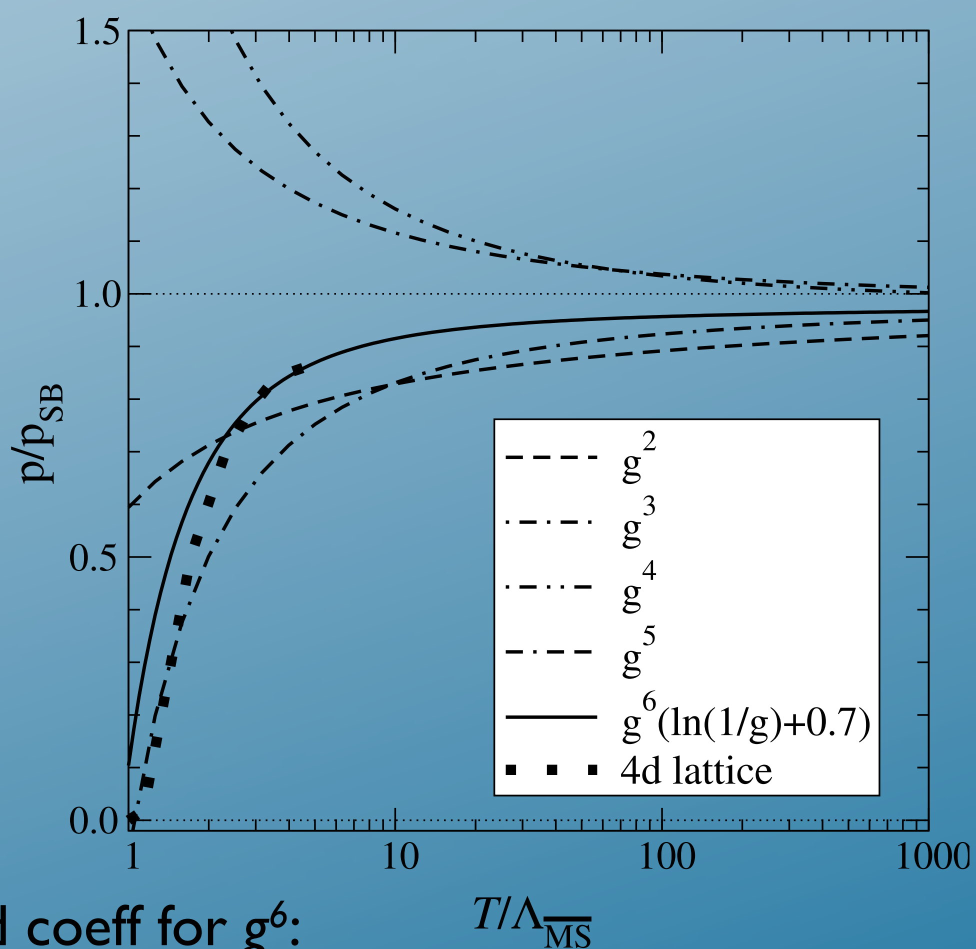
Gergely Endrődi
Zoltán Fodor
Antal Jakovác
Sándor Katz
Kálmán Szabó
*(and myself)*

# I. Quenched simulations

G. Boyd et al., Nucl. Phys. B469, 419 (1996) [hep-lat/9602007]; A. Papa, Nucl. Phys. B478, 335 (1996) [hep-lat/9605004]; B. Beinlich, F. Karsch, E. Laermann and A. Peikert, Eur. Phys. J. C6, 133 (1999) [hep-lat/9707023]; M. Okamoto et al. [CP-PACS Collaboration], Phys. Rev. D60, 094510 (1999) [hep-lat/9905005].

*Quenched is clean and cheap:*
so why don't lattice result come close to the SB limit?
where are high T results?




$g^6 \log(1/g)$ + fitted coeff for $g^6$:

Kajantie, Laine, Rummukainen, Schroder: **Phys.Rev.D67:105008,2003**

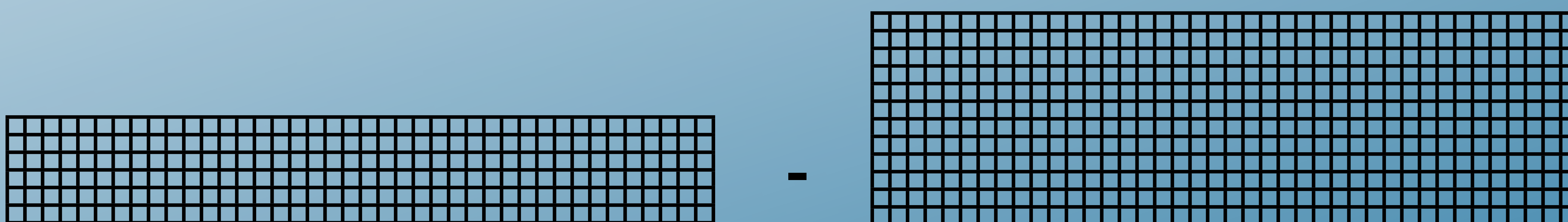
# Renormalization

Free energy (or the trace anomaly) has a *T*-independent quartic divergence.

Standard approach: remove p(T=0) *T=0 is too expensive*

Our approach:

We determine $p(T)-p(T/2)$:



48x6 48x12

$$p(T)=[p(T)-p(T/2)] + [p(T/2)-p(T/4)] + \ldots$$

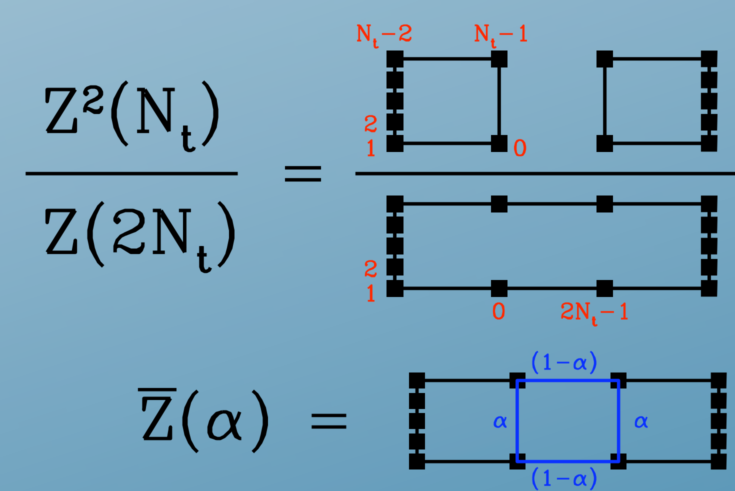
# Direct approach

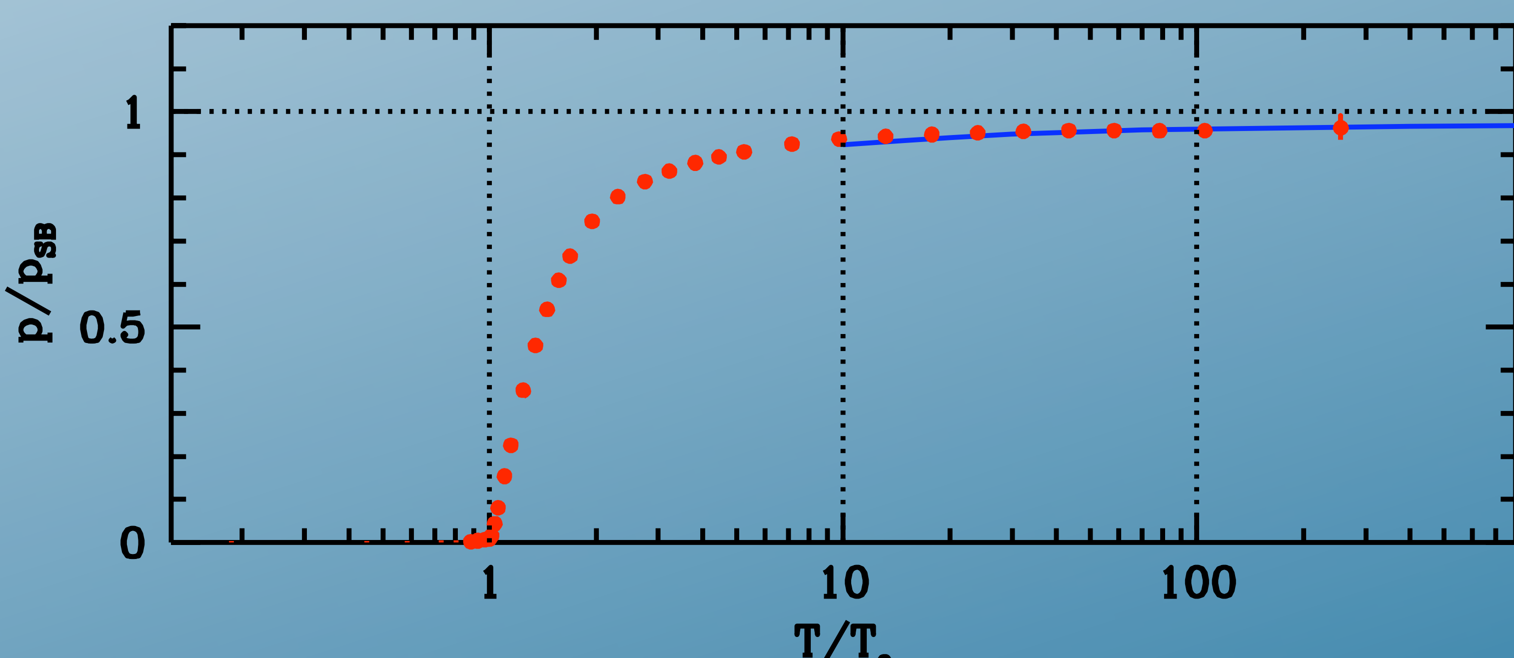
In the standard approach the entire EoS can be calculated as an integral, but not individual pressure values.
The direct approach calculates p(T)-p(T/2) at a single T temperature.

[Endrodi *et al.* 0710.4197]

$$\bar{p} = \frac{1}{N_t N_s^3} \log Z(N_t) - \frac{1}{2N_t N_s^3} \log Z(2N_t) = \frac{1}{2N_t N_s^3} \log \left( \frac{Z(N_t)^2}{Z(2N_t)} \right)$$

$$\bar{p} \sim \log \left( \frac{Z(N_t)^2}{Z(2N_t)} \right) = \log \left( \frac{\bar{Z}(1)}{\bar{Z}(0)} \right) = \int_0^1 d\alpha \frac{d \log \bar{Z}(\alpha)}{d\alpha} = \int_0^1 d\alpha \langle S_{1b} - S_{2b} \rangle$$



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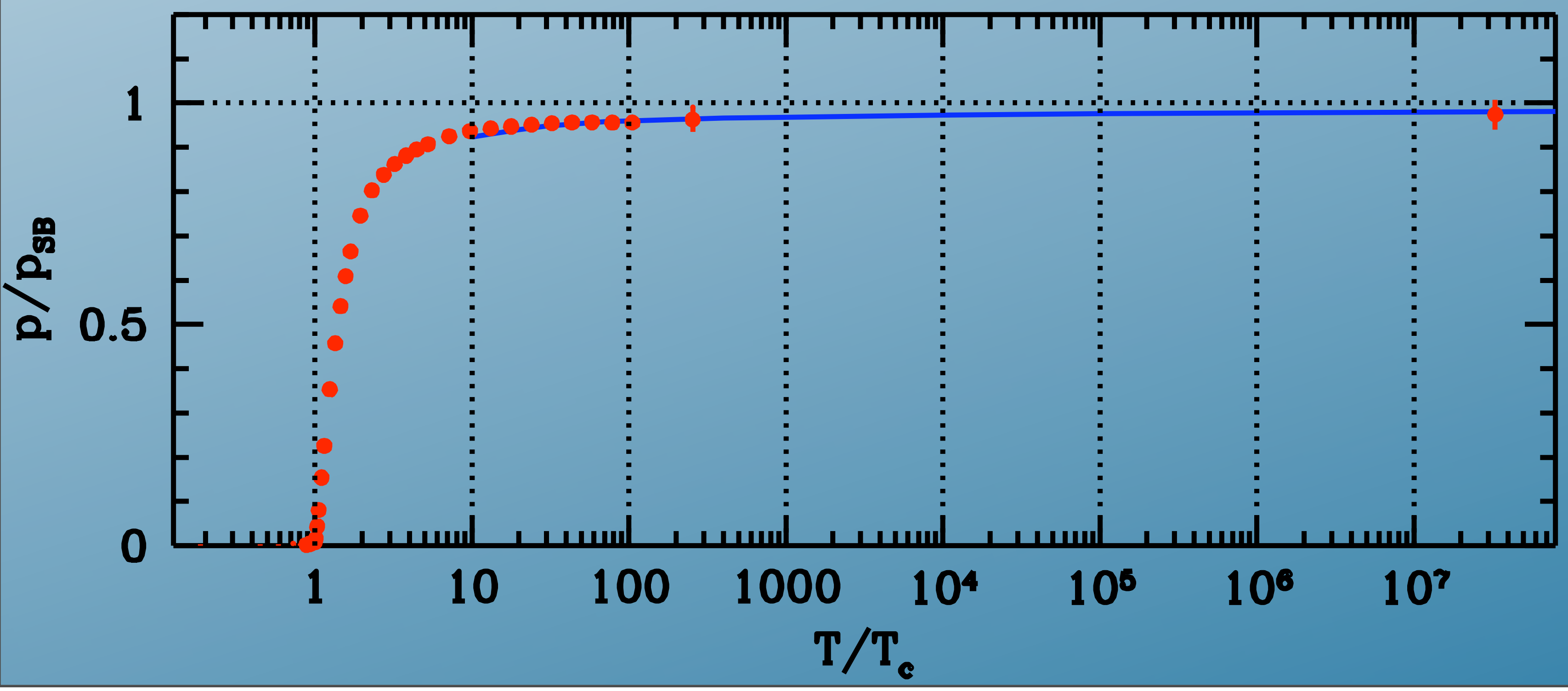


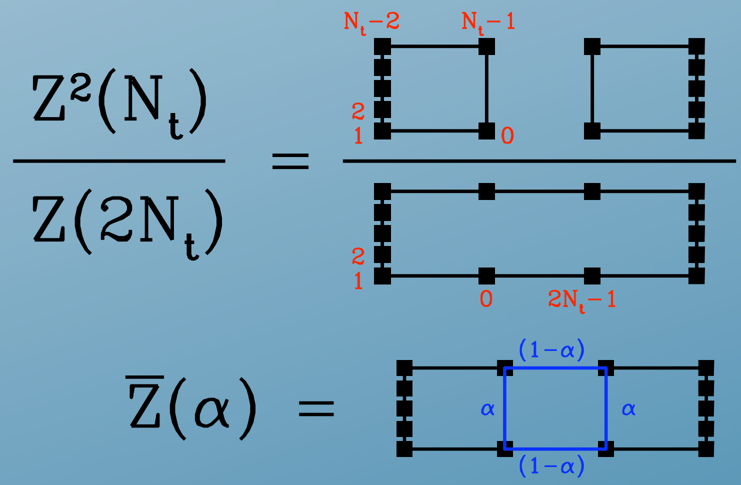

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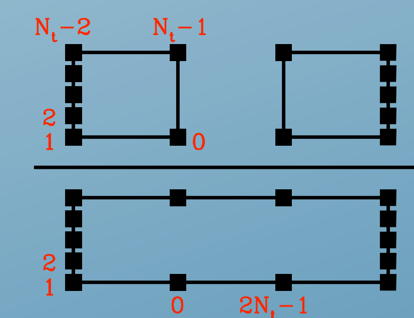
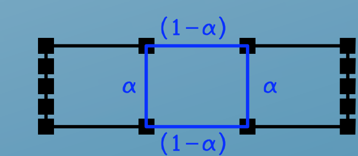
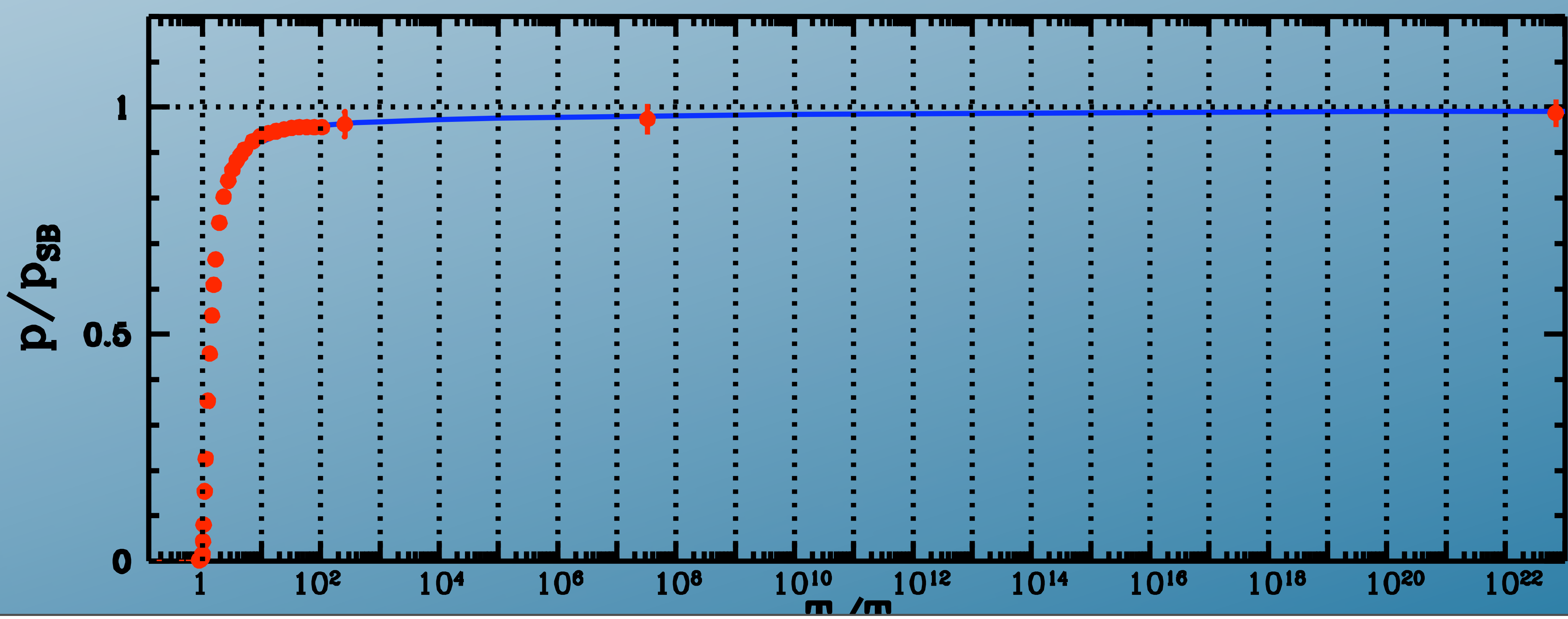
$$\frac{Z^2(N_t)}{Z(2N_t)} =$$

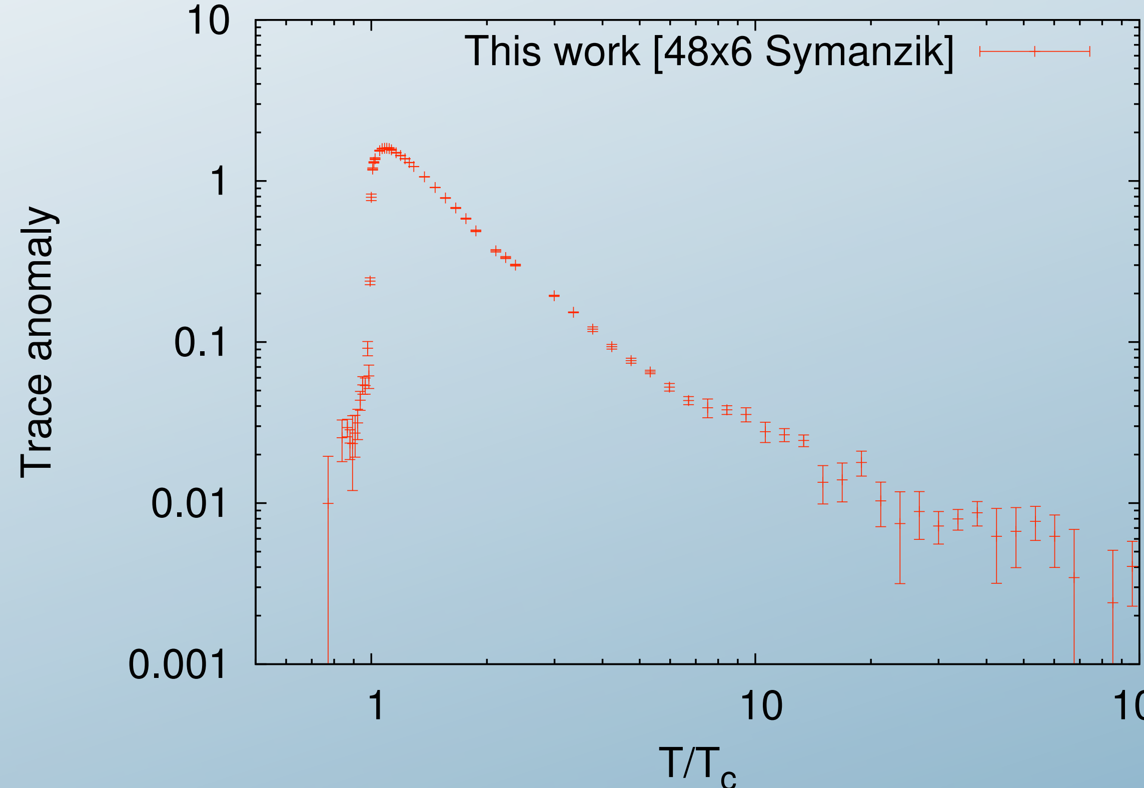



$$\bar{Z}(\alpha) =$$



# The costs of high-T

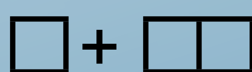


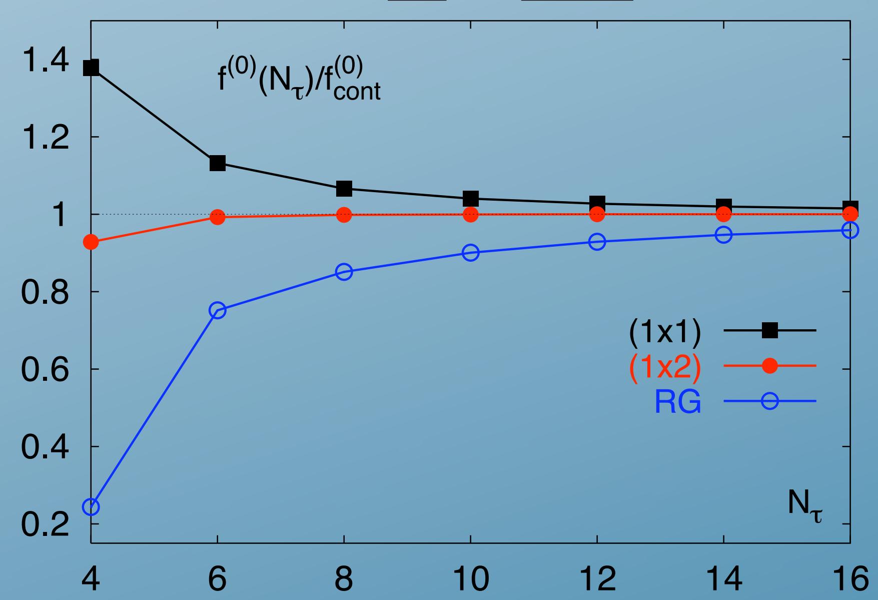

<u>*Renormalization in practice:*</u>

*Trace anomaly is obtained as the difference of two $O(N_t^4)$ numbers*

$$\frac{\Theta^{\mu\mu}(T)}{T^4} \equiv \frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}(p/T^4)$$

Our choice: Symanzik action



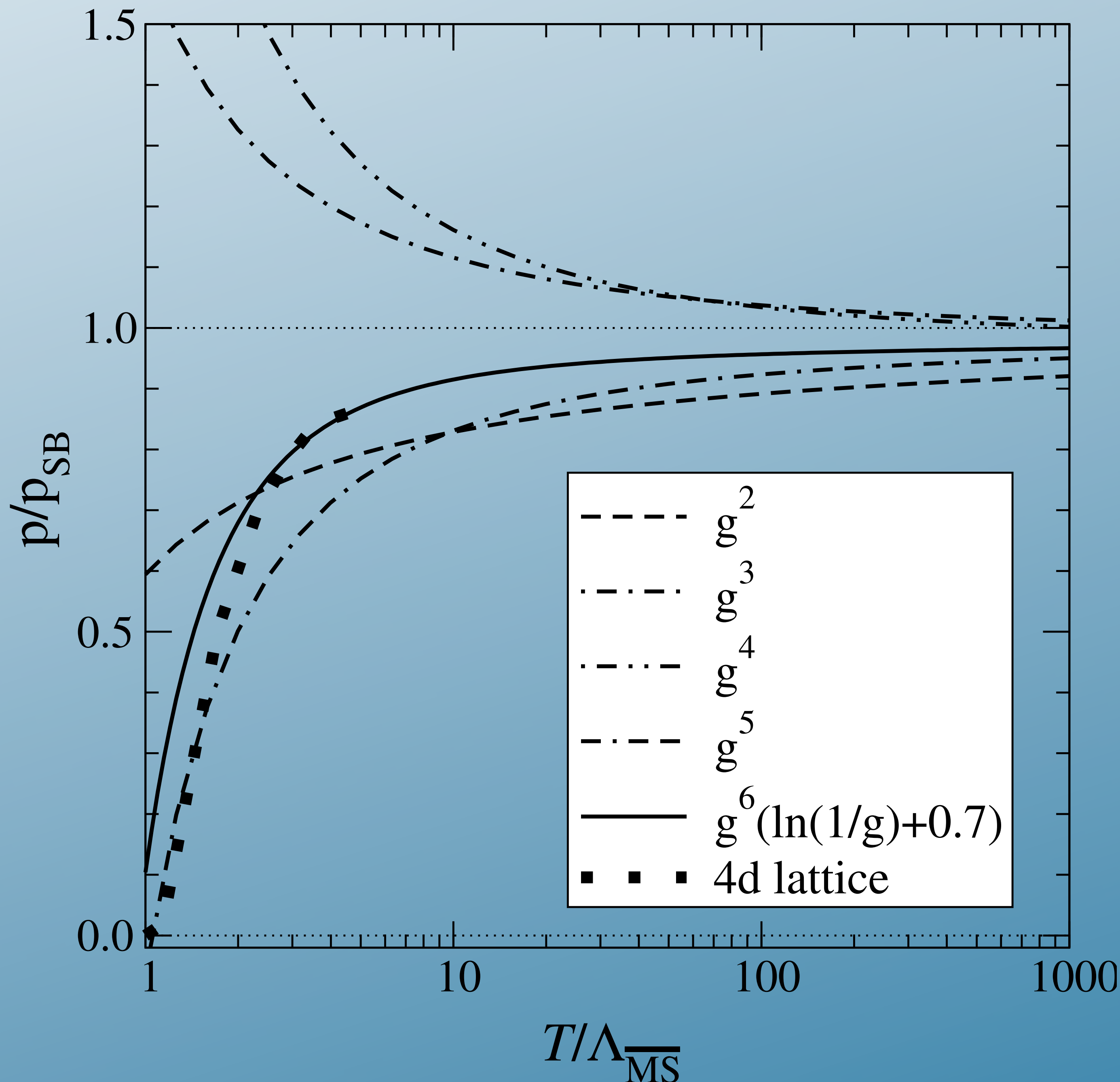



Quenched code: 80-100 Gflop/s on a GPU


≡

# Lattice vs perturbation theory

$g^6 \log(1/g)$ + fitted coeff for $g^6$:

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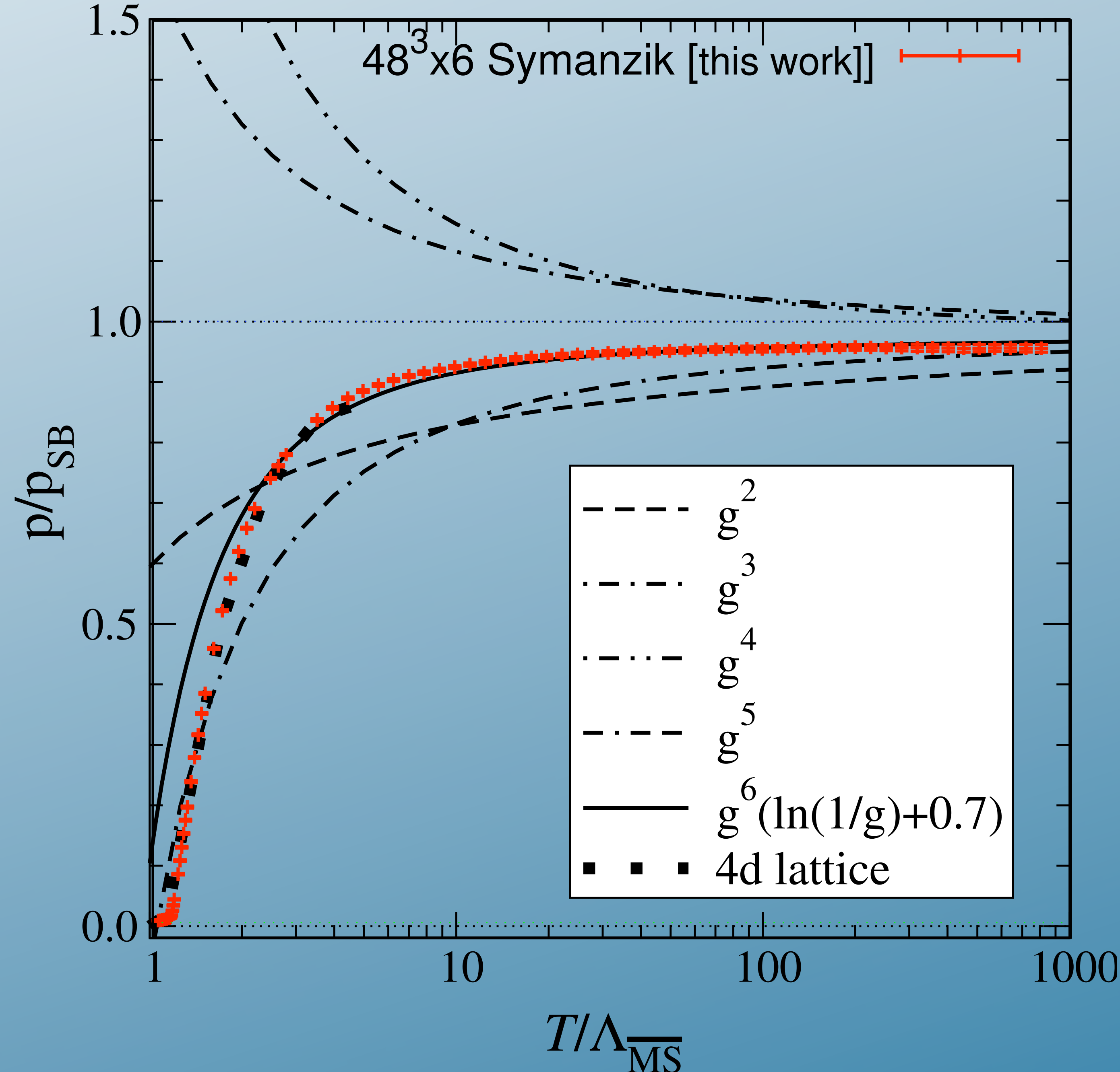


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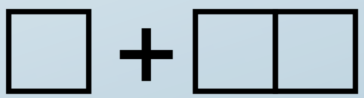
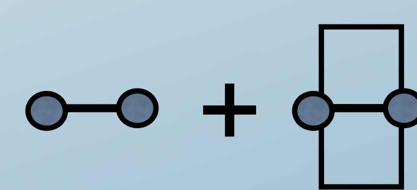
# II. Nf=3 QCD

## Action: <u>staggered fermions with fat links</u>

$S_g =$  $\qquad S_f =$ 

stout smearing parameters $\rho=0.15$, $N_{smr}=2$

*(oversimplified)*

Is staggered formulation appropriate? *Zero T physics matches experiment, UV physics matches perturbation theory*

Is the spectrum physical? *Pion splitting scales close to the continuum limit.*

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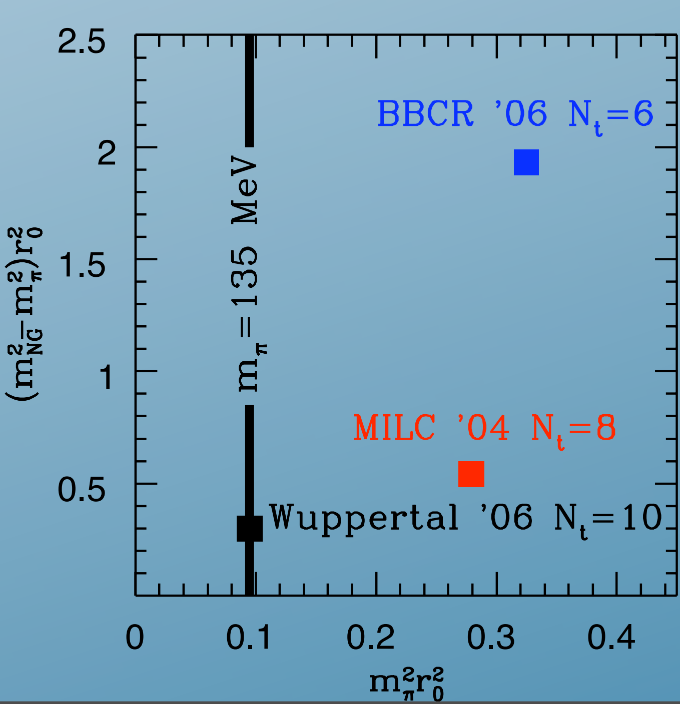
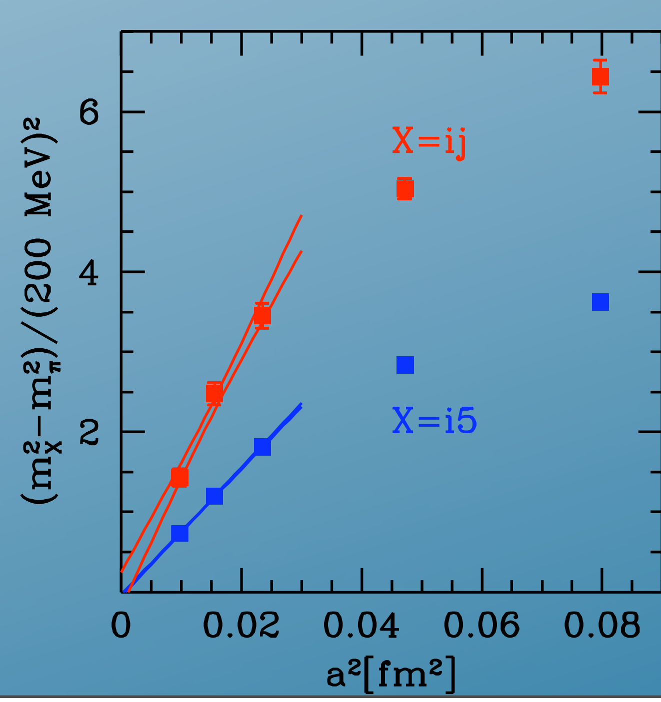
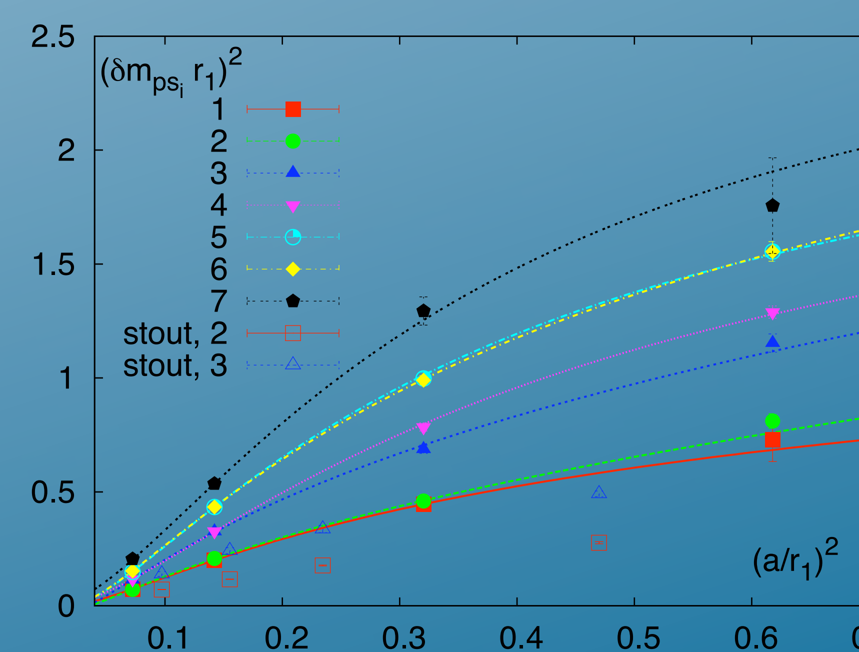
*Stout smearing results in a balanced improved action: reduced taste symmetry breaking*



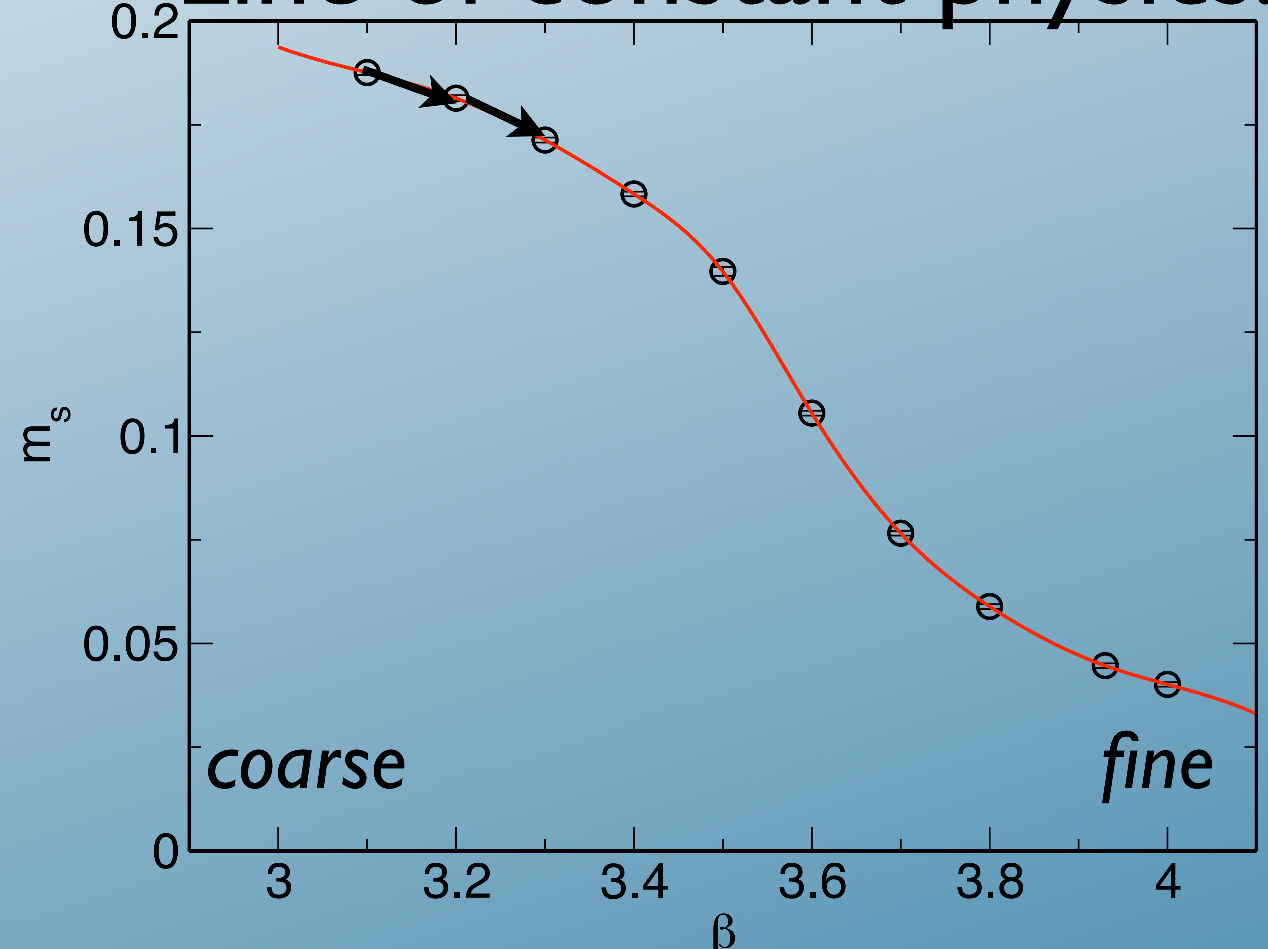



# Pressure is an integral in theory space

$$\frac{\Delta p}{T^4} = N_t^4 \int_{(\beta_0,m_{q0})}^{(\beta,m_q)} d(\beta,m_q)\left[\frac{1}{N_t N_s^3}\begin{pmatrix}\partial \log Z/\partial\beta \\ \partial \log Z/\partial m_q\end{pmatrix} - \frac{1}{N_{t0}N_{s0}^3}\begin{pmatrix}\partial \log Z_0/\partial\beta \\ \partial \log Z_0/\partial m_q\end{pmatrix}\right]$$

*with* $\langle\bar\psi\psi\rangle_q = \frac{T}{V}\frac{\partial \ln Z}{\partial m_q}\ ,\ \ q = l,\ s,$ $\qquad \langle S_g\rangle = -\frac{T}{V}\frac{\partial \ln Z}{\partial \beta}$

Line of constant physics:




beta ➙ -log(a) ➙ log(T)

Integration along the LCP

one integrates the trace anomaly.

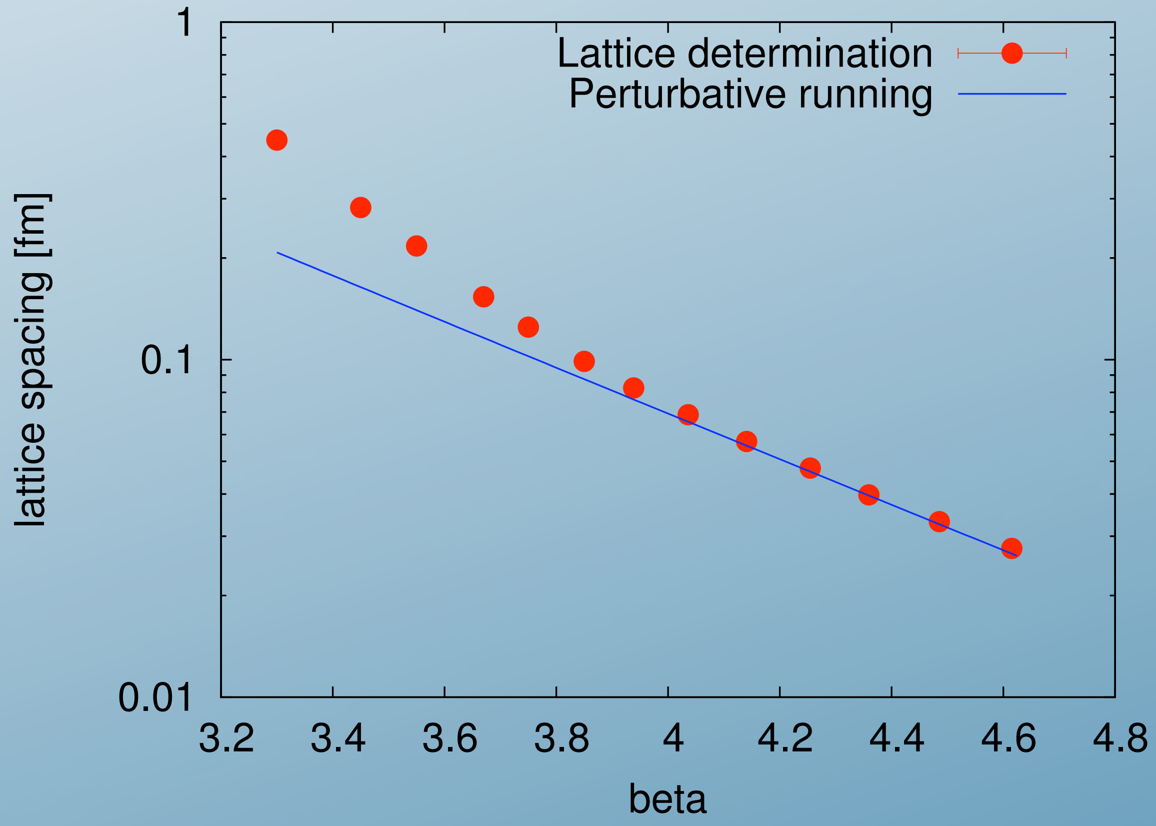
gives $p(T)/T^4$ - $p(T_0)/T_0^4$

# Renormalization

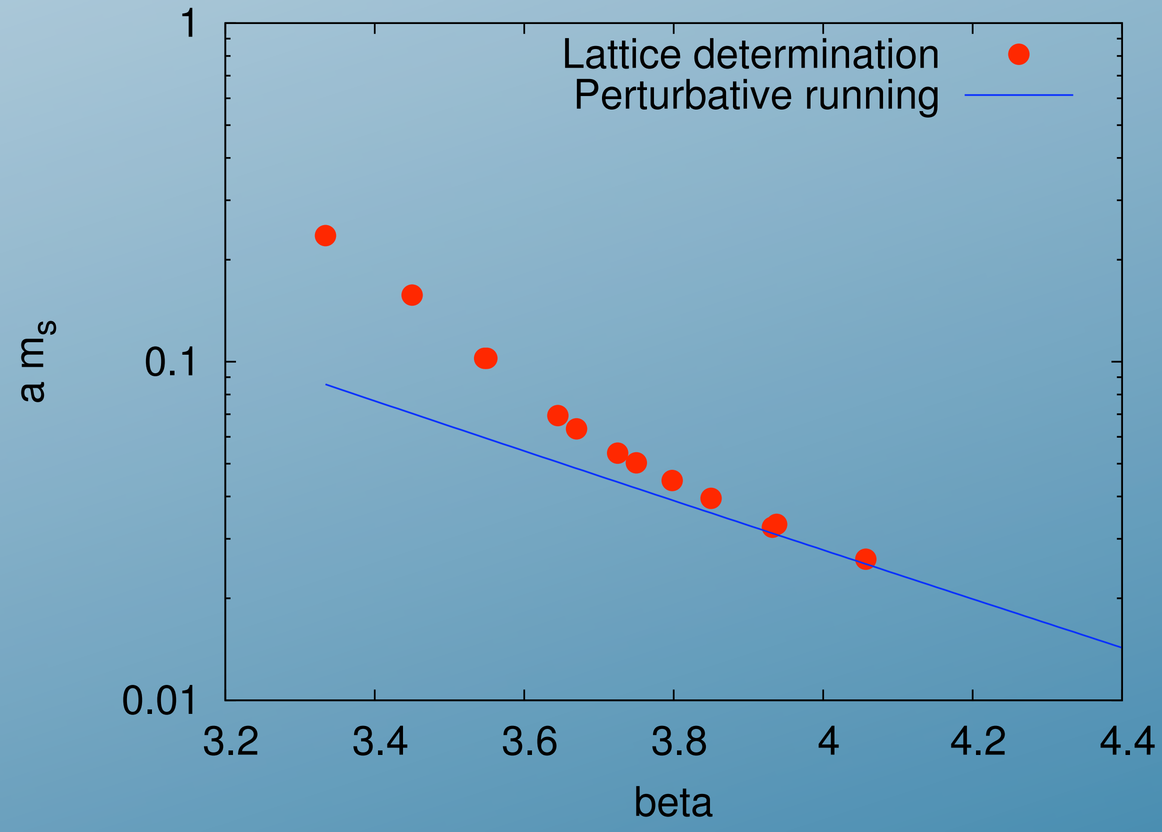
Renorm. condition: $m_K/f_K$=495/155.5 $m_\pi/f_K$=135/155.5 *Nf*=2+1

$m_{PS}/f_{PS} \approx 3.6$ $m_{PS} \approx 720MeV$ *Nf*=3

bare coupling




bare mass

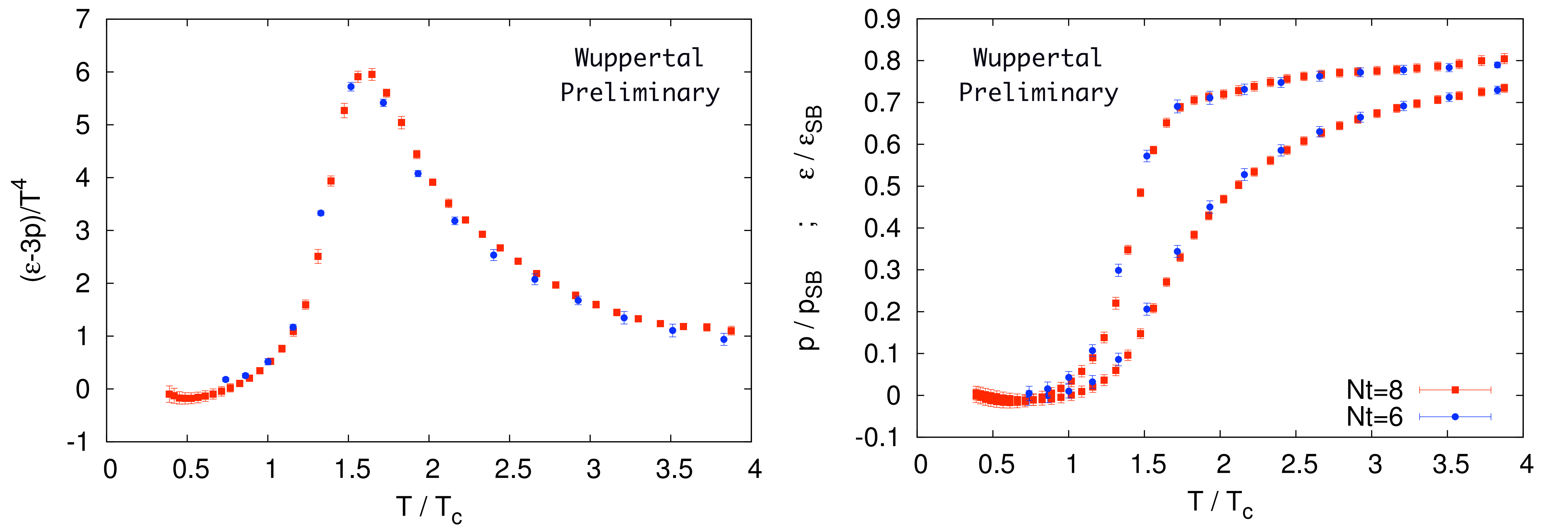



We reached perturbative running:
we completed the renormalization for arbitrary high cut-off.
This enables us to simulate an arbitrary high temperature.

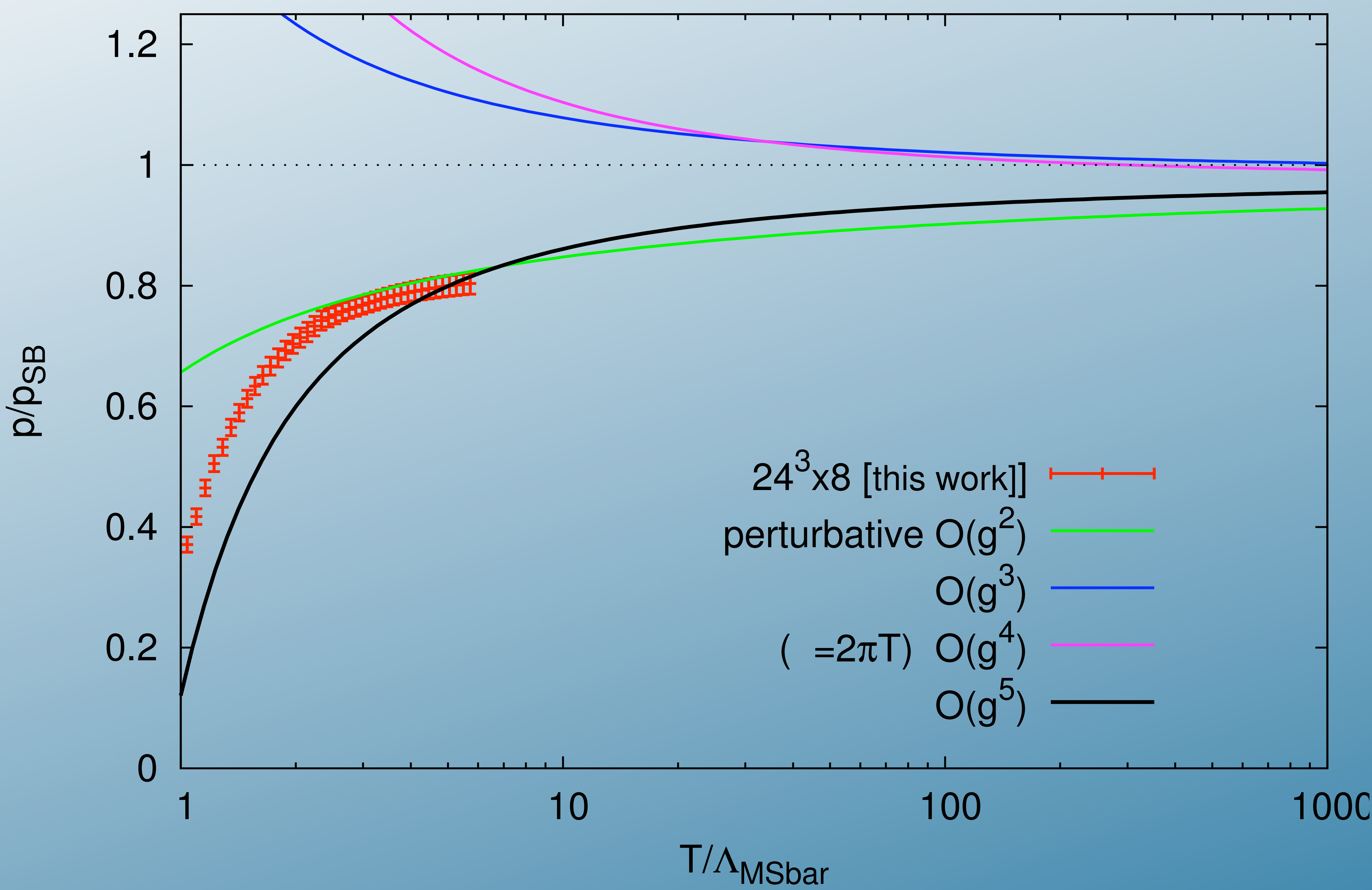
# Nf=3 equation of state

$$\Omega(T,V) = T\ln Z(T,V)$$

$$\frac{\Theta^{\mu\mu}(T)}{T^4} \equiv \frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}(p/T^4) \qquad p = \frac{1}{V}\Omega(T,V) \qquad \epsilon = \frac{T^2}{V}\frac{\partial \Omega(T,V)/T}{\partial T}$$

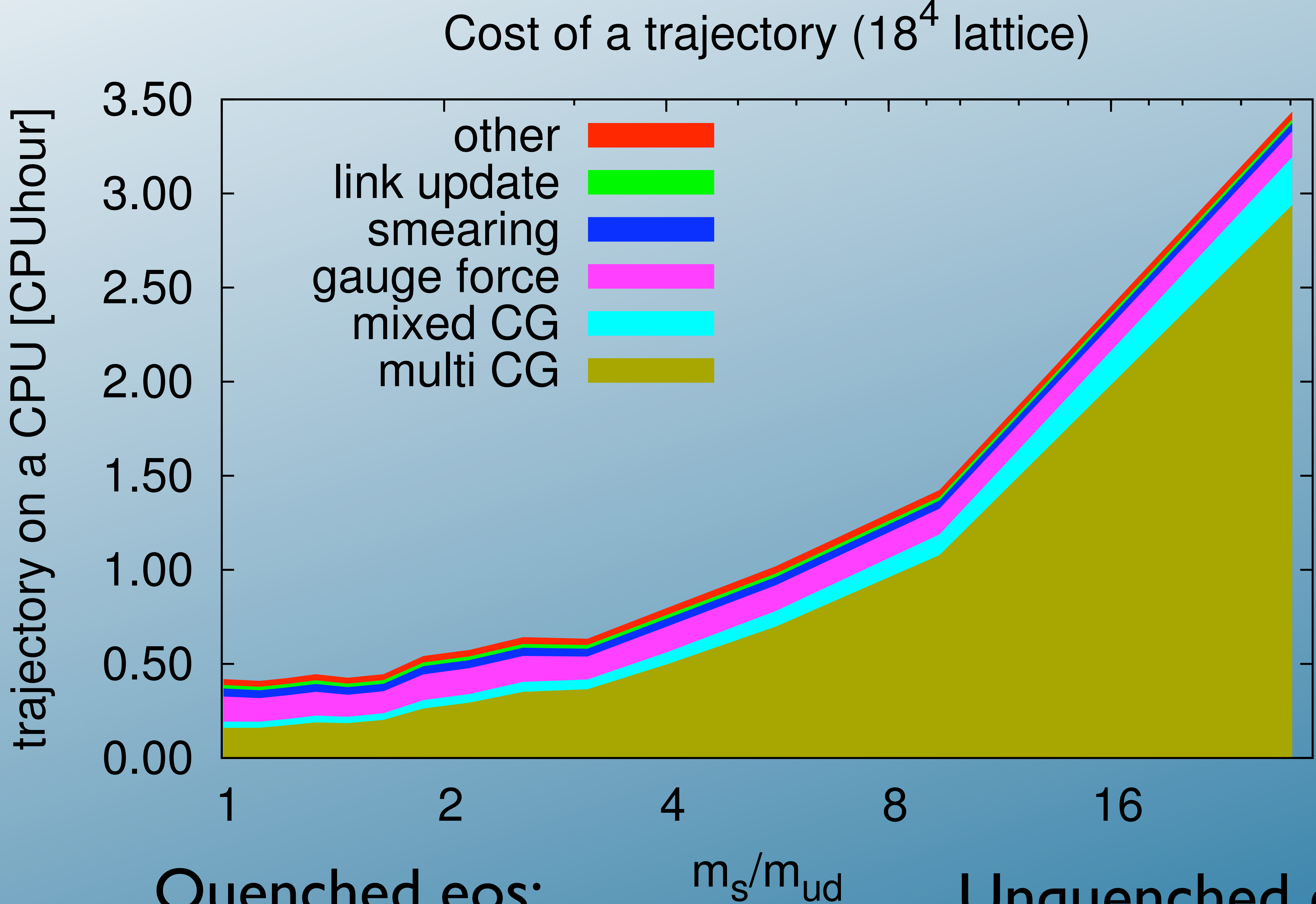


# Towards the perturbative limit




Note that the perturbative curves are very sensitive to:
a) $\Lambda_{QCD}$ b) renormalization. scale

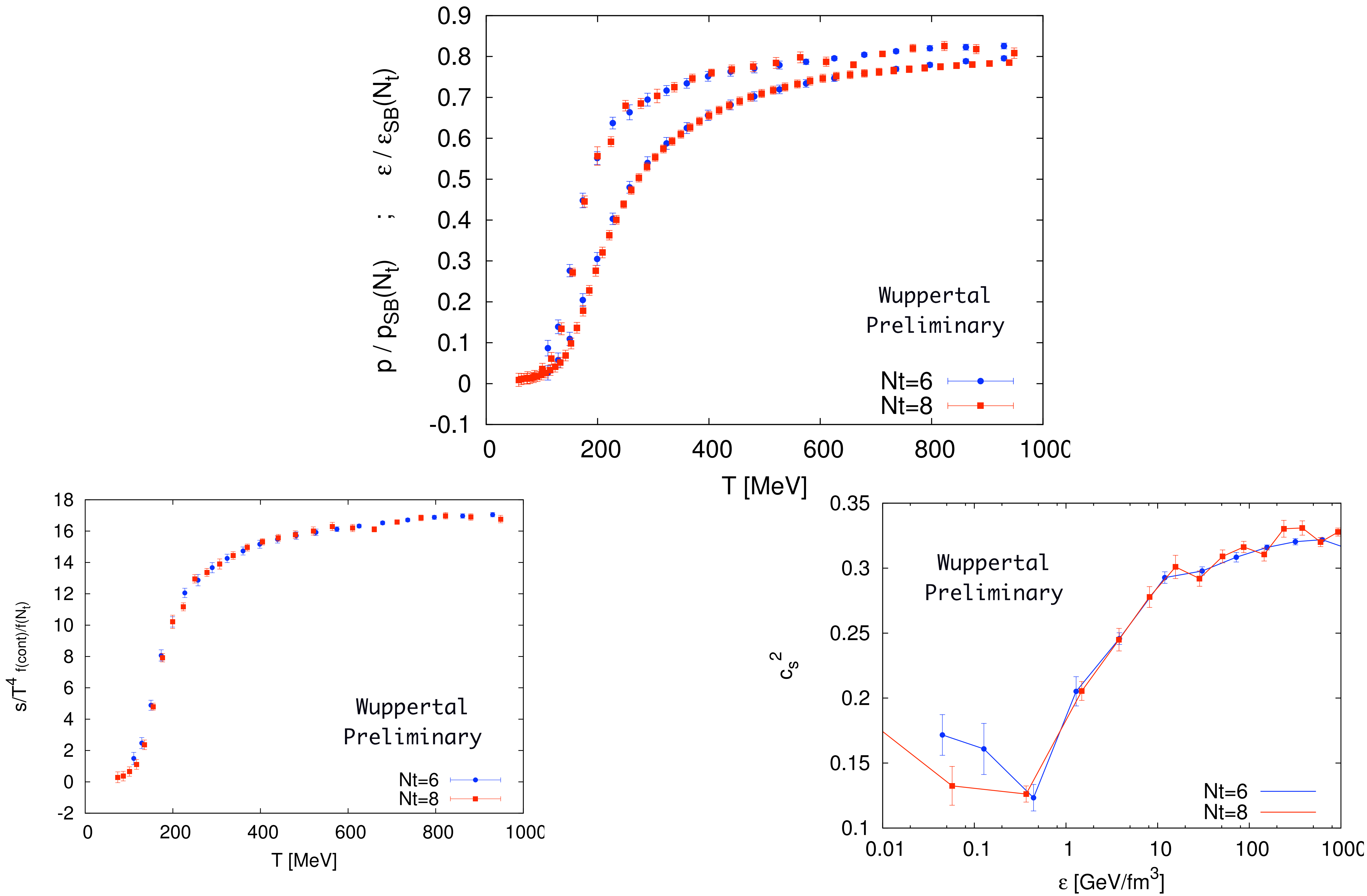
# III. At physical quark mass




Quenched eos:
difficulty ~ $N_t^8$

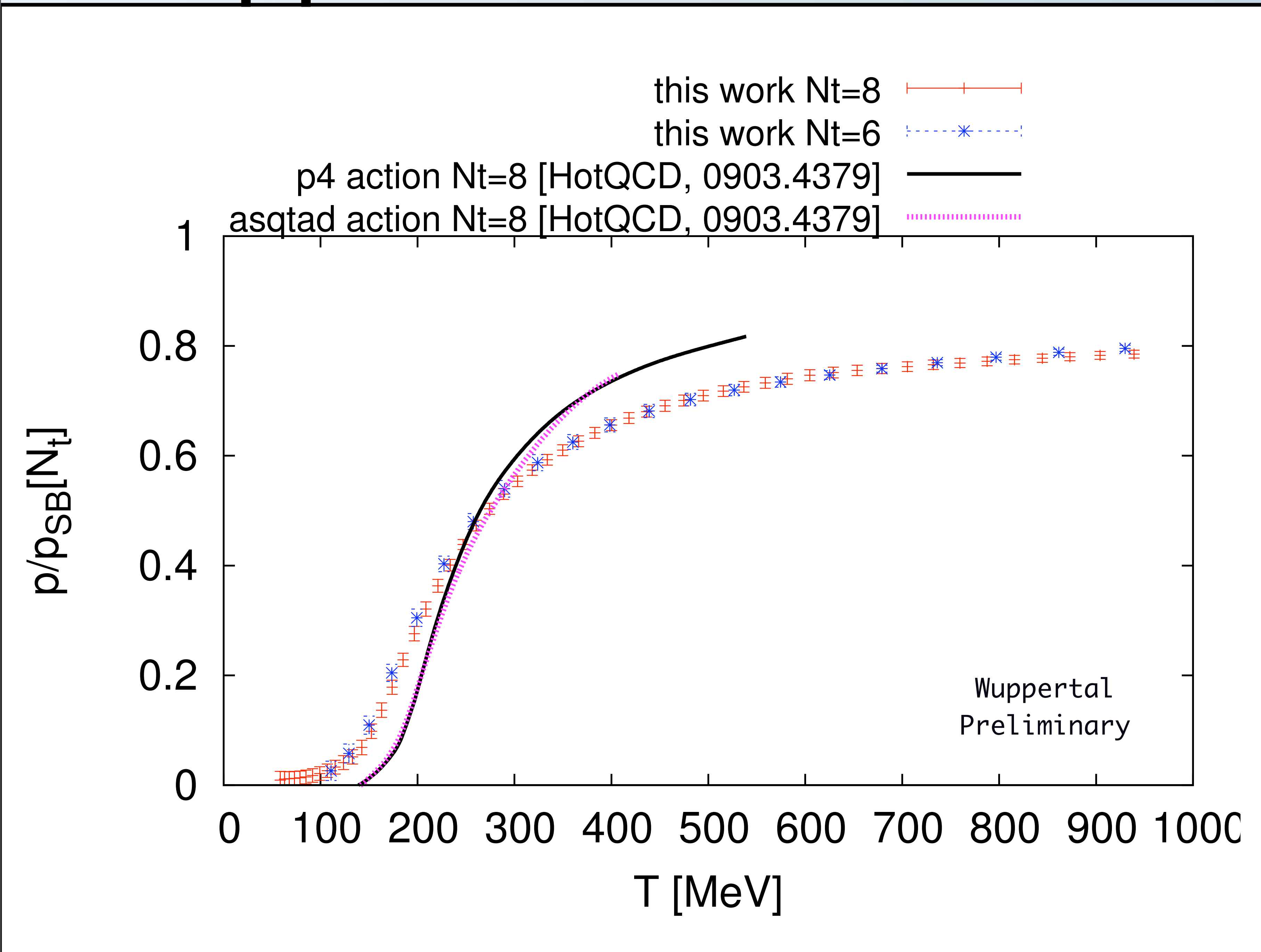
Unquenched eos:
difficulty ~ $N_t^{9-13}$

# The QCD equation of state



# Wuppertal vs HotQCQ

Equation of state




a) Tc discrepancy is manifest in EoS

b) hotQCD EoS shoots up steeper

**p4**: optimized for infinite temperature

*(pert. improvement helps reaching the SB limit)*

**stout**: optimized for phase with broken chiral symmetry

*(smearing helps towards correct spectrum)*

# IV What sets the pion mass?

Illustration:

How to match Wuppertal and hotQCD results?

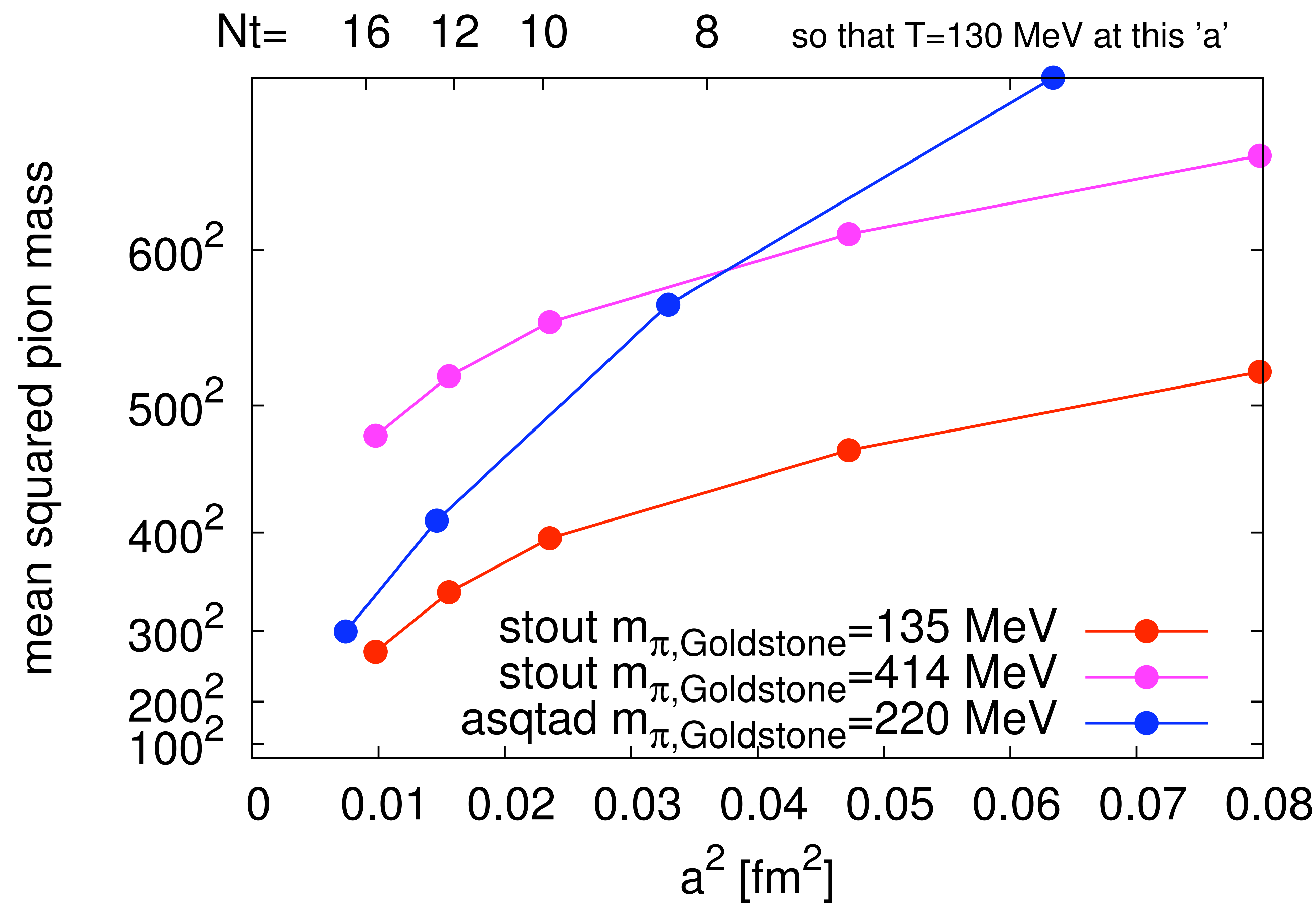
- Lattice artefacts for taste violation

*stout (Wuppertal) < asqtad (MILC) < p4 (Bielefeld)*

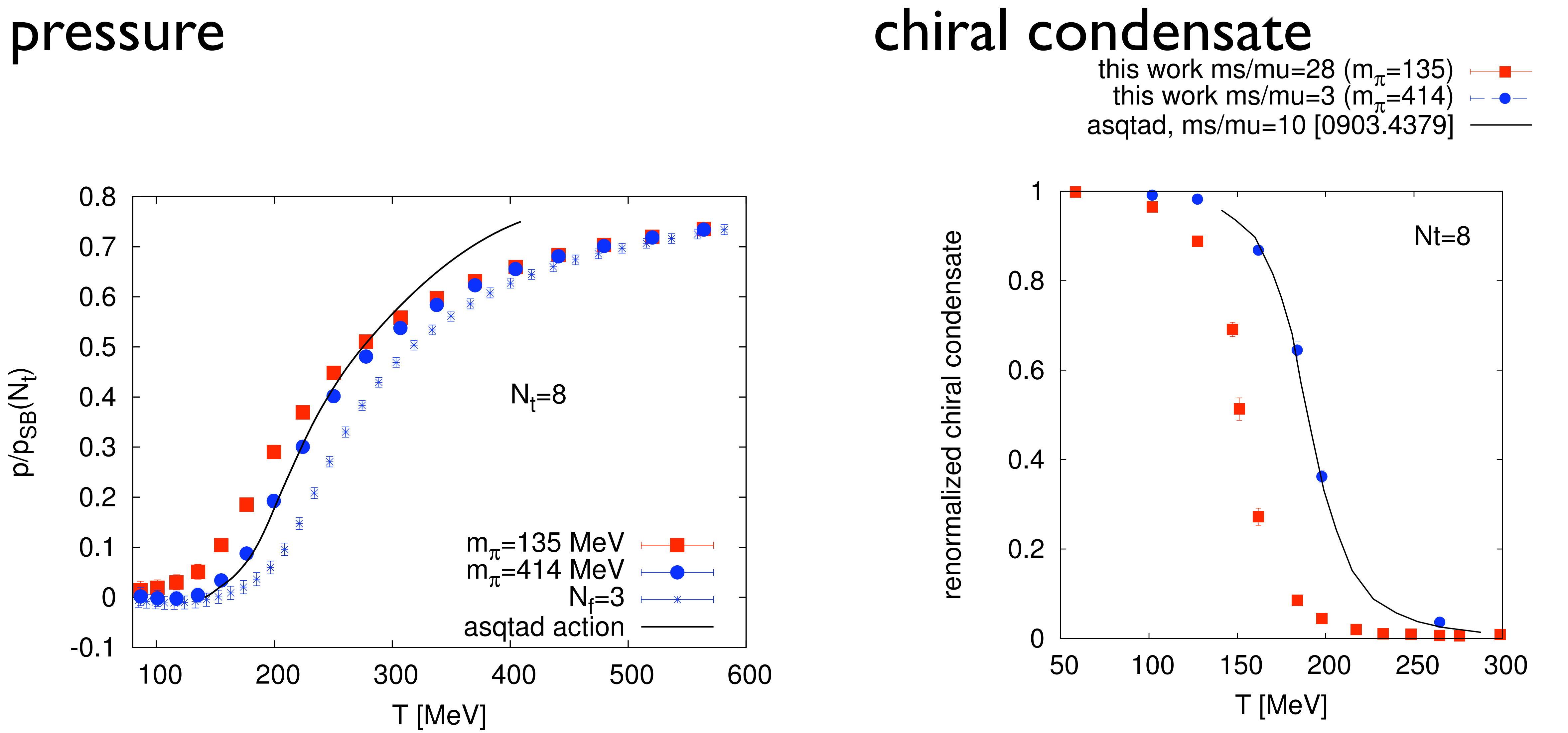
- We try to match asqtad's average pion mass by tuning our $m_\pi$ (no perfect matching is possible)

- We repeat the Tc analysis with this heavier pion

# Matching the average pion mass



# Transition temperature vs pion mass




We reproduce the hotQCD transition temperature with a heavier pion mass.

At that mass we see chiral and confinement transition at the same Tc

# Message:

- We push the Nf=0 and Nf=3 equation of state towards the perturbative limit.
- Our Nf=2+1 equation of state at Nt=4,6 and 8 scales
- The discrepancy in Tc manifests in the equation of state p4 has a steeper and later (30 MeV) rise in the pressure.
- Our pion mass spectrum is significantly closer to physical than our competitor's;
puts confidence in our simulations also below 200 MeV
- The transition pattern observed by the hotQCD collaboration might be reproduced with a "heavier pion".