Large chemical potentials at low temperatures in dense fermionic systems

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collaboration with:

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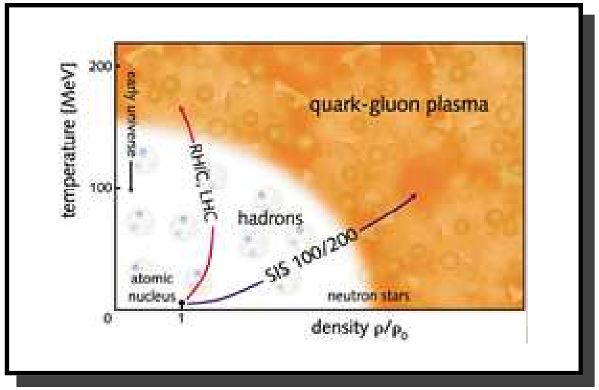
B. Wellegehausen, A. Wipf

Dense Fermionic systems:

- Ultimate aim: study QCD at $\mu \gg T$:

  μ: chemical potential
  T: temperature
  McLerran, Pisarski, (2007):
  $\Rightarrow$ *quarkyonic phase*



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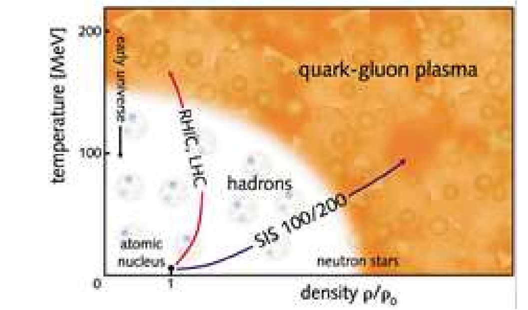

- Explorative study: Gross-Neveu model: $\mu \gg T$

  q: quarks, σ: scalar field

  interaction: $L(x) = \ldots + \bar{q}(x) \left[-(\partial\!\!\!/ + \mu\gamma_0) - i\sigma(x) \right] q(x)$

Dense Fermionic systems:

- Quark determinant for $\mu \gg T$:

$$S_{\rm fer} = -\frac{N_f}{2} \ln Det_{\rm AP} D(\mu) D^\dagger(-\mu)$$
$$= -\frac{N_f}{2} \mathrm{Tr} \ln \left[-(\partial + \mu)^2 + \sigma^2 - i\partial\!\!\!/ \sigma \right]$$

AP: anti-periodic boundary conditions

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AP: anti-periodic boundary conditions

- Quark density:

$$d = \int dx \, \langle q^\dagger q \rangle = T \, \frac{\partial S_{\rm fer}}{\partial \mu}$$

Dense Fermionic systems:

- Free quark theory: $\sigma(x) =$ constant (mass):

  $DD^\dagger$ real, but $DD^\dagger < 0$ for $\mu/T > \pi$

  choose $\mu/T < \pi$, Schwinger-proper time regularisation, Poisson resummation,

  + analytical continuation (if $\mu/T > \pi$), ...

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- Final result: (Fermi functions)

$$d = \int dE\ E \left[\frac{1}{\exp\{(E-\mu)/T\}+1} - \frac{1}{\exp\{(E+\mu)/T\}+1} \right]$$

particle contr. anti-particle contr.

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- Interacting theory: $\sigma(\vec{x})$?
- $\Rightarrow$ World line formalism:

$$d = \int dE\ E\ \left[\frac{\rho(E)}{\exp\{(E-\mu)/T\}+1} - \frac{\rho(E)}{\exp\{(E+\mu)/T\}+1} \right]$$

$$U(\tau) = \frac{1}{\sqrt{4\pi\ \tau}} \int dx_c\ \langle \exp\{-\int_0^\tau d\tau\ \sigma^2\}\ \mathrm{tr}_d\ \mathcal{P} \cosh\left(i \int_0^\tau d\tau\ \partial\!\!\!/ \sigma\right) \rangle\ .$$

$$U(\tau) = 2 \int_0^\infty dE\ E\ \exp[-\tau E^2]\ \rho(E)\ .$$

τ proper time, $\langle \ldots \rangle$ worldline expectation value, x_c: loop centre of mass

(Dunne, Gies, Klingmüller, Langfeld, JHEP 0908:010 (2009))

Dense Fermionic systems:

- What have we gained?
  (still need to invert the Laplace transform)

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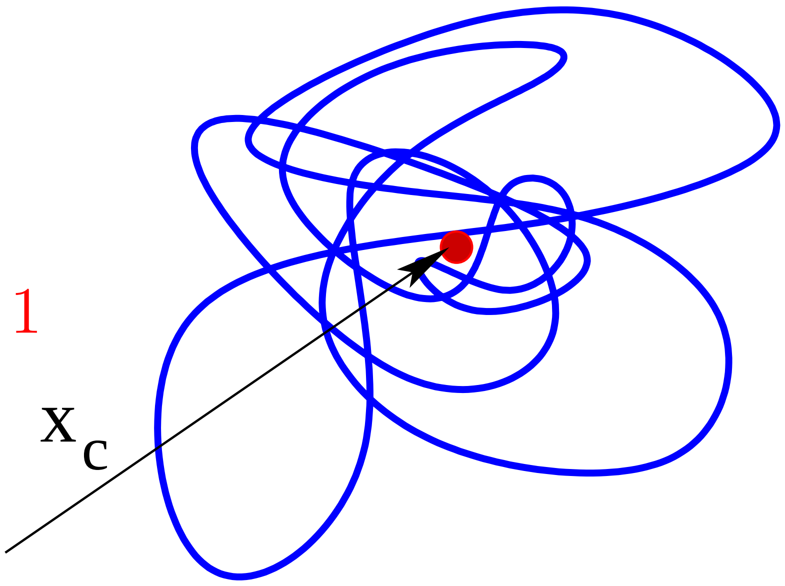
- Achievements:
  - $\rho(E) \;\Rightarrow$ interacting case
  - analytic continuation for the ultra-dense case $\mu/T \gg 1$ is done analytically



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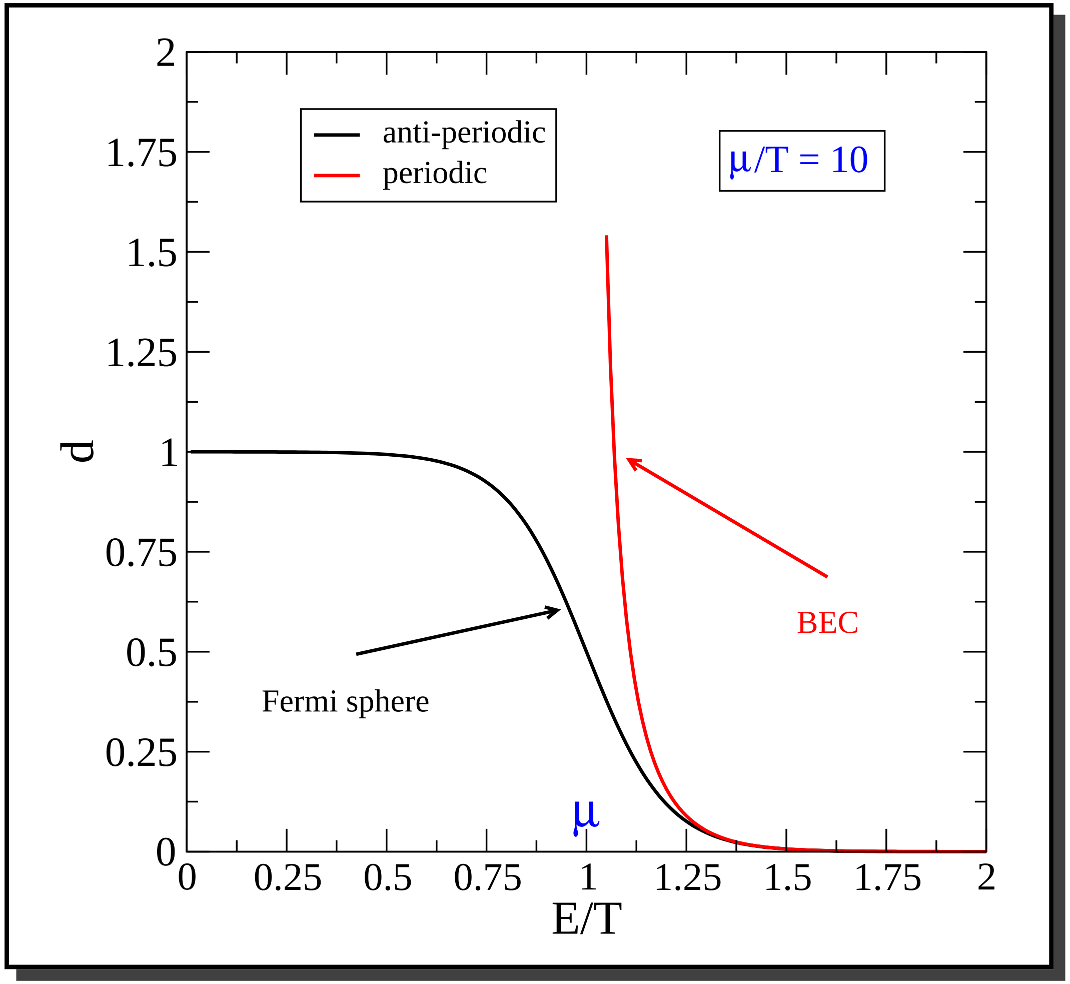

Ultra-dense N-colour QCD

- condsider:
  $SU(N)$ Yang-Mills theory
  quarks in the fundamental representation
  (potentially large) values for the chemical potential μ
  (potentially small) values for the temperature T

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  quarks in the fundamental representation
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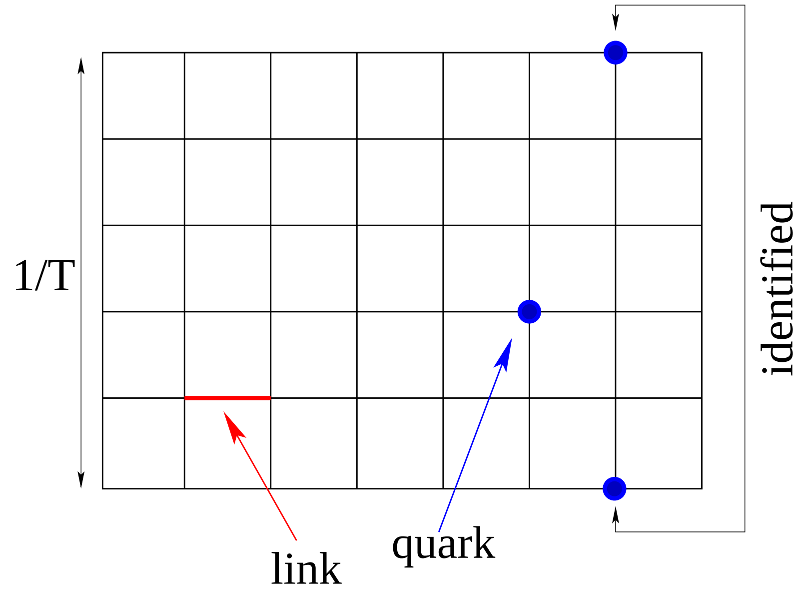
- use lattice regularisation:

  $U_\mu(x) \in SU(N)$: gauge fields
  for later use: the centre element

$$Z_N = \exp\{2\pi i\, m/N\}\mathbb{1}\ , \qquad -N/2 < m \leq N/2$$

Ultra-dense N-colour QCD

- space-time lattice:



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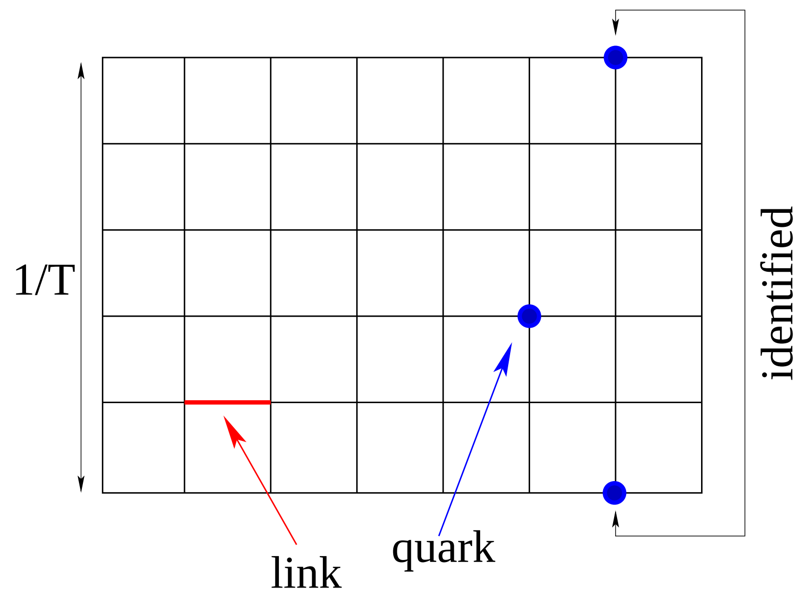


- integrating out the quarks:

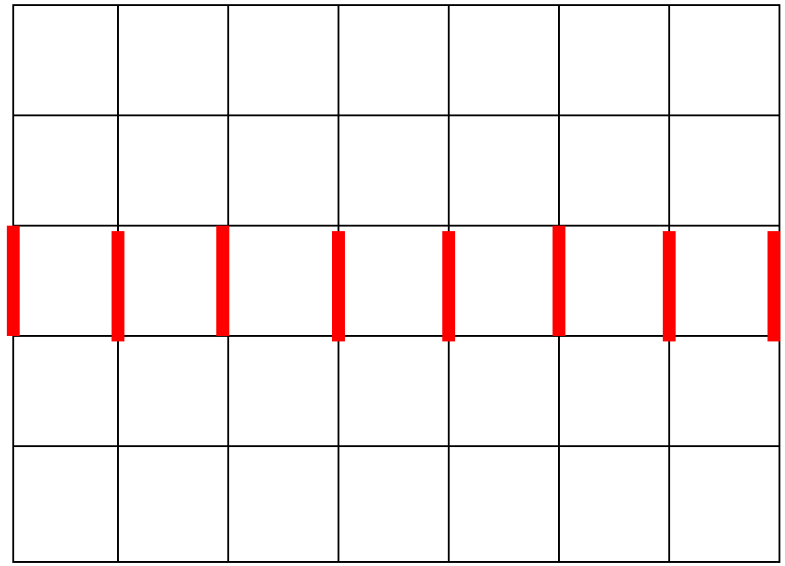
$$\mathcal{Z}_{\mathrm{QCD}} = \langle \, \mathrm{Det}_{AP} D[U] \rangle_U$$

AP: anti-periodic boundary conditions

D(U): quark operator (μ dependent)

Ultra-dense N-colour QCD

- consider: $U_0 \rightarrow^{Z} U_0$:



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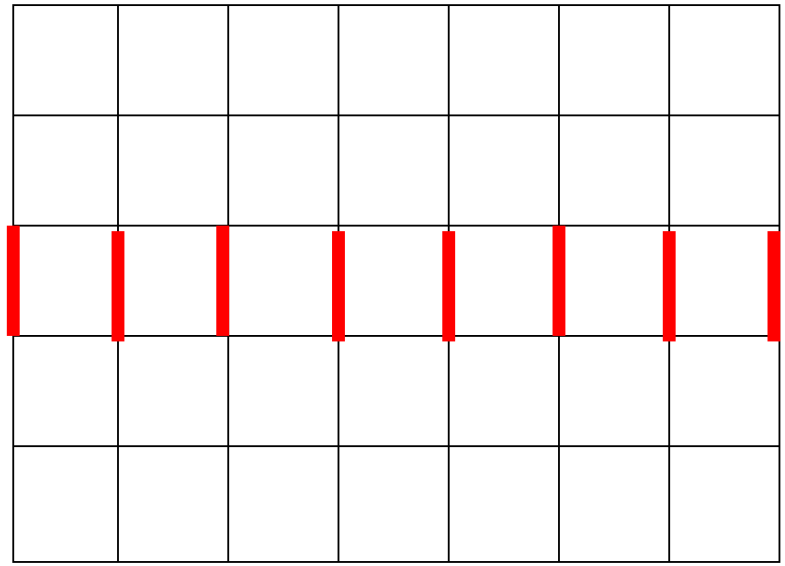
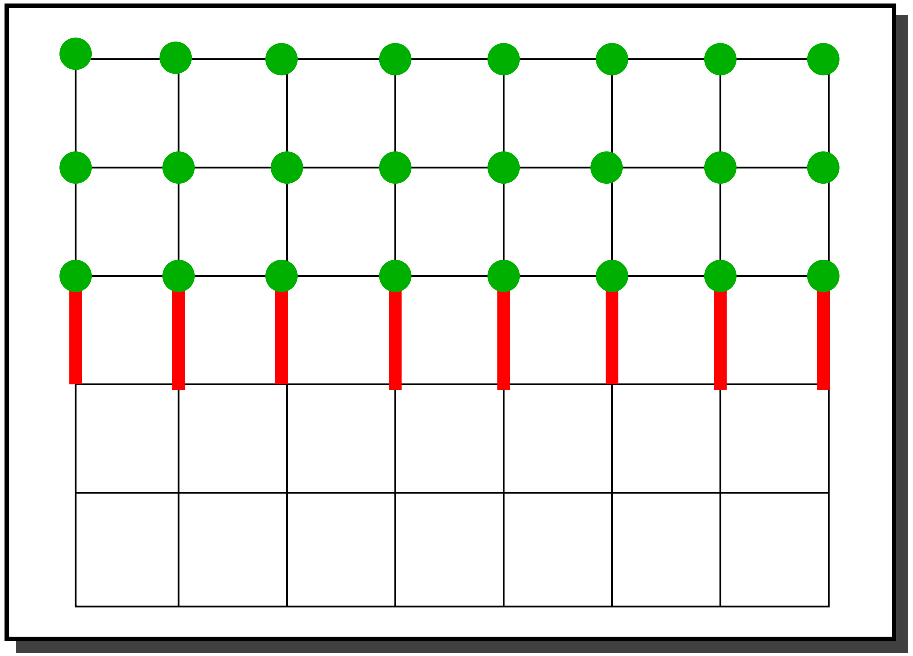


- What is $\mathrm{Det}_{AP} D[^{Z}U]$?

Ultra-dense N-colour QCD

- consider variable substitution: [$Z \in SU(N)$]

$$q'(x) = Z(x) q(x), \qquad U'_\mu(x) = Z(x)\ ^Z U_\mu(x) Z^\dagger(x+\mu)$$



(Roberge, Weiss, NPB 275 (1986) 734)

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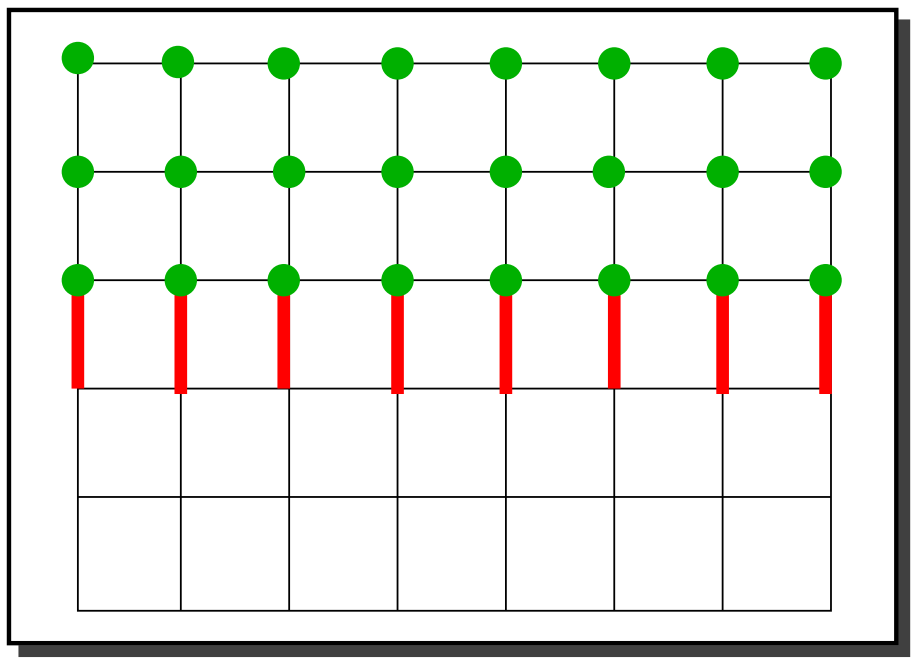
$$q'(x) = Z(x)q(x), \quad U'_\mu(x) = Z(x)\ {}^{Z}U_\mu(x)Z^\dagger(x+\mu)$$

- results:

$U'_\mu = U_\mu$
q': ZA periodic



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Ultra-dense N-colour QCD

- We therefore find:

$$\left\langle \, \mathrm{Det}_{AP} D[{}^{Z}U] \right\rangle_U \;=\; \left\langle \, \mathrm{Det}_{ZAP} D[U] \right\rangle_U$$

Ultra-dense N-colour QCD

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- Confinement phase $\Rightarrow$ centre symmetry unbroken

$$\left\langle \, \mathrm{Det}_{AP} D[^{Z}U] \right\rangle_U = \langle \, \mathrm{Det}_{AP} D[U] \rangle_U$$

and therefore:

$$\mathcal{Z}_{\mathrm{QCD}} = \langle \, \mathrm{Det}_{AP} D[U] \rangle_U = \langle \, \mathrm{Det}_{ZAP} D[U] \rangle_U$$

Ultra-dense N-colour QCD

- Choice of the centre element:
  for N even, there is: $Z_N = (-1)\,\mathbb{1}$
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  for N even, there is: $Z_N = (-1)\,\mathbb{1}$
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- Final result:

$$\mathcal{Z}_{\mathrm{QCD}} = \langle\, \mathrm{Det}_{AP} D[U] \rangle_U \;=\; \langle\, \mathrm{Det}_{P} D[U] \rangle_U$$

  we can use periodic bc for quarks equally well !

  (Langfeld, Wellegehausen, Wipf, arXiv:0906.5554 (hep-lat))

Ultra-dense N-colour QCD

- Interpretation for: number of colours even in the confinement phase

  No Fermi surface at all

  Bose-Einstein condensation of quarks ??

Ultra-dense N-colour QCD

- Numerical results:
  probability distribution for

$$\ln\left(\frac{\mathrm{Det}_{P}\,D[U]}{\mathrm{Det}_{AP}\,D[U]}\right)$$

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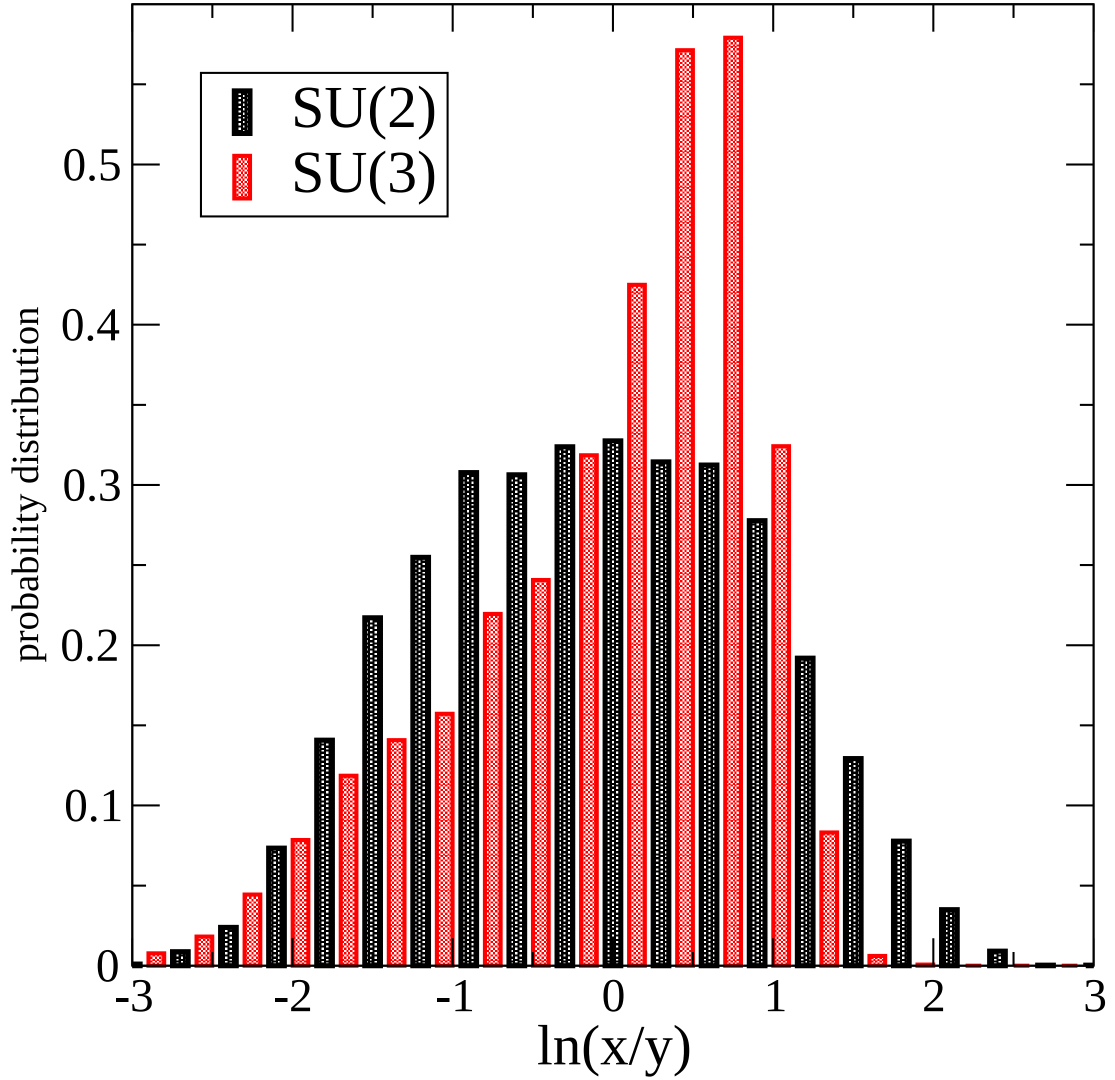
- simulations done in Jena for $N = 2, 3, 4, 5$
  4^4 lattice, $\sigma a^2 = 0.467(10) =$ constant ('t Hooft limit!)

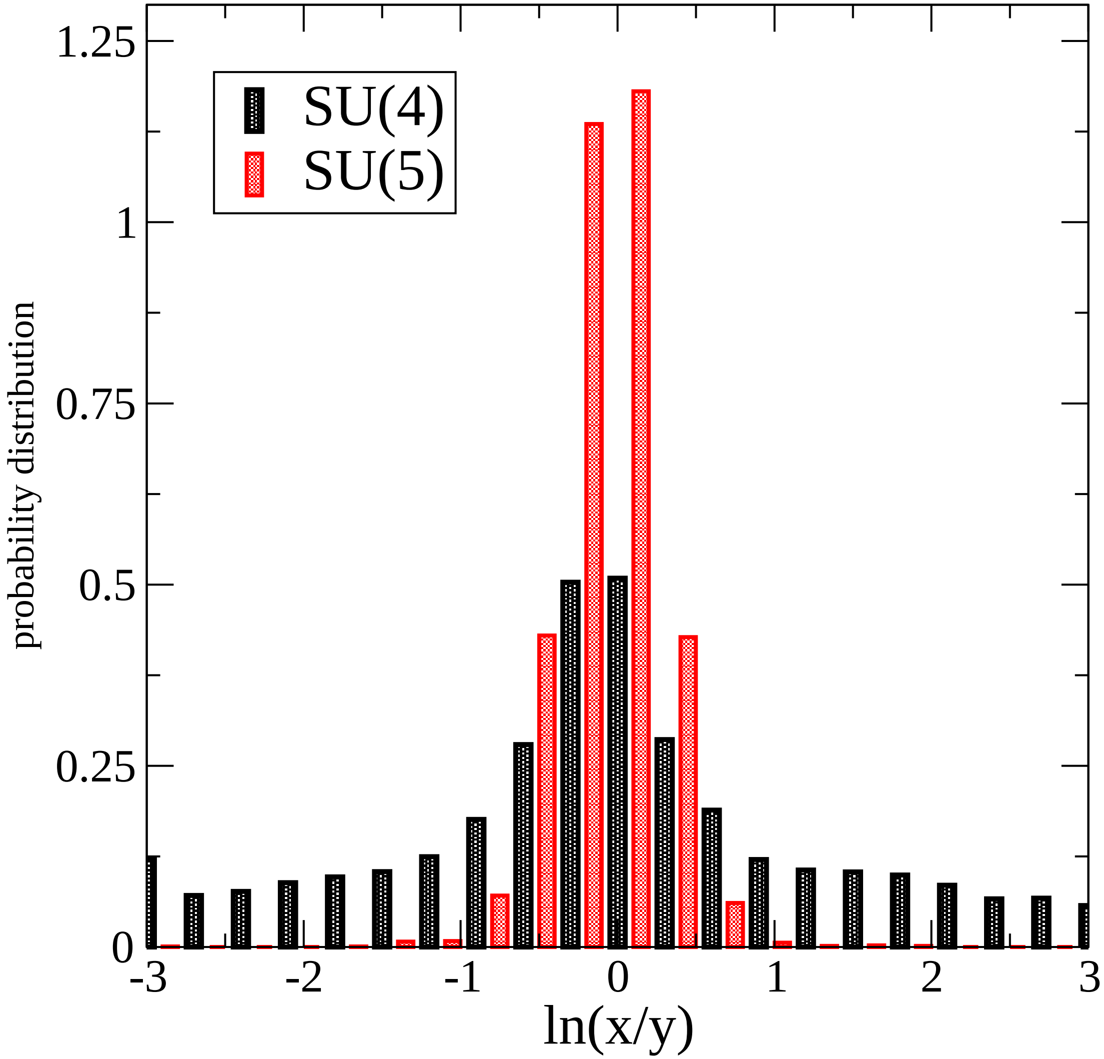
Ultra-dense N-colour QCD

- SU(2) versus SU(3):



Ultra-dense N-colour QCD

- SU(4) versus SU(5):



Conclusions:

- Impact of bc:

periodic	$\leftrightarrow$	anti-periodic
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- Worldline Numerics (GN model)

  $\Rightarrow$ ultra-dense $(\mu/T > \pi)$ regime

Conclusions:

- Impact of bc:

  periodic $\leftrightarrow$ anti-periodic

  Bose-Einstein condesation $\leftrightarrow$ Fermi sphere

- Worldline Numerics (GN model)

  $\Rightarrow$ ultra-dense $(\mu/T > \pi)$ regime

- SU(N) QCD-like theories

  (N even, confinement phase)

  $\Rightarrow$ no Fermi surface, BEC of quarks?