QCD Plasma Instabilities and Nonthermal Fixed Points

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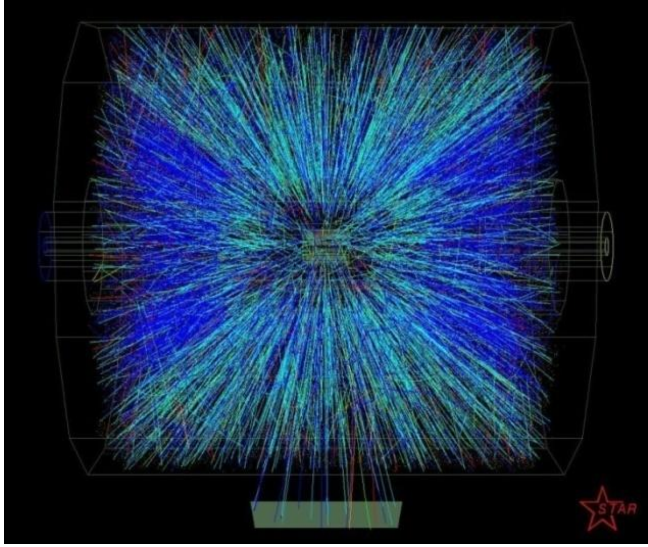
Content

- Nonequilibrium instabilities (Weibel, Nielson-Olesen)
- Classical-statistical lattice gauge theory
- Characteristic time scales of SU(2), SU(3) pure gauge
- Kolmogorov wave turbulence vs. infrared fixed points
- Instability-induced fermion production

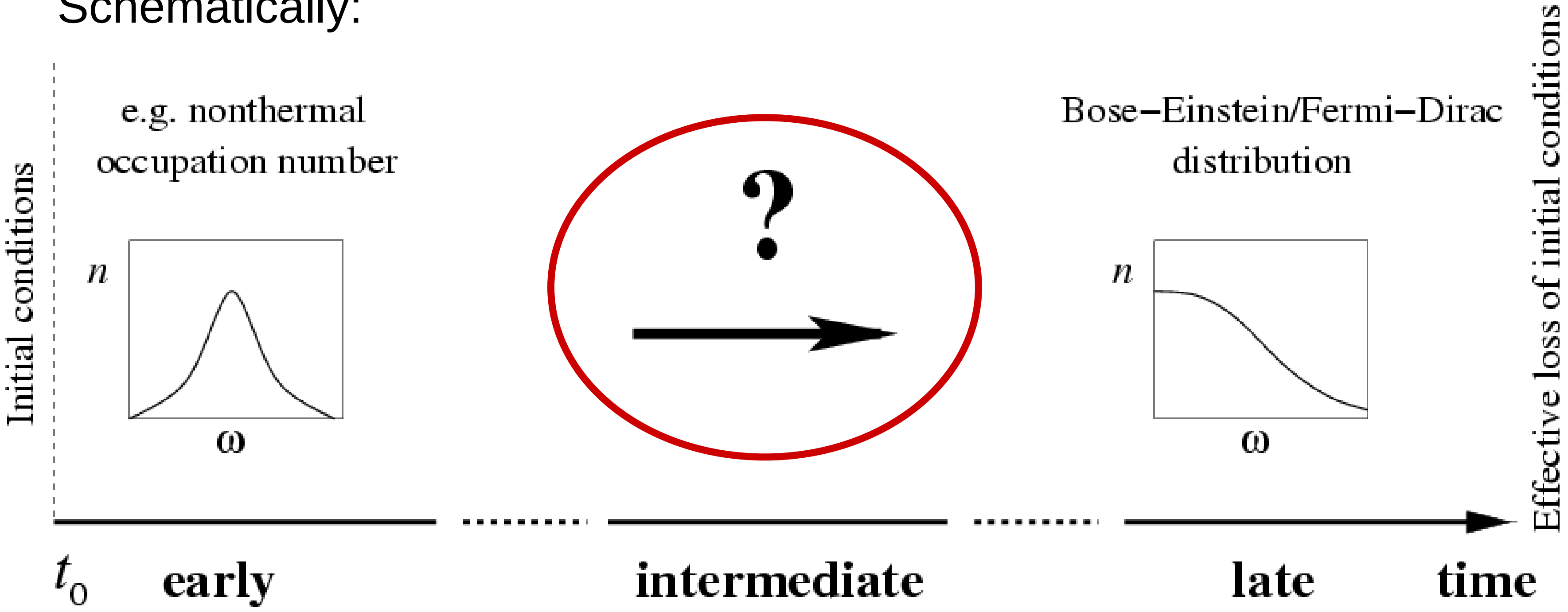
Nonequilibrium dynamics



Relativistic heavy-ion collisions explore strong interaction matter starting from a transient ***nonequilibrium*** state

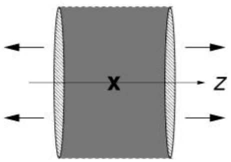
Thermalization process?

Schematically:



- **Characteristic nonequilibrium time scales? Relaxation? Instabilities?**

Short-time dynamics



- ***Anisotropy*** of the stress tensor T_{ij} in a local rest frame:

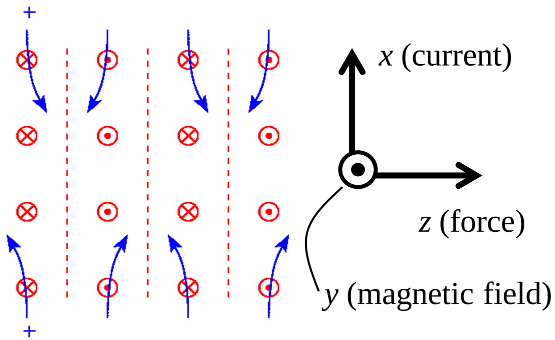
oblate anisotropy $\rightarrow \; T_{xx} \sim T_{yy} \gg T_{zz}$

Isotropization time t_{iso}? In the absence of nonequilibrium instabilities:

$$t_{iso} \sim \mathcal{O}(1/g^4 T)$$

↑ characteristic momentum of typical excitation

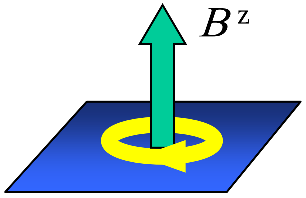
- ***Weibel instability:***



Weibel '59; … Mrowczynski '88, '93, '94; Arnold, Lenaghan, Moore '03; Romatschke, Strickland '03; *very many since then*…

$$t_{iso} \sim \mathcal{O}(1/gT)$$

- ***Nielsen-Olesen instability:***



Nielsen, Olesen '78; Chang, Weiss '79; … Iwasaki '08; Fujii, Itakura '08 …

$$t_{iso} \sim \mathcal{O}(1/g^{1/2} B^{1/2})$$

↑ "homogeneous" background field

…

Nonperturbative description that contains all possible mechanisms:

Real-time lattice QCD

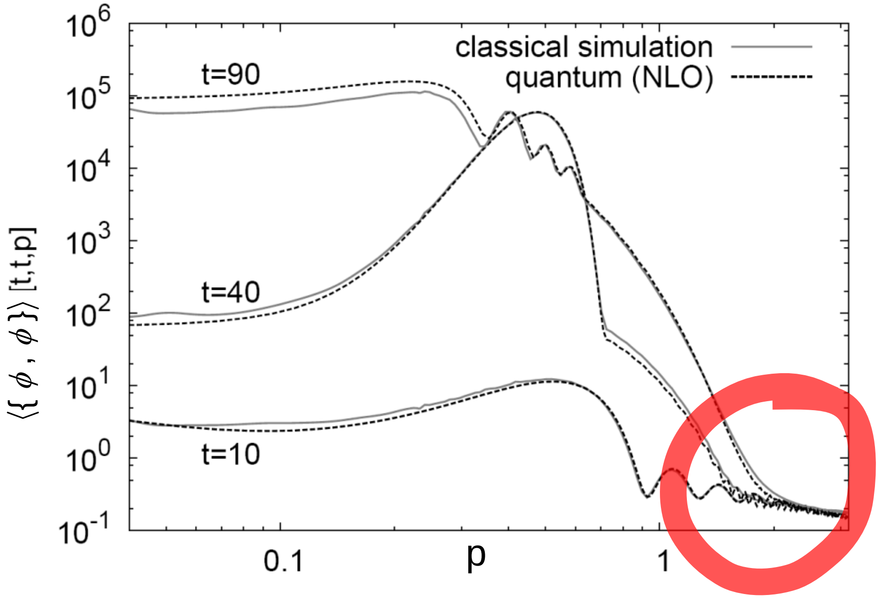
- ***Quantum simulation*** not possible so far (stochastic quantization?)
- ***Classical-statistical simulation:*** includes all fluctuations with

$$\langle\{ A(x) , A(y) \}\rangle \gg \langle[A(x) , A(y)]\rangle$$

i.e. "occupation numbers" $\gg 1$ (includes, in particular, HTL / Vlasov)

physics of nonequilibrium instabilities → high occupation numbers

- Classical simulations well tested for quantum evolutions in scalar theories:



(N=4)-component $\lambda\phi^4$

parametric/spinodal instability

Berges, Rothkopf, Schmidt, *PRL* 101 (2008) 041603

Classical-statistical lattice gauge field simulations

Romatschke, Venugopalan (CGC); Berges, Gelfand, Scheffler, Sexty

Wilson action:

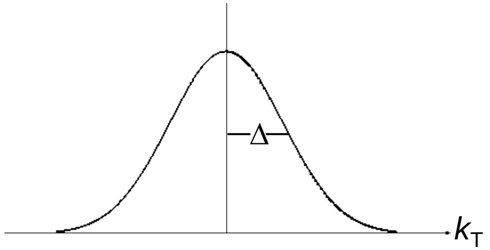
$$S[U] = -\beta_0 \sum_x \sum_i \left\{ \frac{1}{2\mathrm{Tr}\,\mathbf{1}} \left(\mathrm{Tr}\, U_{x,0i} + \mathrm{Tr}\, U^{-1}_{x,0i}\right) - 1 \right\}$$
$$+\beta_s \sum_x \sum_{\substack{i,j \\ i<j}} \left\{ \frac{1}{2\mathrm{Tr}\,\mathbf{1}} \left(\mathrm{Tr}\, U_{x,ij} + \mathrm{Tr}\, U^{-1}_{x,ij}\right) - 1 \right\}$$

Here: $\beta = \beta_0 / \gamma = \beta_s \gamma = 4$, axial-temporal/Coulomb gauge

Initial conditions: Normalized Gaussian probability functional

$P[A(t{=}0),\partial_t A(t{=}0)]$: $\langle\, A(t)\, A(t') \,\rangle = \int DA(0)\, D\partial_t A(0)\, P[A(0),\partial_t A(0)]\, A(t)\, A(t')$

$$\langle\, |A_j^a(t=0,\vec{k})|^2 \,\rangle \sim C \exp\left\{ -\frac{k_x^2 + k_y^2}{2\Delta^2} - \frac{k_z^2}{2\Delta_z^2} \right\}$$

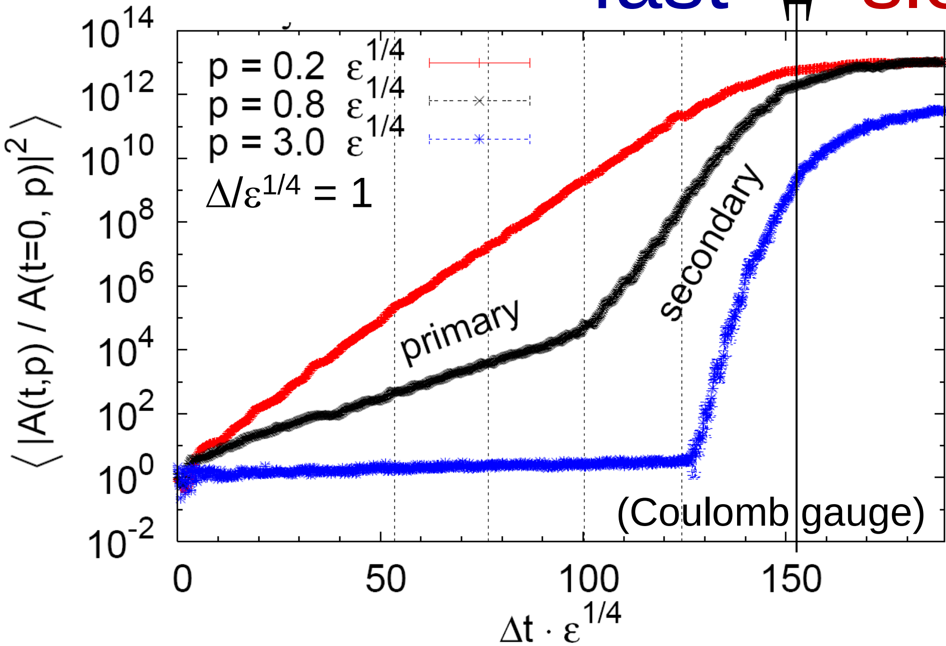


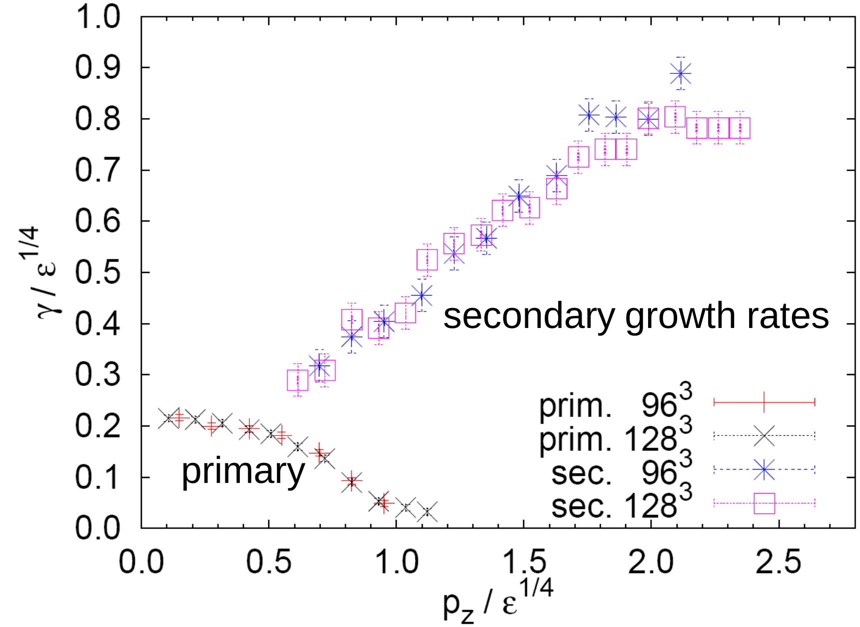
with $\Delta \gg \Delta_z$ (extreme anisotropy)

C is adjusted to obtain a given energy density ε, e.g. $\varepsilon_{\mathrm{RHIC}} \sim 5$–$25$ GeV/fm^3

Characteristic time scales

fast ↔ slow





Berges, Scheffler, Sexty, *PRD* 77 (2008) 034504

Inverse primary growth rates: e.g. $\varepsilon_{\rm RHIC} \sim 5\text{–}25$ GeV/fm^3, $\varepsilon_{\rm LHC} \sim 2 \times \varepsilon_{\rm RHIC}$

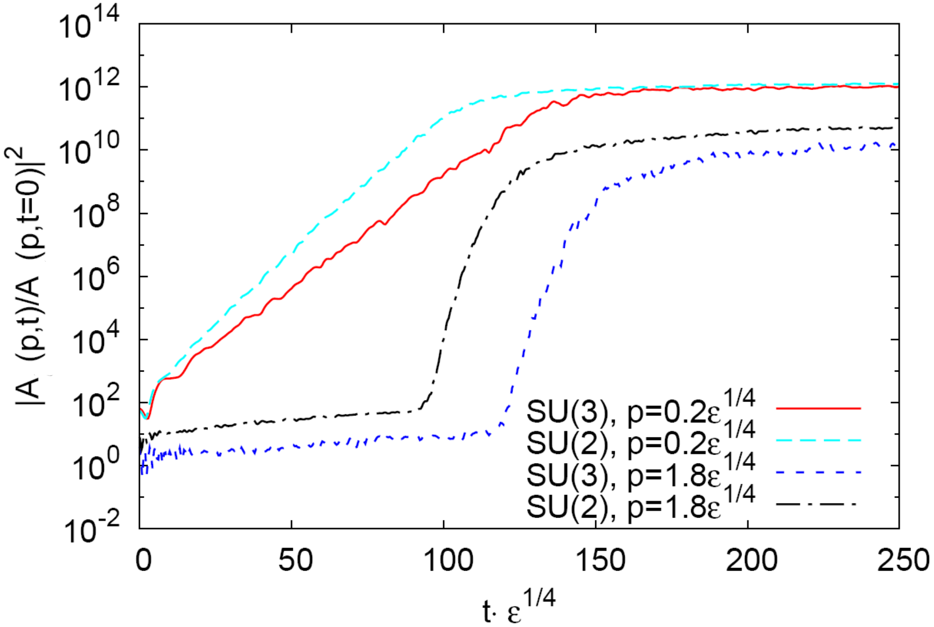
fast:

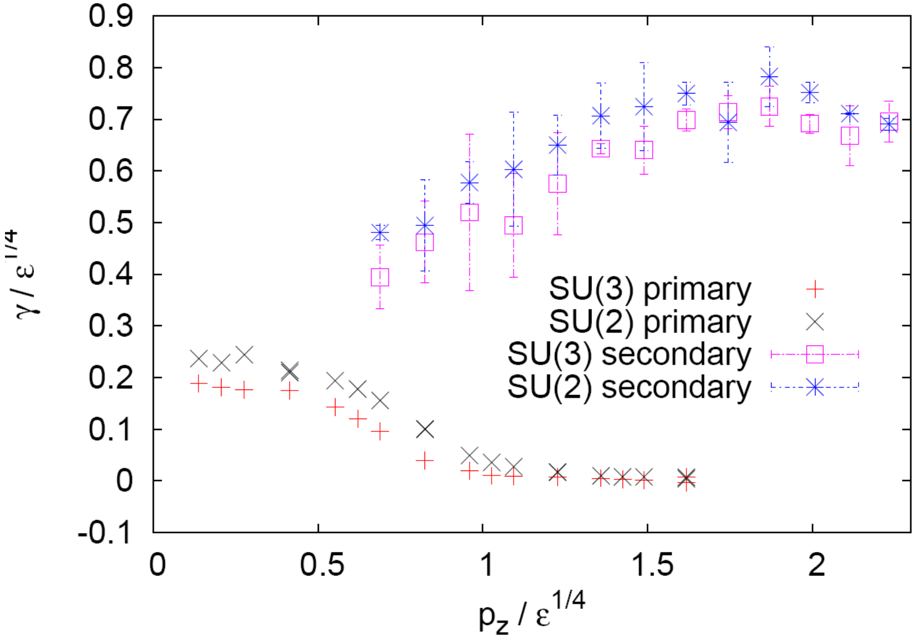
$$\gamma_{\rm max}^{-1} \simeq 1.2 - 1.8\,{\rm fm}/c \;\; ({\rm RHIC})$$

$$\gamma_{\rm max}^{-1} \simeq 1.0 - 1.5\,{\rm fm}/c \;\; ({\rm LHC})$$

SU(2)

Comparison SU(2) vs. SU(3)





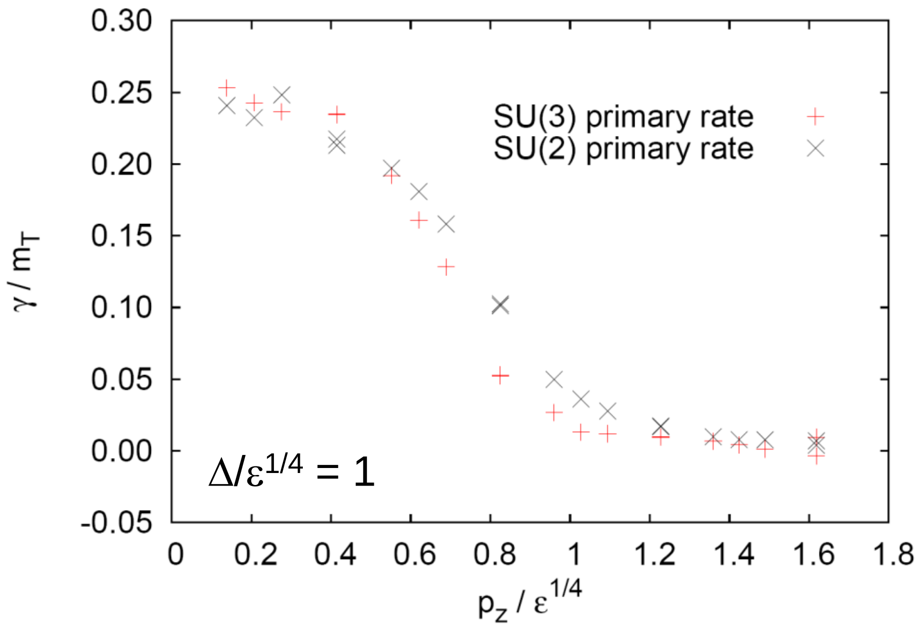
Berges, Gelfand, Scheffler, Sexty, *PLB 677* (2009) 210

$$\gamma^{-1}_{\text{max. pr.}} \simeq 1.6 - 2.4\,\text{fm/c} \qquad (\text{RHIC}),$$
$$\gamma^{-1}_{\text{max. pr.}} \simeq 1.3 - 2.0\,\text{fm/c} \qquad (\text{LHC}).$$

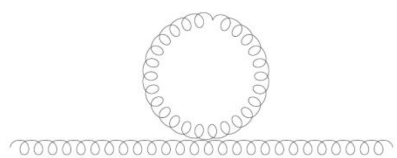
SU(3)

⇒ *reduced* primary growth rates by about 25% for given ε with $\Delta/\varepsilon^{1/4} = 1$

Comparison SU(2) vs. SU(3)



⇒ Measured in units of the characteristic screening masses $m_{T,SU(2)}$ and $m_{T,SU(3)}$, respectively, the primary growth rate is independent of N_c



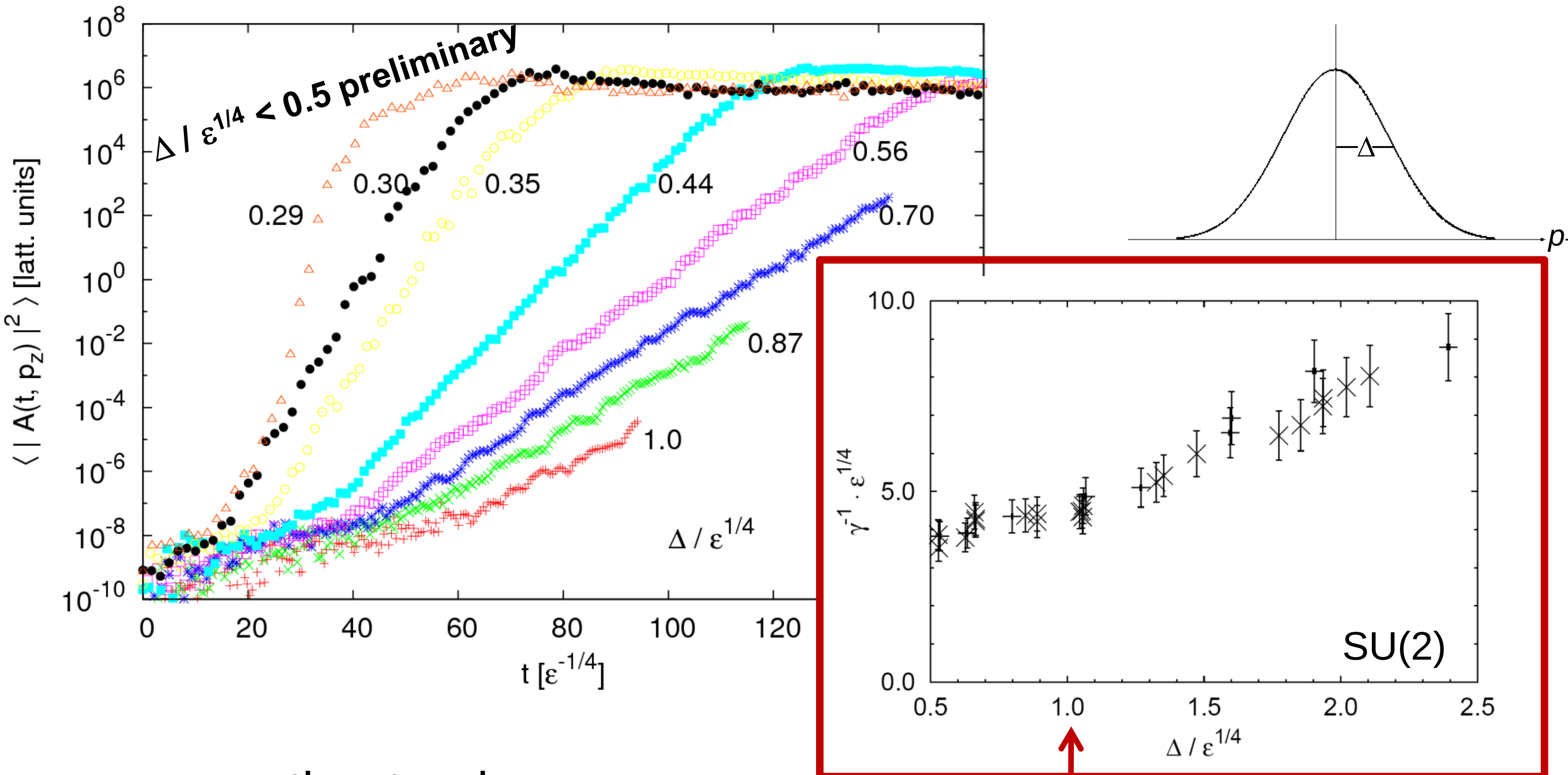
⇒ $\gamma_{SU(3)} / \gamma_{SU(2)} = (3/4) \times (\Delta_{SU(2)} / \Delta_{SU(3)})$:

$C \sim \Delta^2$

$\downarrow$

$\Delta_{SU(2)} = \Delta_{SU(3)}$ i.e. 25% reduction, $\Delta_{SU(2)} / \Delta_{SU(3)} = (8/3)^{1/4}$ i.e. only ~4%

- Initial conditions with **faster isotropization/thermalization**?

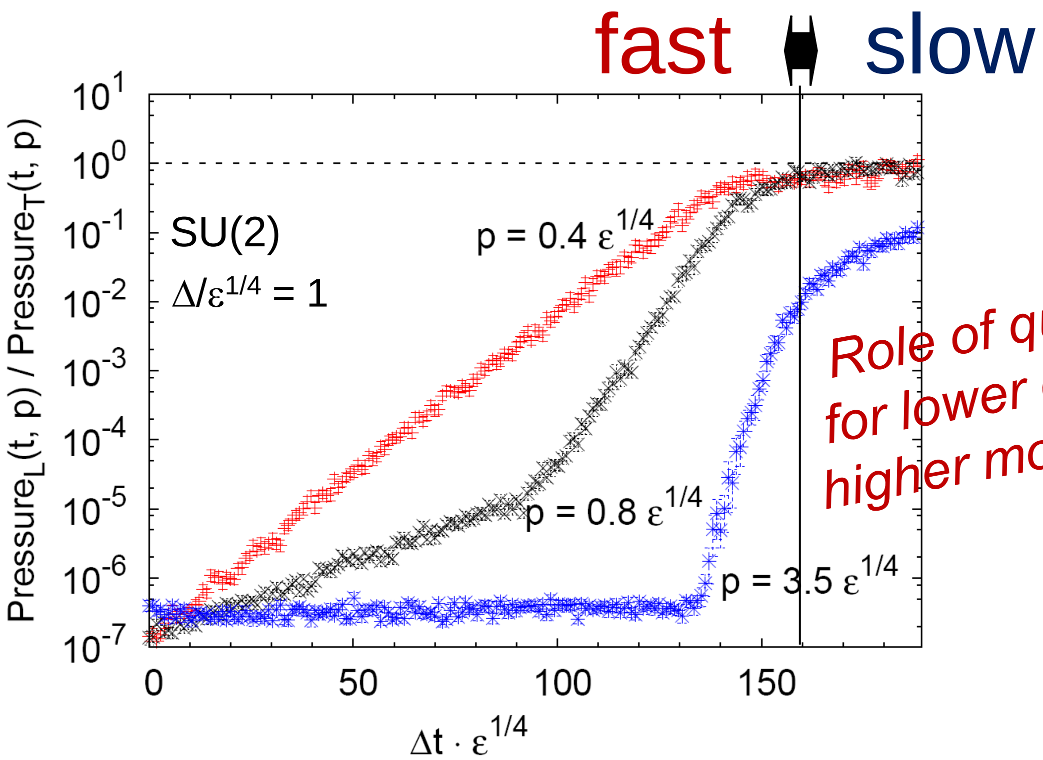


⇒ growth rate γ increases
for smaller $\Delta / \varepsilon^{1/4}$

$\Delta / \varepsilon^{1/4} \ll 1$ corresponds to rather “homogeneous“ field configurations ($\Delta_z / \varepsilon^{1/4} \ll 1$)
$\rightarrow$ Nielsen-Olesen?

Pressure

Spatial Fourier transform of stress tensor $T^{\mu\nu}(t,\mathrm{x})$: $P_L(t,\mathrm{p})$ for $\mu=\nu=3$, $P_T(t,\mathrm{p})$



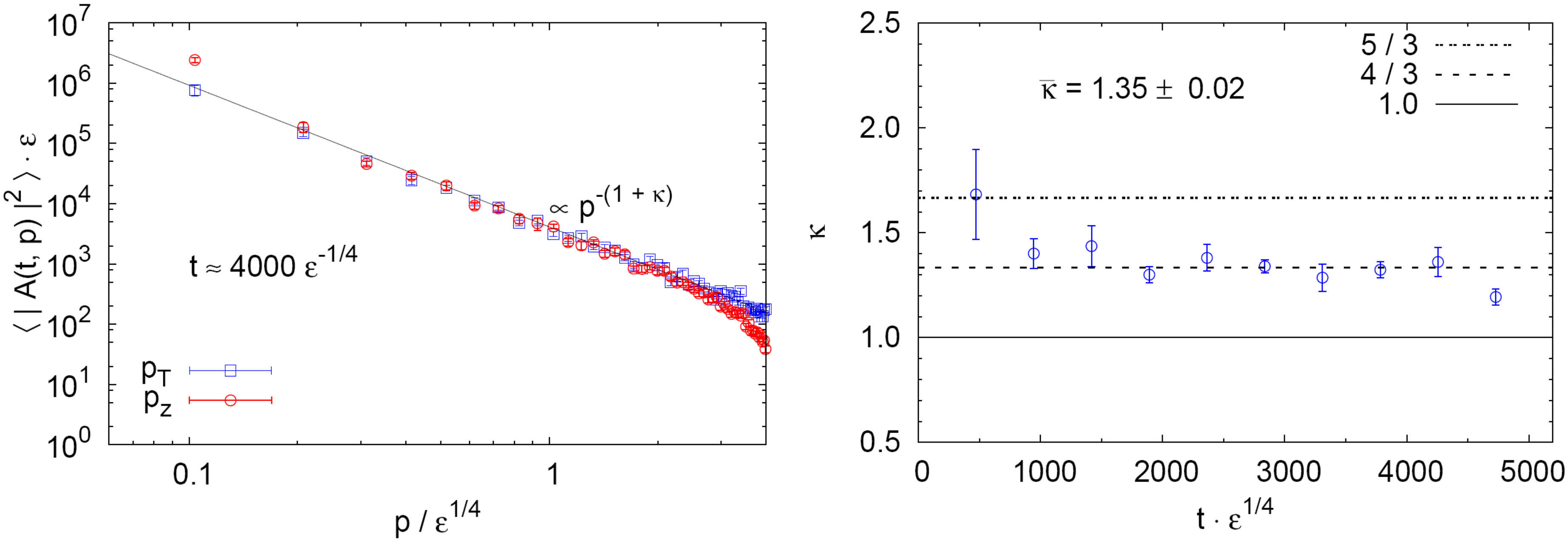
Role of quantum corrections for lower occupied modes at higher momenta?

Fast bottom-up isotropization for momenta:

$p_z \lesssim 1$ GeV

'enough' for hydro?

Kolmogorov wave turbulence



Berges, Scheffler, Sexty, arXiv:0811.4293 [hep-ph]

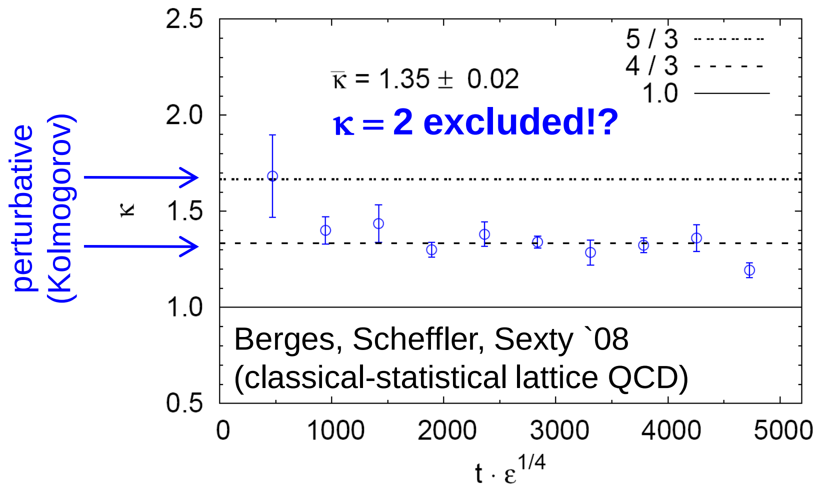
- Scaling exponent κ close to the perturbative value κ = 4/3

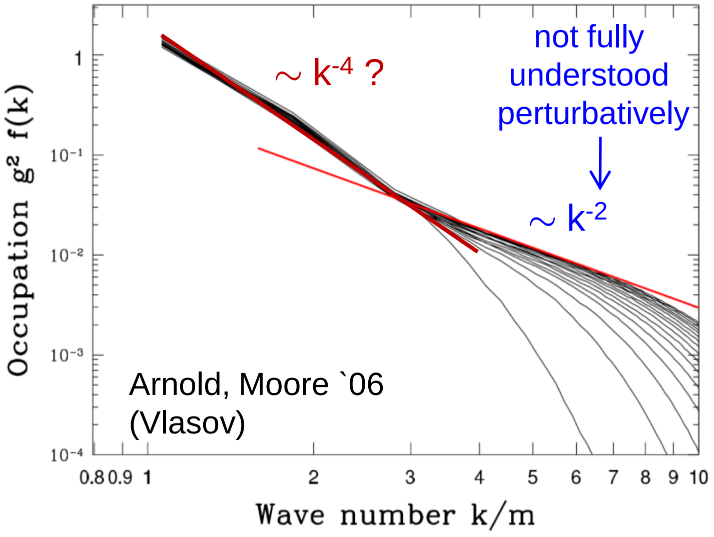
See however: Arnold, Moore *PRD* 73 (2006) 025006; Mueller, Shoshi, Wong, *NPB* 760 (2007) 145

- Different infrared behavior? Nonthermal IR fixed point?

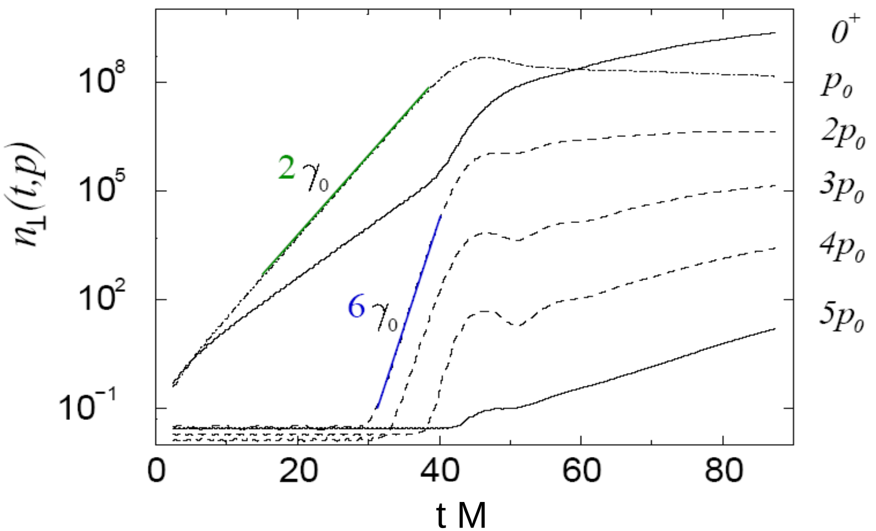
(Infrared occupation number ~ $1/g^2$ ⇒ strongly correlated)

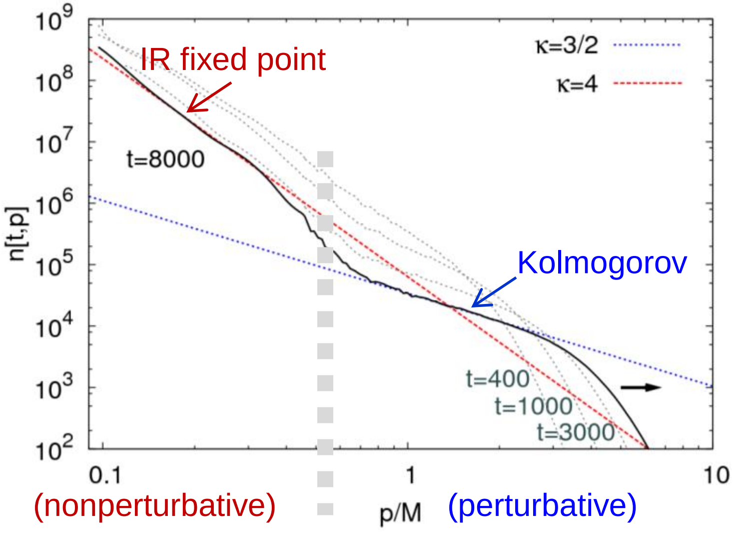
• Apparent discrepancy:





• Compare scalar instability dynamics (parametric resonance):





Berges, Rothkopf, Schmidt, *PRL* 101 (2008) 041603

Slow dynamics: Nonthermal RG fixed points

RG: ‘microscope‘ with varying resolution of length scale

~ 1/*k*

Fixed point: physics looks the same for ‘all‘ resolutions (in rescaled units)

scaling form, e.g.

$$F_k = \frac{1}{2}\langle\{\Phi,\Phi\}\rangle_k \sim \frac{1}{k^{2+\kappa}} \quad \longleftrightarrow \quad \frac{\partial}{\partial k}\Big(\overbrace{F_k\, k^{2+\kappa}}^{\text{‘length rescaled‘ correlator}}\Big) = 0$$

κ: ‘occupation number‘ exponent

similarly, retarded propagator:

$$G_k^{\mathrm{R}} \sim \frac{1}{k^{2-\eta}}$$

η ⟵ anomalous dimension

Typically not for *all* resolutions:

- IR fixed point for k $\to$ 0
- UV fixed point for k $\to$ ‘∞‘

A) Hierarchy of infrared fixed point solutions for scalar *N*-component QFT:

Berges, Hoffmeister, *NPB* 813 (2009) 383

- vacuum: $\kappa = -\eta$
- thermal: $\kappa = -\eta + z$

Fluctuation-dissipation relation:

$$\frac{F_k(\omega,\mathbf{p})}{(G_k^{\mathrm{R}} - G_k^{\mathrm{R}\star})(\omega,\mathbf{p})} \sim n_{\mathrm{BE}}(\omega) + \frac{1}{2}$$

dynamical exponent *z*

- nonequilibrium: $\kappa = -\eta + z + d$

No fluctuation-dissipation relation:

$$\text{"}n(\omega,\mathbf{p})\text{"}$$

spatial dimension *d*

, ... (NLO 1/*N*)

Relativistic ($z \simeq 1$) scalar inflaton in $d = 3$, $\eta \simeq 0$ $\Rightarrow$ $\kappa \simeq 4$, …

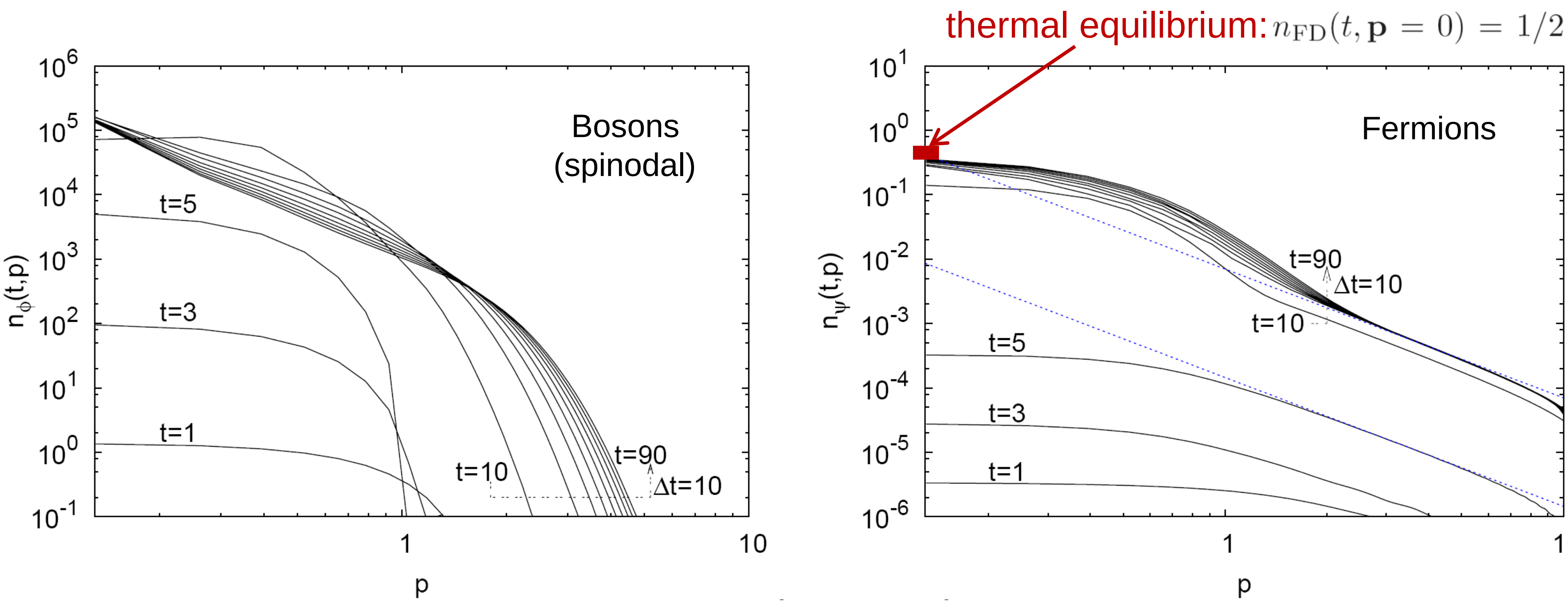
B) Ultraviolet fixed point solutions (classical-statistical limit, Kolmogorov):

Relativistic scalar inflaton in $d = 3$ $\Rightarrow$ $\kappa \simeq 3/2$, $4/3$, …

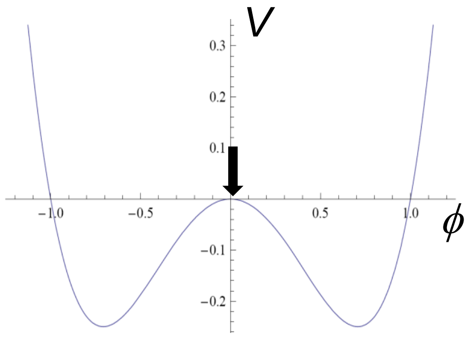
No Kolmogorov turbulence in the far UV due to quantum corrections

Instability-induced fermion production

Quantum evolution of $SU(2)_L \times SU(2)_R$ linear sigma model (2PI *1/N* to NLO):

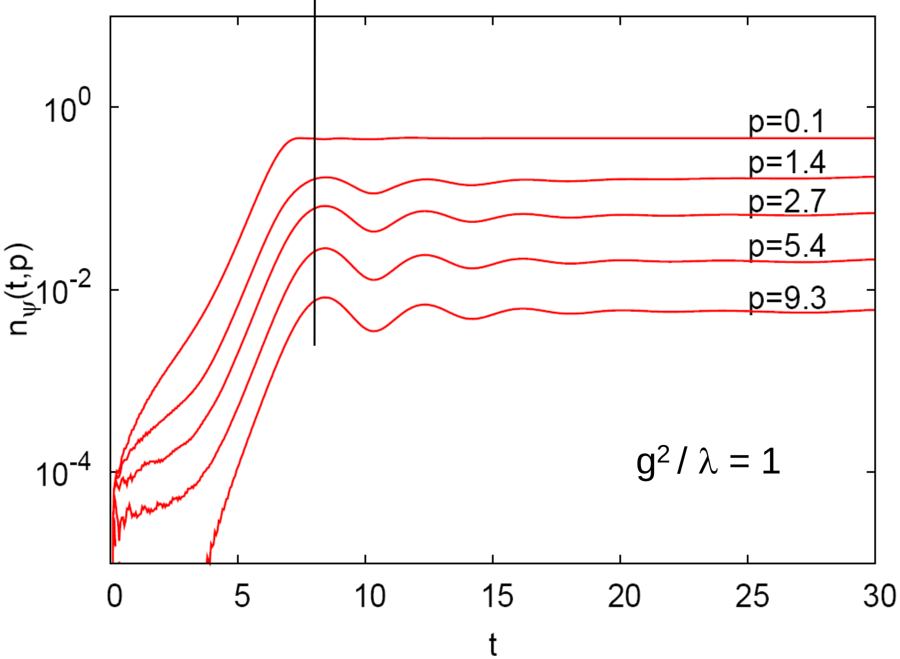


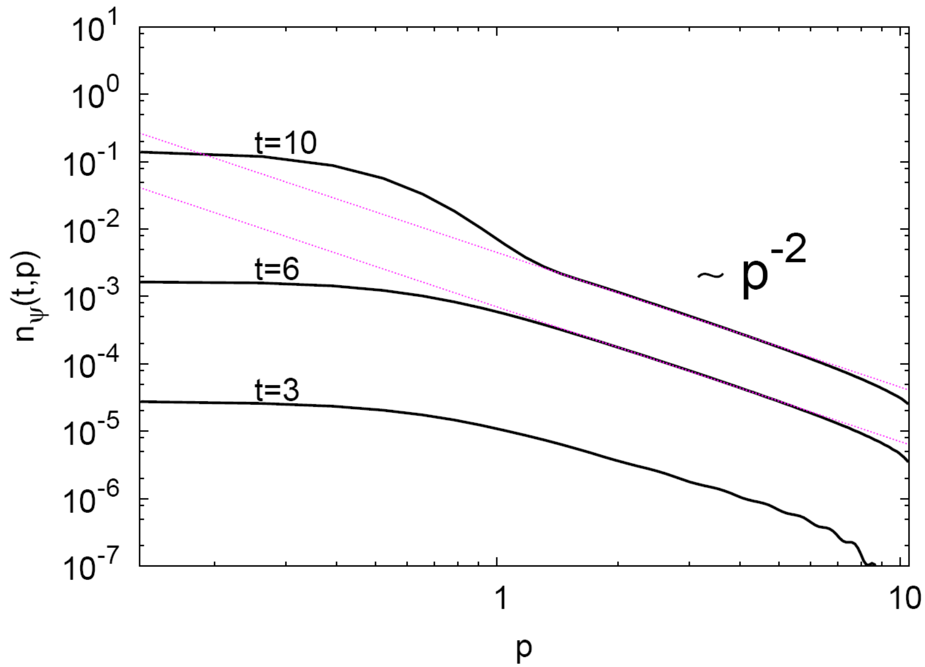
Berges, Pruschke, Rothkopf, *PRD* 80 (2009) 023522



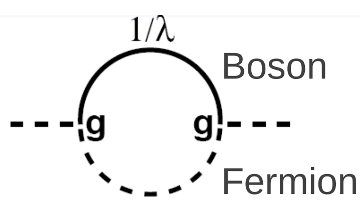
- Fermion production proceeds with the maximum primary boson growth rate!
- Fast approach to Fermi-Dirac distribution in the infrared!

fast ↔ slow





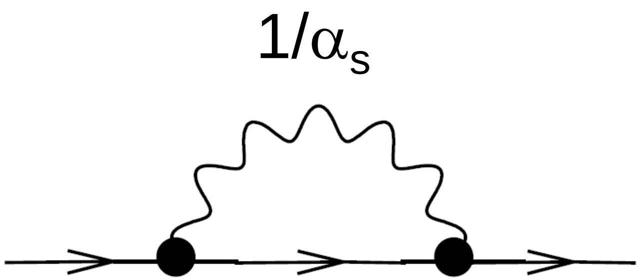
- Fast fermion growth induced via boson-fermion loop:



$\sim \frac{g^2}{\lambda}$

- Scaling behavior at higher momenta (no IR scaling $\rightarrow$ Pauli principle)
- Bosons practically unaffected for Yukawa couplings $\lesssim O(1)$

$\rightarrow$ **Instability-induced fermion production** can lead to substantial deviations from standard production processes



$\sim \; O(1)$

Quantitatively: **Classical-statistical lattice QCD with** (quantum) **fermions** can be simulated with well established techniques!

Conclusions

- **Plasma instabilities** for CGC type initial conditions ($\Delta / \varepsilon^{1/4} = 1$)

$$\gamma^{-1} \simeq 1 - 2 \text{ fm/c}$$

- **'Bottom-up' isotropization** for $\boldsymbol{p \lesssim 1}$ **GeV**,
  i.e. (optimistically) about the range where hydro 'works'

- Initial conditions with **faster isotropization/thermalization**?
  $\rightarrow \quad \Delta \sim O(\Lambda_{QCD})$?

- (Perturbative) **Kolmogorov wave turbulence** with $\kappa \simeq 4/3$
  Nonthermal **IR fixed point** in QCD?

- **Instability-induced fermion production** can lead to substantial deviations from standard production processes

  $\rightarrow$ **fast thermalization of low-momentum fermions** on time scale of maximum boson growth rate seen in linear sigma model