Universal Aspects of Many-Flavor QCD

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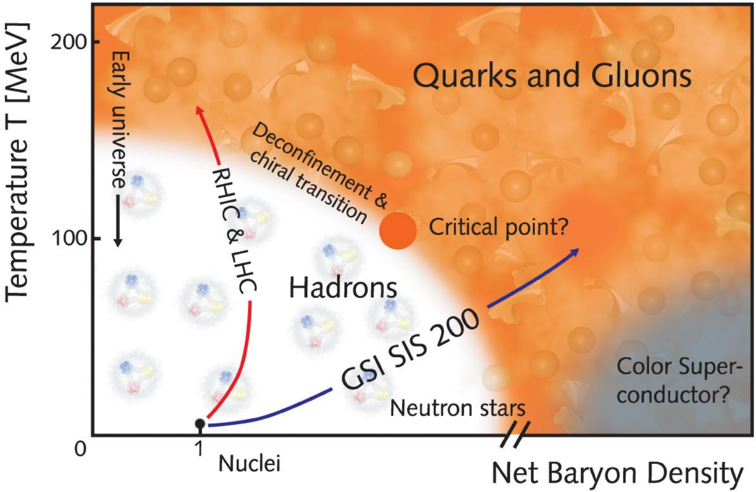
& J. Jaeckel: Eur.Phys.J.C46:433,2006 [hep-ph/0507171]

& J. Braun: Phys.Lett.B645:53,2007 [hep-ph/0512085], JHEP 0606:024,2006 [hep-ph/0602226], arXiv:0909.XXXX

QCD Phase Diagram



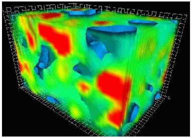
[FAIR@WWW.GSI.DE]

“Learning by Doing”

“Learning by Deforming”

QCD

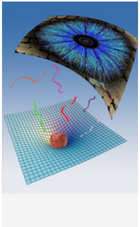
Lattice QCD



[LEINWEBER@ WWW.PHYSICS.ADELAIDE.EDU.AU]

Large N_c

AdS/QCD



[KELLEY@ WWW.NONEQUILIBRIUM.NET]

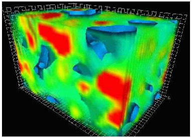
“Learning by Deforming”

QCD

Large N_f

AdS/QCD

Lattice QCD



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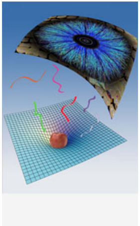

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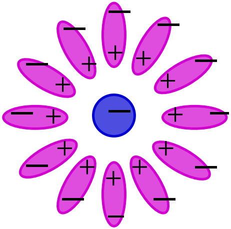
“many-flavor QCD”

for $N_f \longrightarrow \infty$, see (BLAIZOT ET AL.’06)

Many-Flavor QCD

Many-flavor QCD

▷ charge screening:



▷ β function

$$\beta = -2\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)\frac{g^4}{16\pi^2} - 2\left(\frac{34{N_c}^3 + 3N_f - 13{N_c}^2 N_f}{3N_c}\right)\frac{g^6}{(16\pi^2)^2} + \ldots$$

Many-flavor QCD

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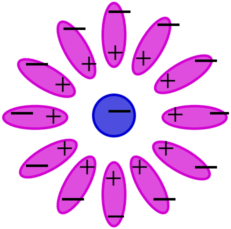

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$$\beta = -2\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)\frac{g^4}{16\pi^2} \underbrace{-2\left(\frac{34N_c{}^3 + 3N_f - 13N_c{}^2N_f}{3N_c}\right)}_{>0}\frac{g^6}{(16\pi^2)^2} + \ldots$$

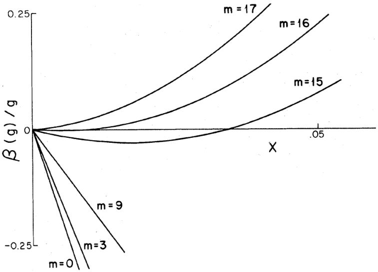
$$\text{for } N_f > \frac{34N_c{}^3}{13N_c{}^2 - 3} \overset{\mathrm{SU(3)}}{\simeq} 8.05$$

Many-flavor QCD

▷ β function

$$\beta = -2\left(\frac{11}{3}N_c - \frac{2}{3}N_f\right)\frac{g^4}{16\pi^2} - 2\left(\frac{34{N_c}^3 + 3N_f - 13{N_c}^2 N_f}{3N_c}\right)\frac{g^6}{(16\pi^2)^2} + \ldots$$

▷ e.g., SU(3): IR fixed point α_* (CASWELL'74; BANKS&ZAKS'82)



[CASWELL@PHYS.REV.LETT.33:244,1974]

Many-flavor QCD

▷ β function

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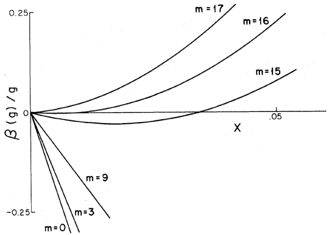
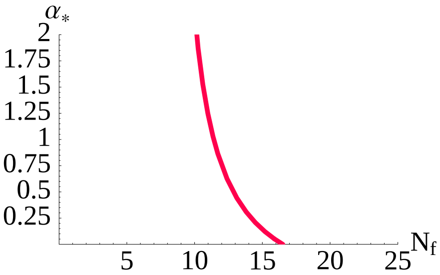
▷ N_f dependence of α_*





Many-flavor QCD

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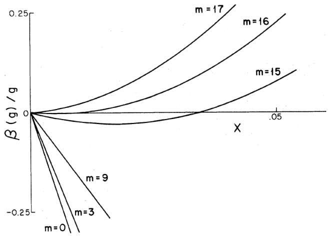
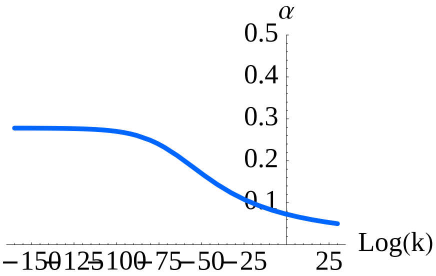
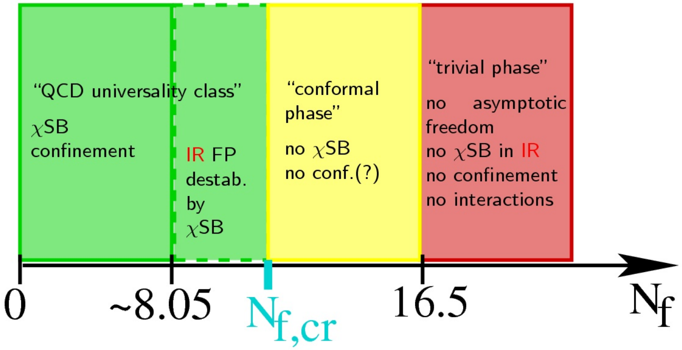
▷ e.g. $N_f = 14$ $\Longrightarrow$ IR fixed point





Many-flavor QCD

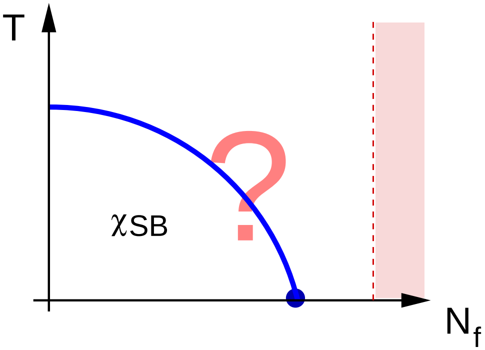


▷ Caswell-Banks-Zaks fixed point destabilized by χSB:

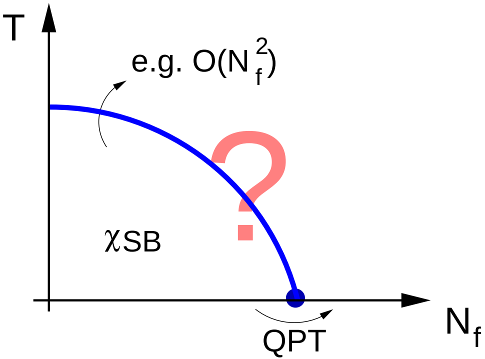
$$\text{for} \quad g^2 > g^2_{\text{cr}}: \quad \text{fermions aqcuire mass} \quad \rightarrow \quad N_{\text{f, eff}} \rightarrow 0$$

▷ similar quantum phase transitions, e.g., in 3d QED (PISARSKI'84)

Many-flavor Phase Diagram



Many-flavor Phase Diagram



Shape of the Phase Boundary: Small N_f

▷ unified scale dependence of observables in QCD (in the χ limit):

$$T_\mathrm{c},\ f_\pi,\ \langle\bar\psi\psi\rangle^{1/3},\ m_{c.q.},\dots\sim\Lambda_\mathrm{QCD}$$

▷ perturbatively: $\Lambda_\mathrm{QCD}\sim$ position of the Landau pole

$$0\leftarrow\frac{1}{\alpha(\Lambda_\mathrm{QCD})}=\frac{1}{\alpha(\mu_0)}+4\pi b_0\ln\frac{\Lambda_\mathrm{QCD}}{\mu_0},\quad b_0=\frac{1}{8\pi}\left(\frac{11}{3}N_\mathrm{c}-\frac{2}{3}N_\mathrm{f}\right)$$

pert. RG scale: $\mu_0=m_Z,\ m_\tau,\ \dots$

CAVEAT: comparison of theories!

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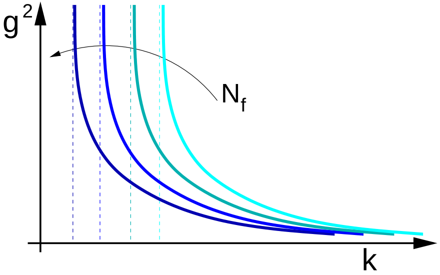

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$$T_\mathrm{c},\ f_\pi,\ \langle\bar{\psi}\psi\rangle^{1/3},\ m_{c.q.},\cdots \sim \Lambda_\mathrm{QCD}$$

▷ T_c scaling at small N_f: (BRAUN, HG'05,'06)

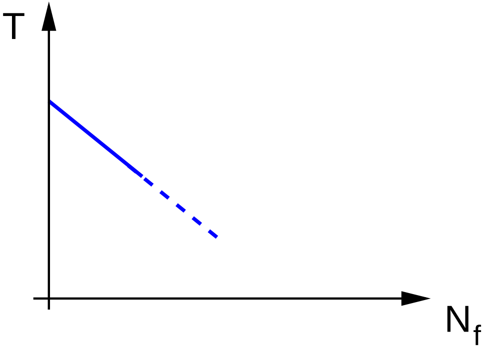
$$\begin{aligned} T_\mathrm{c} &\sim \Lambda_\mathrm{QCD} \simeq \mu_0\, \mathrm{e}^{-\frac{1}{4\pi b_0 \alpha(\mu_0)}} \\ &\simeq \mu_0\, \mathrm{e}^{-\frac{6\pi}{11 N_\mathrm{c} \alpha(\mu_0)}} \left(1 - \epsilon N_\mathrm{f} + \mathcal{O}((\epsilon N_\mathrm{f})^2)\right) \end{aligned}$$

where $\epsilon = \frac{12\pi}{121 {N_\mathrm{c}}^2 \alpha(\mu_0)} \simeq 0.107$ for $N_\mathrm{c} = 3$ and $\mu_0 = m_\tau$

$\Longrightarrow$ linear small-N_f behavior

Shape of the Phase Boundary: Small N_f

(Braun, HG'05,'06)



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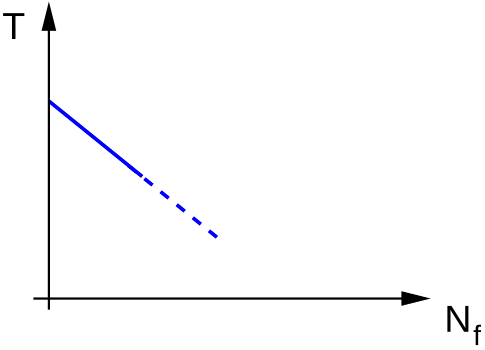
Shape of the Phase Boundary: Small N_f

(Braun, HG'05,'06)

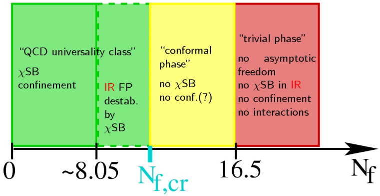


▷ application: N_f-scaling of PNJL / PQM model parameters

$\Longrightarrow$ significant improvement of thermodynamics predictions

(Schaefer, Pawlowski, Wambach '07)

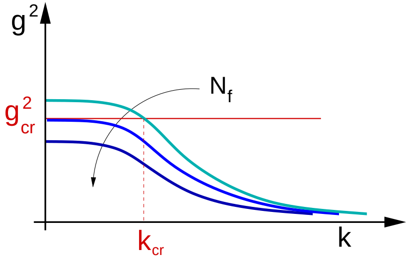
Shape of the Phase Boundary: Large N_f



▷ lower end of conformal window

=

onset of χSB



▷ assumption:

onset of χSB requires

$$g^2 > g^2_{\rm cr}$$

$\Longrightarrow$ fixed-point regime is relevant

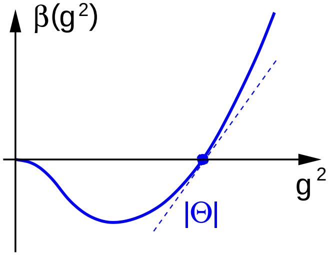
Shape of the Phase Boundary: Large N_f

▷ RG flow in the fixed-point regime:

governed by universal

critical exponent Θ

$$\beta(g^2) \simeq -\Theta\,(g^2 - g_*^2) + \dots$$



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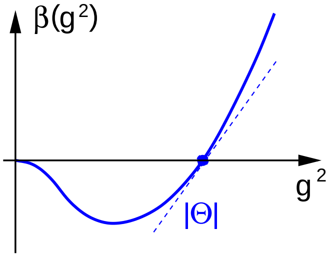
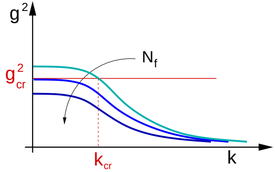
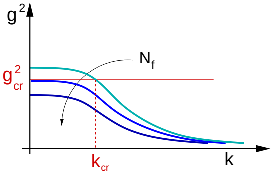


▷ solution in the fixed-point regime:

$$g^2(k) = g_*^2 - \left(\frac{k}{\mu_0}\right)^{|\Theta|}$$



Shape of the Phase Boundary: Large N_f



▷ χSB dynamics sets in at:

$$k_{cr} \simeq \mu_0 \left(g_*^2 - g_{cr}^2\right)^{\frac{1}{|\Theta|}}$$

▷ unified scale dependence of χSB observables in QCD:

$$T_c,\ f_\pi,\ \langle\bar{\psi}\psi\rangle^{1/3},\ m_{c.q.},\cdots \sim k_{cr}$$

▷ approx. proportionality $g_*^2 \sim N_f$

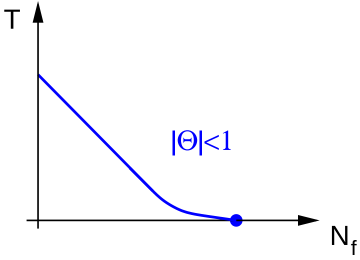
$\Longrightarrow$ scaling relation: (BRAUN, HG'06)

$$T_c \sim \mu_0 \left|N_f - N_{f,cr}\right|^{\frac{1}{|\Theta|}}$$

Scaling Relation near the Conformal Window

(BRAUN, HG'06)

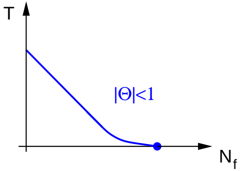
$$T_c \sim |N_f - N_{f,cr}|^{\frac{1}{|\Theta|}}$$

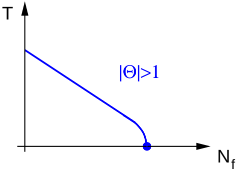


Scaling Relation near the Conformal Window

(BRAUN, HG'06)

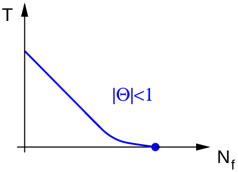
$$T_c \sim \left|N_f - N_{f,cr}\right|^{\frac{1}{|\Theta|}}$$

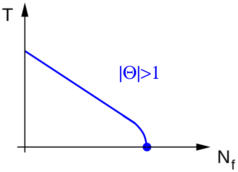




- relates two universal quantities
- relates chiral structure to IR gauge dynamics
- parameter-free prediction

Scaling Relation near the Conformal Window



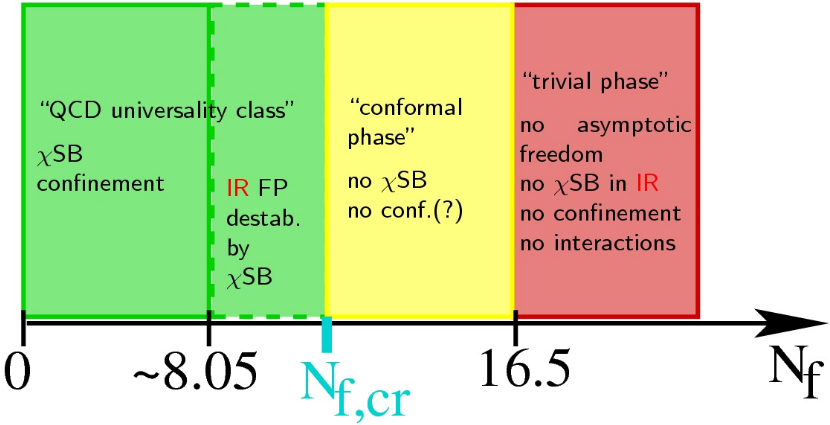


▷ generalizes to generic chiral observables $\mathcal{O}$
with mass dimension $d_{\mathcal{O}}$: (Braun, HG in prep.)

$$\mathcal{O} \sim |N_f - N_{f,cr}|^{\frac{d_{\mathcal{O}}}{|\Theta|}}$$

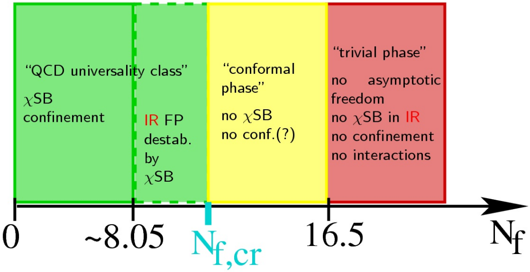
e.g., $\mathcal{O} = T_c, f_\pi, \langle\bar{\psi}\psi\rangle, m_{c.q.}, \ldots$

Many-Flavor QCD, Quantitatively . . . ?



$$N_{f,cr} = ?, \quad \Theta(N_{f,cr}) = ?$$

Many-flavor QCD, Quantitatively . . . ?



$$N_{\mathrm{f,cr}}(@2003) \quad = \quad \begin{cases} 5 & \text{(Harada\&Yamawaki'00)} \\ 6 & \text{(Iwasaki et al.'03)} \\ \gtrsim 6 & \text{(Velkovsky\&Shuryak'97, Appelquist\&Selipsky'97)} \\ \gtrsim 10 & \text{(Sannino\&Schechter'99)} \\ \simeq 12 & \text{(Miransky\&Yamawaki'96, Appelquist et al.'96)} \end{cases}$$

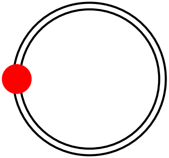
LGT: (Kogut&Sinclair'88; Brown et al.'92; Iwasaki et al.'96; Damgaard et al.'97)

Many-Flavor QCD with Functional RG

▷ RG flow equation

(Wilson'71; Wegner&Houghton'73; Polchinski'84; Wetterich'93)

$$\partial_t \Gamma_k \equiv k\partial_k \Gamma_k = \frac{1}{2}\,\mathrm{Tr}\,\frac{1}{\Gamma_k^{(2)} + R_k}\,\partial_t R_k =$$



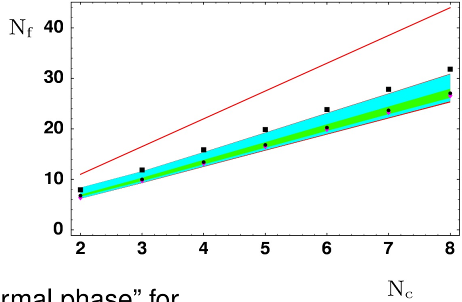
[cf. talks by B.J. Schaefer, J. Braun, J.M. Pawlowski]

▷ computation of effective action in a gauge-covariant derivative expansion: $\mathrm{SU}(N_c)$, $\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$

$$\Gamma_k = \int \frac{Z_F}{4} F^z_{\mu\nu} F^z_{\mu\nu} + \ldots + \bar\psi\,(\mathrm{i} Z_\psi \partial\!\!\!/ + Z_1 \bar g A\!\!\!/)\,\psi$$
$$+\frac{1}{2}\frac{\lambda_\sigma}{k^2}\,(\text{S-P}) + \frac{1}{2}\frac{\lambda_{VA}}{k^2}\left[2(\text{V-A})^{\text{adj.}} + (1/N_c)(\text{V-A})\right]$$
$$+\frac{1}{2}\frac{\lambda_+}{k^2}\,(\text{V+A}) + \frac{1}{2}\frac{\lambda_-}{k^2}\,(\text{V-A})$$

(HG, Jaeckel, Wetterich'04)

Critical flavor number



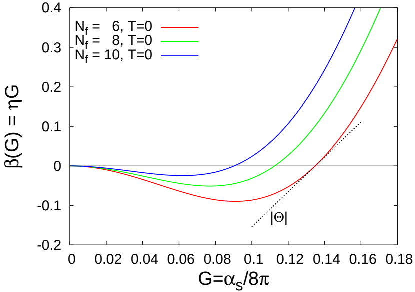
▷ SU(3) "conformal phase" for (HG,JAECKEL'05)

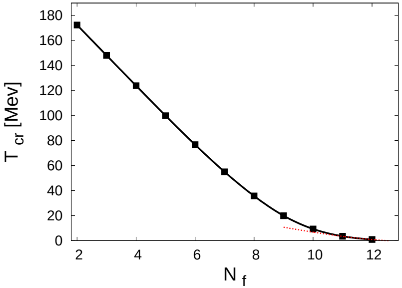
$$N_{f,cr} = 10.0 \pm 0.29(\text{fermion})\,^{+1.55}_{-0.63}(\text{gluon}) \lesssim N_f < 16.5$$

▷ error analysis includes:

- check for truncation dependencies
  (higher-derivative operators, higher-vertex operators, momentum dependencies, anomalous dimensions, ...)
- check for regulator and scheme dependencies

Chiral Phase Boundary $T - N_f$





▷ fixed-point regime: critical exponent Θ

$$\beta_{g^2} \simeq -\Theta\,(g^2 - g_*^2)$$

▷ shape of the phase boundary for $N_f \simeq N_f^{cr}$: (Braun,HG'05,'06)

$$T_c \sim |N_f - N_{f,cr}|^{\frac{1}{|\Theta|}}, \quad \Theta \simeq -0.60$$

Recent Results from the Lattice

- ▷ $N_f = 8$, $N_c = 3$ QCD is in the χSB phase:

(Deuzemann,Lombardo,Pallante'08; Appelquist,Fleming,Neil'08&'09)

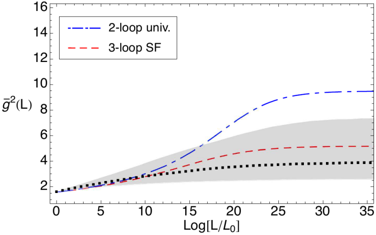
(Jin, Mawhinney'09; Fodor,Holland,Kuti,Nogradi,Schroeder'09)

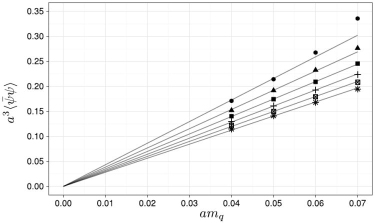
- ▷ $N_f = 9$(rooted staggered), $N_c = 3$ QCD is in the χSB phase:

(Fodor,Holland,Kuti,Nogradi,Schroeder'09)

- ▷ $N_f = 12$, $N_c = 3$ QCD is in the conformal phase:

(Appelquist,Fleming,Neil'08&'09; Deuzemann,Lombardo,Pallante'09)





$$\Longrightarrow \quad 9 < N_{f,cr} \leq 12$$

Conclusions

▷ “conformal window” in many-flavor QCD:

$$N_{\mathrm{f,cr}} \simeq 10 - 12 \leq N_{\mathrm{f}} < 16.5 \text{ for SU(3)}$$

...lessons on chiral structure

...quantum phase transition

...applications to walking technicolor

(Dietrich et al.'06; Terao et al.'07; Sannino'09)

▷ universal aspects:

shape of the phase boundary $\Longleftrightarrow$ IR critical exponent

▷ functional RG for $\Gamma[\phi]$

- systematic and consistent expansion schemes for QCD
- chiral symmetry ✓
- calculations “from first principles”

Chiral Criticality at Finite Temperature

▷ quark modes:

$$m_T^2 = m_f^2 + (2\pi T(n+\frac{1}{2}))^2$$

$\Longrightarrow$ T-dependent

critical coupling:

$$\alpha_{cr}(T) \gtrsim \alpha_{cr} \simeq 0.85$$

(BRAUN,HG'05)