The finite-T phase structure of lattice QCD with twisted-mass Wilson fermions

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Outline of the talk

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3. Anticipating the phase structure
4. Aoki phase
5. First order bulk transition
6. Thermal transition
7. Feasibility of locating β_t at maximal twist
8. Check of $O(a)$ improvement
9. Conclusions

Reference:

E.-M. I., K. Jansen, M.-P. Lombardo, M. Müller-Preussker,

M. Petschlies, O. Philipsen, L. Zeidlewicz, arXiv:0905.3112.

1. Motivation

Why Wilson fermions ?

- advantages/disadvantages

  + locality satisfied

  + almost competing algorithms developed

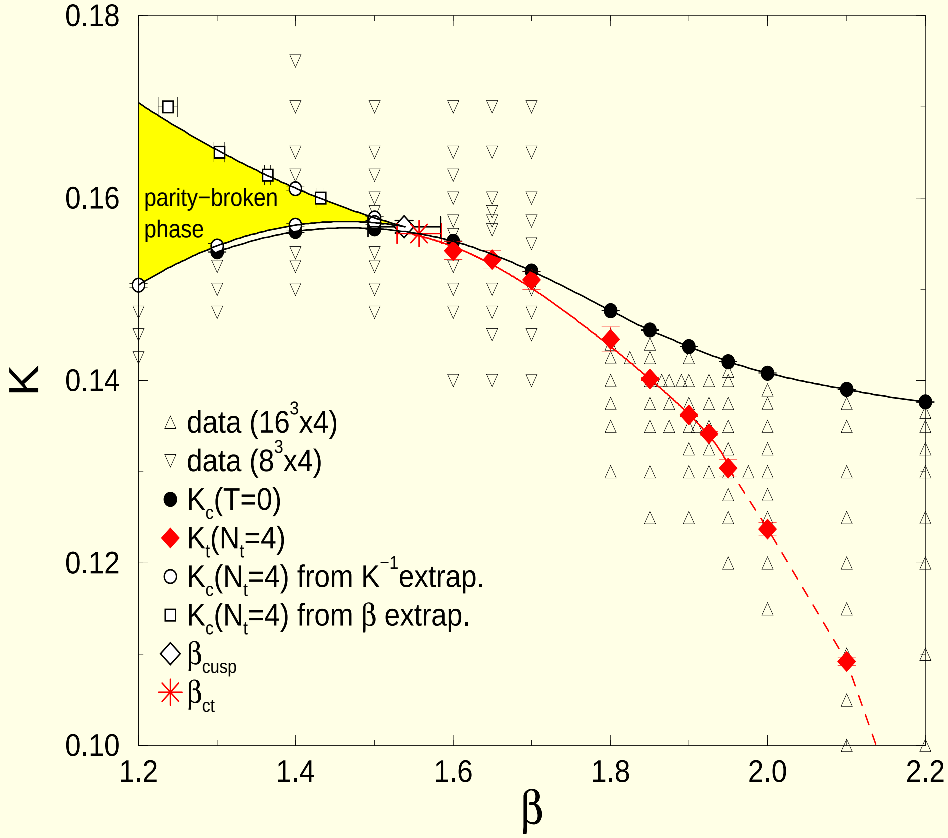
  – chiral symmetry explicitely broken, subtle chiral behavior

  – complicated phase structure, both at $T = 0$ and finite T

  – continuum limit slow

  + the latter can be cured by improvement

Phases of Wilson fermions: early lattice results



A. Ali Khan et al., CP-PACS Collaboration

Phys. Rev. D 63, 034502 (2001)

For $N_f = 2$ Wilson fermions apparently compatible with $O(4)$ scaling (cf. F. Karsch's talk).

Main arguments for the twisted-mass approach

- Prevents the occurrence of small eigenvalues of the Dirac operator by lifting eigenvalues, also zero modes. (Gattringer, Solbrig 2005).

- This should allow to work at smaller quark masses.

- With the hopping parameter κ tuned to its critical value $\kappa_c(\beta)$ ("maximal twist"), the twisted-mass term behaves as a conventional quark mass.

- In this case automatic $O(a)$ improvement is guaranteed (See Farchioni et al. '05, Urbach '07).

Goal of the tmfT collaboration: Extensive study of the phase diagram of tmQCD in the (κ,β,μ_0)-space at non-zero temperature – a prerequisite for QCD finite-T simulations at maximal twist.

Price to pay: A 3-dimensional phase diagram with a complicated structure due to $O(a^2)$ parity/flavor violating effects.

2. Set-up

The gauge action :

$$S_G = \beta \sum_x \left[c_0 \sum_{\mu<\nu} \left(1 - \frac{1}{3} \text{Re Tr } U^{1\times1}_{x\mu\nu} \right) + c_1 \sum_{\mu\neq\nu} \left(1 - \frac{1}{3} \text{Re Tr } U^{1\times2}_{x\mu\nu} \right) \right]$$

tree-level Symanzik action with (inverse) gauge coupling $\beta = 6/g_0^2$, $c_1 = -1/12$ and $c_0 = 1 - 8\ c_1$

The fermion action :

$$S_F = a^4 \sum_x \left\{ \overline{\psi}(x)\ \left[\left(D[U] + m_0 \right) \mathbb{I}_{2\times2} + \text{i}\ \mu\ \tau_3\ \gamma_5 \right]\ \psi(x) \right\}$$

$$D[U] = \frac{1}{2} \left[\gamma_\mu \left(\nabla_\mu + \nabla^*_\mu \right) - a\ \nabla^*_\mu\ \nabla_\mu \right]$$

Wilson-Dirac fermion action with twisted-mass term for $N_f = 2$ light flavors (in the physical basis $\Psi = (u, d)$)

[Frezzotti, Grassi, Sint, Weisz 2001; Frezzotti, Rossi 2004]

- Hopping parameter κ and twisted mass μ_0 combine into a bare quark mass:

$$m_q = \sqrt{\frac{1}{4}\left(\frac{1}{\kappa} - \frac{1}{\kappa c}\right)^2 + \mu_0^2} \qquad (1)$$

- For maximal twist, i.e. $\kappa = \kappa_c(\beta)$ (obtained at $T = 0$), automatic $\mathcal{O}(a)$ improvement is expected.
  (See Farchioni et al. '05, Urbach '07)

- HMC simulations were performed in a wide range
  $\beta = 1.80, \ldots, 3.90$ on various spatial lattice sizes
  $N_s = 16$ (24, 32) and $N_t = 8$.

- For $\mu_0 = 0$ and $N_t = 8$ the phase diagram clearly divides into three regions:
  – the Aoki-Phase [Aoki '84] for strong coupling, i.e. at small β,
  – a bulk transition region at intermediate β,
  – and the thermal transition and scaling region at larger β.

- This phase structure is embedded in full (β, κ, μ_0)-space.

- As we will see, in the scaling region, according to eq. (1), the thermal transition forms a conical surface, eventually a closed ellipse in the (κ, μ_0)-plane at each β.

3. Anticipating the phase structure

Twisted mass - an irrelevant rotation in continuum,

not so on the lattice !

An effective potential study for $T \neq 0$ [Creutz '07]

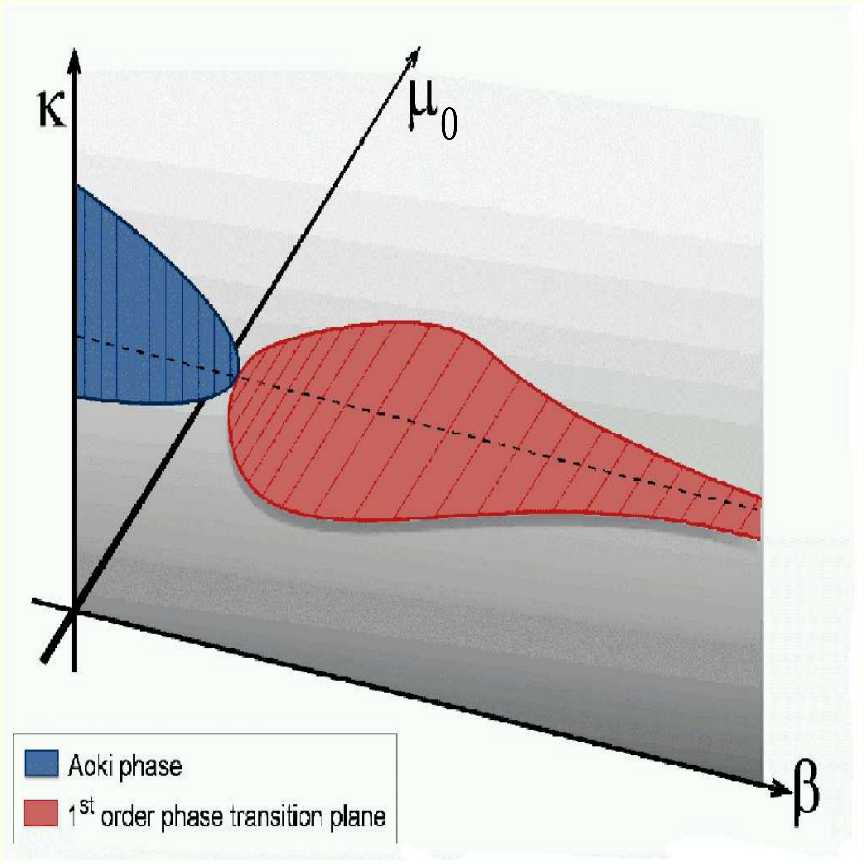
basic order parameter fields : $\sigma = \overline{\psi}\psi \qquad \vec{\pi} = i\overline{\psi}\gamma_5\vec{\tau}\psi$

effective potential including lattice artifacts :

$$V(\vec{\pi},\sigma) = \lambda\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 + c_1\left(\frac{1}{\kappa} - \frac{1}{\kappa_c(\beta)}\right)\sigma + c_2\sigma^2 - \mu\pi_3$$

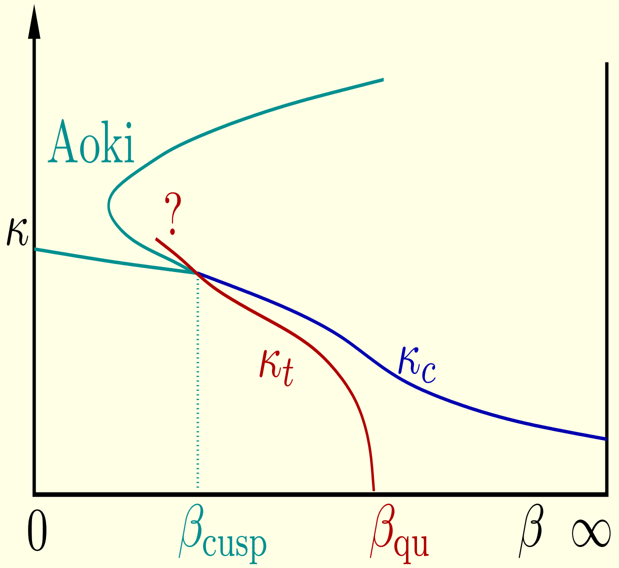
- λ : $O(4)$ symmetric linear σ model
- c_1 : mass term (changing sign at $\kappa_c(\beta)$)
- c_2 : chiral symmetry breaking lattice artifact (changing sign at some β)
- μ : the twisted-mass term

$T = 0$: Phase diagram for twisted-mass Wilson fermions.

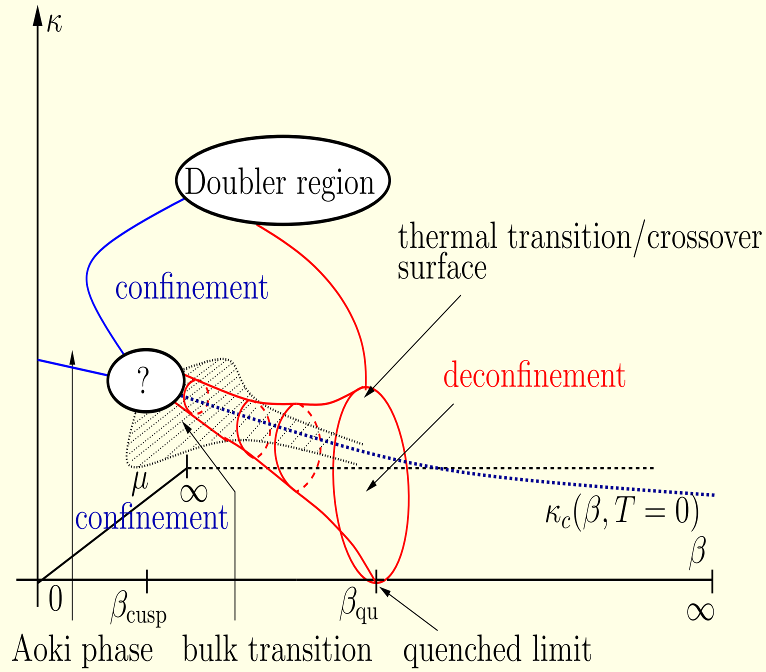


A. Shindler, Phys. Rept. 461 (2008) 37

$T \neq 0$



Standard view of the phase diagram for non-twisted Wilson fermions in the β-κ plane (cf. CP-PACS).

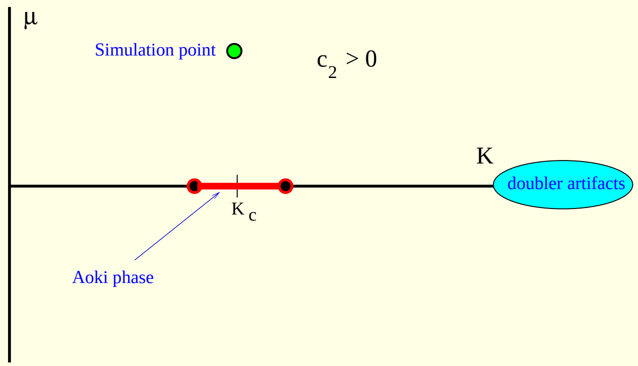


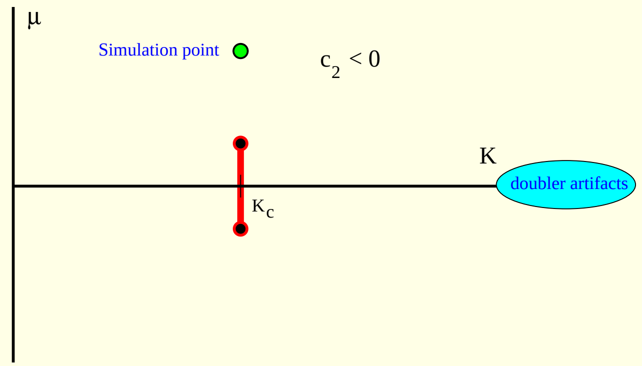
Schematic phase diagram for twisted-mass QCD in the β-κ-μ_0 space, basically as proposed by Creutz [Creutz '07].

Chiral effective action proposes a unified view of the phase diagram embedded in the β-κ-μ diagram.

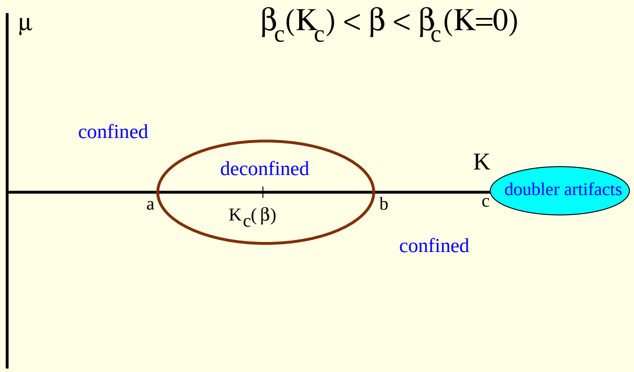
(Sharpe, Singleton, Creutz)

viewed in the κ-μ plane, going from low β to higher β [Creutz '07]

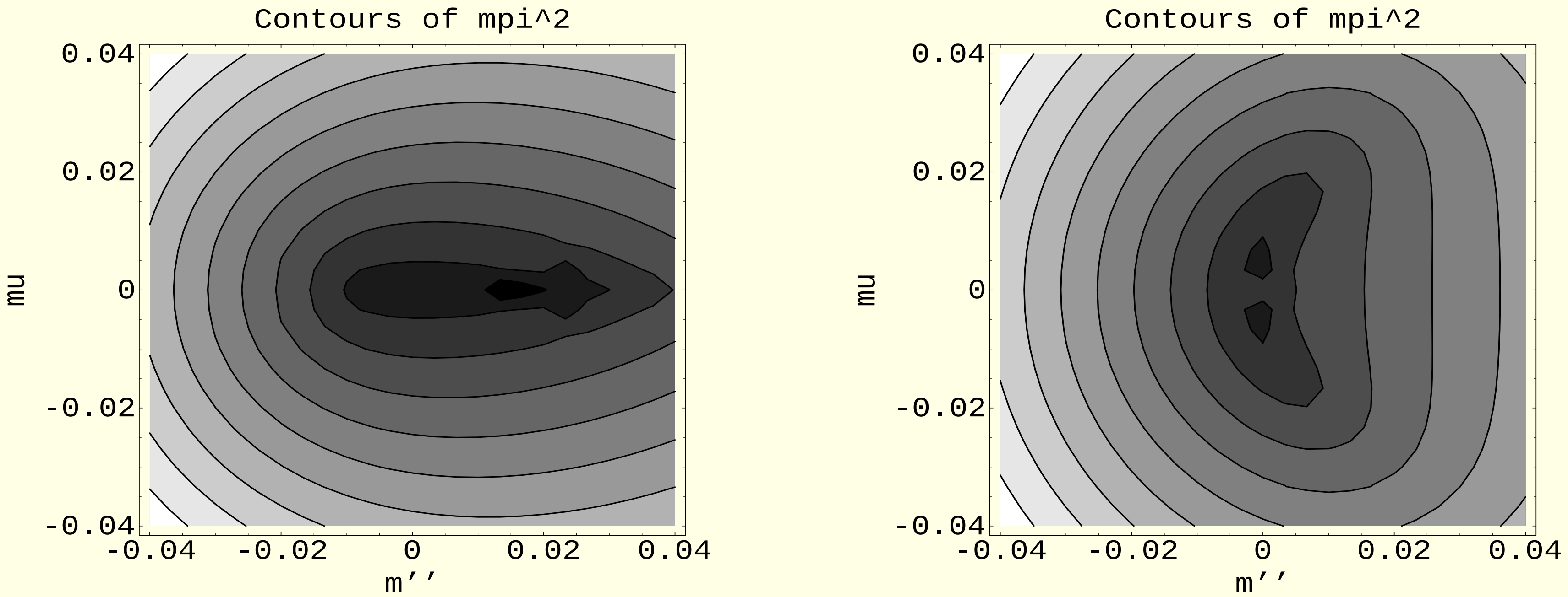




at low β: discontinuity in μ at intermediate β: discontinuity in κ



deconfinement inside a disk around $\kappa_C(\beta)$ in the scaling region of β



Contour plots of $m^2_{\pi^\pm}$ in the twisted-mass plane (from NLO Lattice χPT) are locating the Aoki phase (left) and the first order bulk transition scenario (right).

4. Aoki phase

An example of an "unphysical phase pocket":

- An external "magnetic field" $h = 2\ \kappa\ \mu_0 \Rightarrow$ induces spontaneous breaking of combined flavor-parity symmetry [Aoki 1984,1987] in some κ interval
  $\Rightarrow$ order parameter $\lim_{\mu_0 \to 0} \lim_{V \to \infty} \langle \overline{\psi} i \gamma_5 \tau_3 \psi \rangle \neq 0$. (jumps across $\mu_0 = 0$)

- No phase transition at $\mu_0 \neq 0$ (cf. Ising model at $H \neq 0$)

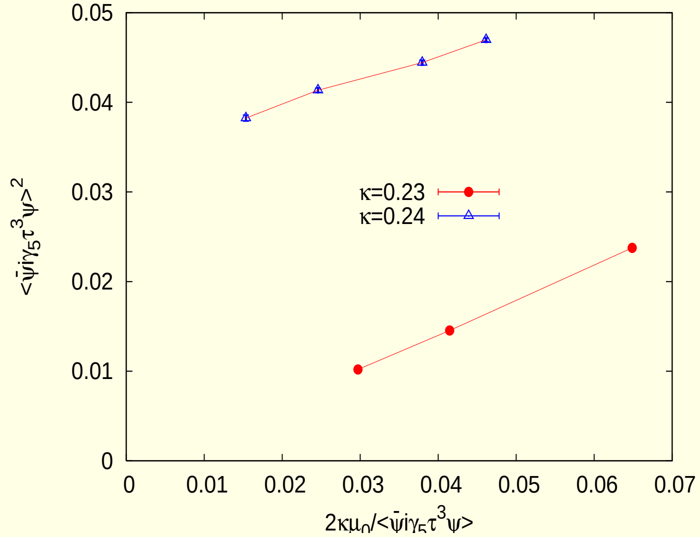
- Extrapolation to $\mu_0 = 0$ can be studied by Fisher plots. Based on an equation of state

$$h = A_0 \sigma^3 + A_1 (\kappa - \kappa_{c,\text{low}}) \sigma \qquad (2)$$

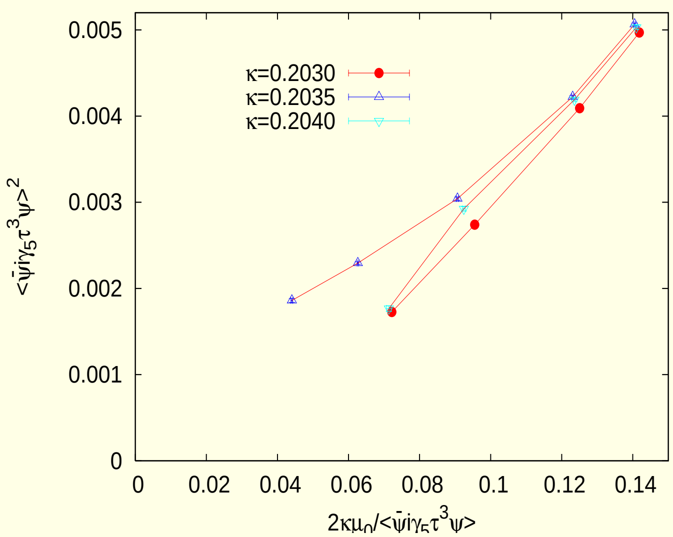
  with $\sigma = \langle \overline{\psi} i \gamma_5 \tau_3 \psi \rangle$, the would-be order parameter σ^2 is plotted vs. h/σ (requesting a positive intercept).

Evidence for the Aoki phase

$\beta = 1.8$



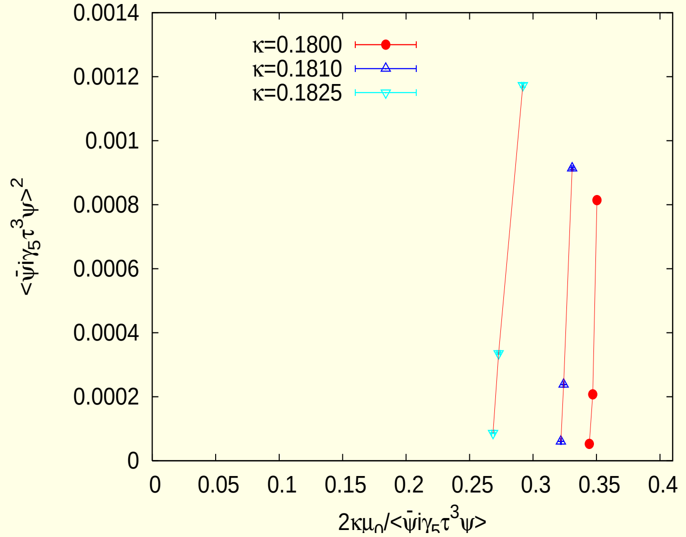
$\beta = 3.0$



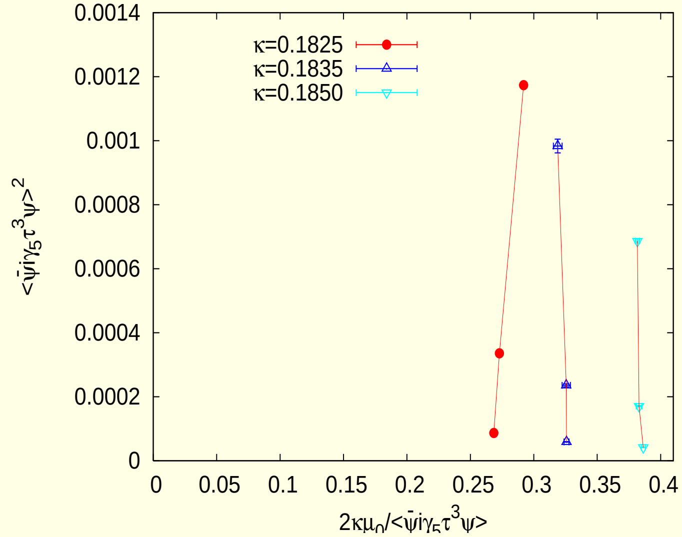
$\Longrightarrow$ Cusp of the Aoki phase extends at least to $(\beta, \kappa) = (3.0, 0.2035)$.

No evidence for the Aoki phase beyond $\beta = 3.4$

$\beta = 3.4,\ \kappa \leq \kappa_c \simeq 0.1825$



$\beta = 3.4,\ \kappa \geq \kappa_c \simeq 0.1825$



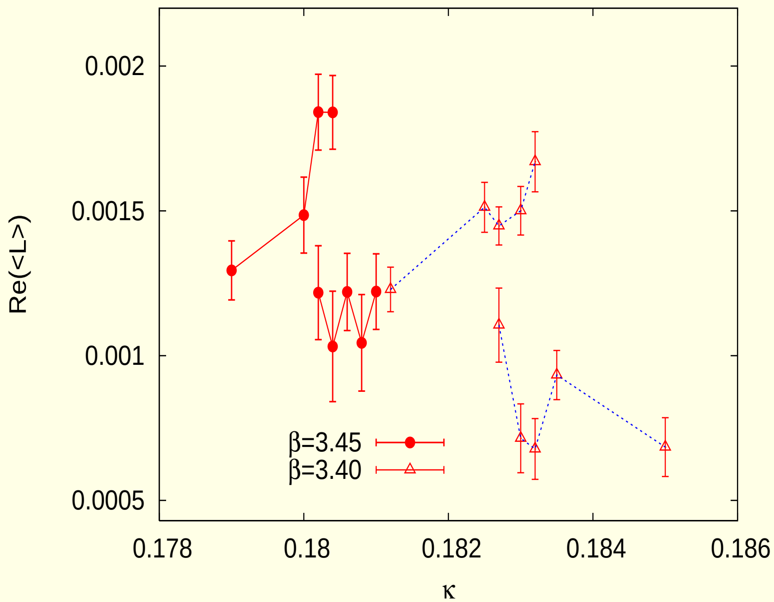
$\Longrightarrow$ No signatures for Aoki phase exist any more for $\beta \geq 3.4$.

5. First order bulk transition

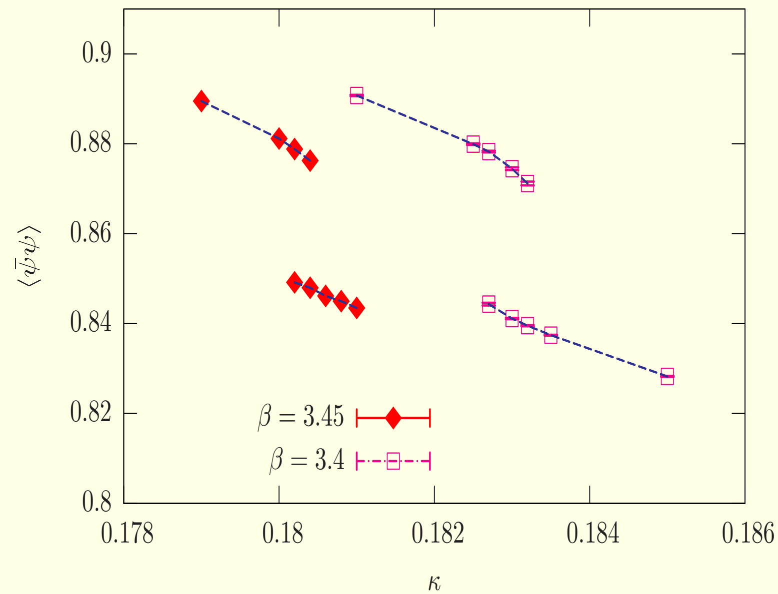
The Sharpe-Singleton scenario [Sharpe-Singleton, '98]

- At intermediate β metastabilities occur signalling a possible 1-st order transition. Represents a remnant of the bulk transition occurring at $T = 0$ [Münster '04].
- Visible in several observables at $\kappa_c(\beta; T = 0)$ and not too large μ_0. Data obtained for $\mu_0 \simeq 0.007,\ \beta = 3.40, 3.45$.

Polyakov loop

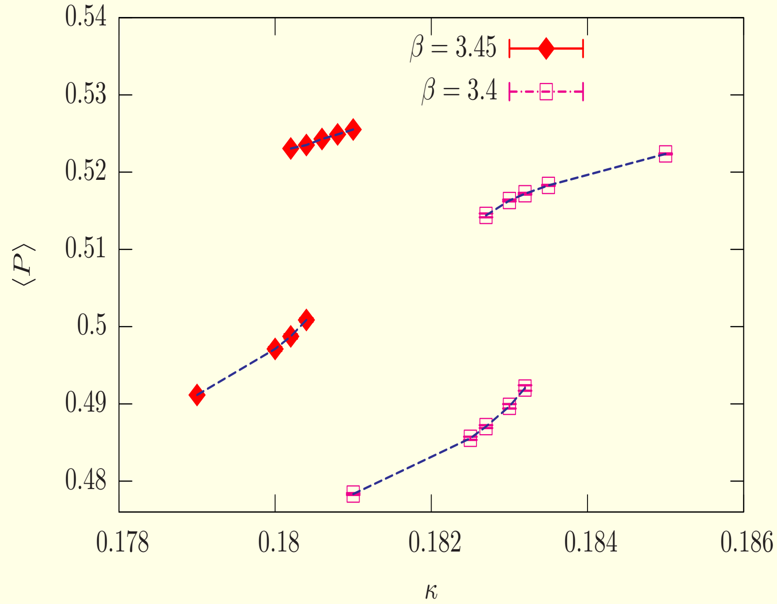


scalar condensate

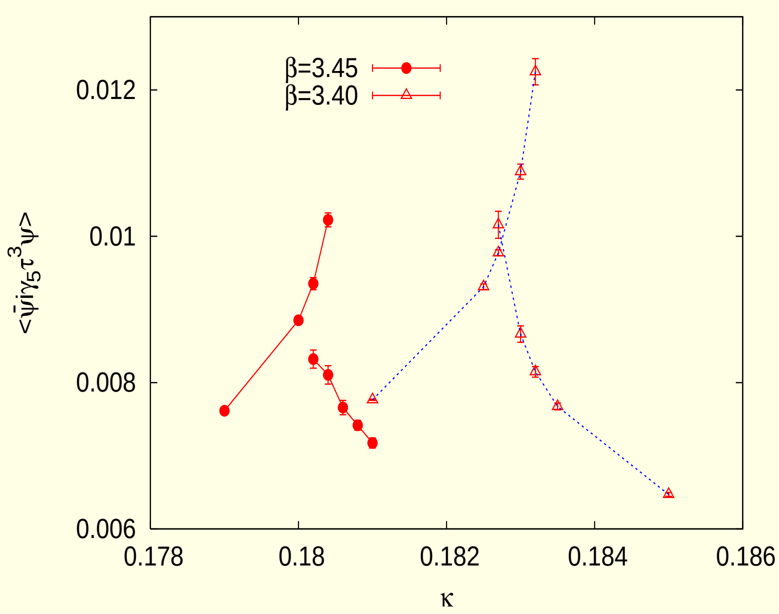


Other observables go discontinuous

plaquette



Aoki order parameter



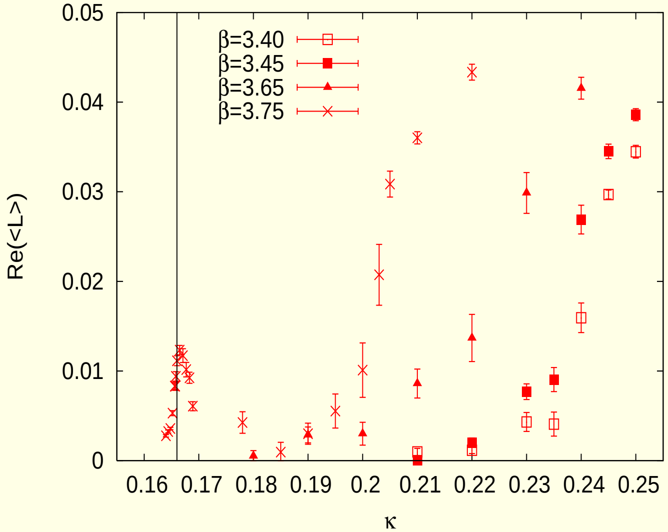
6. Thermal transition

- Observables: average plaquette, real part of the Polyakov loop (PL), the pion norm as well as their respective susceptibilities and integrated autocorrelation times.
- Our κ–scans at $\beta \in \{3.4, \ldots, 3.9\}$ and at fixed μ_0 reached statistics of $\mathcal{O}(10^4)$ HMC trajectories per point in the phase diagram.

Consider a κ–scan concentrating on κ above $\kappa_c(\beta)$ (the vertical lines mark $\kappa_c(\beta)$) :

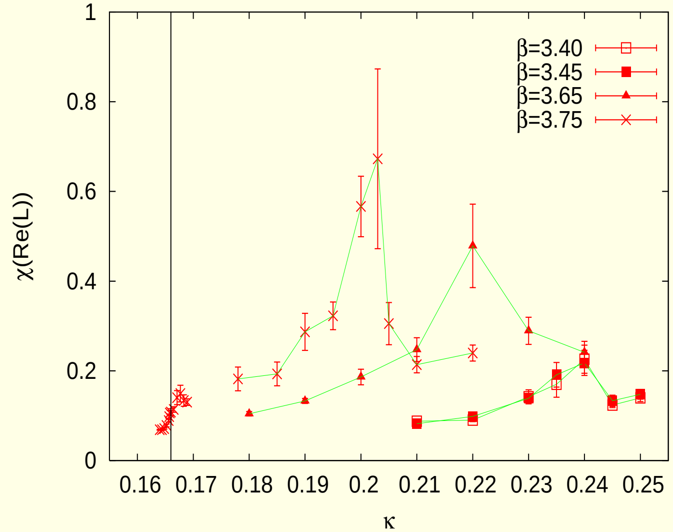
Polyakov Loop :

with $\mu_0 = 0.0068$ for $\beta = 3.4, 3.45, 3.65$;
with $\mu_0 = 0.005$ for $\beta = 3.75$.



Polyakov Loop susceptibility :

with the same μ_0 values



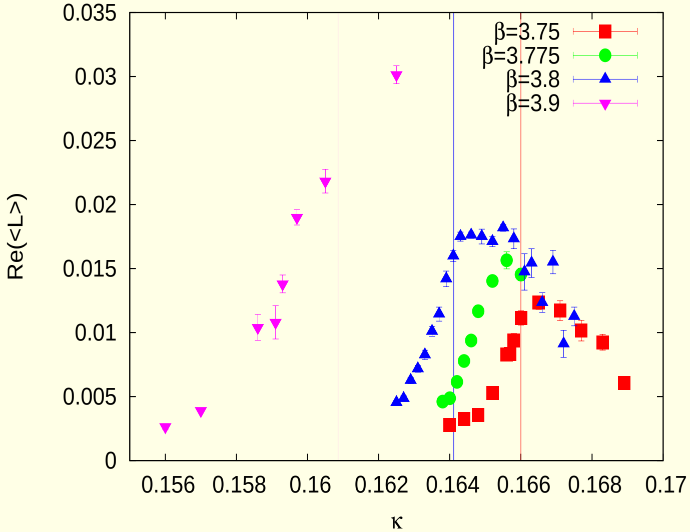
$\Longrightarrow$ At $\kappa > \kappa_c$ the cone is separated from the doubler range.

Next zooming into the cone around $\kappa_c(\beta)$:

(the vertical lines mark $\kappa_c(\beta)$)

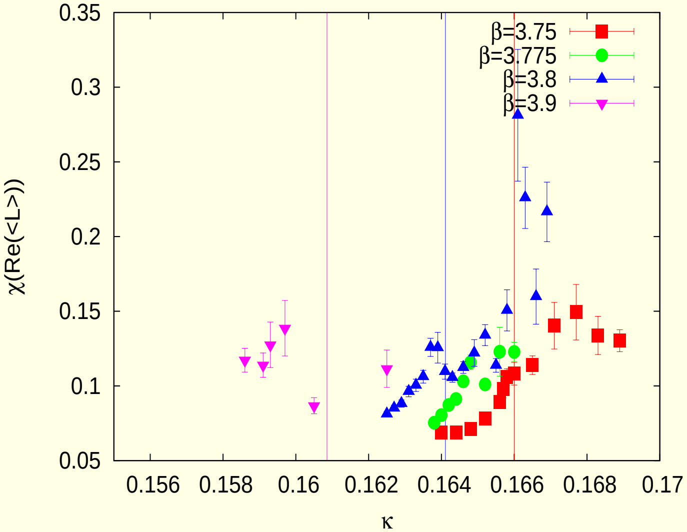
Polyakov Loop:

for $\beta = 3.9, 3.8, 3.775, 3.75$ with $\mu_0 = 0.005$.

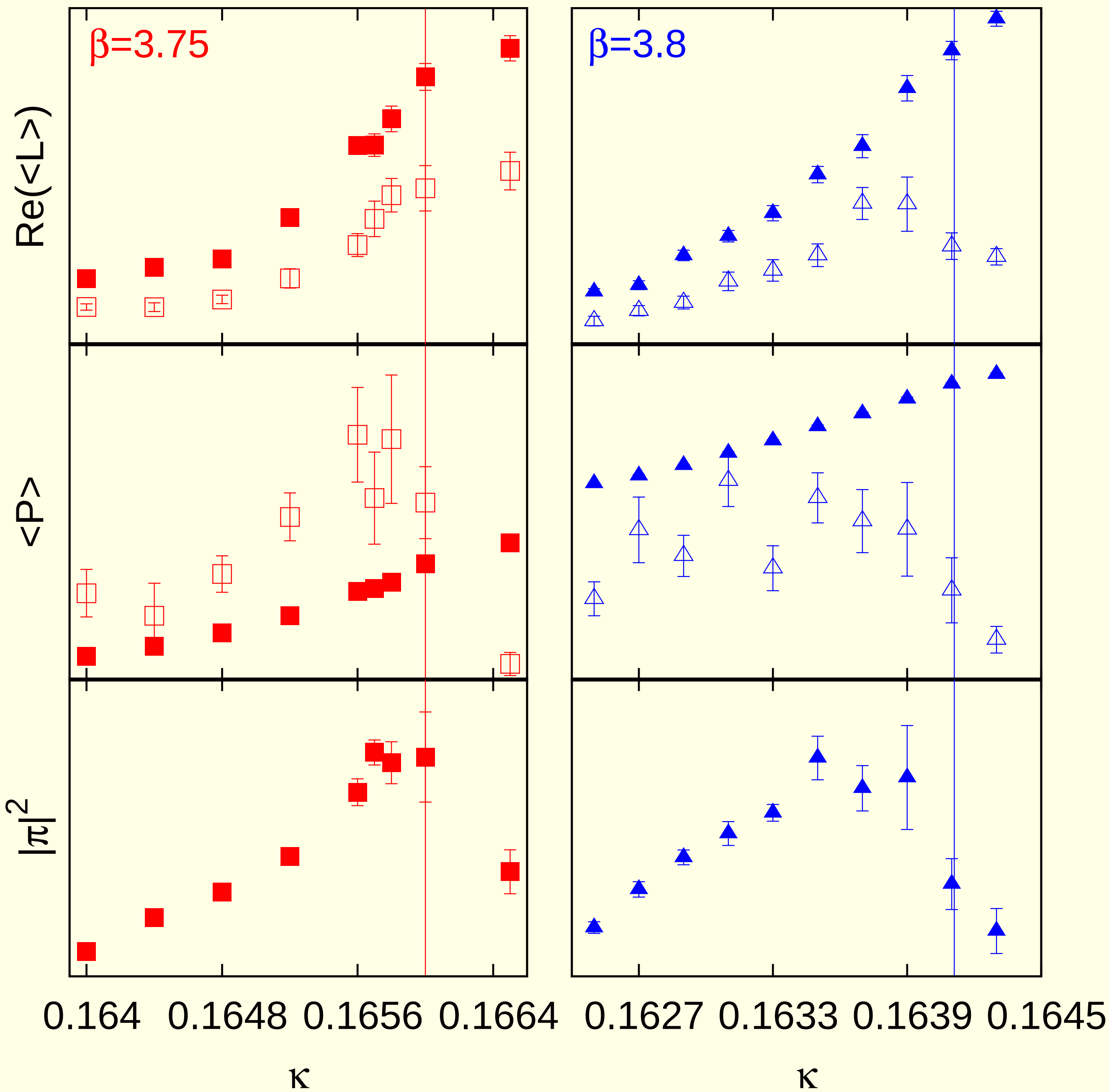


Polyakov Loop susceptibility:

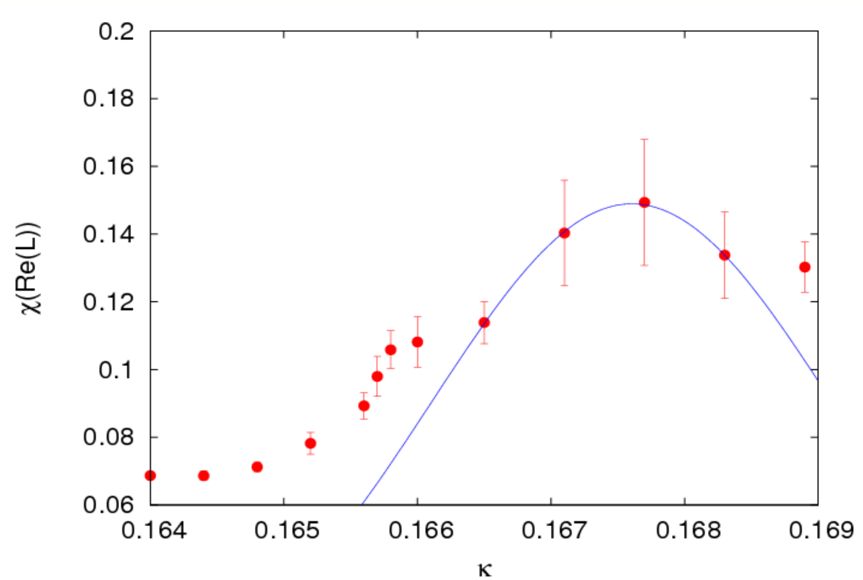
for $\beta = 3.9, 3.8, 3.75$ from left to right.



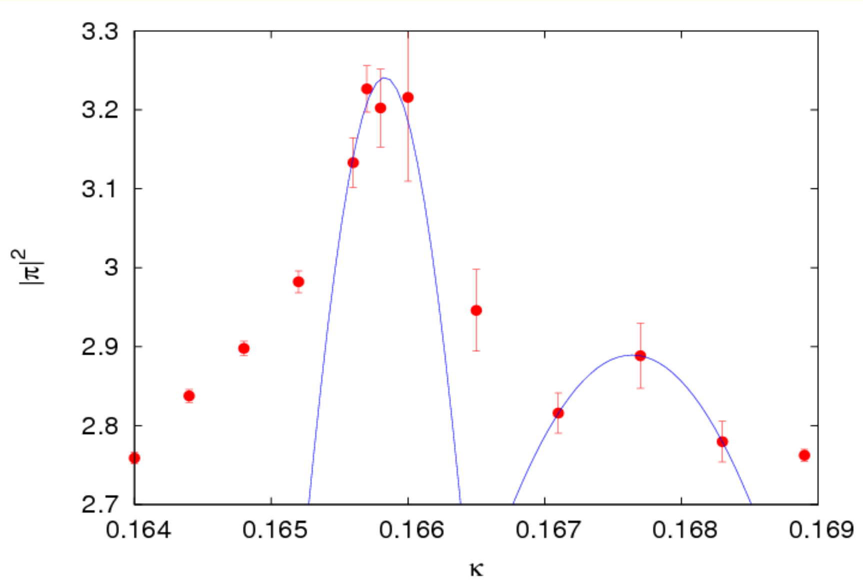
$\Longrightarrow$ The narrow cone itself (around $\kappa_c(\beta)$) can be resolved with large-statistics runs only.


β=3.75
β=3.8
Re(<L>)
<P>
$|\pi|^2$
0.164
0.1648
0.1656
0.1664
0.1627
0.1633
0.1639
0.1645
κ
κ

Fit the peaks $\beta = 3.75$, $\mu_0 = 0.005$ $(\kappa_c(3.75) = 0.166,\ m_\pi \simeq 400$ MeV, $r_0 T \simeq 0.5)$:



Polyakov Loop susceptibility



Pion norm

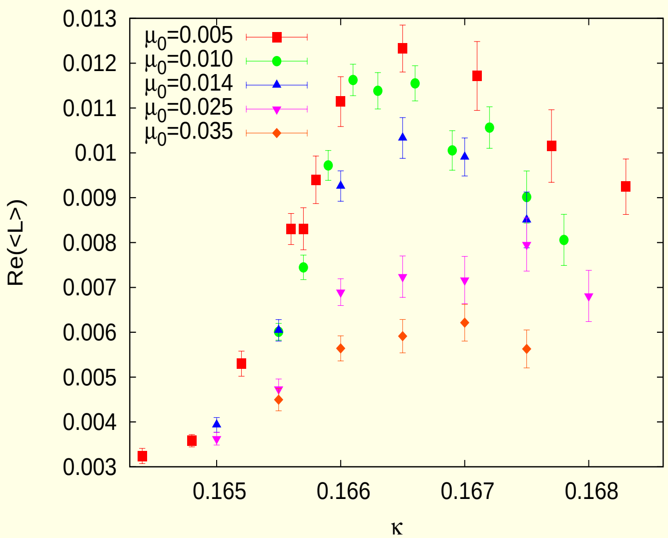
$\Longrightarrow$ Polyakov Loop susceptibility and Pion norm separate the entrance from the exit transition

$\Longrightarrow$ The lower transition is slightly below $\kappa_c(3.75)$.

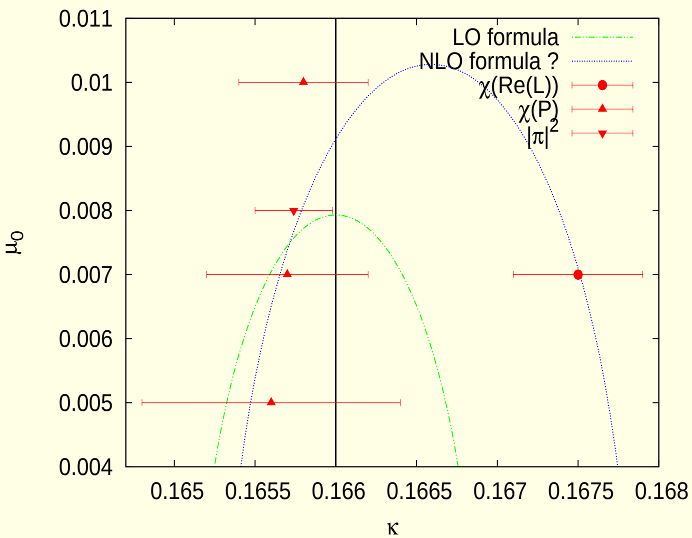
$\Longrightarrow$ The chiral and deconfinement signals are seen at the same κ.

Extension of the thermal cone ?

Polyakov Loop expectation values versus κ for $\beta = 3.75$ at various μ_0:



Expected extension of the ellipse in the $\kappa - \mu_0$ plane at $\beta = 3.75$:



$\Longrightarrow$ No transition seen beyond $\mu_0 = 0.025$ for $\beta = 3.75$.

Adapting the lower and upper (entrance and exit) crossovers to chiral perturbation theory.

Higher order correction are important !

A closer fit of the cone shape:

- Strength and nature of the transition/crossover are subject to discretisation effects, i.e. depend on the twist angle ω. This leads to a distortion of the ellipse and conical shape.

- For sufficiently light quarks, lattice chiral perturbation theory can be applied. The NLO expression for the pion mass allows to estimate the parameters of the ellipse [Sharpe '04]:

$$m_{\pi^\pm}^2 = M' + \frac{16}{f^2}\left((2L_{68} - L_{45})(M')^2 + M'\hat{a}\cos(\omega)(2W - \tilde{W}) + 2\hat{a}^2\cos^2(\omega)W'\right) + \frac{(M')^2}{2\Lambda_\chi^2}\ln\left(\frac{M'}{\Lambda_R}\right) .$$

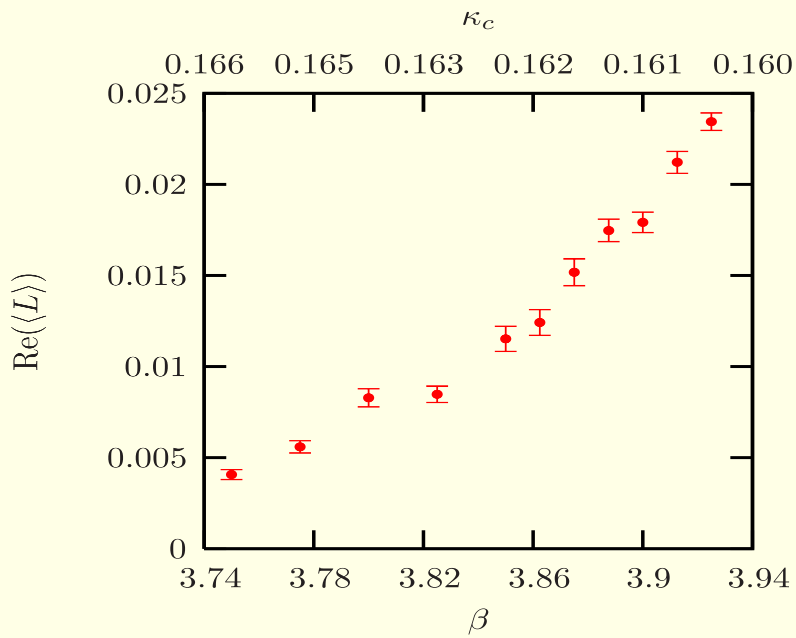
Quark masses are renormalised, $\mu = Z_\mu \mu_0$, $m = Z_m(m_0 - m_c)$;
$M' = \sqrt{\hat{\mu}^2 + (\hat{m}')^2}$, $\hat{a} = 2W_0 a$, $\hat{\mu} = 2B_0 Z_\mu \mu_0$, $\hat{m}' = 2B_0 Z_m(m_0 - m_c)$.
The constants B's, W's, L's can in principle be obtained from fits to lattice results at $T = 0$.

7. Feasibility of a β-scan at maximal twist

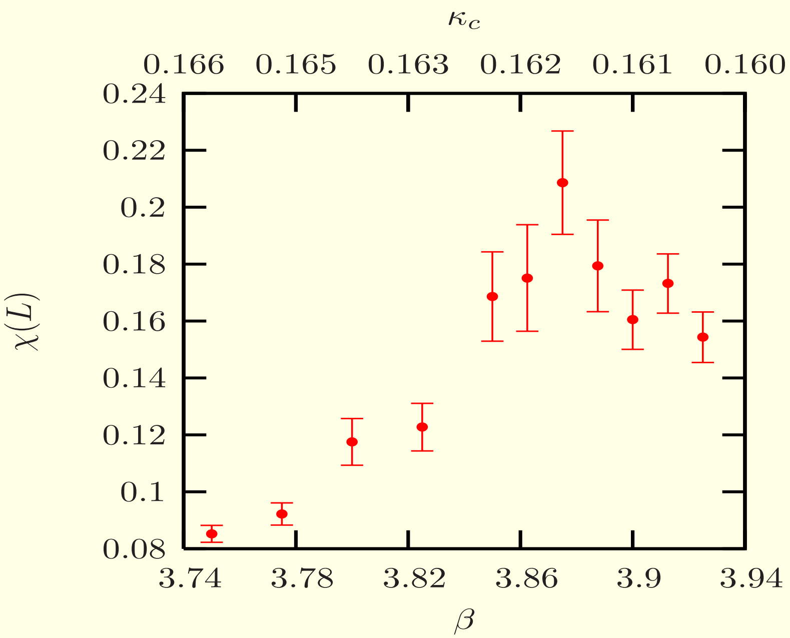
Again with $N_t = 8$, however with large pion mass, $m_\pi \simeq 1$ GeV

Consider a β–scan at large π-mass (huge $\mu_0 = 0.040$) with $\kappa \simeq \kappa_c(\beta)$, i.e. at maximal twist :

Polyakov Loop:



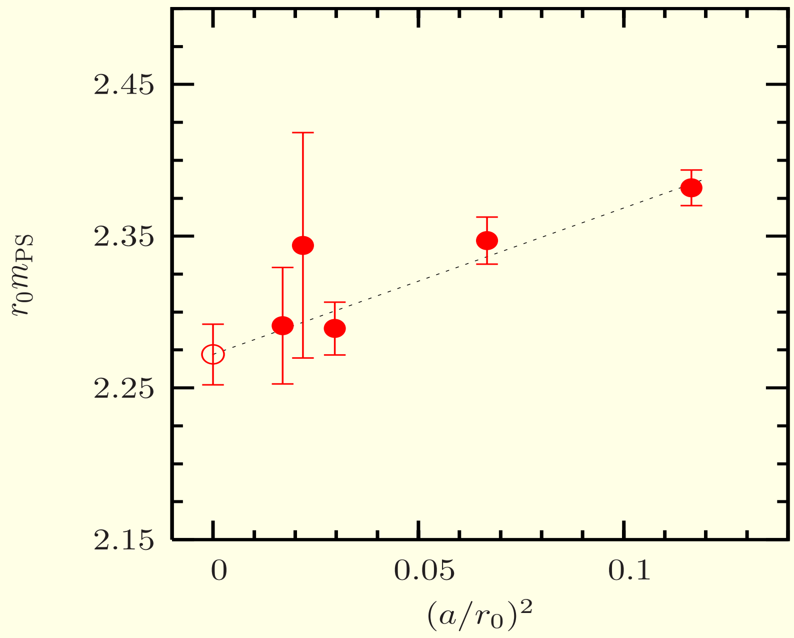
Polyakov Loop susceptibility:



$\Longrightarrow$ Critical point at $\beta_t \simeq 3.88$ (corresponding to $T_c r_0 \simeq 0.66$) where it should be for this m_π.

8. Check of $O(a)$ improvement

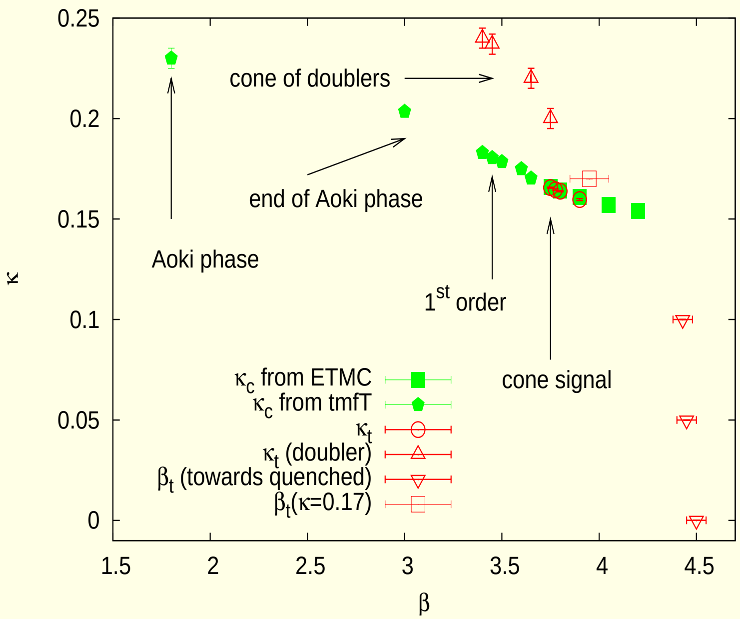
- In the quenched case with Wilson plaquette action the mesonic pseudoscalar screening mass m_{PS} is determined versus $(a/r_0)^2$ for various lattice sizes $N_s = 24, \ldots, 32$ and $N_t = 6, \ldots, 16$ at fixed $T/T_c = 0.655(5)$ and $m_{PS}/m_V \simeq 0.75$.
- The expected linear behavior in $(a/r_0)^2$ is observed.



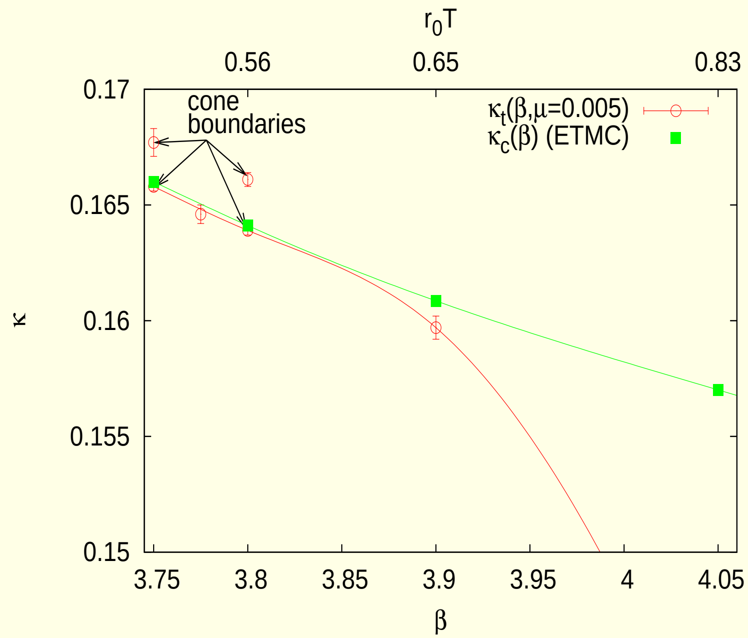
9. Conclusions

Summary of scanning the phase structure

Global view :



Zoomed into the scaling region :



Left: All transition points within $\mu_0 \leq 0.007$. Right: The region $\beta \geq 3.75$.

Results and Perspective

- The complicated global (three-dimensional) phase structure of Wilson twisted mass QCD with $N_f = 2$ and improved gauge action has been resolved.
- The conical shape of the thermal transition surface has been confirmed in the scaling region.
- To find the critical or crossover behavior requires very large statistics. A first test to see the crossover for $N_t = 8$ and, correspondingly, at large pion mass was successful.
- $O(a)$ improvement works, as has been demonstrated for the quenched approximation.
- Next we shall run at maximal twist at $N_t = 10$ and 12, where we expect to see the transition down to $m_\pi = O(300)$MeV.