Fluctuations near the chiral critical end point

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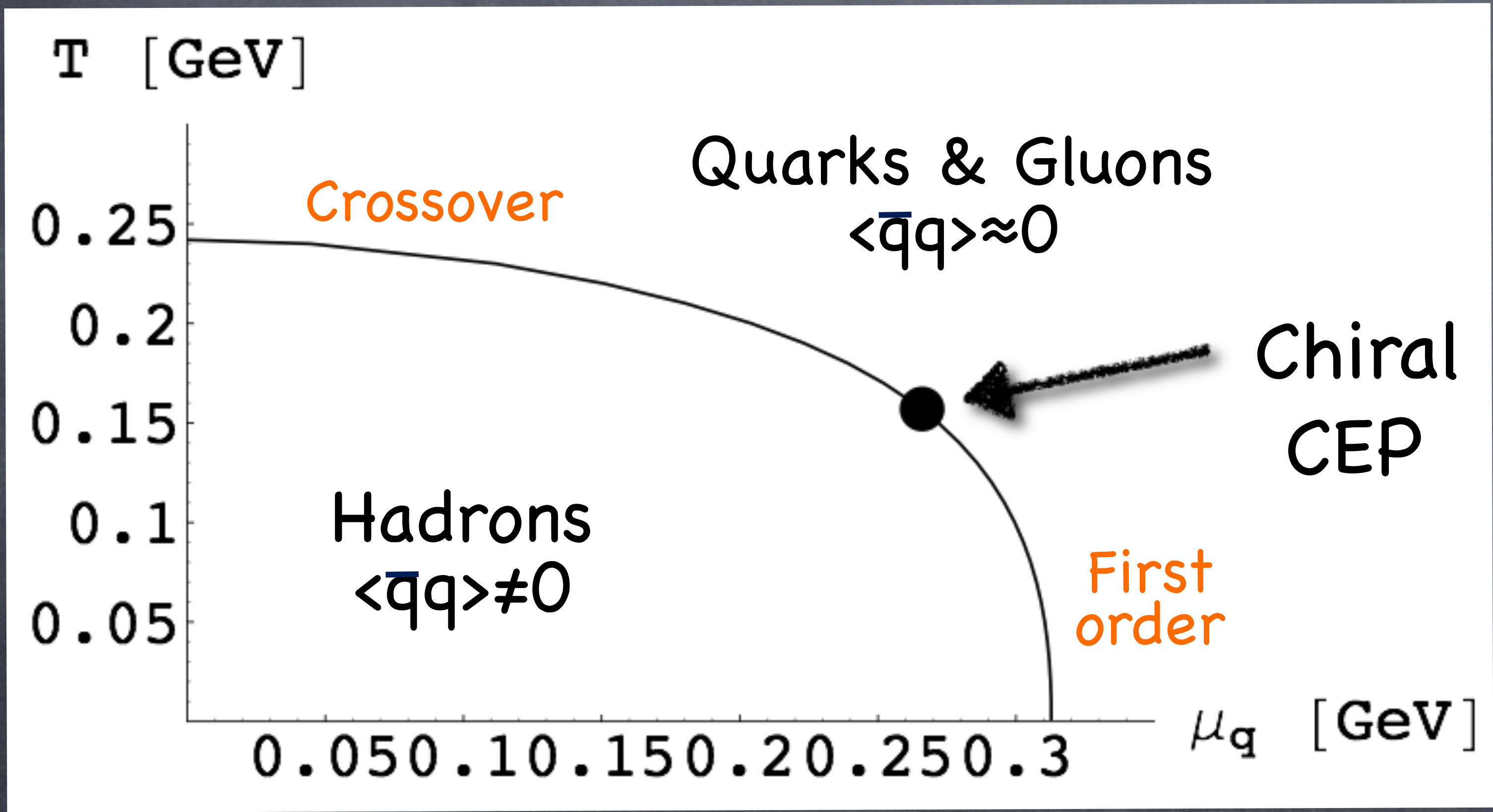
GSI

Schematic QCD Phasediagram



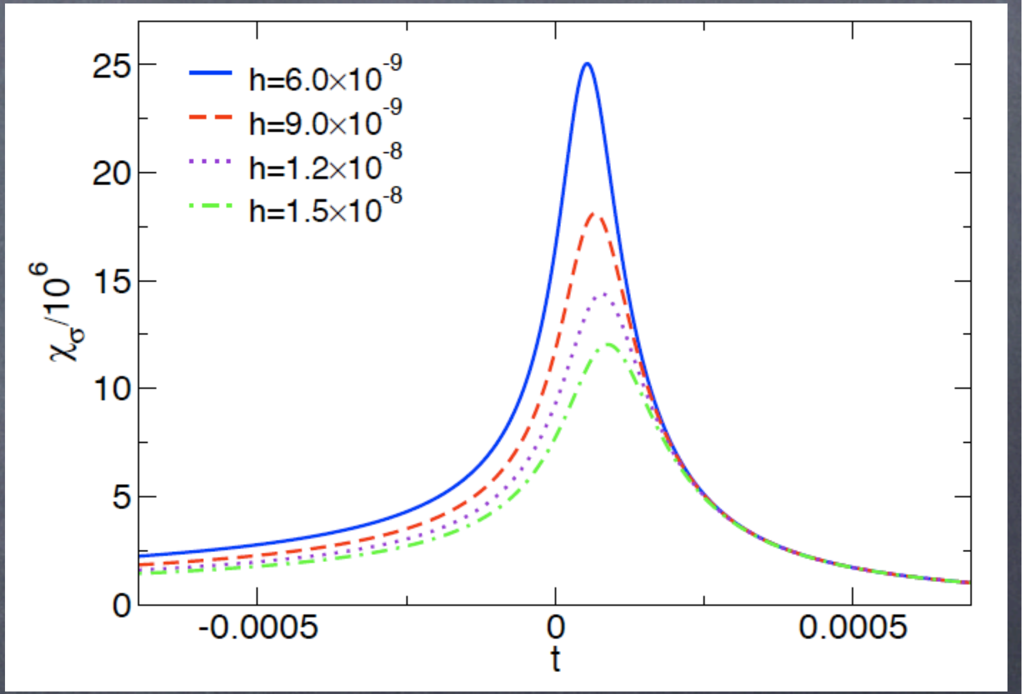
Fluctuations characterize critical endpoint?

Fluctuations & susceptibilities

- General susceptibility

$$\chi_{\mathbf{ab}} = -\frac{1}{\mathbf{V}}\frac{\partial^2\Omega}{\partial \mathbf{a}\partial \mathbf{b}}$$

- Fluctuations of order parameter
  - Chiral suscept. diverges at 2nd order transition ($SU(2)_f$; $\mu = m_\pi = 0$)



$$\chi_{\mathbf{mm}} \sim \langle(\boldsymbol{\Delta}\bar{\psi}\psi)^{\mathbf{2}}\rangle$$
$$\sim \mathbf{m}_\pi^{\mathbf{2(1/\delta-1)}}\ (\mathbf{T} = \mathbf{T_c})$$

- $\chi_{\mu\mu}, \mathbf{C_P}, \mathbf{C_V}$ in general finite

Additional fluctuations at CEP

$$\chi_{\mu\mu} = -\frac{n_q^2}{V}\left[\frac{\partial P}{\partial V}\Big|_T\right]^{-1} \rightarrow \infty$$

$$C_P = T\,\frac{\partial S}{\partial T}\Big|_P \rightarrow \infty$$

$$C_V = T\,\frac{\partial S}{\partial T}\Big|_V \rightarrow \infty$$

$$\langle(\Delta N)^2\rangle \rightarrow \infty$$

$$\langle(\Delta(S/N))^2\rangle \rightarrow \infty$$

$$\langle(\Delta T)^2\rangle \rightarrow 0$$

$$\langle(\Delta P)^2\rangle \rightarrow 0$$

In MFT $C_V < \infty$

- Divergences interdependent:

$$\frac{\partial P}{\partial V}\Big|_T = \frac{C_V}{C_P}\,\frac{\partial P}{\partial V}\Big|_S$$

$$\frac{\partial P}{\partial V}\Big|_T = \frac{\partial P}{\partial V}\Big|_S + \frac{T}{C_V}\left[\frac{\partial P}{\partial T}\Big|_V\right]^2$$

Fluctuations near CEP

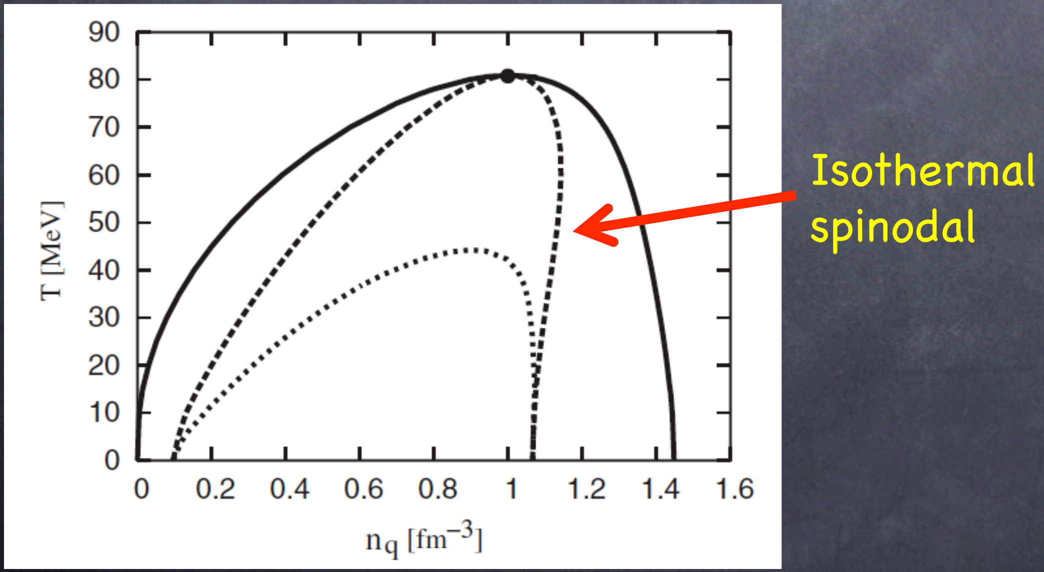
- 1^{st} order transition, spinodal instability

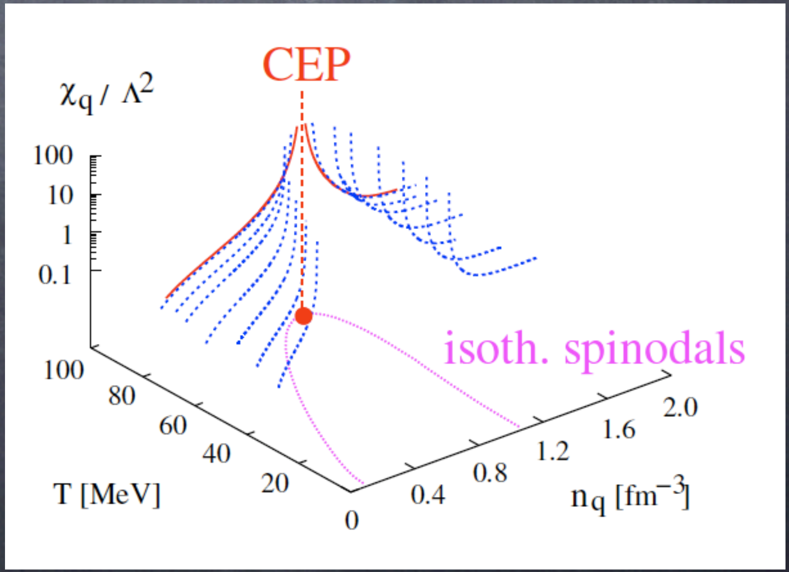
$$V \frac{\partial p}{\partial V}\Big|_T = -\frac{n_q^2}{\chi_{\mu\mu}} = 0$$

$$\chi_{\mu\mu} \longrightarrow \infty$$

- Baryon number fluctuations diverge
  - CEP singularity evolves from spinodal instability
  - Expect large fluctuations not only at CEP!

C. Sasaki, B.F., K. Redlich (QM model)

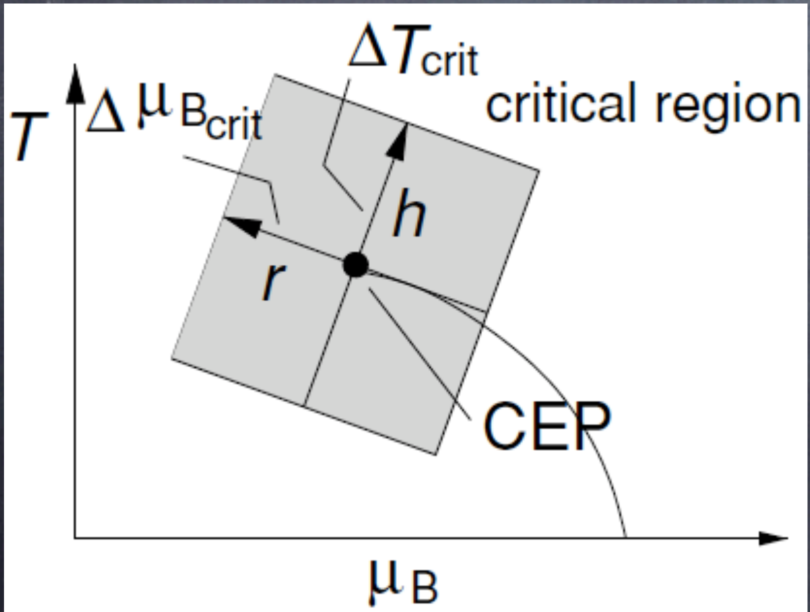




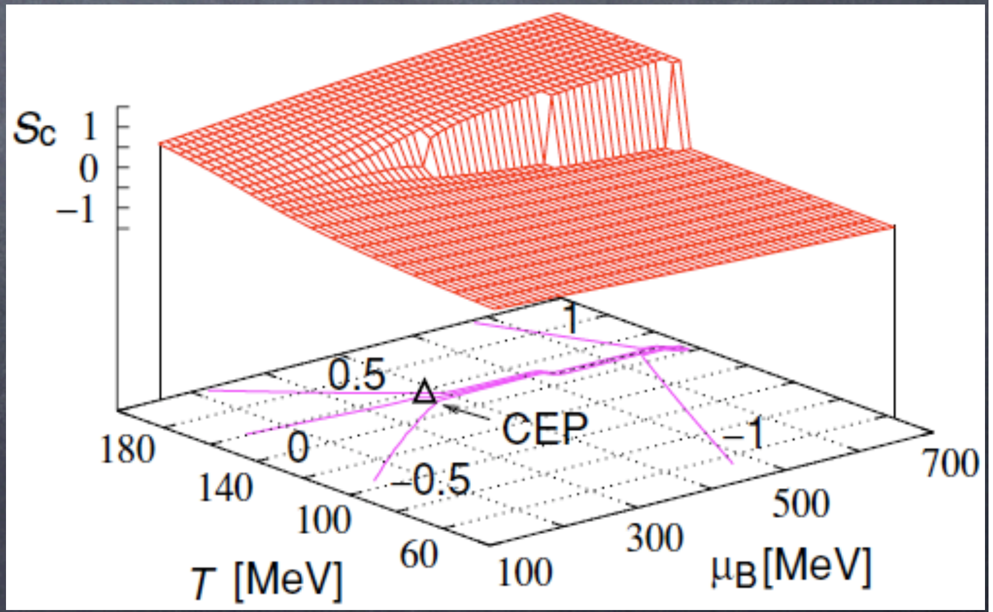
Isentropic expansion: focusing?

- Nonaka & Asakawa (2005):
  Suggestion: Isentropic trajectories universal
  Universality class 3D Ising

Mapping (r,h) ⟶ (T,μ)



Singular part of entropy



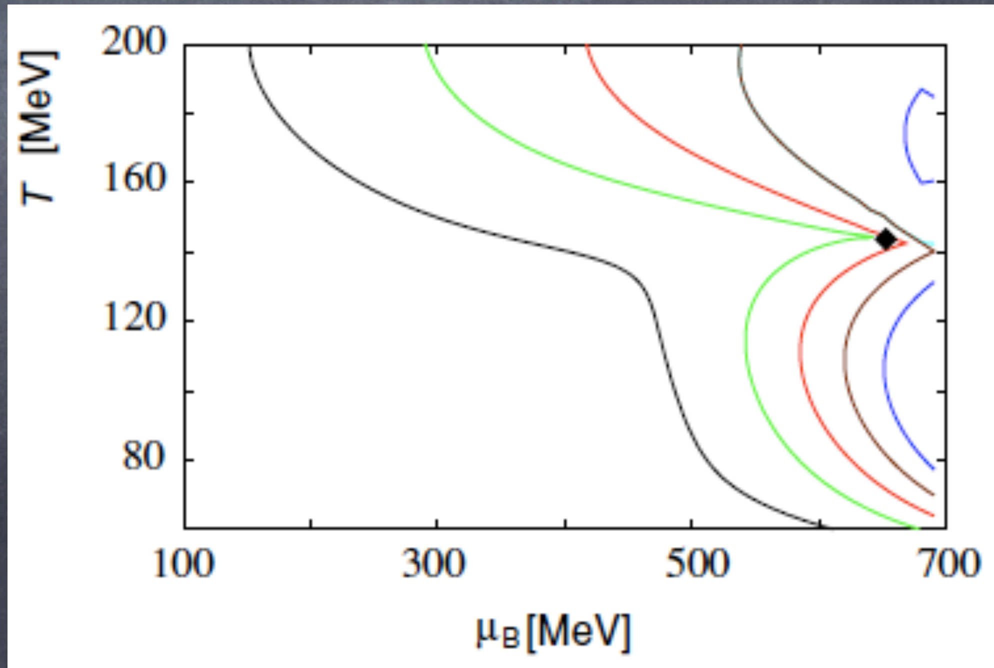
Focusing towards CEP

- Interpolation between phases:

3D Ising model

$$s(T,\mu_B) = \frac{1}{2}(1-\tanh[S_c(T,\mu_B)])s_H(T,\mu_B) + \frac{1}{2}(1+\tanh[S_c(T,\mu_B)])s_Q(T,\mu_B),$$

Isentropic trajectories:



Non-singular background neglected!

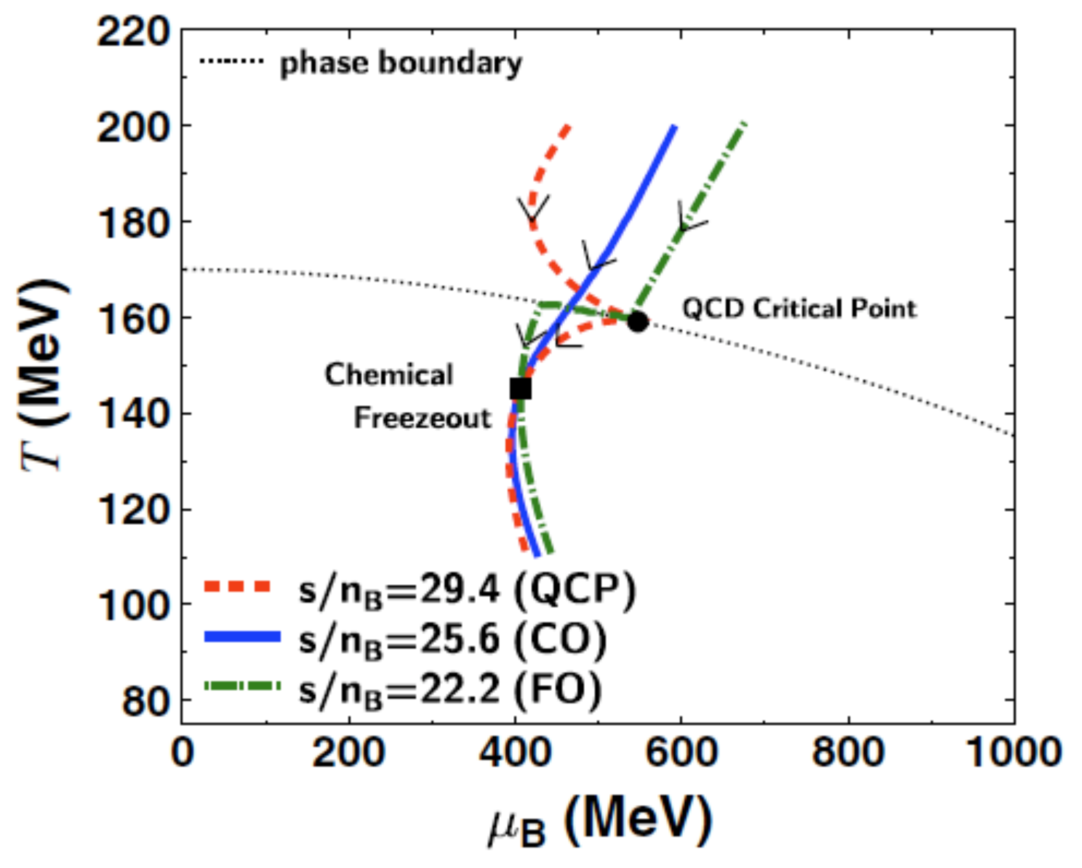
Signature of focusing?

Asakawa, Bass, Müller, Nonaka

- Idea: $\bar{p}/p$ ratio sensitive to μ_B
  large q emitted early, small q later

Isentropic trajectories dependent on EOS

In Equilibrium: momentum-dep. of $\bar{p}/p$ ratio reflects history



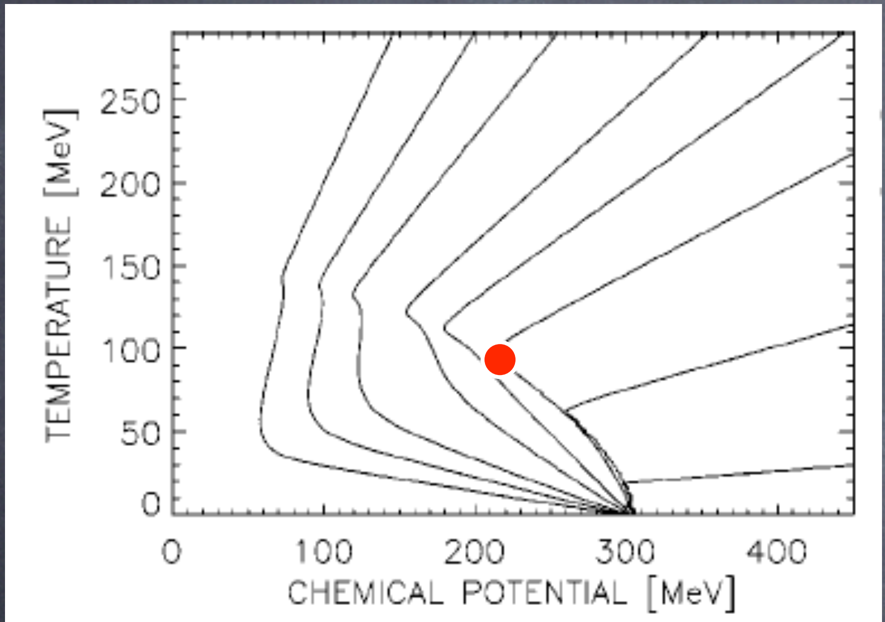
Caveats:

- Critical slowing down
- Focusing universal?

Mean-field NJL vs. QM ($SU(2)_f$)

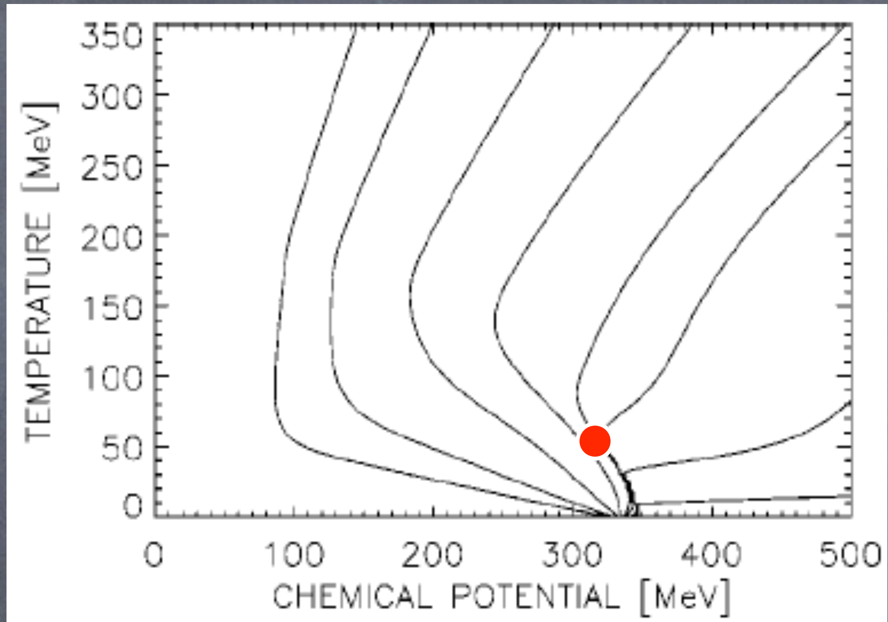
O.Scavenius et al. (2001):

Quark-Meson



Kink

Nambu-Jona-Lasinio



Smooth

No focusing (no fluctuations!)

Cause of kink in QM-model?

The relevance of the vacuum term

- The quark-meson model:

$$\mathcal{L} = \bar{\psi}\left[i\partial\!\!\!/ - g\left(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi}\right)\right]\psi + \frac{1}{2}\left(\partial_\mu\phi\right)^2 - U(\sigma,\vec{\pi}),$$

$$U(\sigma,\vec{\pi}) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\left(\phi^2\right)^2 - h\sigma \qquad \phi = (\sigma, \vec{\pi})$$

- Thermodynamic potential (mean-field appr.):

$$\Omega(T,\mu;\langle\sigma\rangle,\langle\vec{\pi}\rangle) = \Omega_{\bar{q}q}(T,\mu;\langle\sigma\rangle) + U(\langle\sigma\rangle,\langle\vec{\pi}\rangle)$$

$$\Omega_{\bar{q}q}(T,\mu;\langle\sigma\rangle) = \nu_q T \int \frac{d^3k}{(2\pi)^3}\left\{\ln\left(1 - n_F^-(E_q)\right) + \ln\left(1 - n_F^+(E_q)\right)\right\},$$

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Vacuum loop

Singular contribution to Ω

- After integration by parts:

$$\Omega_{\bar{q}q} \sim \int d^3k \frac{k^2}{3E_q}(1 - n^- - n^+) + \text{Surface term}$$

- For $k \to 0, m \to 0$

$$1 - n^- - n^+ \to 0$$

- Dropping the vacuum term:

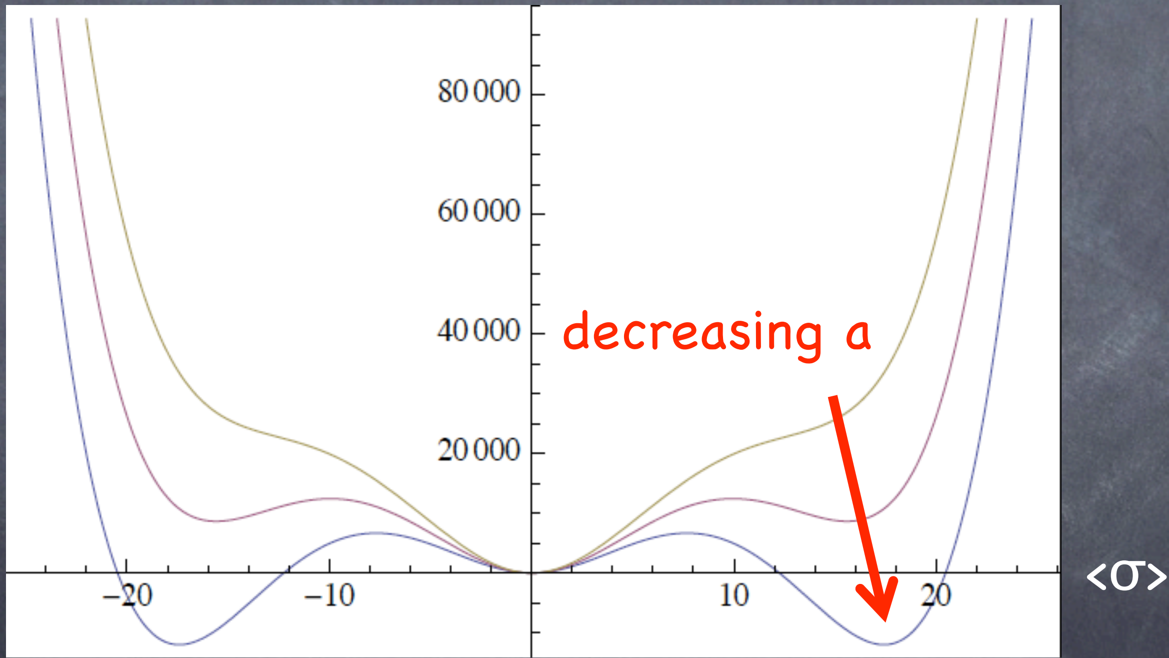
$$\Omega_{\bar{q}q} \sim -\int d^3k \frac{k^2}{3E_q}(n^- + n^+) \sim m^4 \log m^2 + \cdots$$

Cancelled by vacuum term!

Consequences of “no-sea” approximation

- Efective action in chiral limit (h=0)

$$\Gamma(\langle\sigma\rangle) = \frac{a}{2}\langle\sigma\rangle^2 + \frac{b}{4}\langle\sigma\rangle^4 + \frac{c}{4}\langle\sigma\rangle^4 \log(\langle\sigma\rangle^2/\sigma_0^2) + \cdots$$



$$a, b, c > 0$$

$$b_{eff} = b + c\log(\langle\sigma\rangle^2/\sigma_0^2) < 0$$ for small <σ>

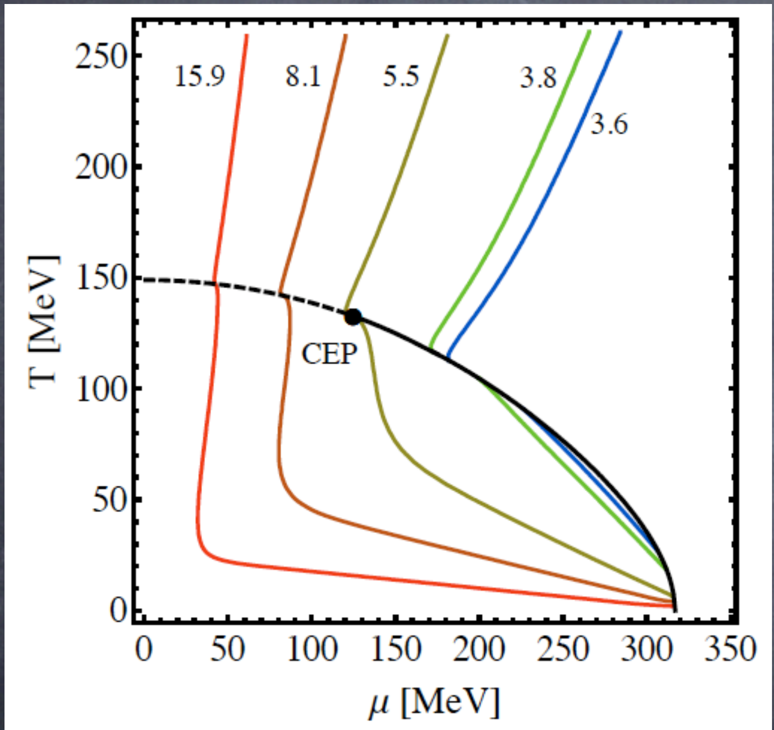
First-order transition in chiral limit!

Baym-Blaizot-Friman-1982
Schaefer-Wambach-2007

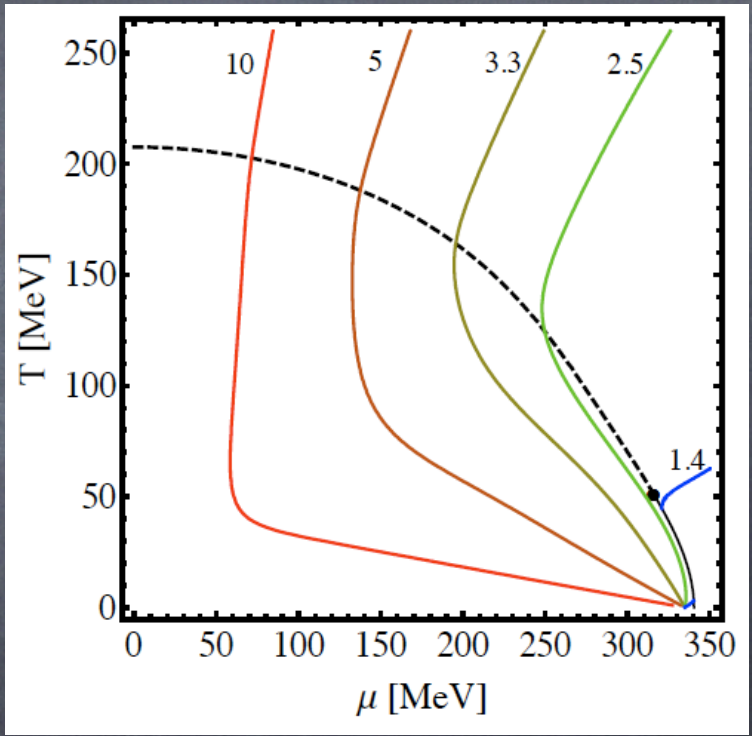
Isentropes w/ and w/o vacuum term

$SU(2)_f$, $m_\pi \neq 0$

No vacuum term



With vacuum term



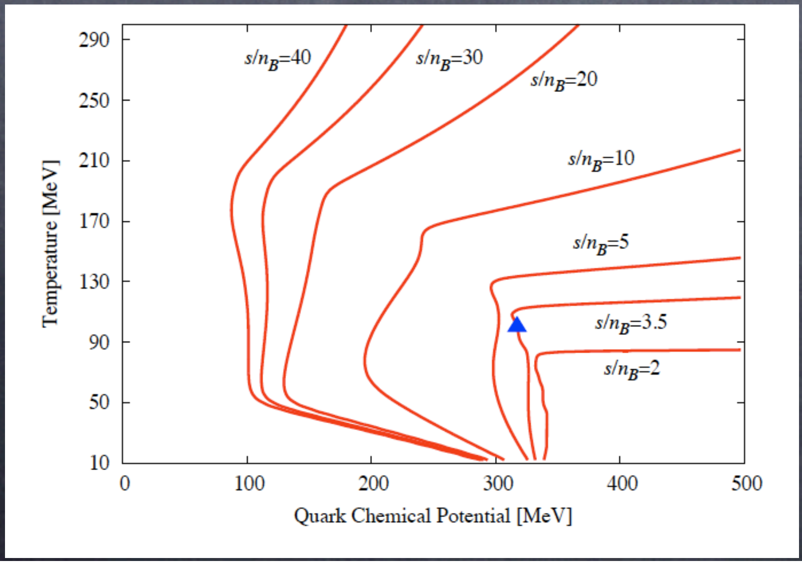
Weak 1[st] order → cross over
Kinks due to latent heat

Smooth isentropes
Shifted $T_c(\mu=0)$ & CEP

Summary mean field

- No focussing (not expected)
- Kinks in crossover artefact of approximation (QM)
- QM w/ vacuum & NJL in qualitative agreement
  - Cancellation of $\mathbf{m^4 \log m^2}$ also with Polyakov loop!
  - Cause of (weaker) kinks in $SU(2)_f$ & $SU(3)_f$ PNJL?

Kähära & Tuominen (2009)
Fukushima (2009)



PNJL $SU(3)_f$
Fukushima (2009)

Soft mode at the CEP

- CEP second order transition:
  - Soft scalar mode
- Enhanced fluctuations:

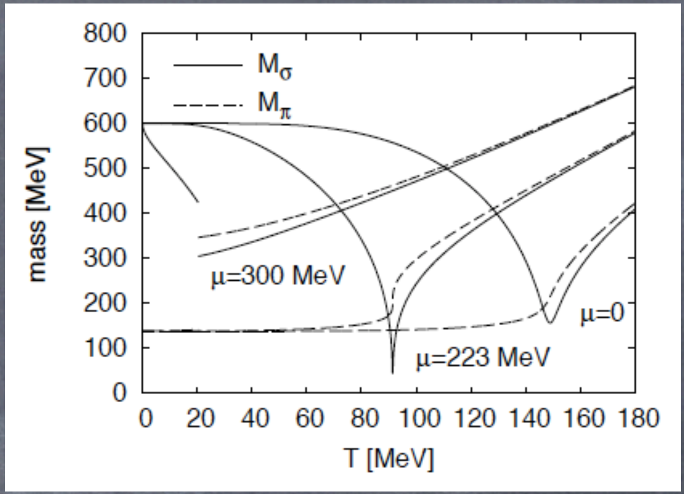
$$\langle(\Delta N)^2\rangle = T \left.\frac{\partial N}{\partial \mu}\right|_{T,V} \rightarrow \infty$$

$$\langle(\Delta(S/N))^2\rangle = \frac{C_p}{N^2} \rightarrow \infty$$

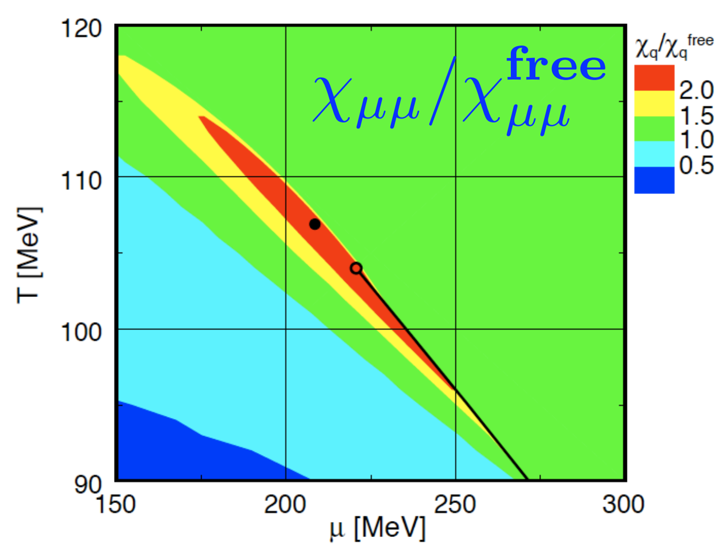
....

- → focusing?

Schaefer & Wambach



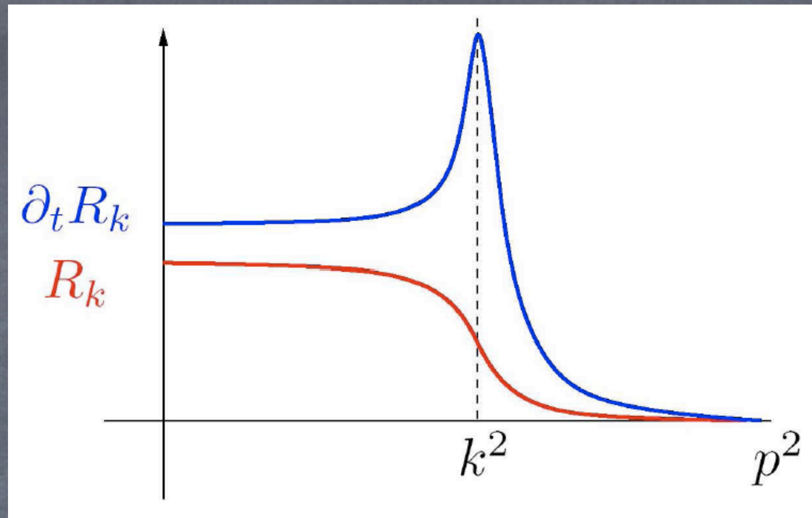
Hatta & Ikeda



Functional RG (bosons)

- Suppress long-wavelength fluctuations using regulator

$$\mathbf{D_0^{-1}(p) = p^2 - m_0^2 - R_k(p^2)}$$

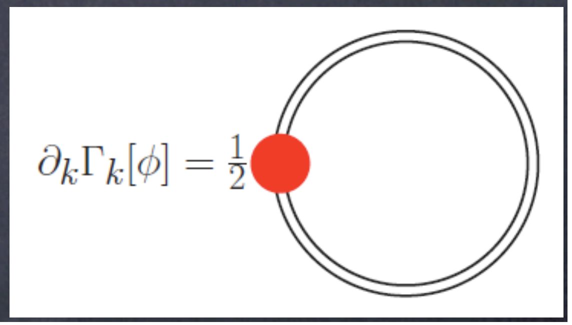


- R_k one-body operator:
  - Compact flow equation for effective action

$$\mathbf{\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr}\left[D \partial_t R_k\right]}$$

$$\mathbf{t = \log\left(k/\Lambda\right)}$$

  - Exact
  - Generates only 1-loop diagrams



FRG for QM-model

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left[(\Gamma^{(2)}_{B,k} + R_{B,k})^{-1}\partial_t R_{B,k}\right.$$
$$\left. - \mathrm{Tr}\left[(\Gamma^{(2)}_{F,k} + R_{F,k})^{-1}\partial_t R_{F,k}\right]\right.$$

$$\Gamma^{(2)}_{B} = \frac{\delta\Gamma}{\delta\phi\delta\phi}$$

$$\phi = (\sigma, \vec{\pi})$$

- LO derivative expansion (η small)
- Optimized regulators
- Thermodynamic potential:

$$\Omega_k = T\,\Gamma_k$$

$$\partial_k \Omega_k(T,\mu;\rho_{0,k}) \sim \left[3\frac{1+2n_\pi}{\omega_\pi} + \frac{1+2n_\sigma}{\omega_\sigma} - 4N_f N_c \frac{1-n_q-n_{\bar{q}}}{E_q}\right]$$

$$\rho_{0,k} = \frac{1}{2}\phi^2 = \frac{1}{2}(\sigma^2 + \vec{\pi}^2)$$

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Vacuum loop → 1

$$\rho_{0,k} = \frac{1}{2}\phi^2 = \frac{1}{2}(\sigma^2 + \vec{\pi}^{\,2})$$

Solving the flow equation

- Two independent methods employed
  - Grid method → B-J Schaefer's talk
  - Taylor expansion around minimum:

$$\Omega_k(T,\mu;\rho) = \sum_{m=0}^{N} \frac{a_{m,k}}{m!} (\rho - \rho_{0,k})^m$$

  - Flow eqns. for coefficients (truncated at N=3):

$$d_k\, a_{0,k} = \frac{h}{\sqrt{2\rho_{0,k}}}\, d_k\, \rho_{0,k} + \partial_k\, \Omega_k$$

Follows minimum →

$$d_k \rho_{0,k} = -\frac{\partial_k \Omega_k'}{h/(2\rho_{0,k})^{3/2} + a_{2,k}}$$

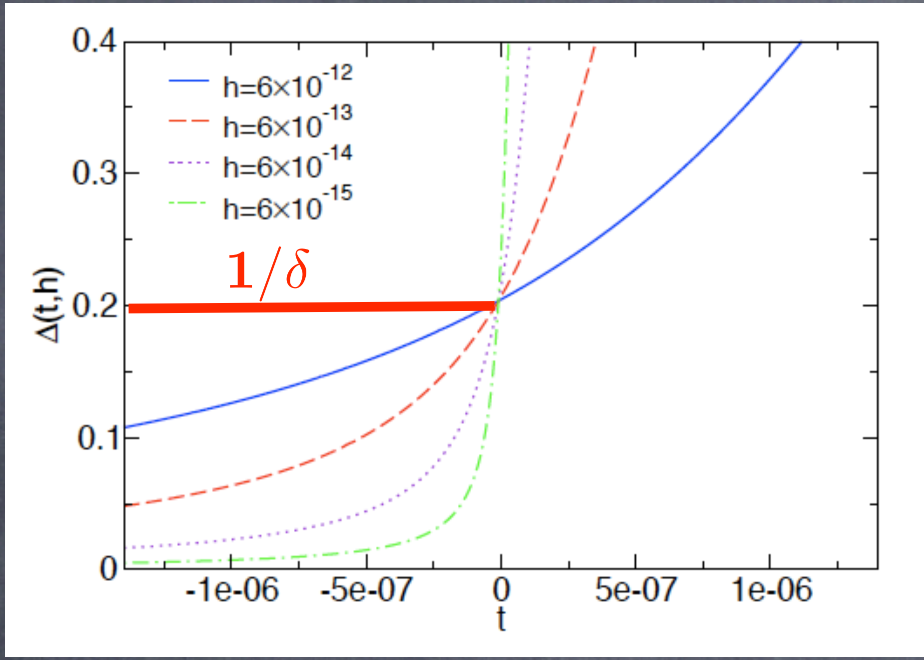
$$(a_{1,k} = h/\sigma_{0,k})$$

$$d_k a_{2,k} = a_{3,k}\, d_k\, \rho_{0,k} + \partial_k\, \Omega_k''$$

$$d_k a_{3,k} = \partial_k\, \Omega_k'''$$

FRG at work: O(4) critical point

- Efficiently accounts for fluctuations



B. Stokic, B.F., K. Redlich

$$\mathbf{\Delta(t,h)} = \chi_\sigma/\chi_\pi$$

$$\lim_{\mathbf{h}\to\mathbf{0}} \mathbf{\Delta(t=0,h)} = \mathbf{1}/\delta$$

$$\delta = \mathbf{4.818(4.851)}$$

$$\beta = \mathbf{0.406(0.3836)}$$

$$\gamma = \mathbf{1.575(1.477)}$$

- LO derivative expansion: $\eta\,(= \mathbf{0.0254})$ **neglected**

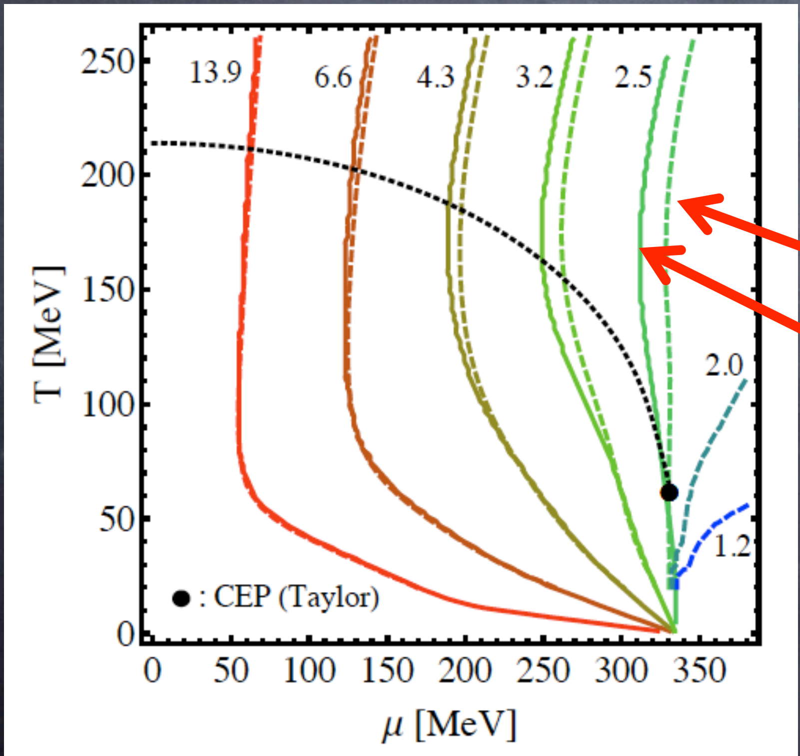
Isentropic trajectories

- Lines of constant s/n

$$\mathbf{s} = \frac{\partial \mathbf{p}}{\partial \mathbf{T}} = -\left.\frac{\partial\left[\mathbf{a_{0,k}} - \mathbf{h}\sqrt{2\rho_{0,k}}\right]}{\partial \mathbf{T}}\right|_{\mathbf{k=0}}$$

$$\mathbf{n} = \frac{\partial \mathbf{p}}{\partial \mu} = -\left.\frac{\partial\left[\mathbf{a_{0,k}} - \mathbf{h}\sqrt{2\rho_{0,k}}\right]}{\partial \mu}\right|_{\mathbf{k=0}}$$

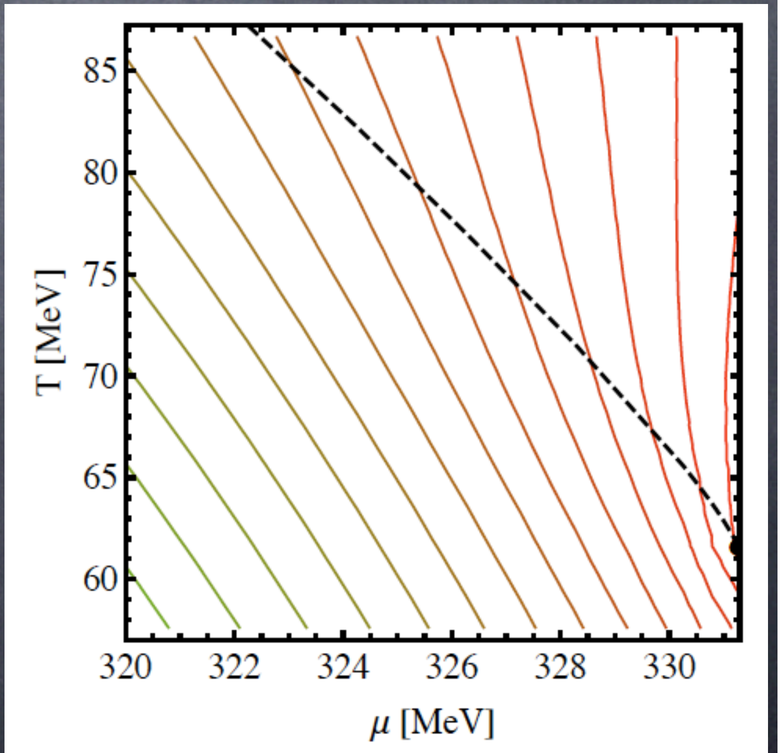
E. Nakano, B.-J. Schaefer, B. Stokic, B.F., K. Redlich



Two indep. calculations

Grid

Taylor



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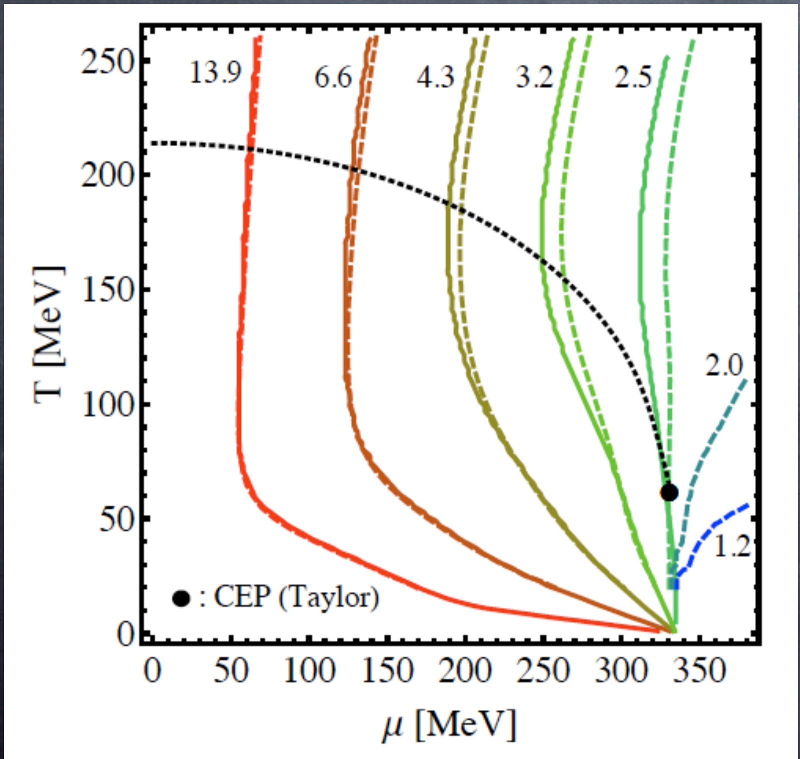
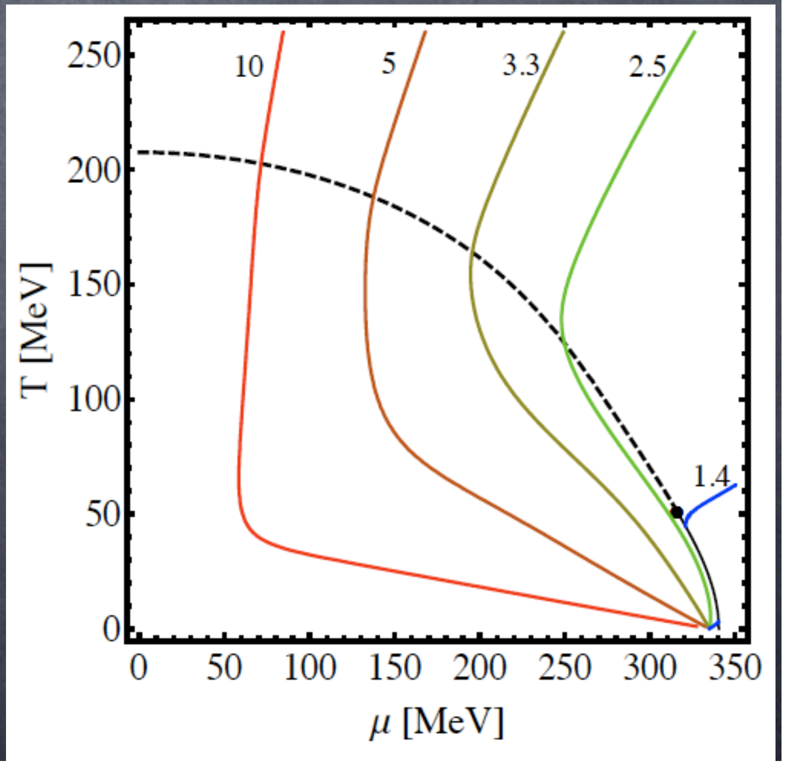


No focusing
No kinks
MF ≈ RG

Mean-field with vacuum term



Summary RG

- Focusing not universal
  - Expected, since s/n finite at CEP
- Fluctuations weak in s/n
- Results model dependent:
  - Do not exclude focusing in QCD
- Probing fluctuations at the chiral transition remain challenging