G_2 Gauge Theories

Effective Polyakov Loop Models, Casimir Scaling and the Gauge-Higgs Phase Diagram

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Outline

- G_2 is the smallest simple and simply connected Lie group with a trivial centre.
- Investigations of G_2 YM help to clarify the relevance of centre symmetry for confinement. →Holland et al. (2003), →Greensite et al. (2008)
- Similarly as in QCD with dynamical quarks is the Polyakov loop an approximate order parameter.
  $\Rightarrow$ Effective theories for Polyakov loop variables include the relevant properties of G_2 gluodynamics.
- New test case for the Svetitsky-Yaffe conjecture:
  The confinement-deconfinement transition in a $d+1$ dimensional pure gauge theory can be described by an effective spin model in d dimensions.
- To high precision we check for Casimir scaling of the string tension in different representations on intermediate scales (before strings breaking occurs). →Liptak and Olejnik (2008)
- When a fundamental Higgs field is coupled there is a transition to $SU(3)$ gluodynamics. All additional degrees of freedom are frozen out for large hopping parameter. →Pepe and Wiese (2006)

- G_2 is the smallest of the five exceptional simple Lie groups.
- It is a subgroup of $SO(7)$ subject to seven independent cubic constraints for the 7-dimensional matrices g representing $SO(7)$,

$$T_{abc} = T_{def}\, g_{da}\, g_{eb}\, g_{fc}$$

with the total antisymmetric tensor T given by

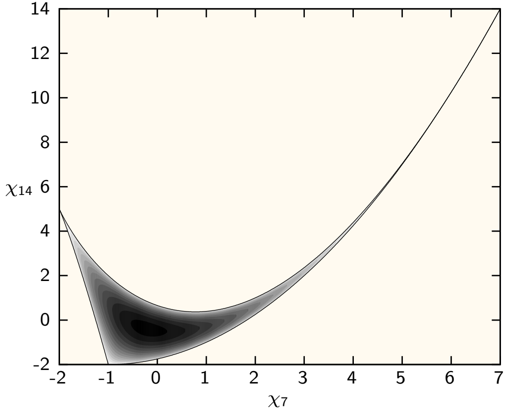
$$T_{127} = T_{154} = T_{163} = T_{235} = T_{264} = T_{374} = T_{576} = 1.$$

- It has 14 generators and is of rank 2.
- Fundamental representations are the defining 7-dimensional and the adjoint 14-dimensional representation.

For the effective theories formulated in gauge invariant (traced) Polyakov loops only the reduced Haar measure $d\mu \propto J\, d\chi_7\, d\chi_{14}$ is needed with

$$J^2 = \left(4\chi_7^3 - \chi_7^2 - 2\chi_7 - 10\chi_7\chi_{14} + 7 - 10\chi_{14} - \chi_{14}^2\right)\left(7 - \chi_7^2 - 2\chi_7 + 4\chi_{14}\right).$$

→Uhlmann et al. (2006)



No symmetry of the fundamental domain. ⇔ Trivial centre!

Situation in $SU(3)$ gluodynamics

- Quarks and anti-quarks transform under fundamental representations 3, $\bar{3}$.
- Their charges can only be screened by particles with non-vanishing 3-ality, especially not by gluons!
- In the confining phase the static quark anti-quark potential is linearly rising up to arbitrary long distances and the Polyakov loop expectation value vanishes.
- Polyakov loop as order parameter for the $\mathbb{Z}_3$ centre symmetry and for confinement.

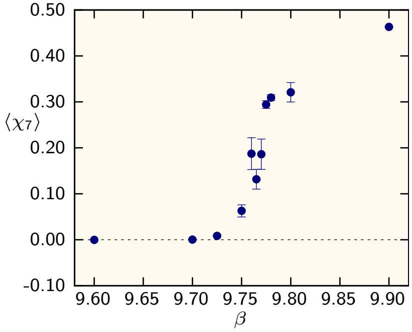
Situation in G_2 gluodynamics

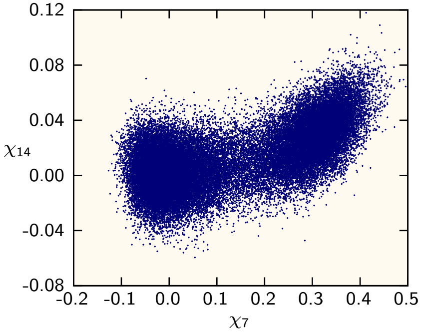
- Quarks transform under the 7-dimensional fundamental/defining representation, gluons under the 14-dimensional fundamental representation.
- Three centre-blind dynamical gluons can screen the colour charge of a single quark,

$$(7) \otimes (14) \otimes (14) \otimes (14) = (1) \oplus \cdots .$$

- The flux tube between two static quarks can break and the Polyakov loop does not vanish even in the confining phase.
  $\Rightarrow$ Polyakov loop is (at best) an approximate order parameter!
- Confinement is defined as confinement at intermediate scales.

We still see a clear signal in the Polyakov loop at the confinement-deconfinement transition:





Effective models for G_2 gluodynamics

Starting with the Wilson action

$$S_W = \beta \sum_{\Box} \left(1 - \frac{1}{N_C} \text{Re tr } U_{\Box} \right), \quad \beta = \frac{2N_C}{a^4 g^2}, \quad N_C = 7$$

we apply a strong coupling expansion (for small β). The truncation scheme combines following features:

- Ordering by powers of β. This is related to the dimension of corresponding representations.
- Ordering by distance of interacting Polyakov loops.

- Now only the leading order is studied.
  →Wellegehausen, Wipf and Wozar (2009)
- This amounts to nearest neighbour interaction.
- Only the two fundamental representations $[1,0]=(7)$ and $[0,1]=(14)$ are involved.
- The action is explicitly given by

$$S_{\text{eff}} = \lambda_7 \sum_{\langle \mathbf{xy} \rangle} \chi_7(\mathcal{P}_{\mathbf{x}})\chi_7(\mathcal{P}_{\mathbf{y}}) + \lambda_{14} \sum_{\langle \mathbf{xy} \rangle} \chi_{14}(\mathcal{P}_{\mathbf{x}})\chi_{14}(\mathcal{P}_{\mathbf{y}}),$$

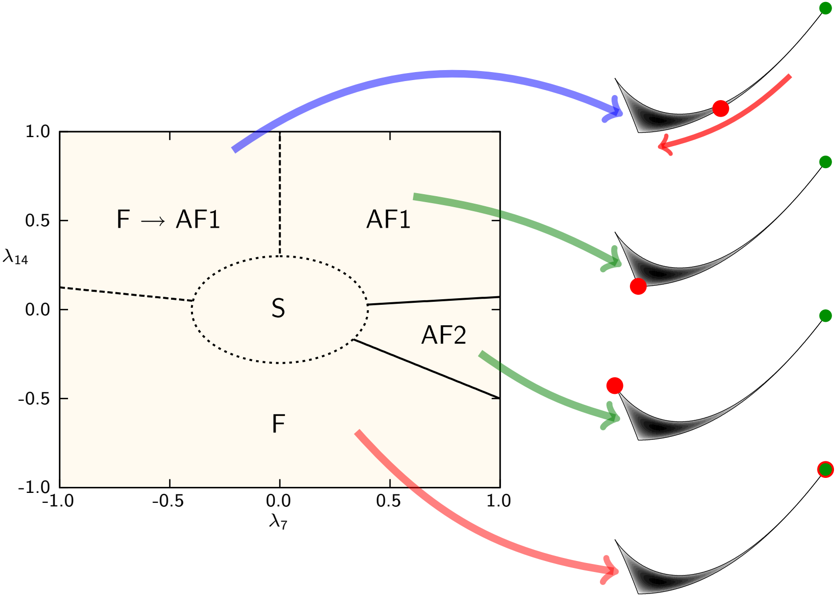
- In next-to leading order there are 6 additional terms with nearest neighbour interaction.

- For large couplings $|\lambda_7|$ and $|\lambda_{14}|$ fluctuations of the Polyakov loop are suppressed.
- We compute the phase diagram by minimising the classical action.
- We anticipate that there are anti-ferromagnetic phases.
- Polyakov loops $\mathcal{P}$ and corresponding characters $\boldsymbol{\chi} = (\chi_7, \chi_{14})(\mathcal{P})$ should take a constant value on each of the sub-lattices

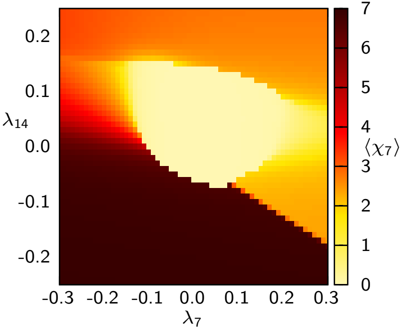
$$\Lambda_\mathrm{o} = \{\mathbf{x} \,|\, x_1 + x_2 + x_3 \text{ odd}\} \quad \text{and} \quad \Lambda_\mathrm{e} = \{\mathbf{x} \,|\, x_1 + x_2 + x_3 \text{ even}\}\,.$$

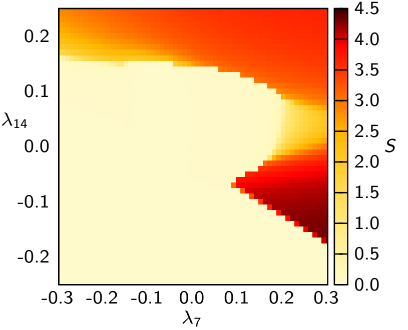
We find that the Polyakov loop on one sub-lattice is equal to the group identity with $\boldsymbol{\chi}_\mathrm{o} = (7, 14)$. There is one ferromagnetic, two anti-ferromagnetic and one phase in transition from ferro- to anti-ferromagnetic.

Classical analysis of the fundamental effective model

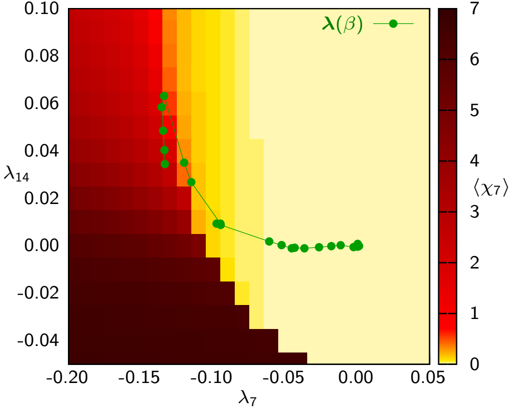


- Simulations were done on an 8^3 lattice.
- We also measure the staggered magnetisation $S = \frac{1}{2}\langle|\chi_{7,\mathrm{e}} - \chi_{7,\mathrm{o}}|\rangle$ in order to gain information about the anti-ferromagnetic phases.



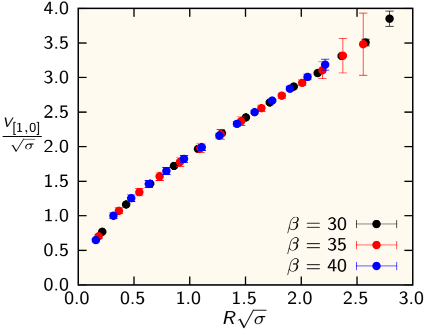


- With inverse Monte-Carlo techniques we can determine the couplings $\boldsymbol{\lambda}(\beta)$.
  →Wozar et al. (2007,2008), →Velytsky (2008)
- We utilize the canonical demon method which has led to stable results $SU(3)$ YM. →Hasenbusch et al. (1995)



Casimir scaling for 3-dimensional G_2 gluodynamics

At zero temperature G_2 gluodynamics is confining and there is a linearly rising static quark anti-quark potential at intermediate distances.
The string tension of gluodynamics with a general gauge group depends on the scale and the representation of the static quarks:



- At intermediate scales we expect Casimir scaling. The string tensions for different representations $\mathcal{R}$ and $\mathcal{R}'$ scale according to $\frac{\sigma_{\mathcal{R}}}{c_{\mathcal{R}}} = \frac{\sigma_{\mathcal{R}'}}{c_{\mathcal{R}'}}$ with $c_{\mathcal{R}}$ being the quadratic Casimir of the representation $\mathcal{R}$.
- At large distances there can be dynamical colour screening and the string tension depends on the transformation properties with respect to the centre subgroup of the gauge group (N-ality).

$\Rightarrow$ Vanishing asymptotic string tension for G_2.

Casimir scaling for 3-dimensional G_2 gluodynamics

- For a given representation $\mathcal{R} = [p, q]$ the quadratic Casimir is

$$c_{[p,q]} = 2p^2 + 6q^2 + 6pq + 10p + 18q.$$

- These values are normalised with respect to the defining representation by $C_\mathcal{R} = c_\mathcal{R}/c_{[1,0]}$.

representation $\mathcal{R}$	$[1,0]$	$[0,1]$	$[2,0]$	$[1,1]$	$[0,2]$	$[3,0]$	$[4,0]$	$[2,1]$
dimension $d_\mathcal{R}$	7	14	27	64	77	77	182	189
Casimir value $c_\mathcal{R}$	12	24	28	42	60	48	72	64
Casimir ratio $C_\mathcal{R}$	1	2	7/3	3.5	5	4	6	16/3

Casimir scaling for 3-dimensional G_2 gluodynamics

- The static quark anti-quark potential is computed using the behaviour of rectangular Wilson loops in representation $\mathcal{R}$,

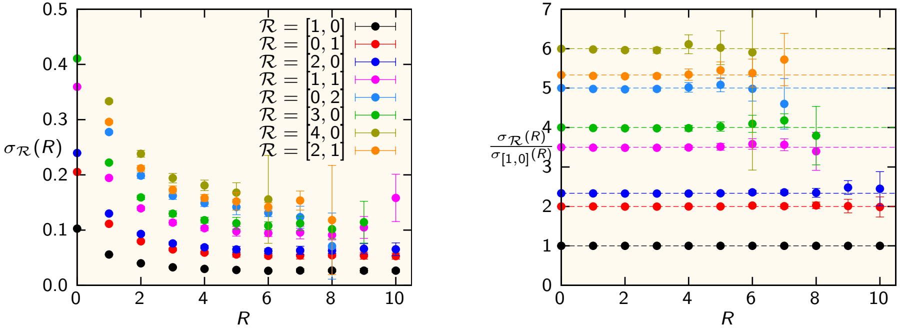
$$\langle W_\mathcal{R}(R,T)\rangle = \exp\big(\kappa_\mathcal{R}(R) - V_\mathcal{R}(R)T\big) \quad \text{with} \quad V_\mathcal{R}(R) \approx \gamma_\mathcal{R} - \frac{\alpha_\mathcal{R}}{R} + \sigma_\mathcal{R} R.$$

- The string tension $\sigma_\mathcal{R}$ is then computed from the Creutz ratio

$$\sigma_\mathcal{R}(R) = \frac{\alpha_\mathcal{R}}{R(R+\rho)} + \sigma_\mathcal{R} = -\frac{1}{\tau\rho}\ln\frac{\langle W_\mathcal{R}(R+\rho,T+\tau)\rangle\,\langle W_\mathcal{R}(R,T)\rangle}{\langle W_\mathcal{R}(R+\rho,T)\rangle\,\langle W_\mathcal{R}(R,T+\tau)\rangle}.$$

  For the evaluation on our 28^3 lattice we used $T = 12$, $\tau = 2$ and $\rho = 1$.
- Monte-Carlo simulations are performed using the Lüscher-Weisz →(2001) exponential error reduction method with multilevel updates.
- The Wilson loops are computed without any smearing.
- Link updates are done via a local version of the hybrid Monte-Carlo algorithm.

Casimir scaling for 3-dimensional G_2 gluodynamics



⇒ Casimir scaling works!

4-dimensional gauge-Higgs model

A Higgs field in the fundamental representation is coupled to the G_2 gauge theory. The corresponding action is given by

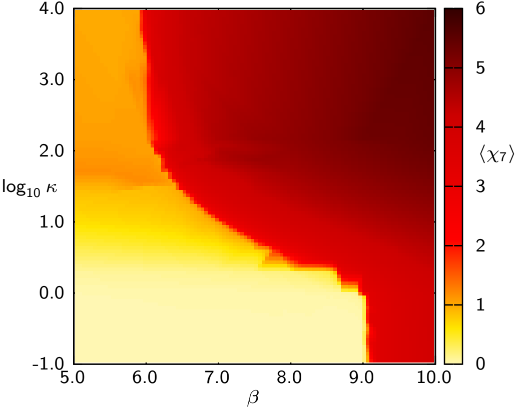
$$S = \beta \sum_{\Box} \left(1 - \frac{1}{7} \operatorname{tr} \operatorname{Re} U_{\Box}\right) - \kappa \sum_{x,\mu} \Phi_{x+\hat{\mu}} U_{x,\mu} \Phi_x$$

with Φ_x as 7-dimensional real vector normalised to $\Phi \cdot \Phi = 1$.
From group theory follows: →Pepe and Wiese (2006)

- For $\beta \to \infty$ all links can be gauge-fixed to $\mathbb{1}$. Then the pure Higgs sector of this model is invariant under a global $SO(7)$ symmetry.
- For large κ the global $SO(7)$ invariance of the Higgs model is spontaneously broken to $SO(6)$ (second order transition).
- Gauging the G_2 subgroup of $SO(7)$ (at finite β) turns this remaining global $SO(6)$ symmetry into a local $SU(3)$ symmetry.
- In this case the 6 Goldstone bosons are eaten and the longitudinal components of G_2 gluons become massive.
- The Higgs mechanism only leaves the $[1,1]_{SU(3)}$ part of the gluons massless and the G_2 gauge theory is reduced to its $SU(3)$ gauge sector.

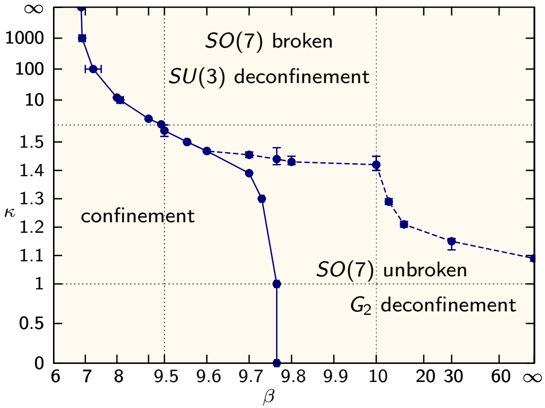
4-dimensional gauge-Higgs model

- We measure the Polyakov loop as an (approximate) order parameter for confinement and investigate the corresponding critical curve in the β-κ plane (here on $12^3 \times 2$ lattice).
- For large κ the confinement phase in $SU(3)$ is characterised by $\langle\chi_7\rangle = 1$.



4-dimensional gauge-Higgs model

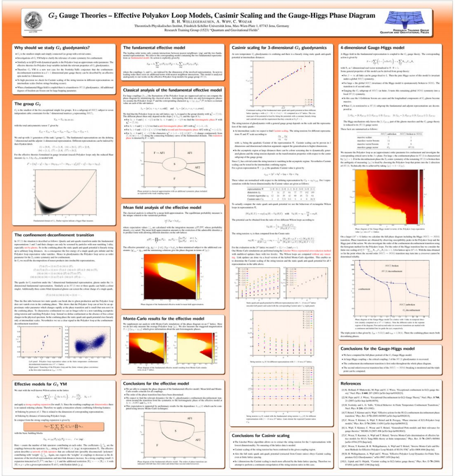
- On larger (up to $20^3 \times 6$) lattices we calculate the full phase diagram including the Higgs $SO(7) \rightarrow SO(6)$ transition.
- Phase transitions are obtained by observing susceptibility peaks in the Polyakov loop and the Higgs part of the action.
- Orders of transitions are determined using histograms for Polyakov loops and finite size scaling for the Higgs part of the action.



Conclusions

- The fundamental effective Polyakov loop model was analyzed extensively.
- The couplings $\boldsymbol{\lambda}(\beta)$ can be monitored and they fully agree with our expectations.
- Casmir scaling was confirmed for the 3-dimensional G_2 gluodynamics for up to 8 representations without smearing.
- The full phase diagram of the gauge-Higgs model was determined.

More information can be found on the poster:



Thank you!