# Chiral and Deconfinement Aspects of (2+1)-flavor QCD

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EMMI Workshop

Quarks, Hadrons, and the Phase Diagram of QCD

St. Goar, Germany

QCD: two phase transitions:

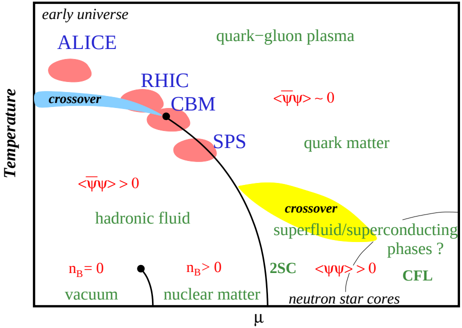
1. restoration of chiral symmetry

   $SU_{L+R}(N_f) \to SU_L(N_f) \times SU_R(N_f)$

   order parameter:

   $$\langle \bar{q}q \rangle \begin{cases} > 0 \Leftrightarrow \text{symmetry broken, } T < T_c \\ = 0 \Leftrightarrow \text{symmetric phase, } T > T_c \end{cases}$$

   associate limit: $m_q \to 0$




chiral transition: spontaneous restoration of global $SU_L(N_f) \times SU_R(N_f)$ at high $T$

QCD: two phase transitions:

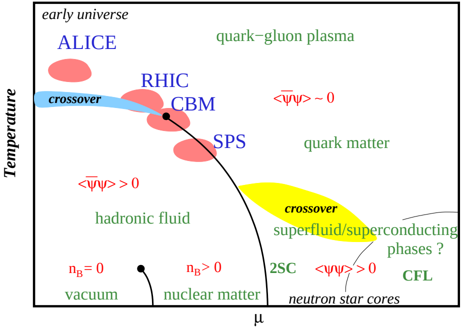
1. restoration of chiral symmetry
2. de/confinement (center symmetry)

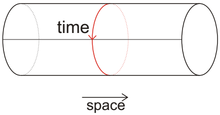
order parameter:

$$\Phi \begin{cases} = 0 \Leftrightarrow \text{confined phase,} & T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase,} & T > T_c \end{cases}$$

$$\Phi = \frac{1}{N_c} \langle \mathrm{tr}_c \mathcal{P} e^{i \int_0^\beta d\tau A_0(\tau, \vec{x})} \rangle$$

associate limit: $m_q \to \infty$




➜ related to free energy of a static quark state: $\Phi = e^{-F_q}$

QCD: two phase transitions:

1. restoration of chiral symmetry
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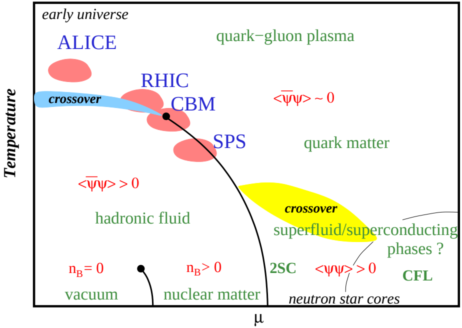
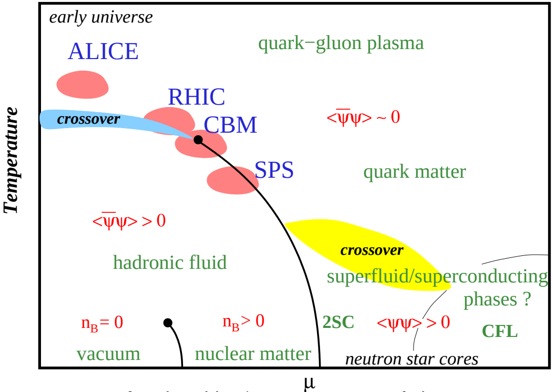



alternative:

➜ dressed Polyakov loop (or dual quark condensate)

it relates chiral and deconfinement transition to spectral properties of Dirac operator

# The conjectured QCD Phase Diagram




At densities/temperatures of interest only model calculations available

Open issues:

related to chiral & deconfinement transition

- existence of CEP?
- its location?
- additional CEPs? How many?
- coincidence of both transitions at $\mu = 0$?
- quarkyonic phase at $\mu > 0$?
- chiral CEP/ deconfinement CEP?
- so far only MFA results effect of fluctuations (e.g. size of crit. reg.)?
- ...

## effective models:

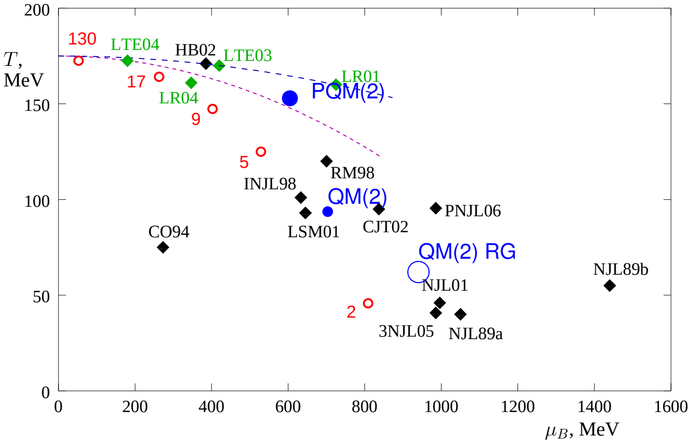
1. Quark-meson model (renormalizable) — or other models e.g. NJL
2. Polyakov–quark-meson model — or PNJL models

model studies vs. lattice simulations

Black points: models

Lines & green points: lattice

Red points: Freezeout points for HIC

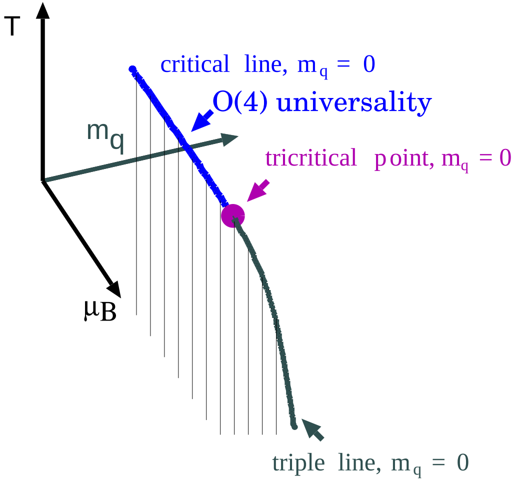



Stephanov '05 & '07

lattice methods:

- reweighting
- imaginary $\mu_B$
- Taylor expansion around $\mu_B = 0$

Chiral limit: $(m_q = 0)$ $SU(2) \times SU(2) \sim O(4)$-symmetry $\longrightarrow$ 4 modes critical $\sigma, \vec{\pi}$



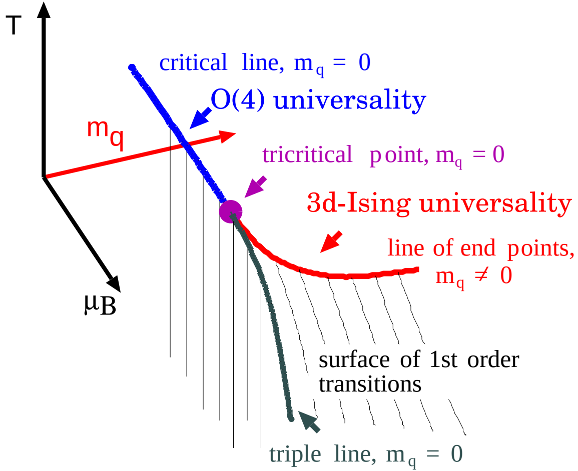

General properties

- chiral limit
  tricritical point (Gaussian fixed point)

# Phase diagram in $(T, \mu_B, m_q)$-space

Chiral limit: $(m_q = 0)$ $SU(2) \times SU(2) \sim O(4)$-symmetry $\longrightarrow$ 4 modes critical $\sigma, \vec{\pi}$

$m_q \neq 0$ : no symmetry remains $\longrightarrow$ only one critical mode $\sigma$ (Ising) ($\vec{\pi}$ massive)




## General properties

- chiral limit
  tricritical point
  (Gaussian fixed point)
- finite $m_q$
  critical endpoints
  (3D-Ising class)

# Outline

◦ Three-Flavor Quark-Meson Model

◦ ...with Polyakov loop dynamics

◦ Finite density extrapolations

# $N_f = 3$ Quark-Meson (QM) model

- Model Lagrangian: $\mathcal{L}_{\text{qm}} = \mathcal{L}_{\text{quark}} + \mathcal{L}_{\text{meson}}$

a) Quark part with Yukawa coupling $g$:

$$\mathcal{L}_{\text{quark}} = \bar{q}(i\partial\!\!\!/ - g\frac{\lambda_a}{2}(\sigma_a + i\gamma_5\pi_a))q$$

b) Meson part: scalar $\sigma_a$ and pseudoscalar $\pi_a$ nonet

fields: $\phi = \sum_{a=0}^{8} \frac{\lambda_a}{2}(\sigma_a + i\pi_a)$

$$\begin{aligned}\mathcal{L}_{\text{meson}} = {} & \text{tr}[\partial_\mu\phi^\dagger\partial^\mu\phi] - m^2\text{tr}[\phi^\dagger\phi] - \lambda_1(\text{tr}[\phi^\dagger\phi])^2 \\ & -\lambda_2\text{tr}[(\phi^\dagger\phi)^2] + c\,[\det(\phi) + \det(\phi^\dagger)] \\ & +\text{tr}[H(\phi + \phi^\dagger)]\end{aligned}$$

explicit sym. breaking matrix: $H = \sum_a \frac{\lambda_a}{2} h_a$

$U(1)_A$ symmetry breaking implemented by 't Hooft interaction

# Mean field approximation

- partition function:

$$\mathcal{Z}(T,\mu) = \int \mathcal{D}\bar{q}\mathcal{D}q \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \exp\left\{ i\int_0^{1/T} dt d^3x \left( \mathcal{L}_{N_f=3} + \sum_f \mu_f \bar{q}_f \gamma_0 q_f \right) \right\}$$

- here SU(2) isospin symmetry: only two condensates $\sigma_x$ and $\sigma_y$
- evaluation fermion determinant

**Grand canonical potential**

$$\Omega(T,\mu) = -\frac{T \ln \mathcal{Z}}{V} = U(\sigma_x,\sigma_y) + \Omega_{\bar{q}q}(T,\mu)$$

with mesonic potential $U(\sigma_x,\sigma_y)$ and

$$\Omega_{\bar{q}q}(T,\mu) = -2N_c T \sum_{f=u,d,s} \int \frac{d^3k}{(2\pi)^3} \left\{ \ln(1+e^{-(E_{q,f}-\mu_f)/T}) + \ln(1+e^{-(E_{q,f}+\mu_f)/T}) \right\}$$

➜ divergent vacuum contribution neglected here $\Rightarrow$ influences phase diagram cf talk B. Friman

EoM determines non-strange $\sigma_x(T,\mu)$ and strange $\sigma_y(T,\mu)$ condensates

$$\frac{\partial\Omega}{\partial\sigma_0} = \frac{\partial\Omega}{\partial\sigma_8}\bigg|_{\sigma_0=\sigma_x,\sigma_8=\sigma_y} = 0$$

[BJS, M. Wagner, '09]

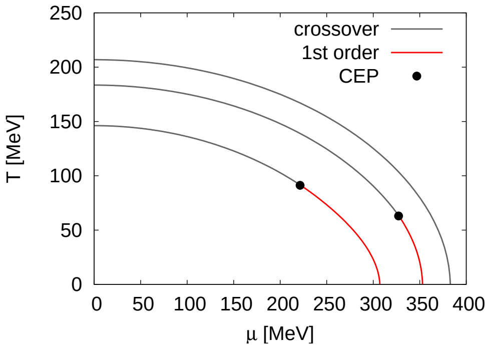
Model parameter fitted to (pseudo)scalar meson spectrum:

PDG: $f_0(600)$ mass=$(400 \ldots 1200)$ MeV $\rightarrow$ broad resonance

➜ existence of CEP depends on $m_\sigma$ value!

Example: $m_\sigma = 600$ MeV (lower lines), $800$ and $900$ MeV (here mean-field approximation)

**with $U(1)_A$**



# In-medium meson masses

[BJS, M. Wagner, '09]

- genuine problem of linear sigma model w/o quarks at finite T

  $\longrightarrow$ negative meson masses

- but not in this approximation

  ➜Ward identities, Goldstone theorem etc. are all valid in-medium e.g.

$$h_x = f_\pi m_\pi^2$$

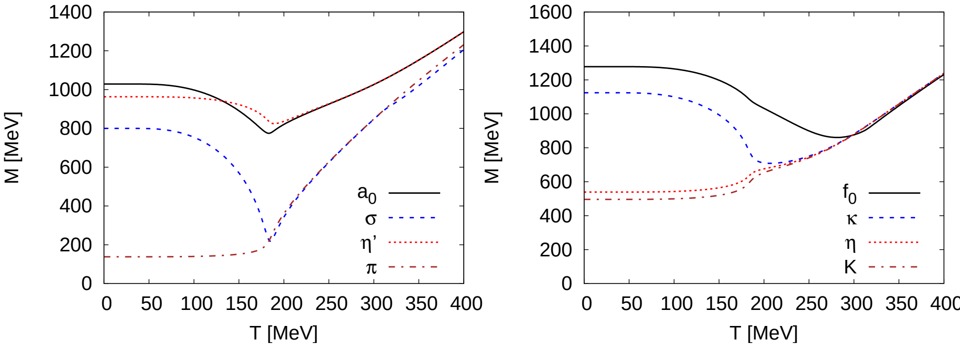
  similar in strange sector

- At low temperatures: mesons dominate

  At high temperatures: quarks dominate

[BJS, M. Wagner, '09]

masses with $U(1)_A$ anomaly



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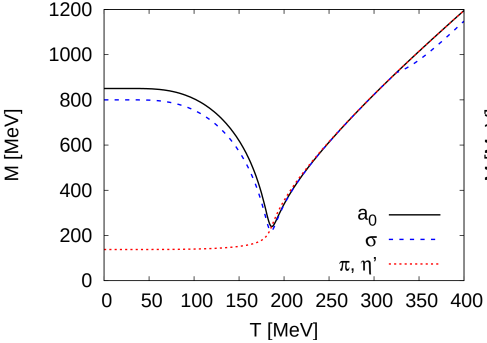


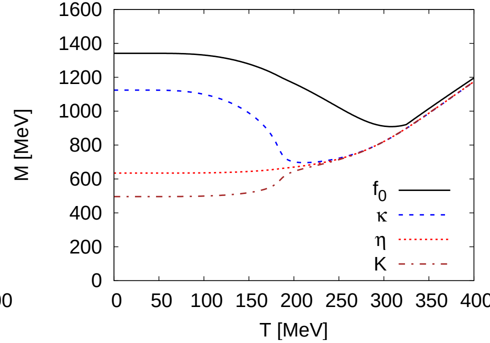

[BJS, M. Wagner, '09]

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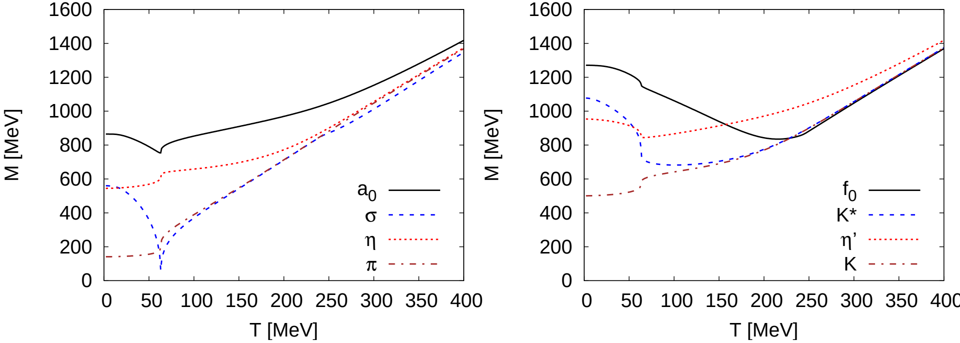
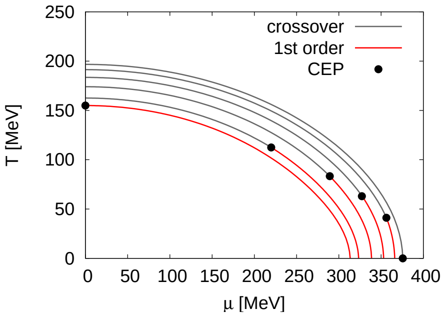


# Mass sensitivity

Chiral limit: RG arguments $\rightarrow$ for $N_f \geq 3$ first-order ✔ [Pisarski, Wilczek '84]

- $m_\pi / m_\pi^* = 0.49$ (lower line), $0.6, 0.8 \ldots, 1.36$ (upper line)
  $m_\pi^* = 138$ MeV , $m_K^* = 496$ MeV , ratio $m_\pi / m_K = m_\pi^* / m_K^*$ fixed

**with $U(1)_A$, $m_\sigma = 800$ MeV**



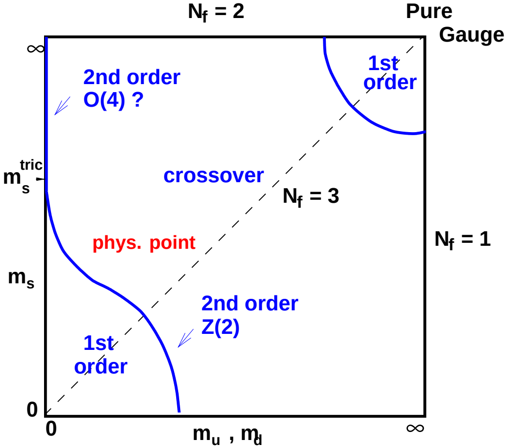
Columbia plot:

[Brown et al. '90]

$T_\chi^{N_f=2} \sim 175$ MeV

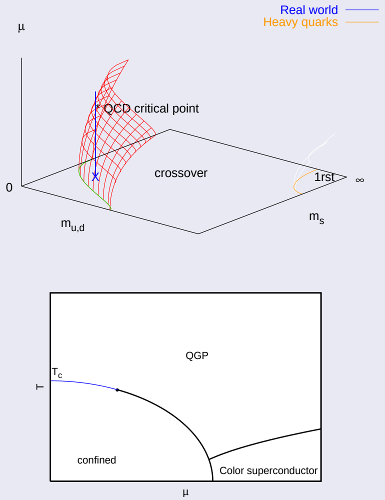
$T_d \sim 270$ MeV

$T_\chi^{N_f=3} \sim 155$ MeV

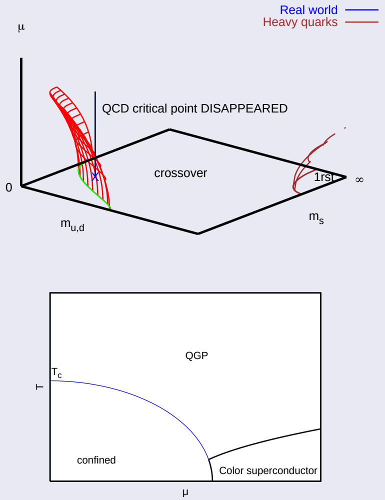


# Mass Sensitivity (lattice, $N_f = 3, \mu_B \neq 0$)

## Standard scenario: $m_c(\mu)$ increasing




## Nonstandard scenario: $m_c(\mu)$ decr.




[de Forcrand, Philipsen: hep-lat/0611027]

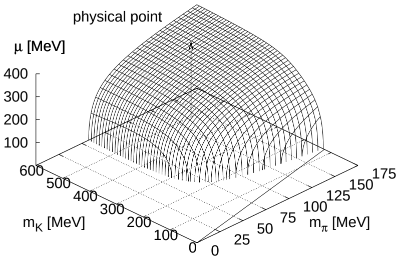
# Chiral critical surface $(m_\sigma = 800$ MeV$)$

[BJS, M. Wagner, '09]

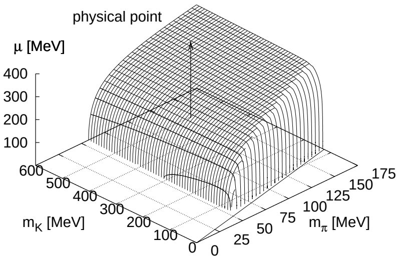
- chiral critical surface in $(m_\pi, m_K)$-plane

  $\rightarrow$ standard scenario for $m_\sigma = 800$ MeV (as expected)

**with $U(1)_A$**




**without $U(1)_A$**




Note: 't Hooft coupling $\mu$-independent

PNJL with (unrealistic) large vector int. $\rightarrow$ bending of surface

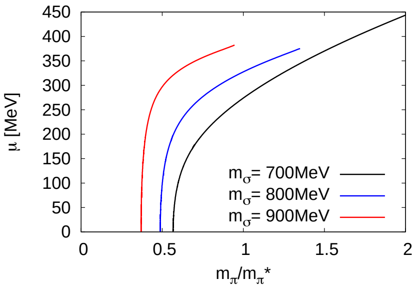
[BJS, M. Wagner, '09]

- chiral critical surface in $(m_\pi, m_K)$-plane for different $m_\sigma$

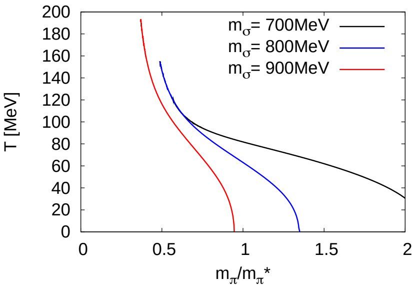
▷ CEP vanishes for $m_\sigma > 800$ MeV $\rightarrow$ possible non-standard scenario?

➜ three cuts of critical surface along fixed $m_\pi/m_K$ ratio through physical point

**critical $\mu_c$**




**critical $T_c$**



# Outline

- Three-Flavor Quark-Meson Model
- ...with Polyakov loop dynamics
- Finite density extrapolations

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

1 polynomial Polyakov loop potential:

Polyakov 1978, Meisinger 1996
Pisarski 2000

$$\frac{\mathcal{U}(\phi,\bar{\phi})}{T^4} = -\frac{b_2(T,T_0)}{2}\phi\bar{\phi} - \frac{b_3}{6}\left(\phi^3 + \bar{\phi}^3\right) + \frac{b_4}{16}\left(\phi\bar{\phi}\right)^2$$

with

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

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with

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

2 logarithmic potential:

Rössner et al. 2007

$$\frac{\mathcal{U}_{\text{log}}}{T^4} = -\frac{1}{2}a(T)\bar{\phi}\phi + b(T)\ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]$$

with

$$a(T) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 \quad \text{and} \quad b(T) = b_3(T_0/T)^3$$

# Polyakov–quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

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with

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

3 Fukushima

Fukushima 2008

$$\mathcal{U}_{\text{Fuku}} = -bT\left\{54e^{-a/T}\phi\bar{\phi} + \ln\left[1 - 6\bar{\phi}\phi + 4\left(\phi^3 + \bar{\phi}^3\right) - 3\left(\bar{\phi}\phi\right)^2\right]\right\}$$

with

$a$ controls deconfinement $b$ strength of mixing chiral & deconfinement

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

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with

$$b_2(T,T_0) = a_0 + a_1(T_0/T) + a_2(T_0/T)^2 + a_3(T_0/T)^3$$

in presence of dynamical quarks: $T_0 = T_0(N_f)$

BJS, Pawlowski, Wambach, 2007

| $N_f$ | 0 | 1 | 2 | $2+1$ | 3 |
|---|---|---|---|---|---|
| $T_0$ [MeV] | 270 | 240 | 208 | 187 | 178 |

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

1 polynomial Polyakov loop potential: Polyakov 1978, Meisinger 1996 Pisarski 2000

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$\mu \neq 0: \quad \bar{\phi} > \phi$

since $\bar{\phi}$ is related to free energy gain of antiquarks

in medium with more quarks → antiquarks are more easily screened.

# Polyakov–quark-meson (PQM) model

- Lagrangian $\mathcal{L}_{\text{PQM}} = \mathcal{L}_{\text{qm}} + \mathcal{L}_{\text{pol}}$ with $\mathcal{L}_{\text{pol}} = -\bar{q}\gamma_0 A_0 q - \mathcal{U}(\phi, \bar{\phi})$

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| $N_f$ | 0 | 1 | 2 | $2+1$ | 3 |
|---|---|---|---|---|---|
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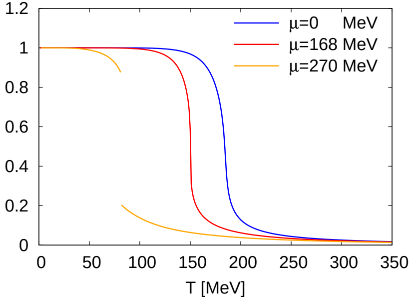
$\mu \neq 0:$ finally $T_0 = T_0(N_f, \mu)$

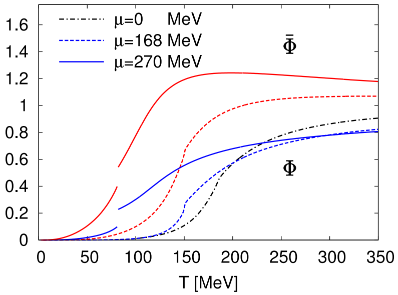
# Finite temperature and finite $\mu$ (PQM $N_f = 2$)

[BJS, Pawlowski, Wambach '07]

without $\mu$-modifications in Polyakov potential:

order parameters



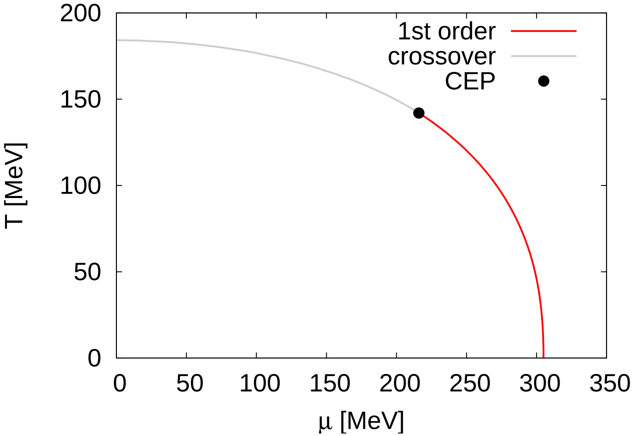



[BJS, Pawlowski, Wambach '07]

- for PQM model $N_f = 2$

in mean field approximation

chiral transition and 'deconfinement' coincide



[BJS, Pawlowski, Wambach '07]

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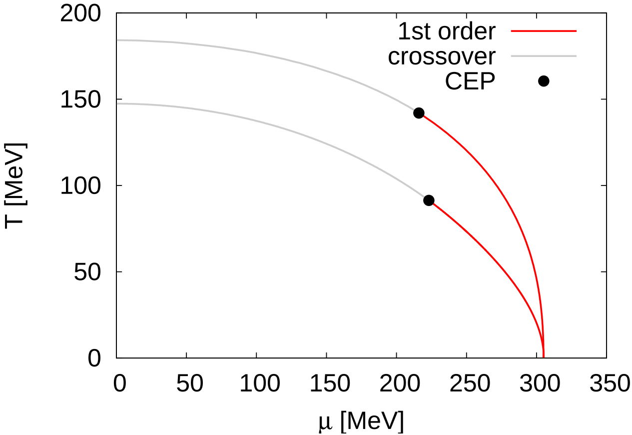


- for PQM model $N_f = 2$
- for QM model $N_f = 2$ (lower lines)

[BJS, Pawlowski, Wambach '07]

in mean field approximation

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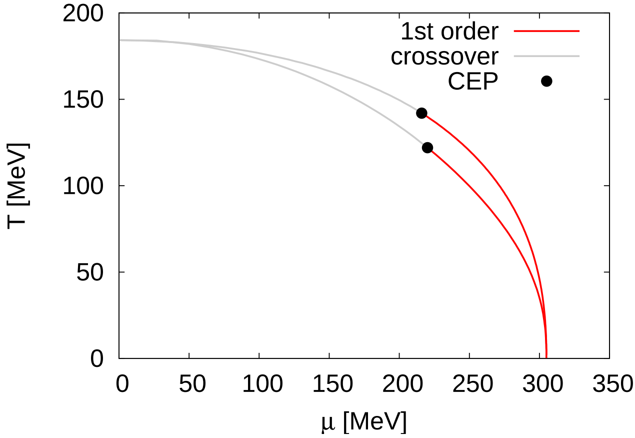



- for PQM model $N_f = 2$
- for PQM model $N_f = 2$ with $\mu$-modification in Polyakov loop potential (lower lines)

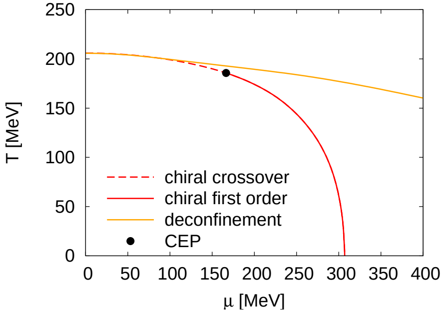
[BJS, M. Wagner; in preparation '09]

influence of Polyakov loop

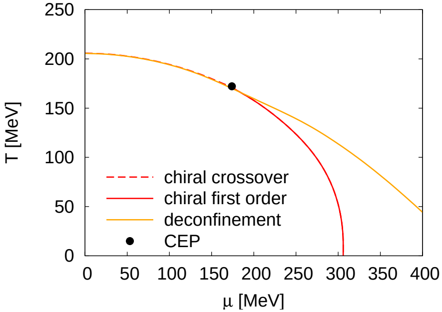
Logarithmic Polyakov loop potential

Mean-field approximation

$T_0 = 270$ MeV (constant)




$T_0(\mu)$ (i.e. with $\mu$ corrections)

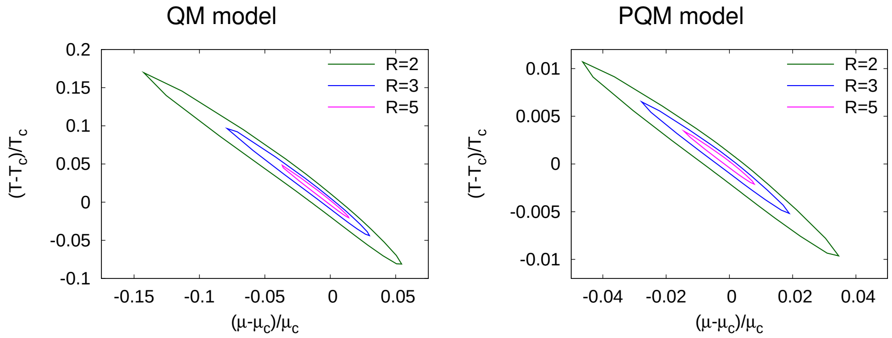


[BJS, M. Wagner; in preparation '09]

contour plot of size of the critical region around CEP

defined via fixed ratio of susceptibilities: $R = \chi_q / \chi_q^{\text{free}}$

➜ compressed with Polyakov loop

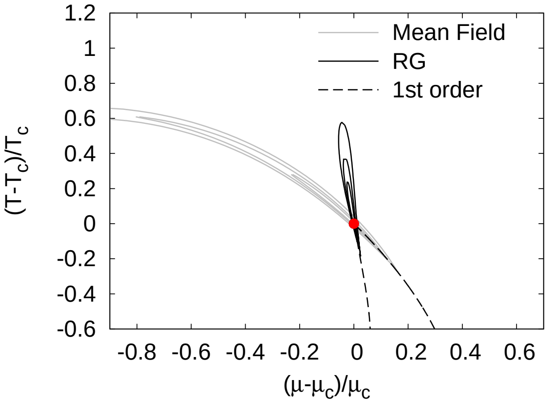


[BJS, J. Wambach '06]

similar conclusion if fluctuations (via RG techniques) are included

example: $N_f = 2$ QM model

Mean Field $\leftrightarrow$ RG analysis

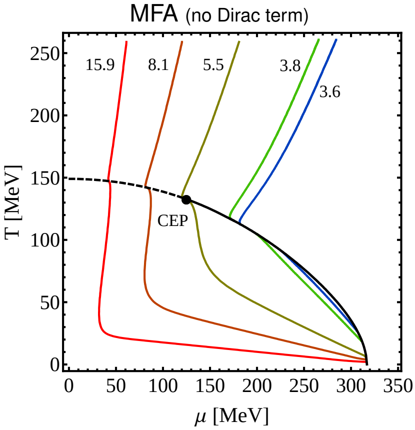


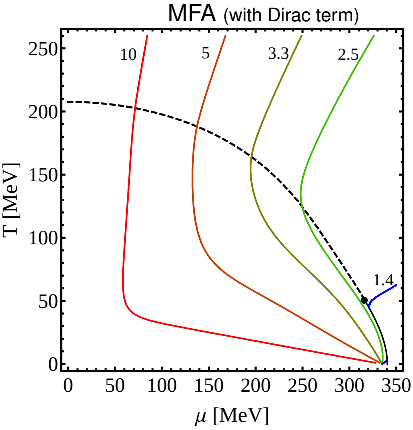
[E. Nakano, BJS, B.Stokic, B.Friman, K.Redlich; arXiv:0907.1344]

here: $N_f = 2$ QM model: kink in MFA are washed out in FRG

$\rightarrow$ no focussing if fluctuations taken into account

a) influence of Dirac term b) smallnest of crit region



# Isentropes $s/n = const$ and Focussing

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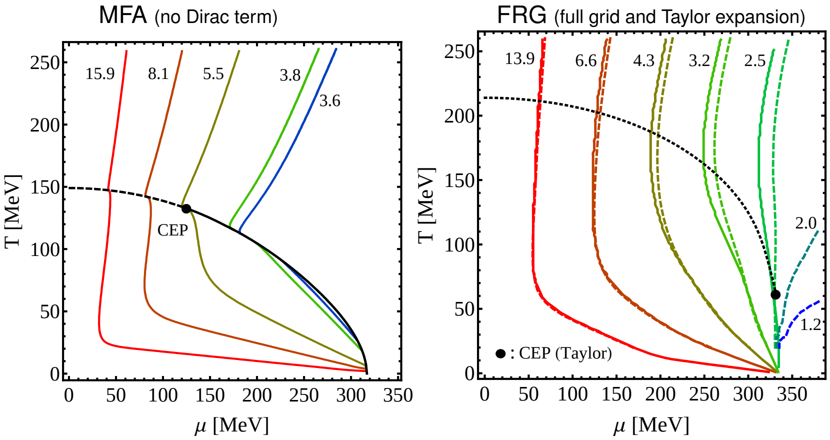

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kink structure at boundary in MFA

$\Rightarrow$ remnant of first-order transition in chiral limit

if Dirac term neglected

see talk by B. Friman

# Functional RG Approach

Consider fluctuations: PQM with the FRG

FRG (average effective action) [Wetterich '93]

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr}\, \partial_t R_k \left( \frac{1}{\Gamma_k^{(2)} + R_k} \right) \quad ; \quad \Gamma_k^{(2)} = \frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi}$$

$\Gamma_k[\phi]$ scale dependent effective action ; $t = \ln(k/\Lambda)$ ; $R_k$ regulators

## PQM $N_f = 2$ [T.K.Herbst, BJS]

$$\partial_t \Omega_k = \frac{k^5}{12\pi^2} \left[ -\frac{2N_f N_c}{E_q} \left( F_q(\Phi, \Phi^*) + F_{\bar{q}}(\Phi, \Phi^*) \right) + \frac{1}{E_\sigma} \text{coth} \left( \frac{E_\sigma}{2T} \right) + \frac{3}{E_\pi} \text{coth} \left( \frac{E_\pi}{2T} \right) \right]$$

with $E_{\sigma,\pi,q} = \sqrt{k^2 + m^2_{\sigma,\pi,q}}$ , $m^2_\sigma = 2\Omega' + 4\sigma^2\Omega''$ , $m^2_\pi = 2\Omega'$ , $m^2_q = g^2\sigma^2$

and

$$F_q(\Phi, \Phi^*) = \frac{-1 - \Phi^* e^{\beta(E_q-\mu)} + \Phi e^{2\beta(E_q-\mu)} + e^{3\beta(E_q-\mu)}}{1 + 3\Phi^* e^{\beta(E_q-\mu)} + 3\Phi e^{2\beta(E_q-\mu)} + e^{3\beta(E_q-\mu)}}$$

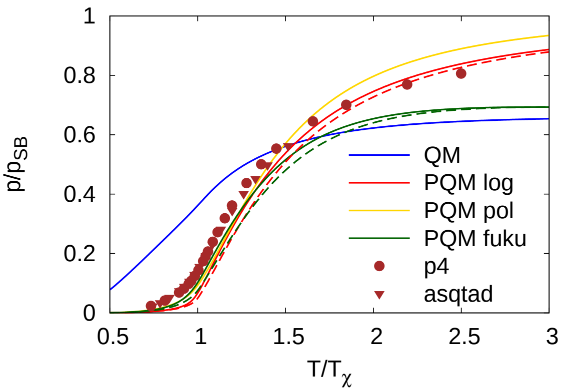
$$F_{\bar{q}}(\Phi, \Phi^*) = F_q(\Phi^*, \Phi)|_{\mu \to -\mu}$$

$\Rightarrow$ see poster by T.K. Herbst

[BJS, M. Wagner, J. Wambach; in preparation '09]

SB limit: $\frac{p_{SB}}{T^4} = 2(N_c^2 - 1)\frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$

(P)QM models (three different Polyakov loop potentials) versus QCD lattice simulations




- dashed lines: PQM with lattice masses $m_\pi \sim 220, m_K \sim 503$ MeV
- solid lines: (P)QM with realistic masses

lattice data: [Bazavov et al. '09]

[BJS, M. Wagner, J. Wambach; in preparation '09]

SB limit: $\frac{p_{\rm SB}}{T^4} = 2(N_c^2-1)\frac{\pi^2}{90} + N_f N_c \frac{7\pi^2}{180}$

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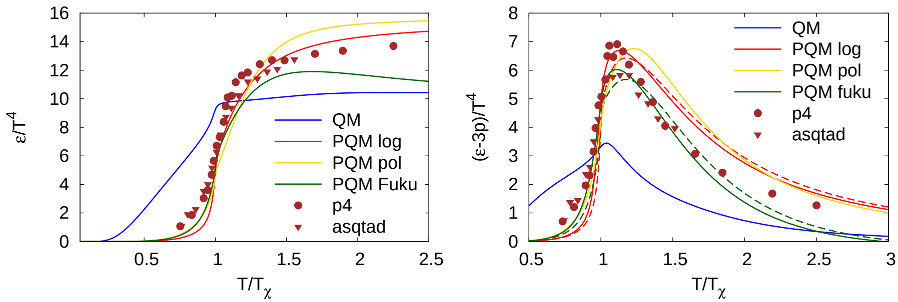



dashed lines: $m_\pi \sim 220, m_K \sim 503$ MeV

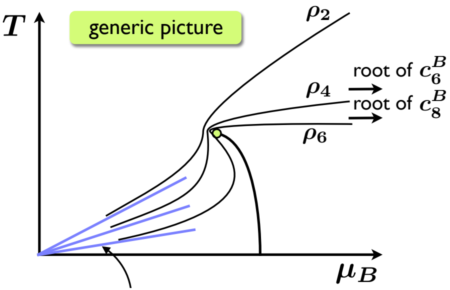
[Bazavov et al. '09]

# Outline

- Three-Flavor Quark-Meson Model
- ...with Polyakov loop dynamics
- Finite density extrapolations

Taylor expansion:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n \left(p(T,\mu)/T^4\right)}{\partial \left(\mu/T\right)^n}\bigg|_{\mu=0}$$




[C. Schmidt '08]

<u>convergence radii:</u>

limited by first-order line?

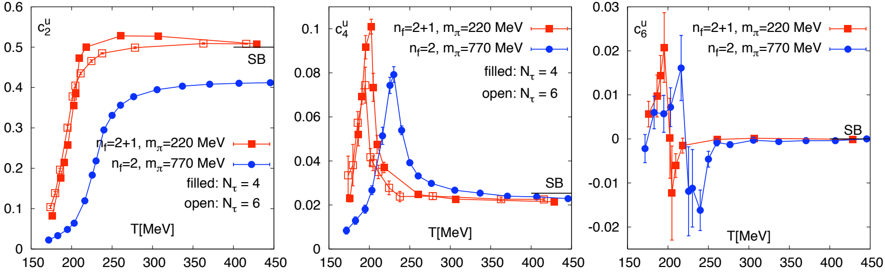
$$\rho_{2n} = \left|\frac{c_2}{c_{2n}}\right|^{1/(2n-2)}$$

or

$$r_{2n} = \left|\frac{c_{2n}}{c_{2n+2}}\right|^{1/2}$$

Taylor expansion:

$$\frac{p(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu}{T}\right)^n \quad \text{with} \quad c_n(T) = \frac{1}{n!} \frac{\partial^n \left(p(T,\mu)/T^4\right)}{\partial \left(\mu/T\right)^n}\Bigg|_{\mu=0}$$

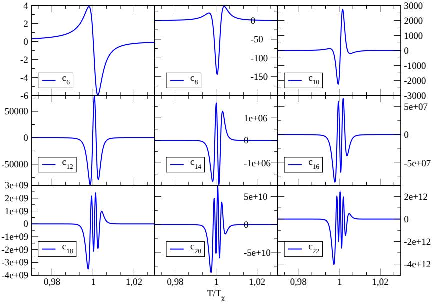



[Miao et al. '08]

New numerical method: based on algorithmic differentiation (AD)

[M. Wagner, A. Walther, BJS subm. to CPC '09]

Taylor coefficients $c_n$ numerically known to high order, e.g. $n = 22$




- ▷ this technique applied to PQM model
- ▷ investigation of convergence properties of series

Can we locate the QCD critical endpoint with the Taylor expansion ?

$\Rightarrow$ talk by M. Wagner

- $N_F = 3$ chiral (Polyakov)-quark-meson model study

  ➜ Mean-field approximation

  with and without axial anomaly

- novel AD technique: high order Taylor coefficients, here: $c_{n=24}(T)$

Findings:

▷ Parameter in Polyakov loop potential: $T_0 \Rightarrow T_0(N_f, \mu)$

▷ Chiral & deconfinement transition coincide for $N_f = 2$ with $T_0(\mu)$-corrections but not for $N_f = 2+1$

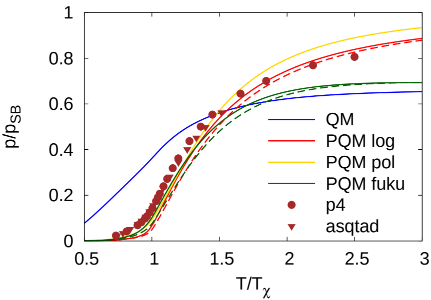
▷ Mean-field approximation encouraging

but effects of Dirac term point to interesting physics if fluctuations are considered

→ FRG with PQM truncation

▷ Taylorcoefficient $c_n(T) \rightarrow$ high order

⇒ convergence properties of Taylor expansion




Outlook:

- include glue dynamics with FRG → full QCD