QCD Phase Diagram and Thermodynamics from the PNJL Model

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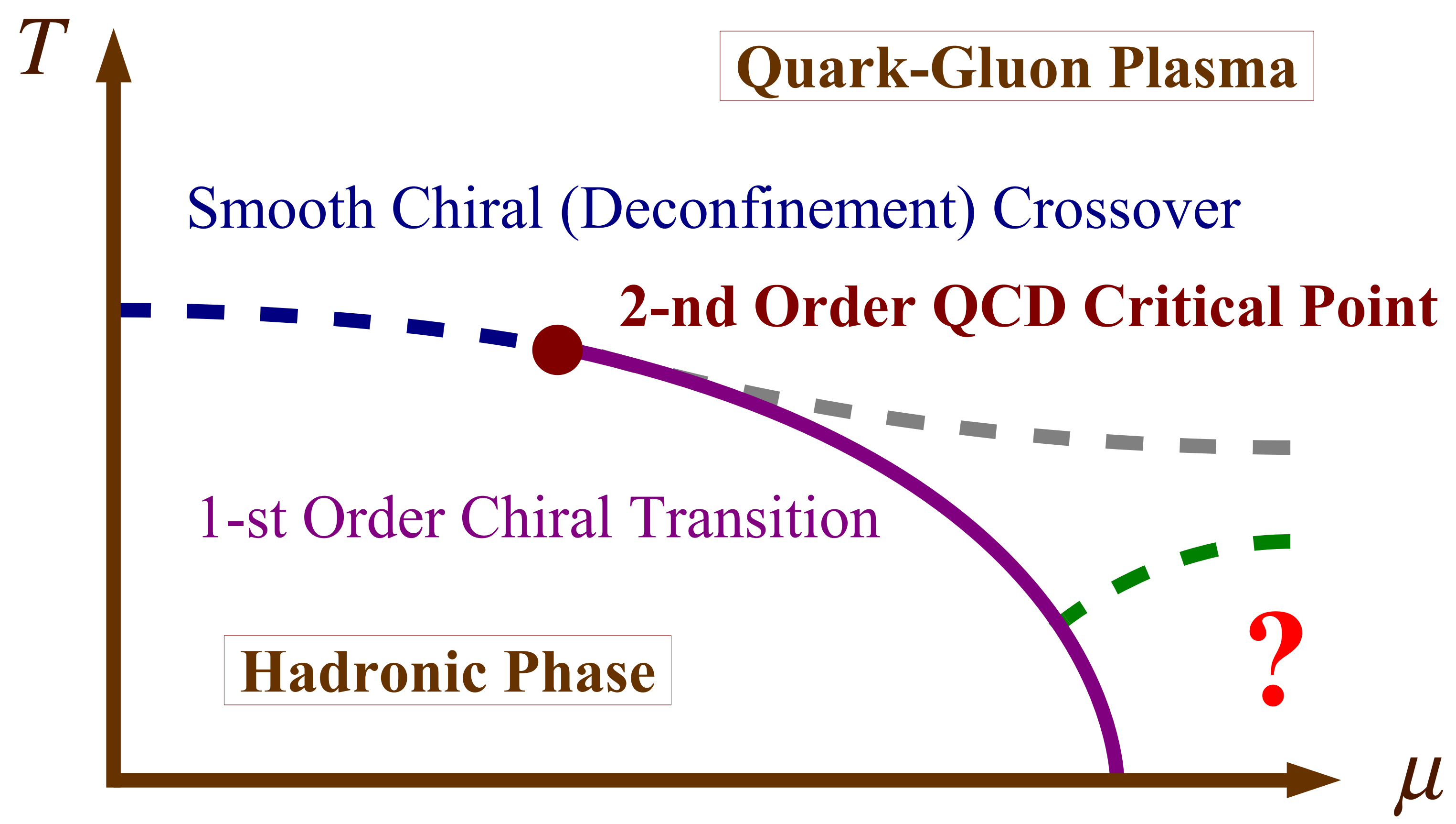
Aug. 2009 in St. Goar

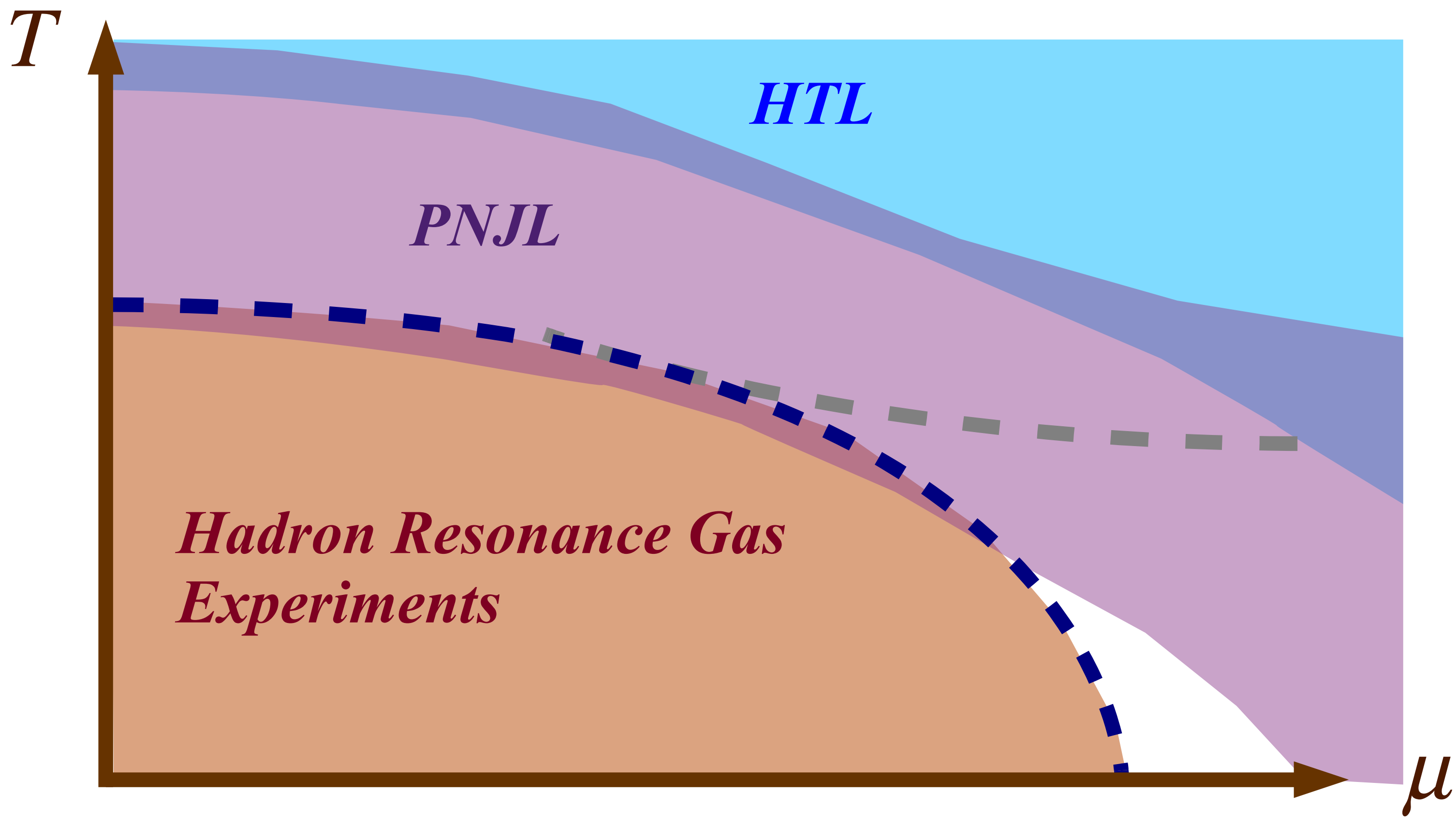
Talk Plan

- **QCD Phase Diagram by Order Parameters**
  - Color deconfinement transition
  - Chiral symmetry restoration transition
- **QCD Phase Diagram by Thermodynamics**
  - Thermal excitations at high temperature
  - Baryon rich matter at high density
- **N_c=2 Phase Diagram**
  - Similar structure to the QCD phase diagram

Conventional QCD Phase Diagram



Validity Region of PNJL Model



Degrees of Freedom Above T_c

Quarks and Gluons

- 2 (spin) x 2 (antiparticle) x 2+1 (flavors) x 3 (colors) x (7/8) = **21 ~ 31.5** quarks
- 2 (polarization) x 8 (colors) = **16** gluons

These degrees of freedom are "smoothly" liberated as *T* is increasing. (No strict confinement at finite *T*.)

Control Variable of Thermal Excitations

- Polyakov loop

$$L = P \exp\left[ig \int dx_4 A_4\right]$$

$$\ell = \langle \mathrm{tr}\, L \rangle$$

Thermal Excitation of Quarks

- **Partition function without the Polyakov loop**

$$2N_f N_c \int \frac{d^3k}{(2\pi)^3} \left[\log\left(1+\mathrm{e}^{-(E-\mu)/T}\right) + \log\left(1+\mathrm{e}^{-(E+\mu)/T}\right) \right]$$

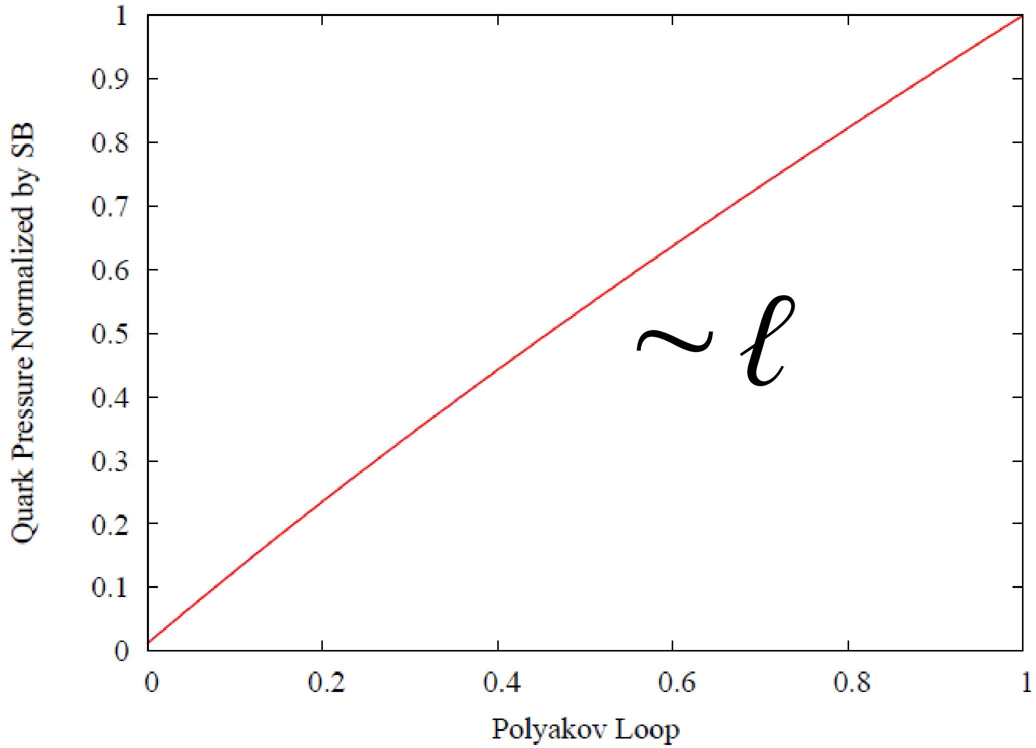
- **Coupling with the fundamental Polyakov loop**

$$2N_f \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}\left[\log\left(1+L\,\mathrm{e}^{-(E-\mu)/T}\right) + \log\left(1+L^\dagger \mathrm{e}^{-(E+\mu)/T}\right) \right]$$

Massless Quark Excitation

Controled by the fundamental Polyakov loop

Confinement
No quarks but color singlets remain



Deconfinement
SB limit of free quarks

Massive Quarks $\rightarrow$ Interplay between chiral and deconfinement

Thermal Excitation of Gluons

- **Partition function without the Polyakov loop**

$$-(N_c^2-1)\int \frac{d^3k}{(2\pi)^3}\log\left(1-\mathrm{e}^{-k/T}\right)$$

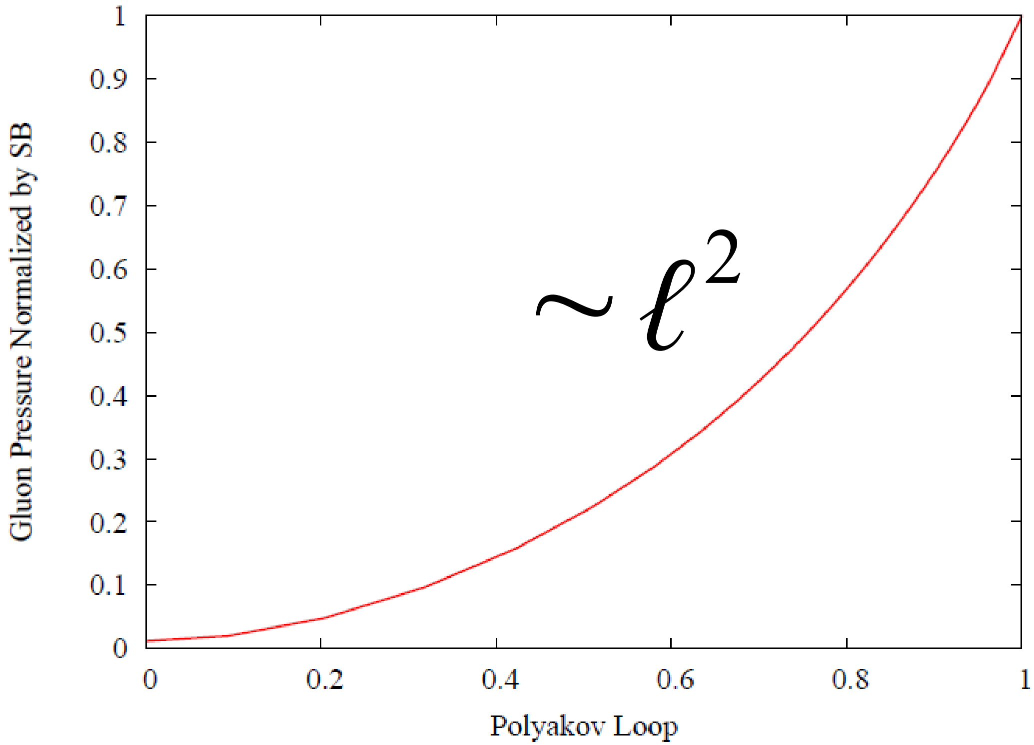
- **Coupling with the adjoint Polyakov loop**

$$-\int \frac{d^3k}{(2\pi)^3}\,\mathrm{tr}\log\left(1-L_A\,\mathrm{e}^{-k/T}\right)$$

Massless Gluon Excitation

Controled by the adjoint Polyakov loop

Confinement
Transverse gluons are effectively suppressed.



Deconfinement
SB limit of free transverse gluons

Fundamental Polyakov loop is *NOT* an order parameter for *GLUON* deconfinement in the color adjoint rep... but effectively behaves like an order parameter.

Tsai-Mueller (2008)

Thermodynamic Potential

In principle $P(T,\mu_B)$ is a function of ℓ with T and μ_B dependent coefficients.

This is, however, just a parametrization and has no predictive power yet.

Separate the quark sector from the parametrization.

- μ_B dependence comes from the quark sector only.
- Another order parameter $\langle \bar{\Psi}\psi \rangle$ is in the quark sector.
- Effects of the presence of quarks are the model *prediction*.

Fixing the Pure Gauge Sector

Ansatz

$$V(\ell) = -\frac{1}{2}a(T)\ell\bar{\ell} + b(T)\log\left[1-6\ell\bar{\ell}+4(\ell^3+\bar{\ell}^3)-3(\ell\bar{\ell})^2\right]$$

Vandermonde determinant

Parameters

$$a(T) = T^4(3.51-2.47t^{-1}+15.2t^{-2})$$
$$b(T) = -1.75t^{-3}\cdot T^4$$
$$t = T/T_c$$

3 parameters

Ratti-Thaler-Weise
Ratti-Roessner-Weise

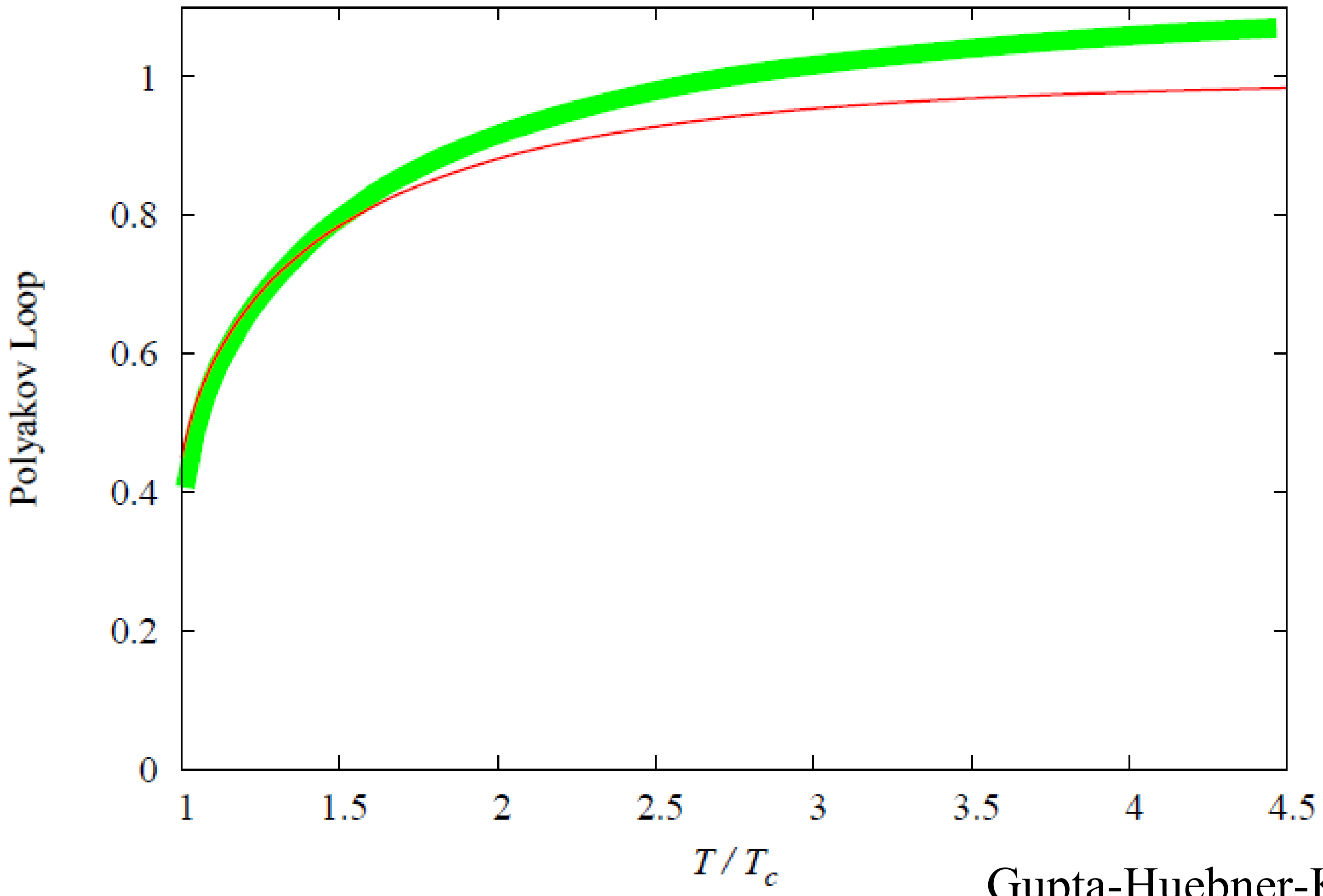
c.f. strong-coupling ansatz

$$a(T) = T\cdot b\cdot 54e^{-a/T}$$
$$b(T) = T\cdot b$$

2 parameters
Fukushima

Polyakov Loop

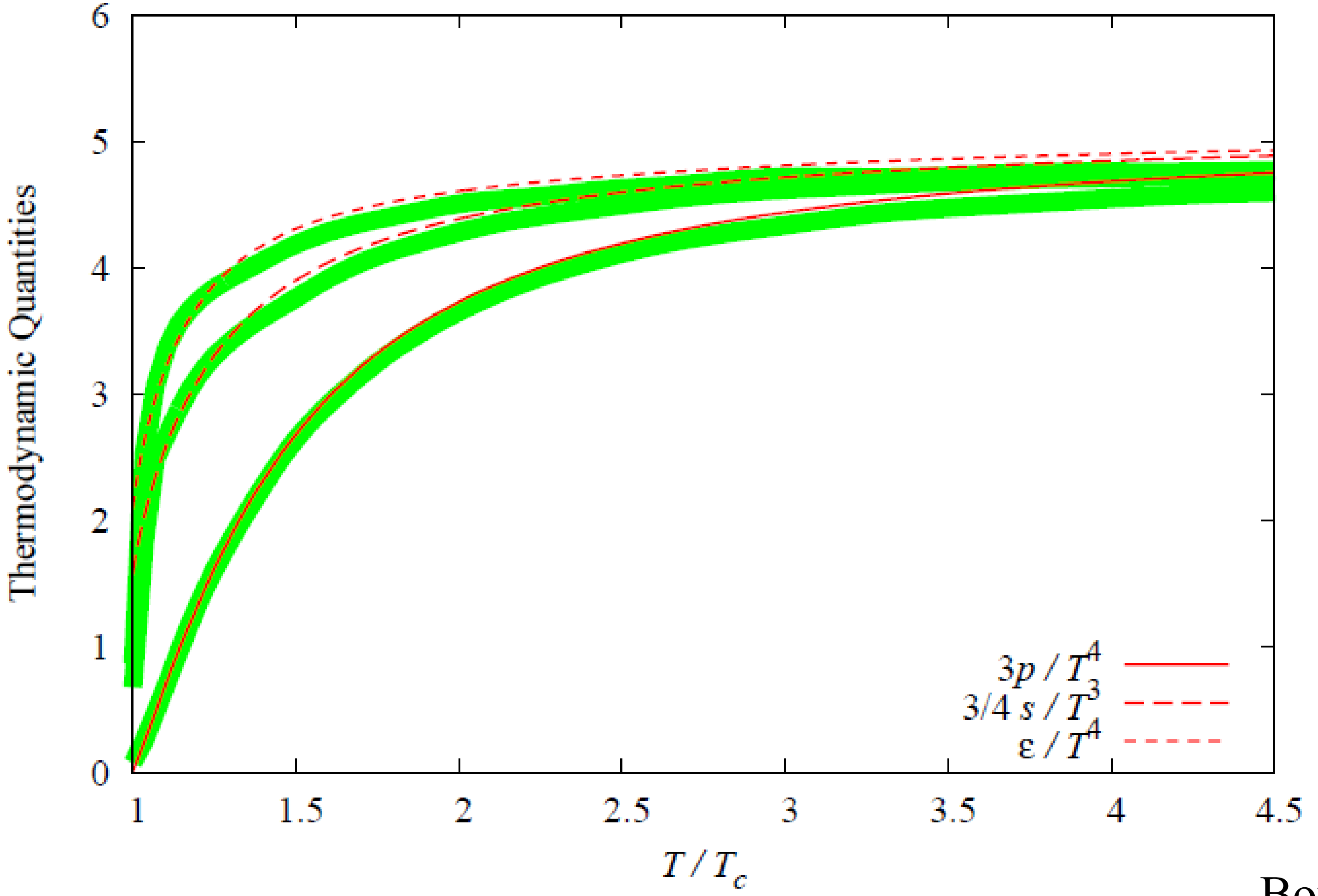
Fit by the "experimental" data



Gupta-Huebner-Kaczmarek (2008)

Thermodynamics

Fit by the "experimental" data



Boyd et al. (1996)

Fixing the Quark Sector

Nambu--Jona-Lasinio (NJL) Model

$$L = \bar{\psi}(i\gamma\cdot\partial - m_f) + \frac{g_S}{2}[(\bar{\psi}\lambda\psi)^2 + (\bar{\psi}i\gamma_5\lambda\psi)^2]$$
$$+ g_D[\det\bar{\psi}(1-\gamma_5)\psi + \text{h.c.}]$$

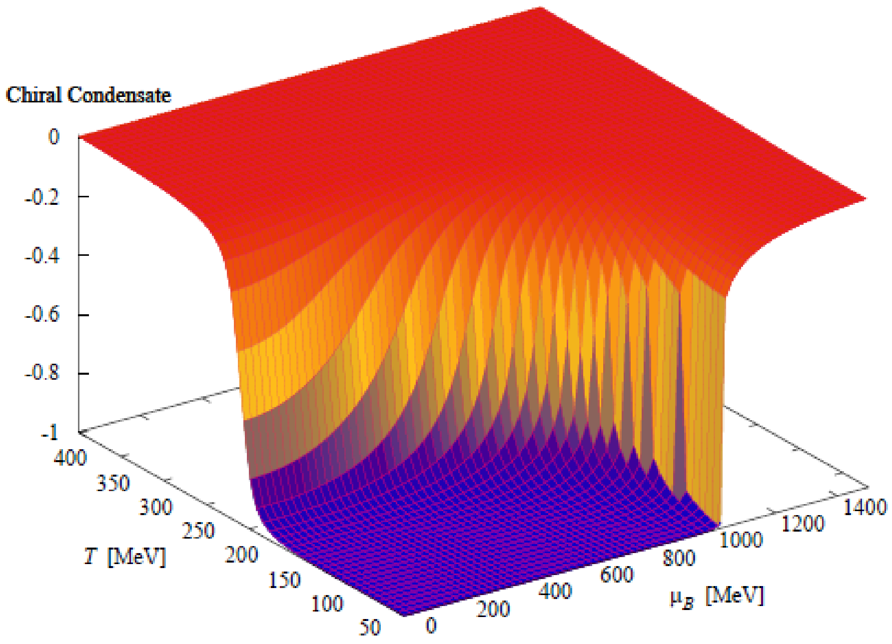
Parameters

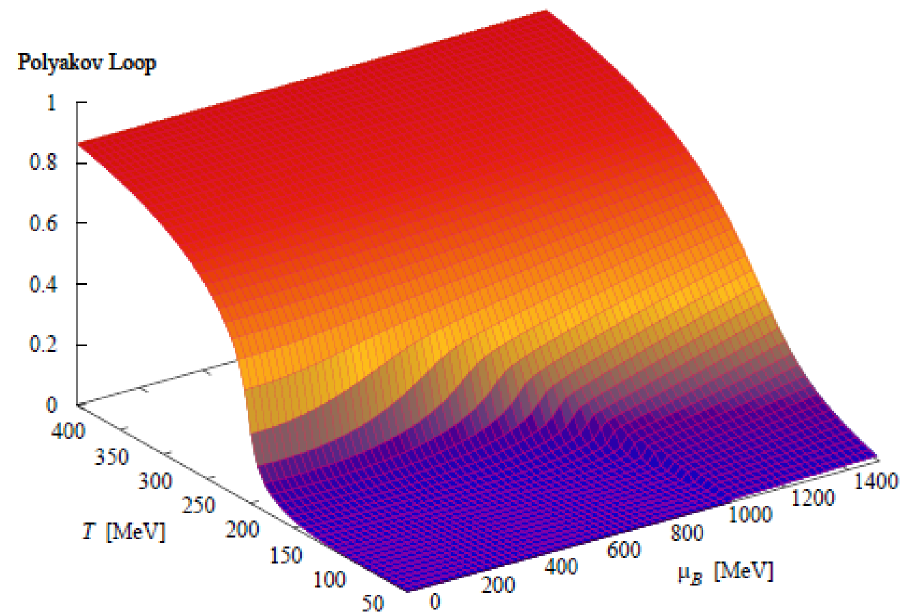
$\Lambda = 631.4\,\text{MeV}$		m_π
$m_{ud} = 5.5\,\text{MeV}$		f_π
$m_s = 135.7\,\text{MeV}$	⟵	m_K
$g_S\Lambda^2 = 3.67$		$m_{\eta'}$
$g_D\Lambda^5 = -9.29$		one more? M_{ud} $\langle\bar{\psi}\psi\rangle$

NJL + Coupling with ℓ + Potential of ℓ → PNJL

Order Parameters

3D Plots – Chiral Condensate and Polyakov Loop

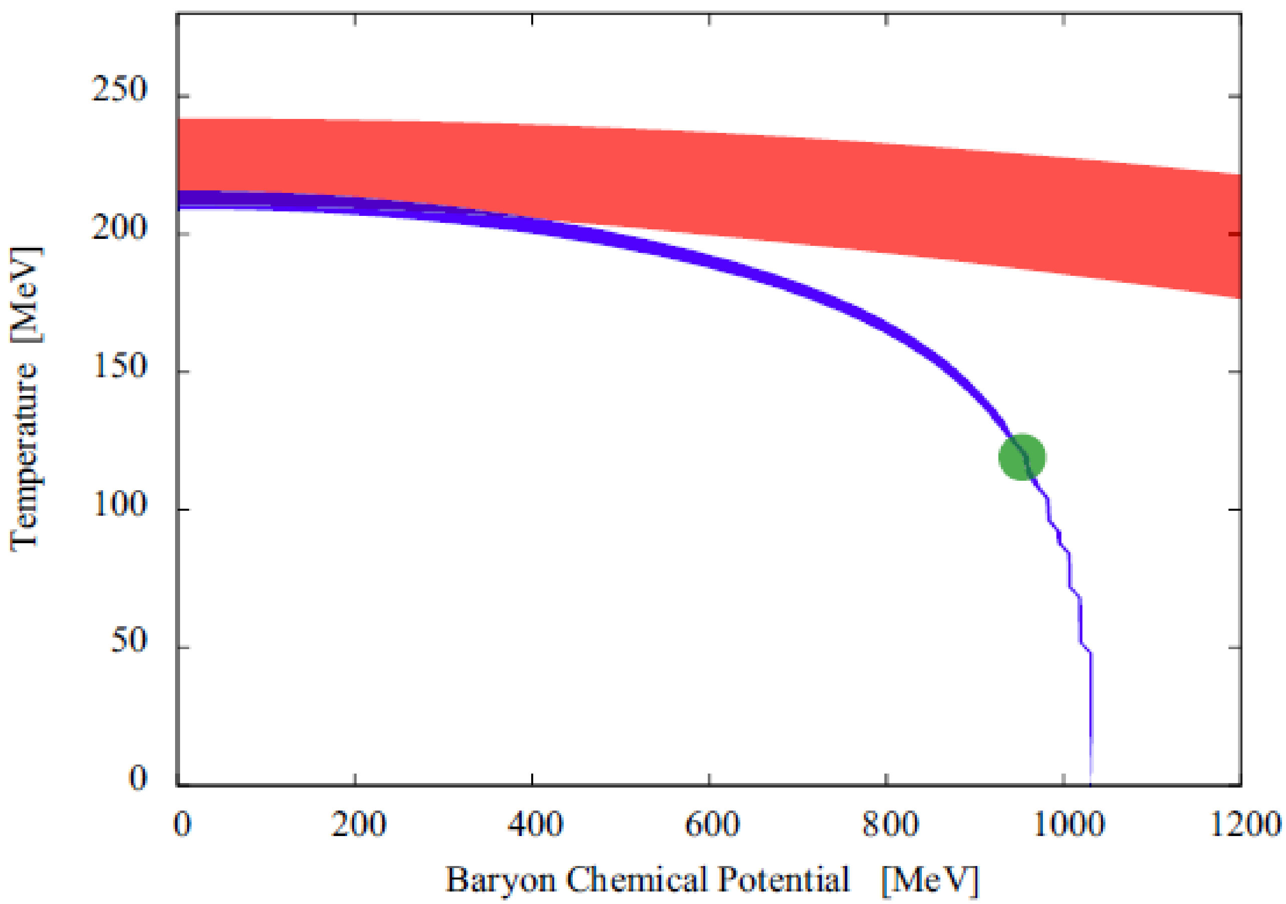




KF (2008)

Phase Diagram

Phase Boundaries by Order Parameters

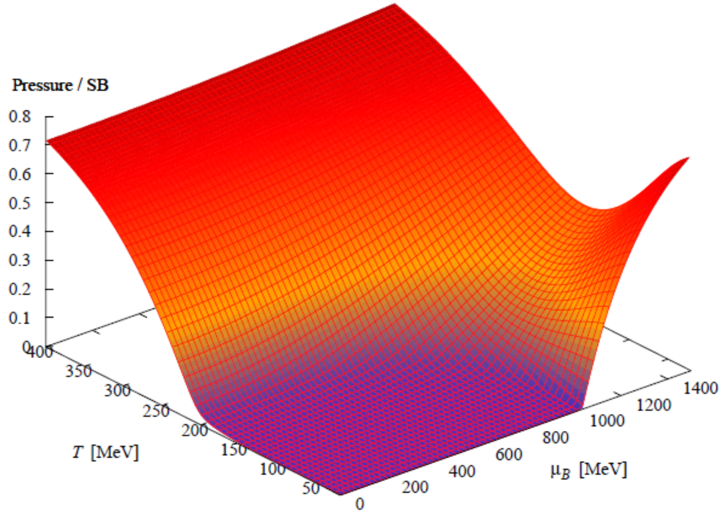


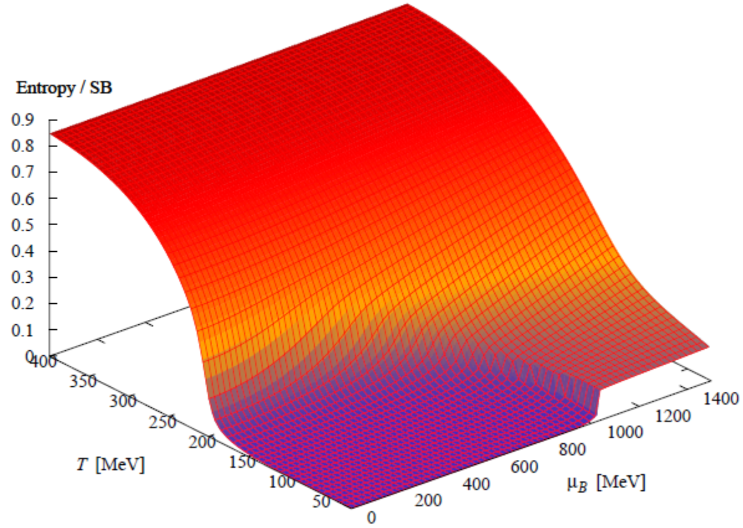
(Normalized) order parameters ranging between 0.4 ~ 0.6

c.f If susceptibility (or slope) is used, two lines meet up to some density...

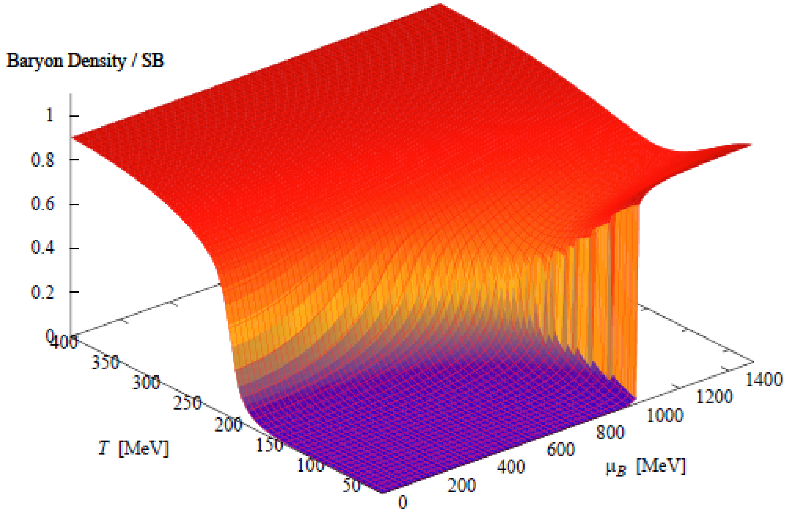
Thermodynamics

3D Plots – Pressure and Entropy (density) /SB



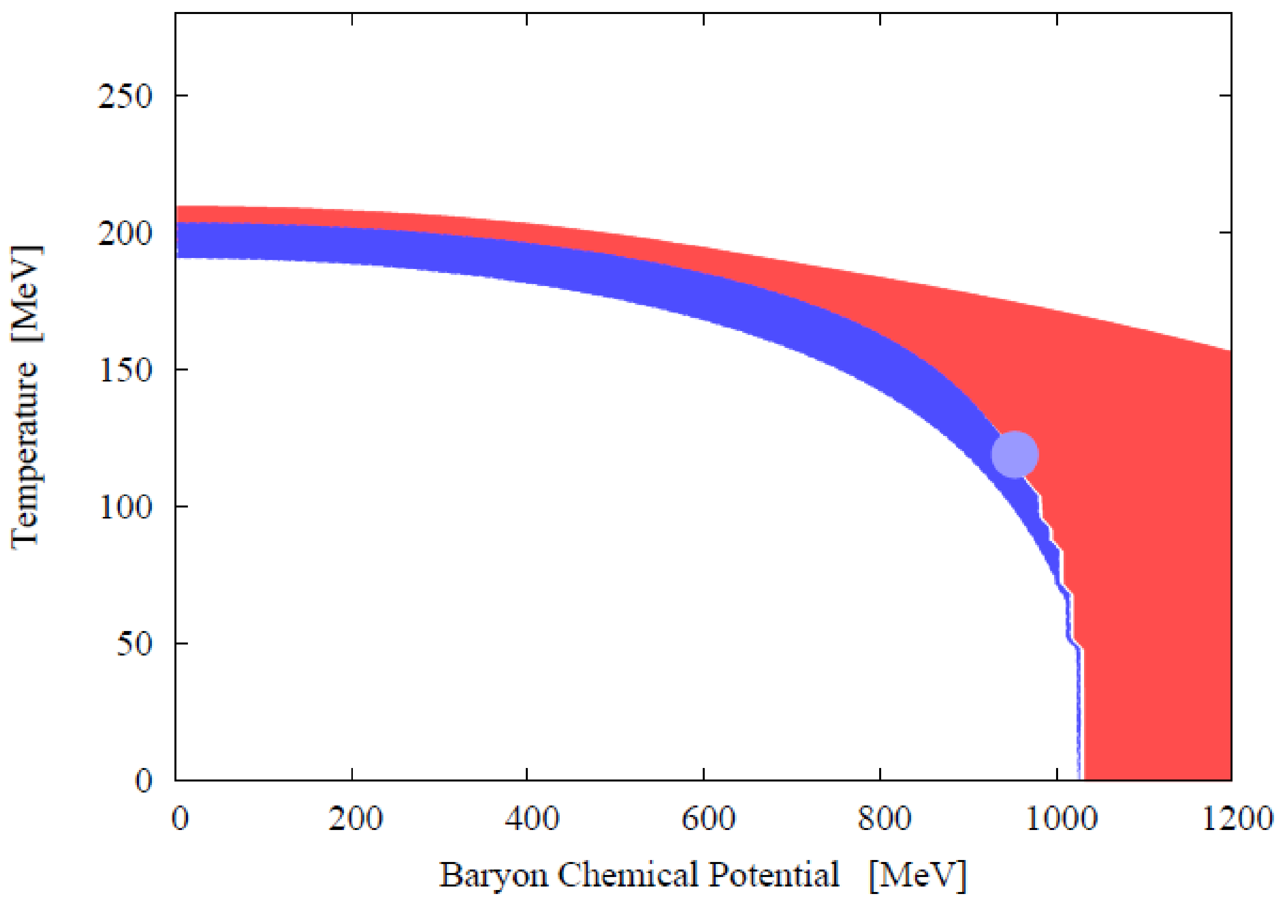


Quark Density /SB



Phase Diagram

Phase Boundaries by Thermodynamics



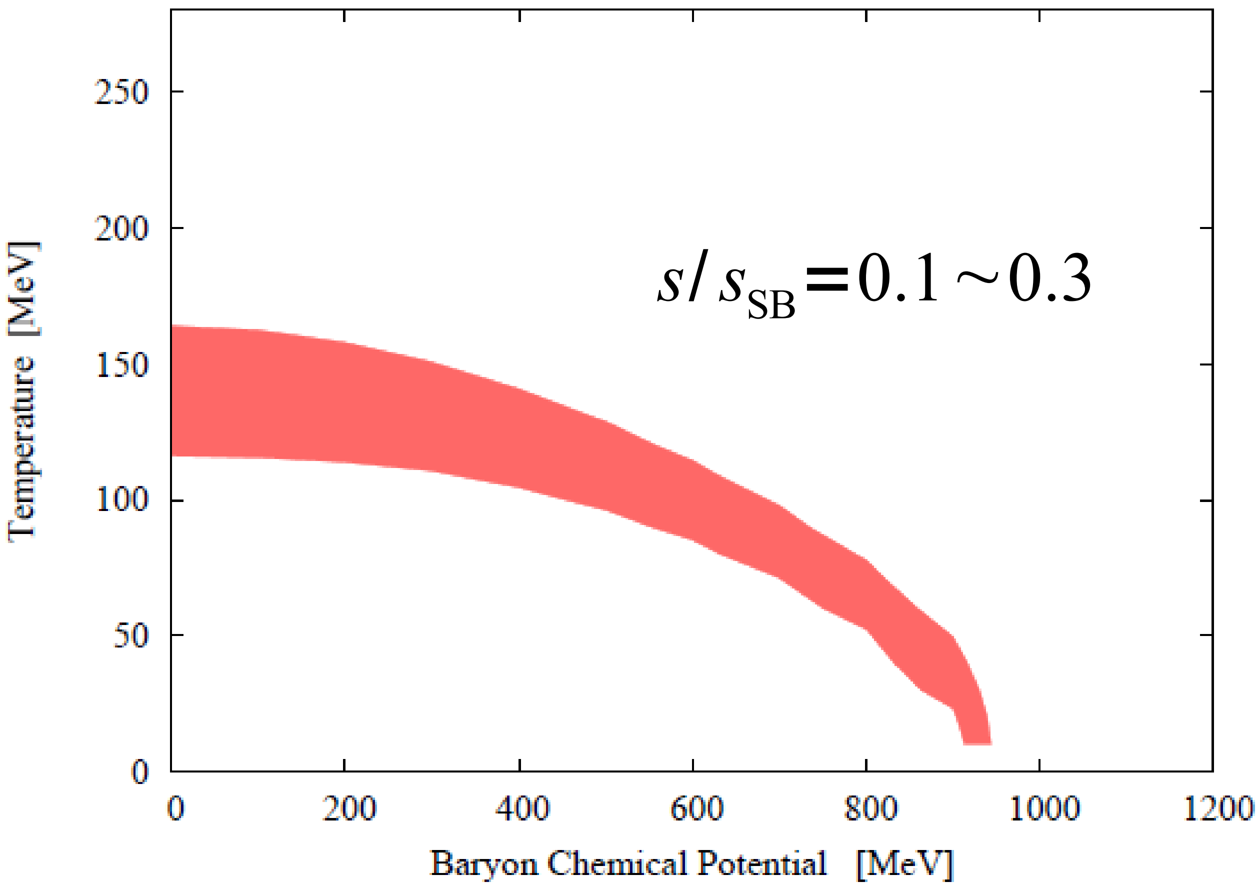
(Normalized) s and n_B ranging between 0.1 ~ 0.3 (Onset)

Similar to the one by order parameters...

Problem at high density where baryons are not correctly taken into account... clear in comparison to Statistical Model. (Fukushima-Hidaka)

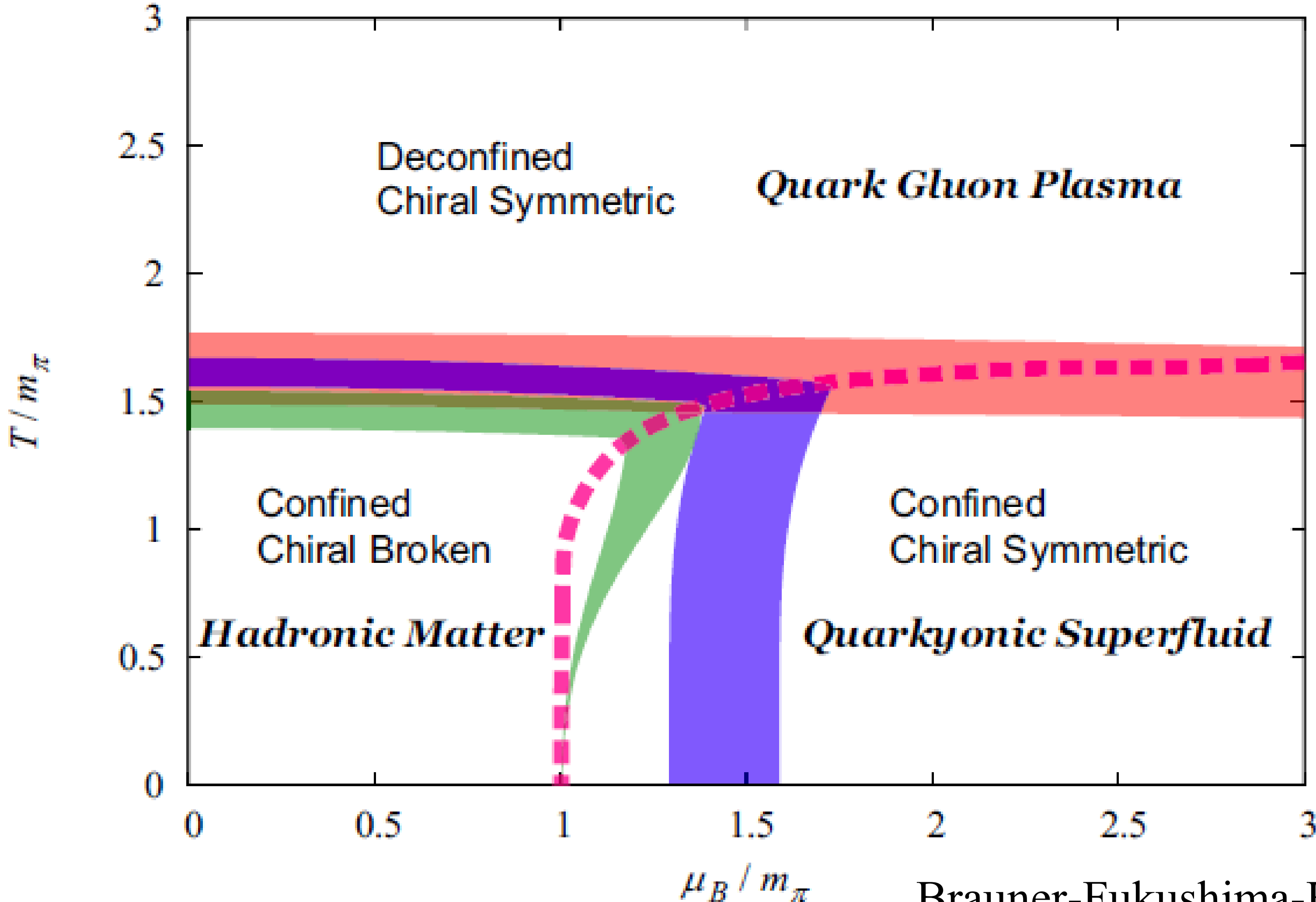
Preliminary Results

Entropy density from THERMUS



Two-Color Phase Diagram

Can be tested in the lattice simulation



Brauner-Fukushima-Hidaka (2009)

Features in QC_2D

- Monte-Carlo simulation on the lattice is possible if even number of flavors are degenerated.
- Baryons are not qqq but qq and can be treated in the same way as mesons. Baryons are correctly taken into account even in a simple mean-field calc.
- Ambiguity in the model building is much less and is a clean environment because of Pauli-Guersey symmetry.
- Phase structure is very similar to that in QCD. Superfluid of baryonic pions corresponds to nuclear matter and "quarkyonic" matter.

Summary

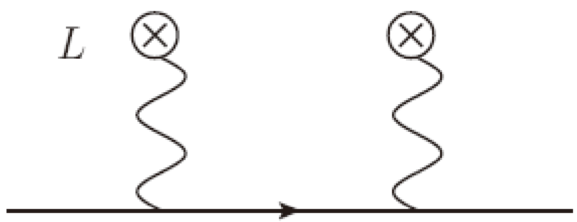
- PNJL model has two order parameters – the chiral codensate and the Polyakov loop – which can nicely describe the "thermal" confinement and deconfinement.

- Order parameters define the phase diagram, and the entropy and baryon number densities define another one, whch looks similar to the former.

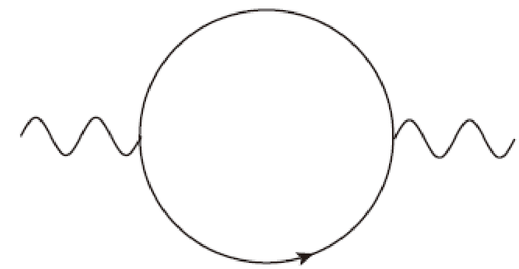
- Two-color system is a nice testing ground.
  (Work in progress with Nakamura)

Mixing through Mean-Fields

One may think that such a treatment takes care of



but lacks for



Quarks do not directly affect the Polyakov loop potential, but through the expectation value of the Polyakov loop, quarks change the pressure from the pure gluonic sector.

Gluon and quark sectors are not separable.