Inhomogeneous Phases and the Chiral Critical Point in NJL-type models

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Quarks, Hadrons, and the Phase Diagram of QCD
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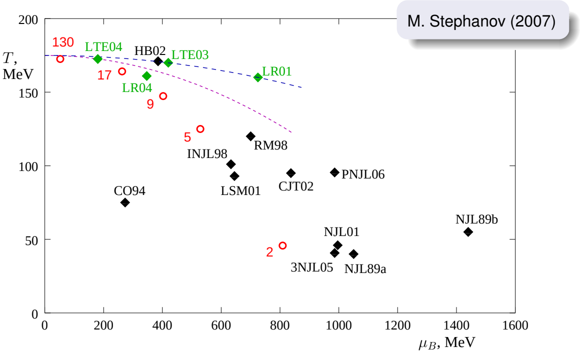
introduction: QCD phase diagram and critical point



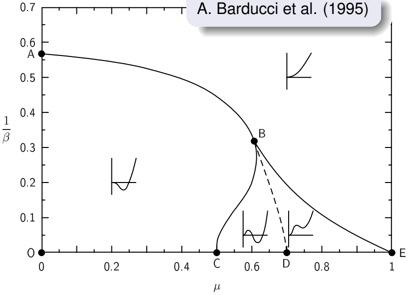
moderate densities and $N_c = 3 \rightarrow$ NJL-type models

won't try to add an additional point, instead ...

inhomogeneous phases

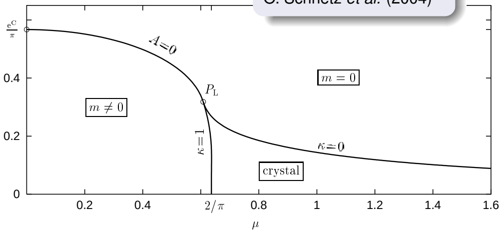
motivation: Gross-Neveu model $(1+1\text{D})$ in mean-field

A. Barducci et al. (1995)



$\xrightarrow{T}$

O. Schnetz *et al.* (2004)



analogues in QCD/NJL:

- large *N* analysis / quarkyonic matter — D. Deryagin *et al.* (1992), . . .
- $T = 0$ — R. Rapp *et al.* (2001), plane-wave — T. Tatsumi *et al.* (2004)

outline

1. generalized Ginzburg-Landau analysis at chiral critical point
2. one-dimensional modulations in the NJL model
3. one-dimensional modulations in the quark-meson model

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NJL model in mean-field approximation

- model+approximation

$$\mathcal{L}_{NJL} = \bar\psi\left(i\gamma^\mu\partial_\mu - m_0\right)\psi + \frac{G}{2}\left((\bar\psi\psi)^2 + (\bar\psi i\gamma^5\tau^a\psi)^2\right)$$
$$\xrightarrow{mean-field} \bar\psi\left(i\gamma^\mu\partial_\mu - M(x)\right)\psi - \frac{1}{4G}\left(M(x) - m_0\right)^2$$

(where $M(x) = m_0 + G\langle\bar\psi(x)\psi(x)\rangle$ and $\langle\bar\psi i\gamma^5\tau^a\psi\rangle = 0$)

- thermodynamic potential

$$\Omega[T,\mu;M(x)] = -\frac{T}{V}\mathrm{Tr\,Log}\left(i\partial_t - \underbrace{\gamma^0(-i\gamma^i\partial_i + M(x))}_{H_{NJL}} + \mu\right) + \frac{1}{V}\int_V \frac{1}{4G}\left(M(x) - m_0\right)^2$$

- evaluation of $\Omega \leftrightarrow$ spectrum of H_{NJL}

homogeneous phases $\rightarrow$ one-parameter problem
inhomogeneous phases $\rightarrow$?

generalized Ginzburg-Landau expansion

expanding in $M(x)$ (here: $m_0 = 0$ for simplicity)

$$\Omega[T,\mu;M(x)] \quad = \quad -\frac{T}{V}\sum_{n>0}\frac{1}{n}\mathrm{Tr}_{D,c,f,V}(S_0(x_i,x_{i+1})M(x_{i+1}))^n + \frac{1}{V}\int_V \frac{1}{4G}M(x)^2\,,$$

gradients of $M(x_i)$ and finally ordering gives ...

$$\begin{aligned}\Omega[T,\mu;M(x)]-\Omega[T,\mu;0] \quad = \quad & \frac{\alpha}{2}M(x)^2 \\ & +\frac{\beta}{4}\left(M(x)^4+(\nabla M(x))^2\right) \\ & +\frac{\gamma}{6}\left(M(x)^6+5M(x)^2(\nabla M(x))^2+\frac{1}{2}(\Delta M(x))^2\right) \\ & +\ldots\end{aligned}$$

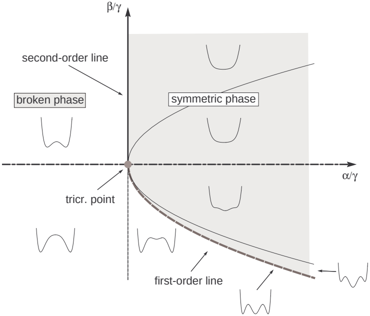
- inhomogeneous phases go up to chiral critical point ($\alpha = \beta = 0$)
- similar result for finite masses
- similar form as in Gross-Neveu model - not coincidental
- modifications of model?

ground state near the critical point

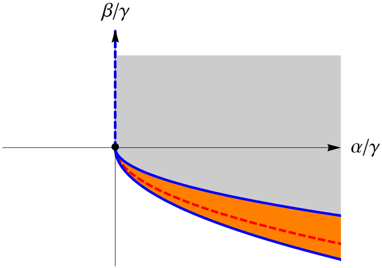
$$\Omega[M] - \Omega[0] = \frac{\alpha}{2}M^2 + \frac{\beta}{2}(M^4 + M'^2) + \frac{\gamma}{6}(M^6 + 5M^2M'^2 + \frac{1}{2}M''^2)$$

ansatz: $M(z) = const.$

ansatz: $M(z) = \sqrt{\nu}\Delta\,\mathrm{sn}(\Delta z, \nu)$



$\longrightarrow$



no first order transition!

outline

lower dimensions → lower dimensional modulations

$$\frac{T}{V}\mathrm{Tr}\,\mathrm{Log}\left(i\partial_t - H + \mu\right) \propto \sum_n \left(E_n + 2T\ \ln(1 + e^{-(E_n-\mu)/T})\right)$$

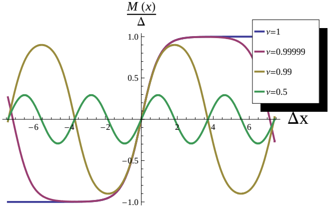
- consider $M(x, y, z) \equiv M(z)$
  translation invariance in xy-direction → conserved $P_\perp$
- Hamiltonian for $p_\perp = 0$

$$H_{NJL} = \begin{pmatrix} H_{GN} & \\ & H_{GN} \end{pmatrix}$$

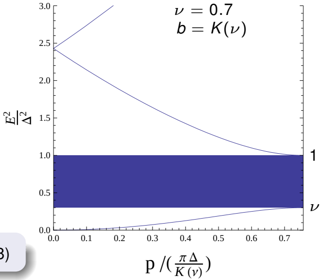
(general feature for lower dimensional modulations) self-consistent solution:

$$M(z) = \nu\Delta \mathrm{sn}(b|\nu)\mathrm{sn}(\Delta z|\nu)\mathrm{sn}(\Delta(z+b)|\nu) + \Delta\frac{\mathrm{cn}(b|\nu)\mathrm{dn}(b|\nu)}{\mathrm{sn}(b|\nu)}$$

three parameters: scale Δ, elliptic modulus ν, explicit breaking b



$\updownarrow$ $b \neq K(\nu)$



$\uparrow$ $b \neq K(\nu)$

O. Schnetz *et al.* (2004); O. Schnetz *et al.* (2005); G. Basar *et al.* (2008)

NJL model and regularization

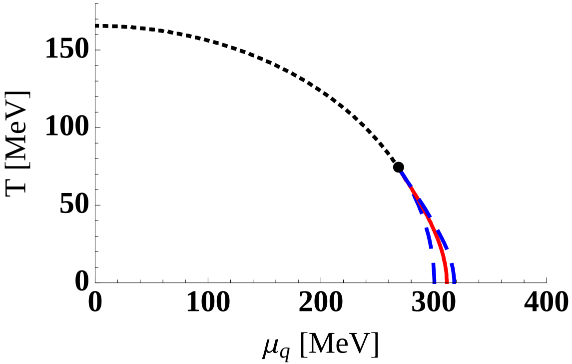
$$\Omega_{NJL}[T,\mu;M(x)] = \underbrace{\Omega_{NJL}[0,0;M(x)]}_{\textit{UV divergent}} + \underbrace{\delta\Omega_{NJL,medium}[T,\mu;M(x)]}_{\textit{finite}}$$

- inhomogeneous phases $\rightarrow$ no momentum cutoff possible (phase diagram phenomenology?!)

$$\mathrm{Tr}\,\mathrm{Log}\,(i\omega_n + H) \rightarrow -\frac{1}{2}\mathrm{Tr}\int_0^\infty \frac{d\tau}{\tau} f(\tau) e^{-\tau(\omega_n^2 + H^2)}$$

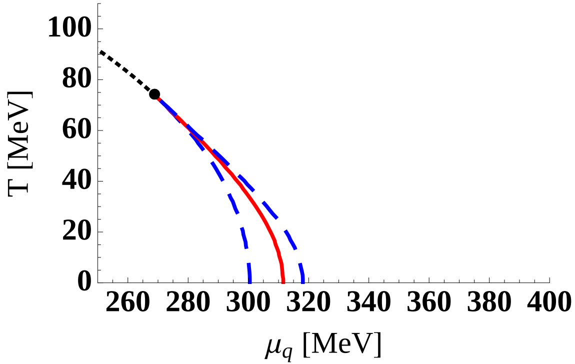
  blocking function $f(\tau) = 1 - 3e^{-\tau\Lambda^2} + 3e^{-\tau 2\Lambda^2} - e^{-\tau 3\Lambda^2}$

- parameters $\{G, \Lambda, m_0 = 0\} \leftrightarrow \{M_q \gtrsim M_N/3, f_\pi = 88\mathrm{MeV}\}$



- $M_q = 300\text{MeV} \quad \Rightarrow \quad \langle\bar{\psi}\psi\rangle = (-193\text{MeV})^3$
- CP at small T, high μ; moderate spinodal region

phase diagram



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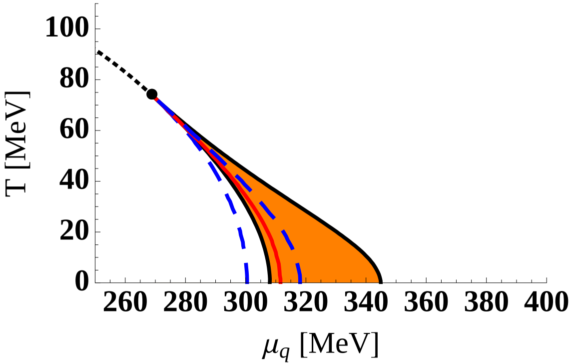
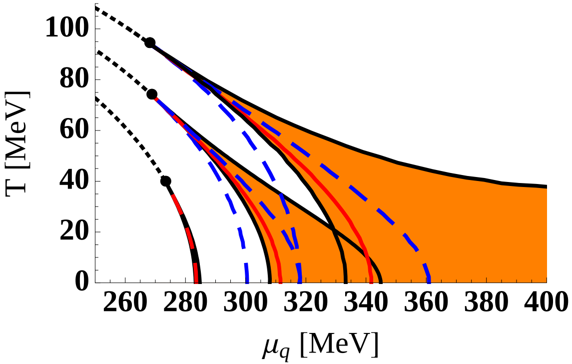
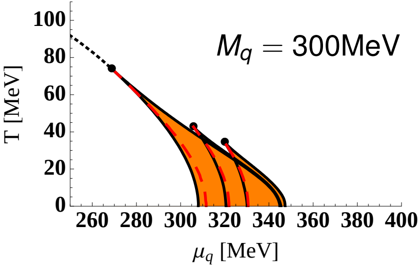


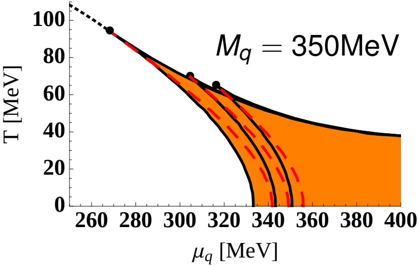
- inhomogeneous phases up to CP
- domain of inhomogeneous phase ↔ spinodal region

phase diagram



- $M_q = 250, 300, 350$MeV
- strength of phase transition ↔ domain of inhomogeneous phase





- finite current quark masses: $m_0 = 0, 5, 10$MeV
- no qualitative change

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quark-meson model in mean-field approximation

$$\mathcal{L}_{QM} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - g(\sigma + i\gamma_5\tau^a\pi^a)\right)\psi - U(\sigma,\pi^a)$$

$$U(\sigma,\pi^a) = -\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi^a\partial^\mu\pi^a\right) + \frac{\lambda}{4}\left(\sigma^2 + \pi^a\pi^a - v^2\right)^2 - c\sigma$$

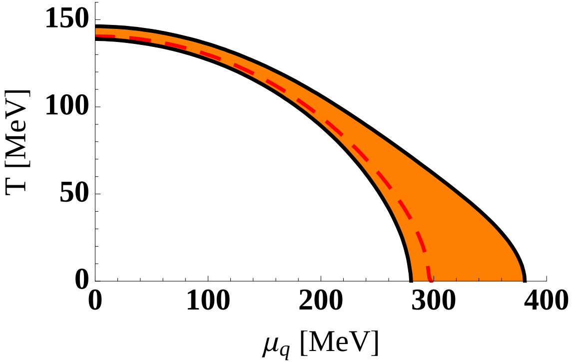
fix $\{g, v, \lambda, c\}$ by $\{M_q = 300\text{MeV},\ f_\pi = 93\text{MeV},\ m_\sigma = 600\text{MeV},\ m_\pi\}$

thermodynamic potential in 'mean-field':

$$\Omega_{QM}[T,\mu; M(x) = g\sigma(x)] = U(\sigma(x),0) + \underbrace{\delta\Omega_{medium}[T,\mu;M(x)]}_{\text{as in NJL}}$$

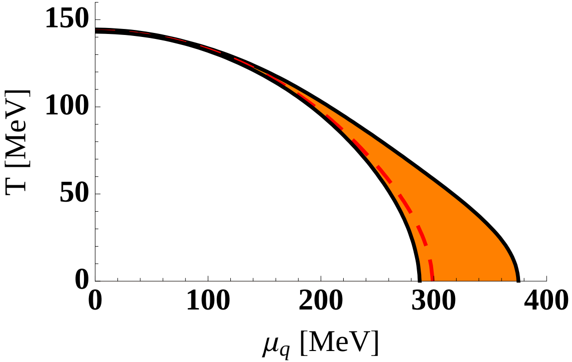
O. Scavenius *et al.* (2000); B. Schaefer *et al.* (2006)

phase diagram



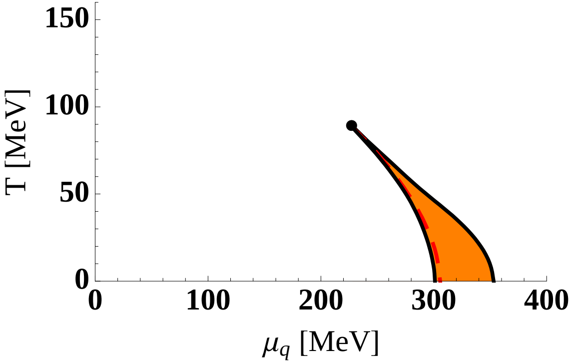
- $m_\pi =$ 0MeV $\rightarrow$ broad domain for inhomogeneous phase

phase diagram



- $m_\pi = 69\text{MeV} \quad \rightarrow$ still no CP, domain shrinks

phase diagram



- $m_\pi = 138\text{MeV}$ $\rightarrow$ similar scenario as in NJL model before

summary

inhomogeneous phases in NJL-type models

- generalized Ginzburg-Landau expansion
  $\longrightarrow$ generically reach out to critical point
- similarity to $1+1D$ models
  $\longleftrightarrow$ lower dimensional solutions to lower dimensional modulations
  $\longrightarrow$ no first-order phase transition
- strong first order phase transition
  $\longrightarrow$ significant window in phase diagram

future prospects

- Ginzburg-Landau analysis: effects of other interactions near CP
- phase diagram: color-superconductivity / other models
- higher dimensional inhomogeneities
- ...