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SU(2) deconfinement transition in 2+1 dimensions: critical couplings from twisted b.c.'s and universality

"Quarks, Hadrons and the Phase Diagram of QCD,"
EMMI Workshop, St. Goar, 2 Sept. 2009

Lorenz von Smekal

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Work together with *Sam Edwards*, arXiv:0908.4030 [hep-lat]

HIC for FAIR
Helmholtz International Center
LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

EMMI

ExtreMe Matter Institute EMMI

European Commission

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Contents

Introduction

The use of boundary conditions:

- Anti-(C-)periodic, monopole mass in compact U(1)

Polley, Wiese, 1991

- Twisted, vortex free energies in SU(N)

't Hooft, 1979; Hasenfratz, Hasenfratz, Niedermayer, 1990

Kajantie, Karkkainen, Rummukainen,1991

Kovacs, Tomboulis, 2000; de Forcrand, LvS, 2001; ...

- C-periodic twists, 't Hooft-Polyakov monopoles in SU(N)+adjoint Higgs

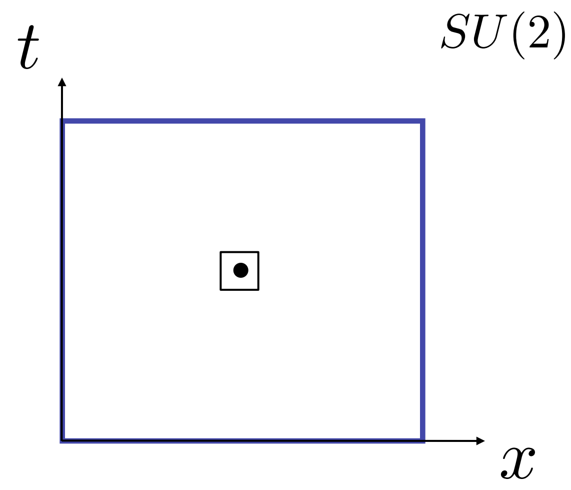
Kronfeld, Wiese, 1991

Davis, Hart, Kibble, Rajantie, 2002

Edwards, Mehta, Rajantie, LvS 2009

't Hooft's Twisted B.C.'s

- twisted b.c.'s for $SU(N)$ fix the total # mod. N of center vortices through each plane



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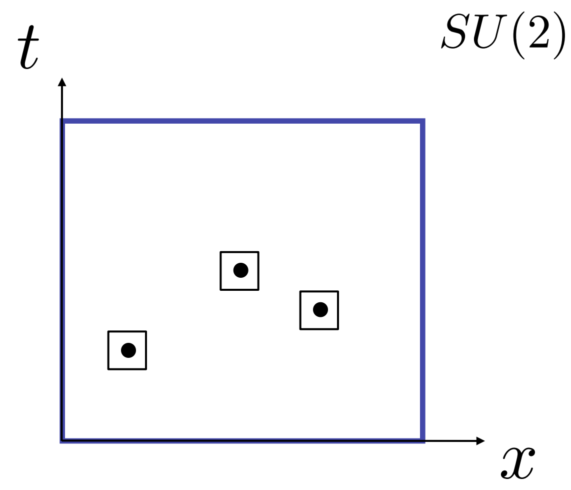

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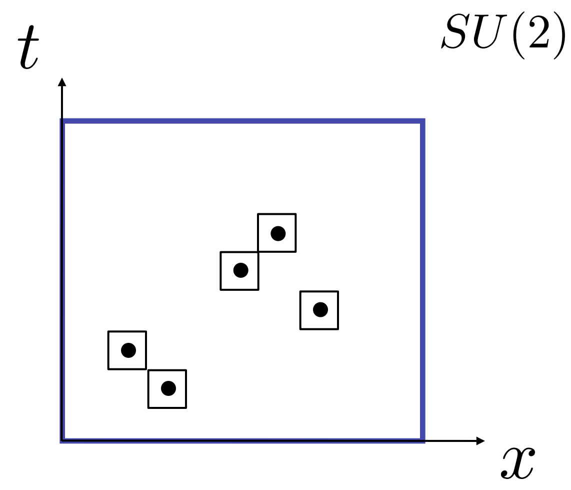

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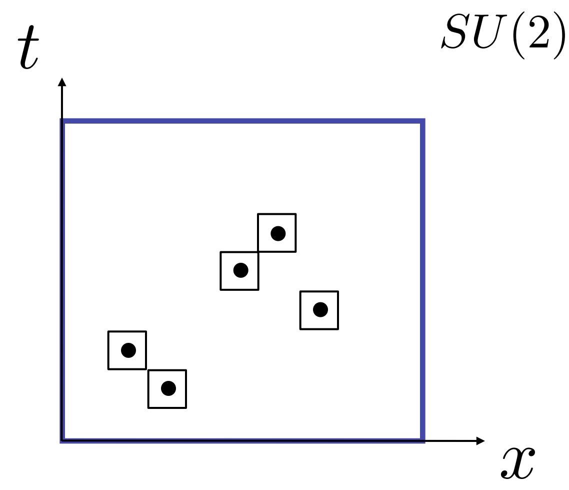

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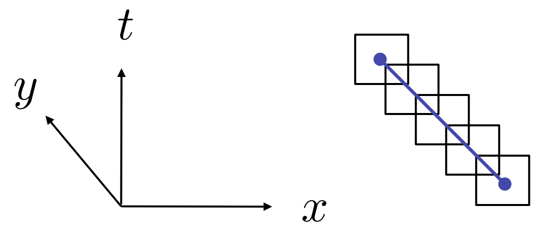

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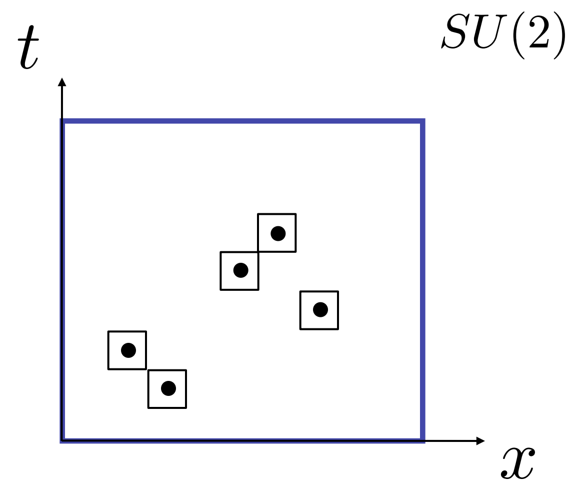

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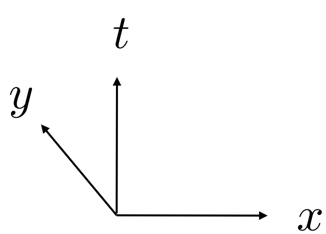
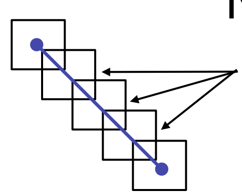
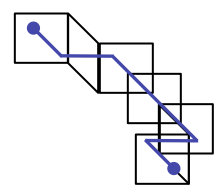




NB: change of variables, e.g., these 3 links $U \to -U$:



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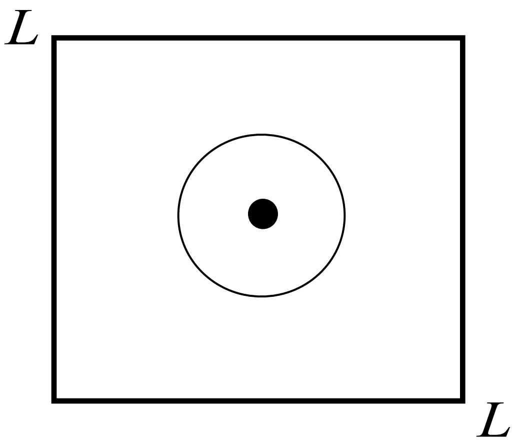
’t Hooft’s Twisted B.C.’s

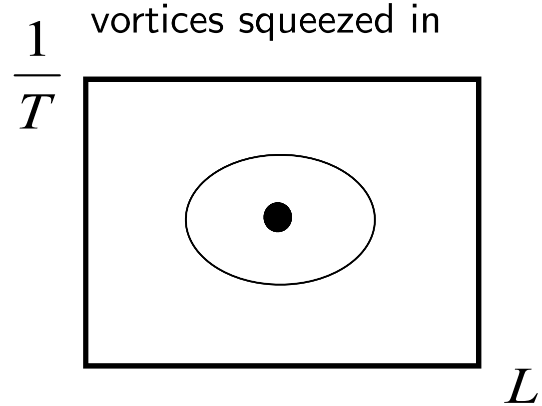
Vortices lower their free energy by spreading

$T > 0$, distinguish:

spatial planes $\vec{m}$,
can spread at all T



d temporal planes $\vec{k}$,
vortices squeezed in



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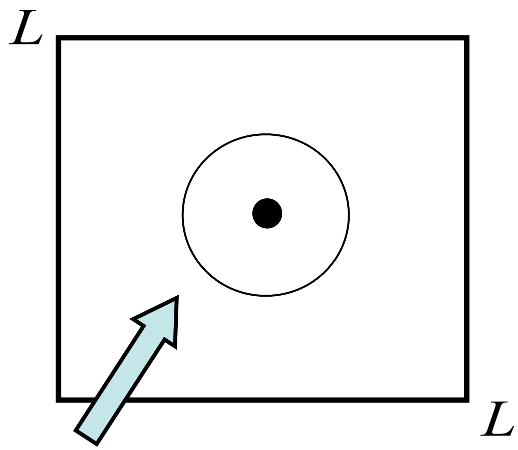

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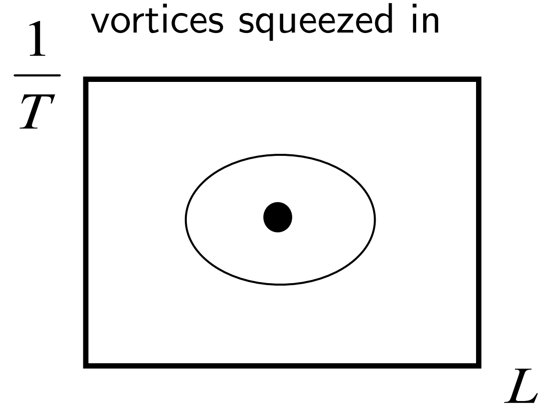

- **spatial twist $\rightarrow$ magnetic Z_N-flux in direction $\vec{m}$ (Z_N-monopole)**

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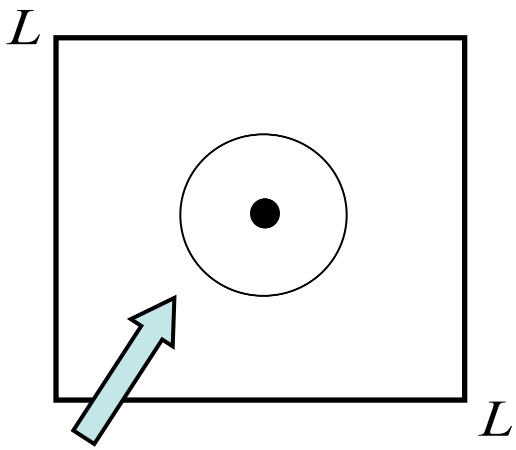

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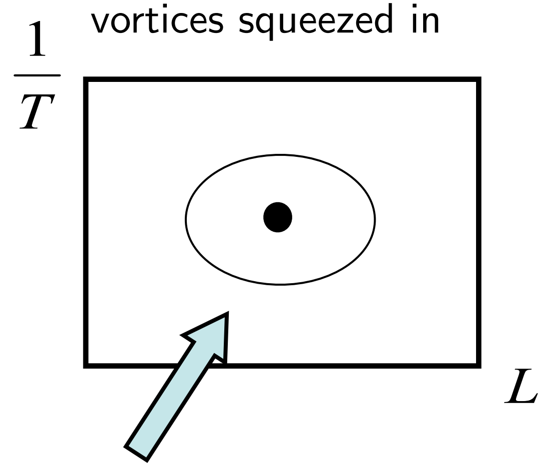

- **temporal twist, dual (Kramers-Wannier) to: electric Z_N-flux in direction $\vec{e}$**
  (by Z_N^d-FT: $\vec{k} \rightarrow \vec{e}$)

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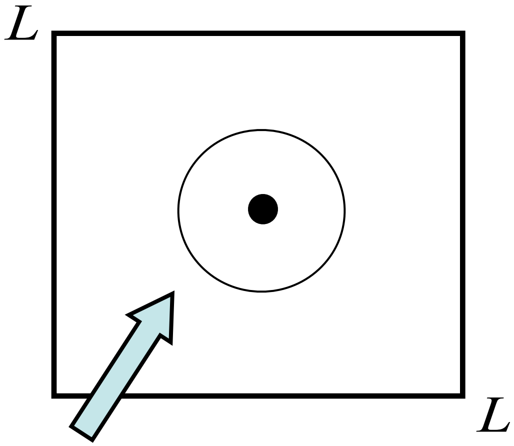

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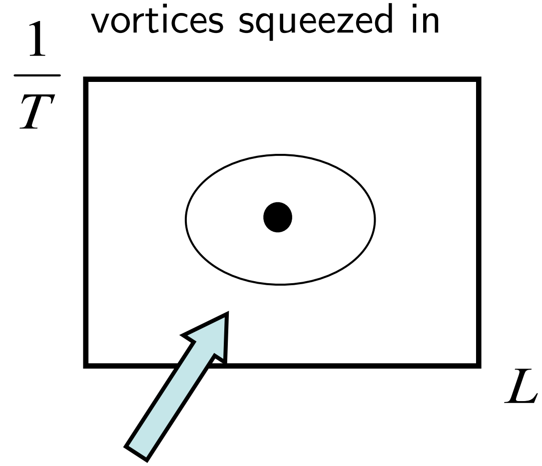

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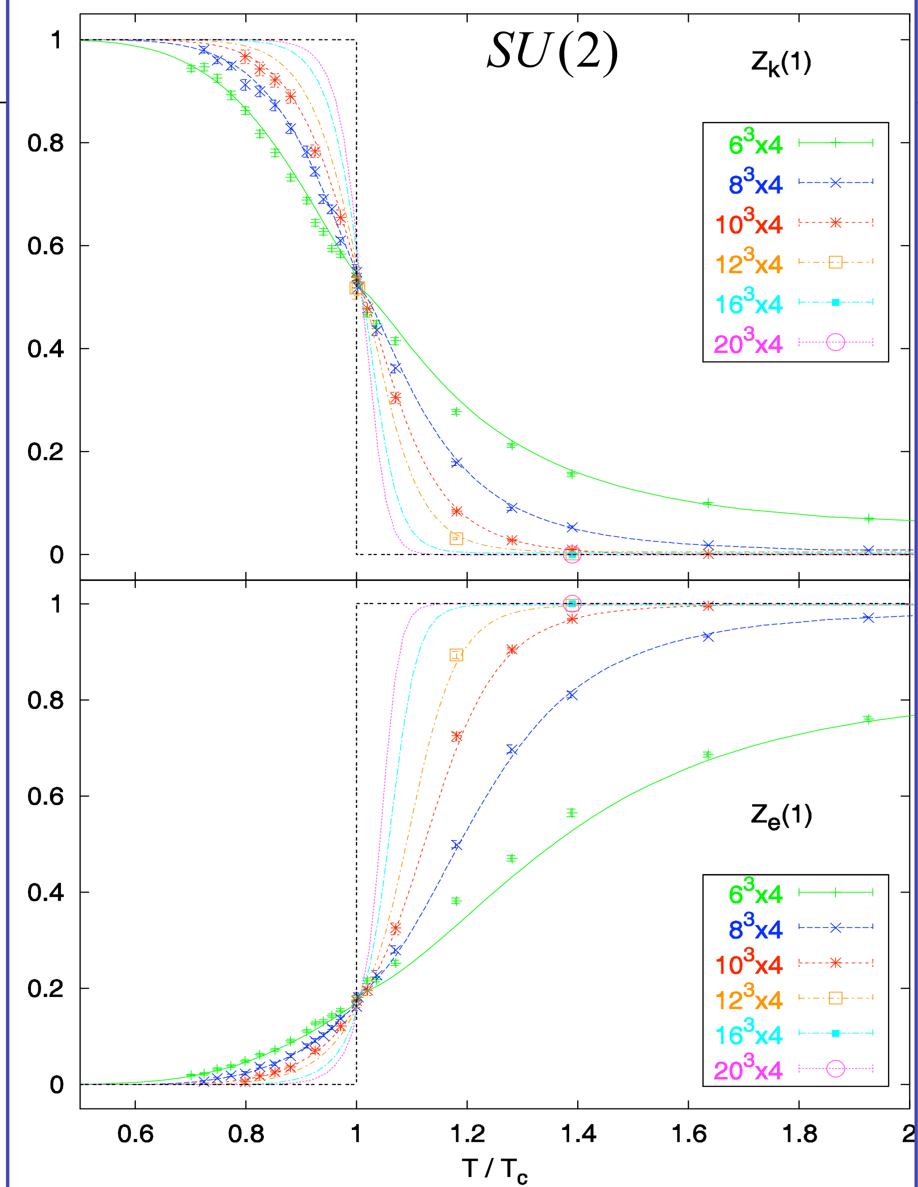
NB: Combine with charge conjugation to prepare ’t Hooft-Polyakov monople in $SU(N)$+adjoint Higgs on the lattice
S. Edwards, D. Mehta, A. Rajantie and LvS, `arXiv:0906:5531 [hep-lat]`

Center Vortex vs. Electric Flux Free Energies

- twisted partition functions / ´t Hooft loops:

$$\frac{Z_k(\vec{k},\vec{m})}{Z_k(0,0)} = \left\langle \widetilde{W}^{\max}_{(\mu,\nu)} \right\rangle$$



Ph. de Forcrand & L.v.S.,
PRD 66 (2002) 011504 (R)

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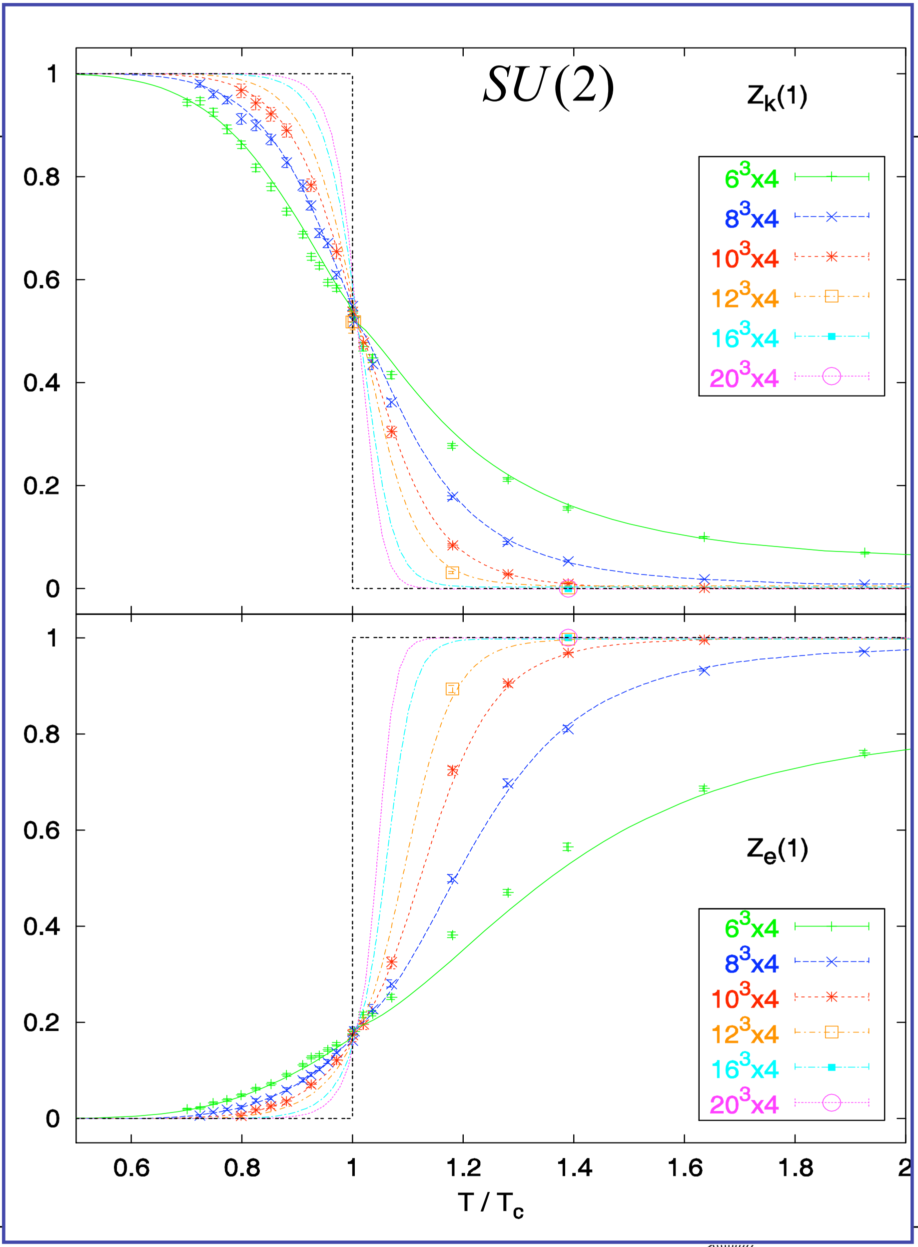

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- **finite free energies (only) at** T_C, ***e.g.*,**

$$Z_k(1,0)\big|_{T_C} = 0.54(1)$$

Ph. de Forcrand & L.v.S.,
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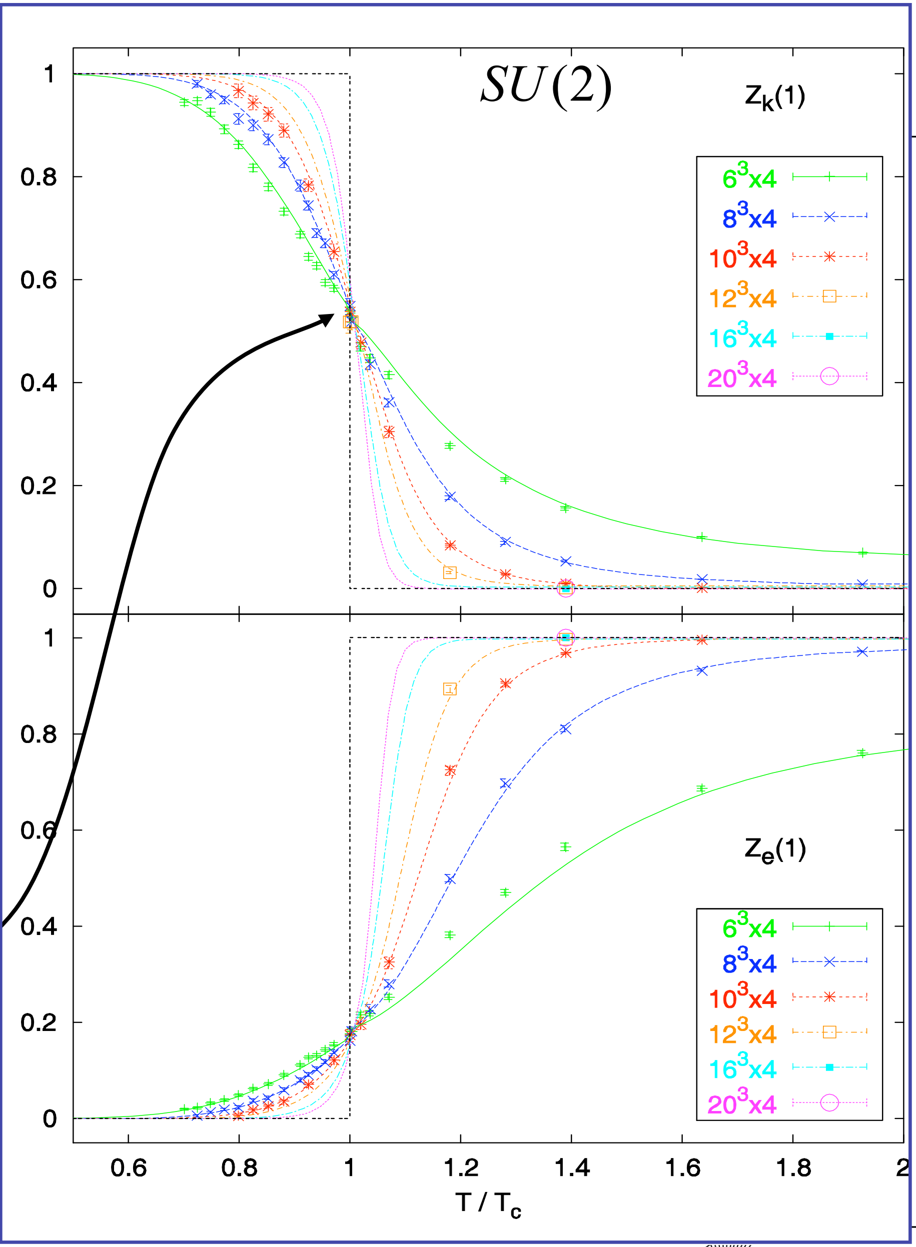

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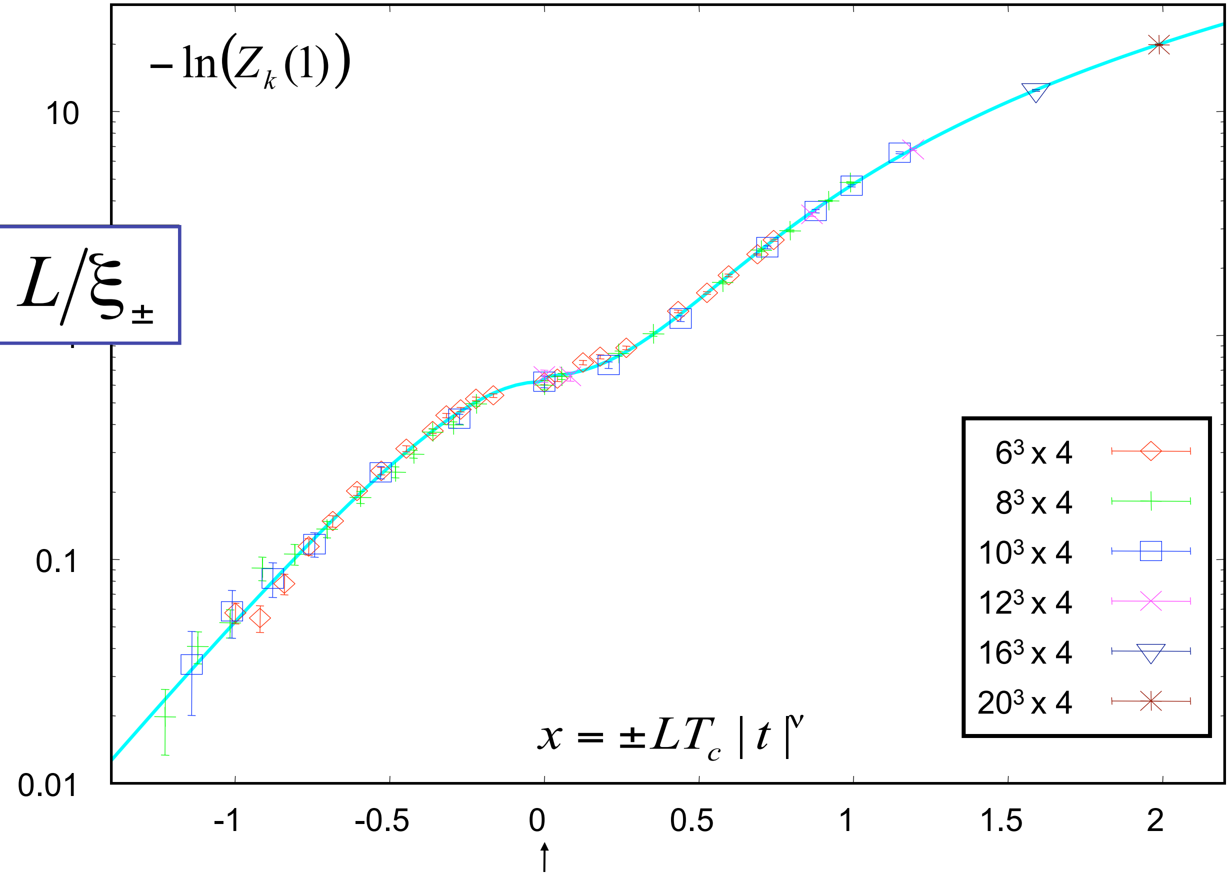
Finite Size Scaling

- relevant lengths: L and $\xi_\pm = \xi_\pm^0 |t|^{-\nu}$, correlation length for $t = (T/T_c - 1) \gtrless 0$

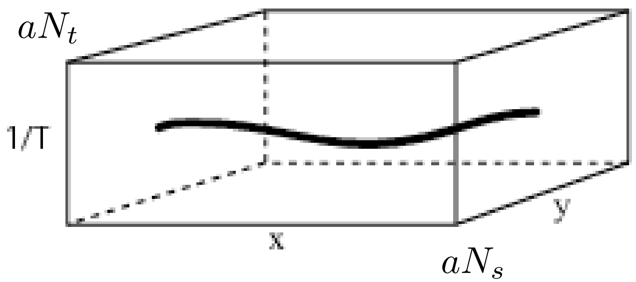
scaling variable:

$$x = \pm L T_c \, |t|^\nu \propto L/\xi_\pm$$

$\nu = 0.63$, 3-d Ising



SU(2) in 2+1 Dimensions



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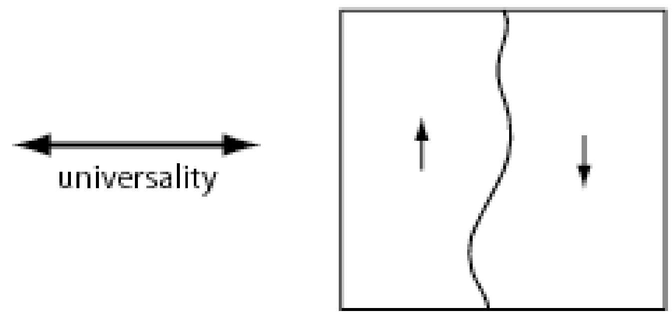

2D Ising model with interfaces ($N \times N$ square):

$$\lim_{N\to\infty} \frac{Z_{ap}(T_c)}{Z_{pp}(T_c)} = \frac{1}{1+2^{3/4}}$$

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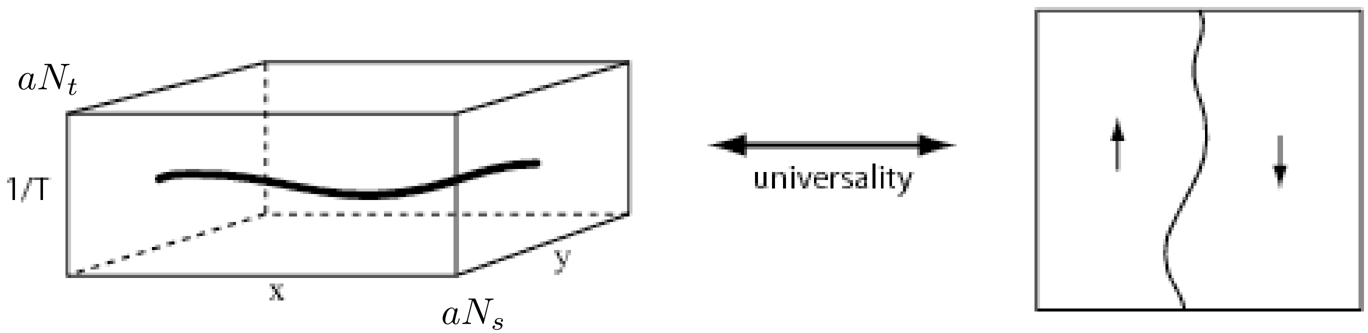

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FSS ansatz:

$$\frac{Z_{tw}}{Z_0} = \frac{1}{1+2^{3/4}} + b(\beta-\beta_c)N_s^{1/\nu} + cN_s^{-\omega} + \dots$$

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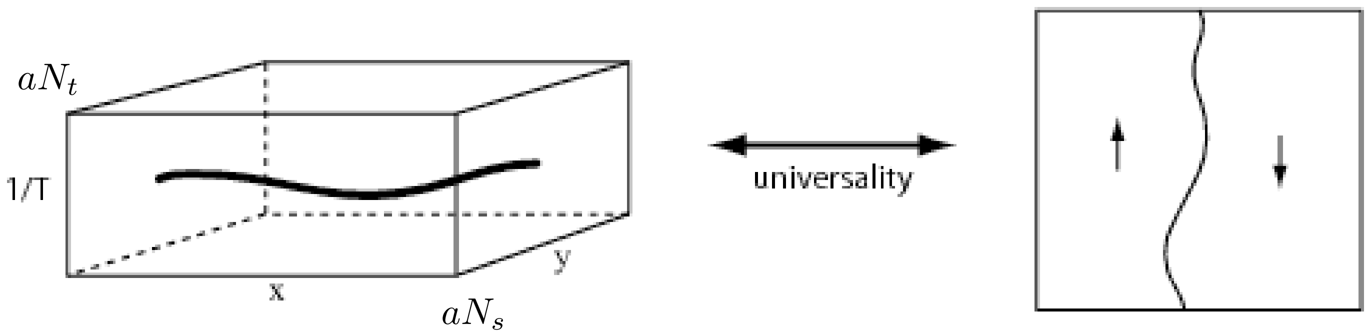

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$$\Rightarrow \qquad \beta_c(N_t, N_s) = \beta_{c,\infty}(N_t) - d(N_t)\, N_s^{-(\omega+1/\nu)} + \dots$$

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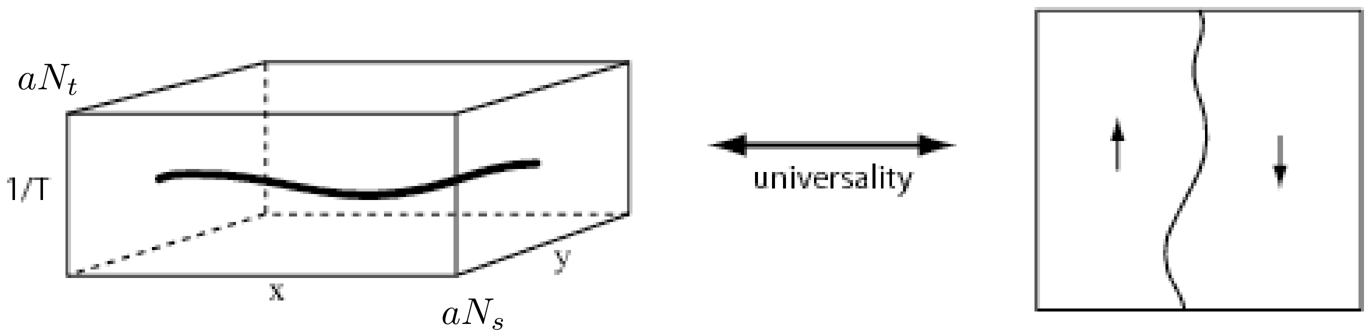

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($\nu = 1$ here)

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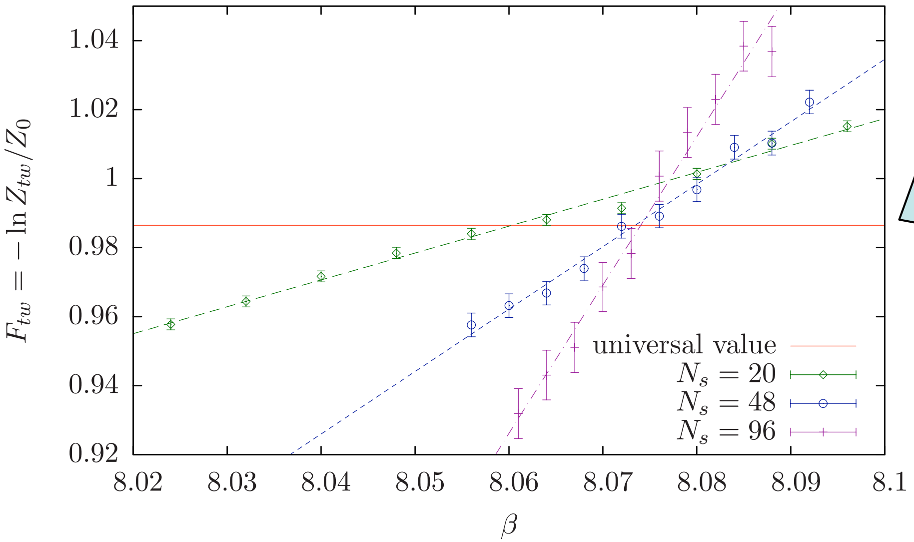
Critical Couplings



$$F_{tw} = \ln(1 + 2^{3/4}) + \text{const.} \times (\beta - \beta_c)$$

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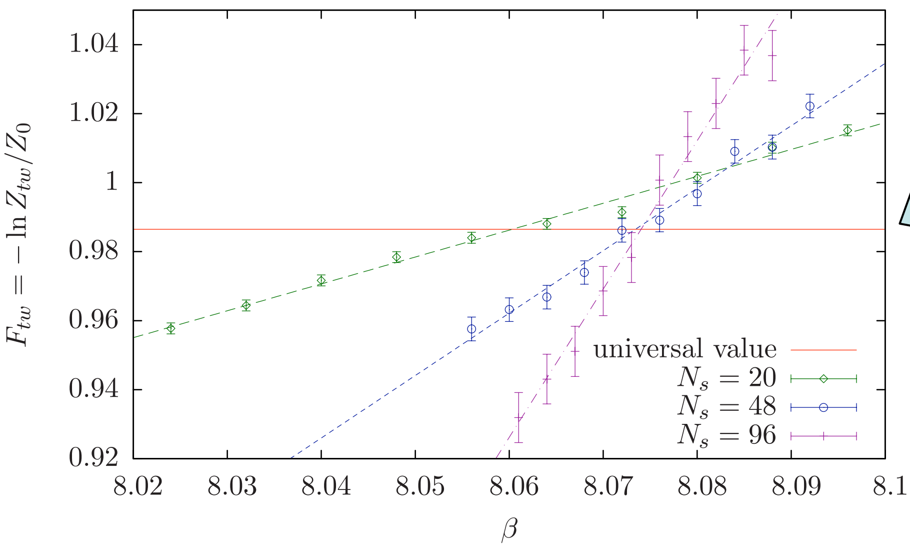

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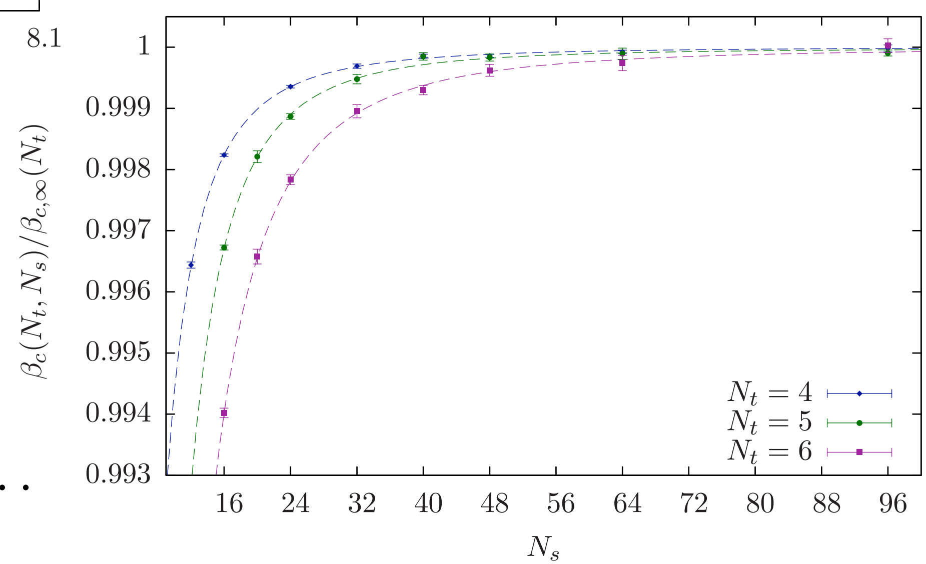

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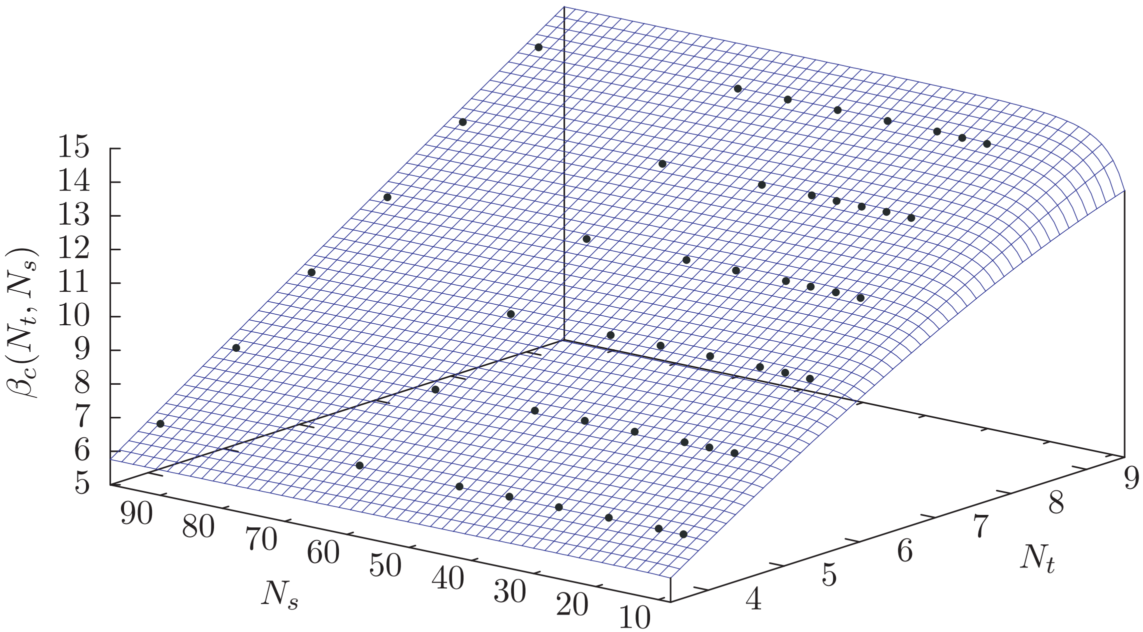
Critical Couplings

$$\frac{\beta_c(N_t)}{4} = \frac{T_c}{g_3^2}\, N_t - c_1 - c_2\, \frac{g_3^2}{T_c}\, \frac{1}{N_t} + \dots$$



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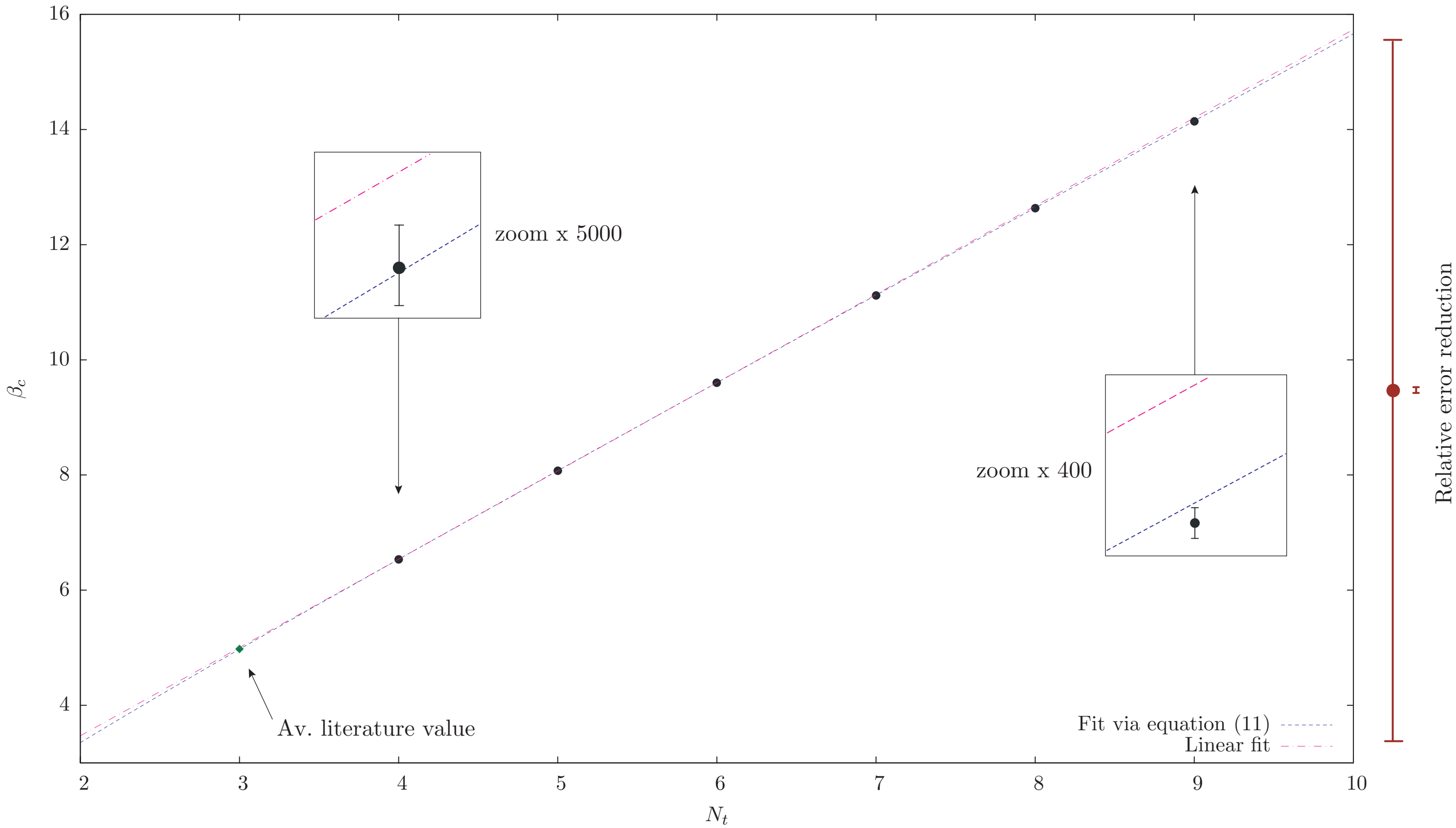
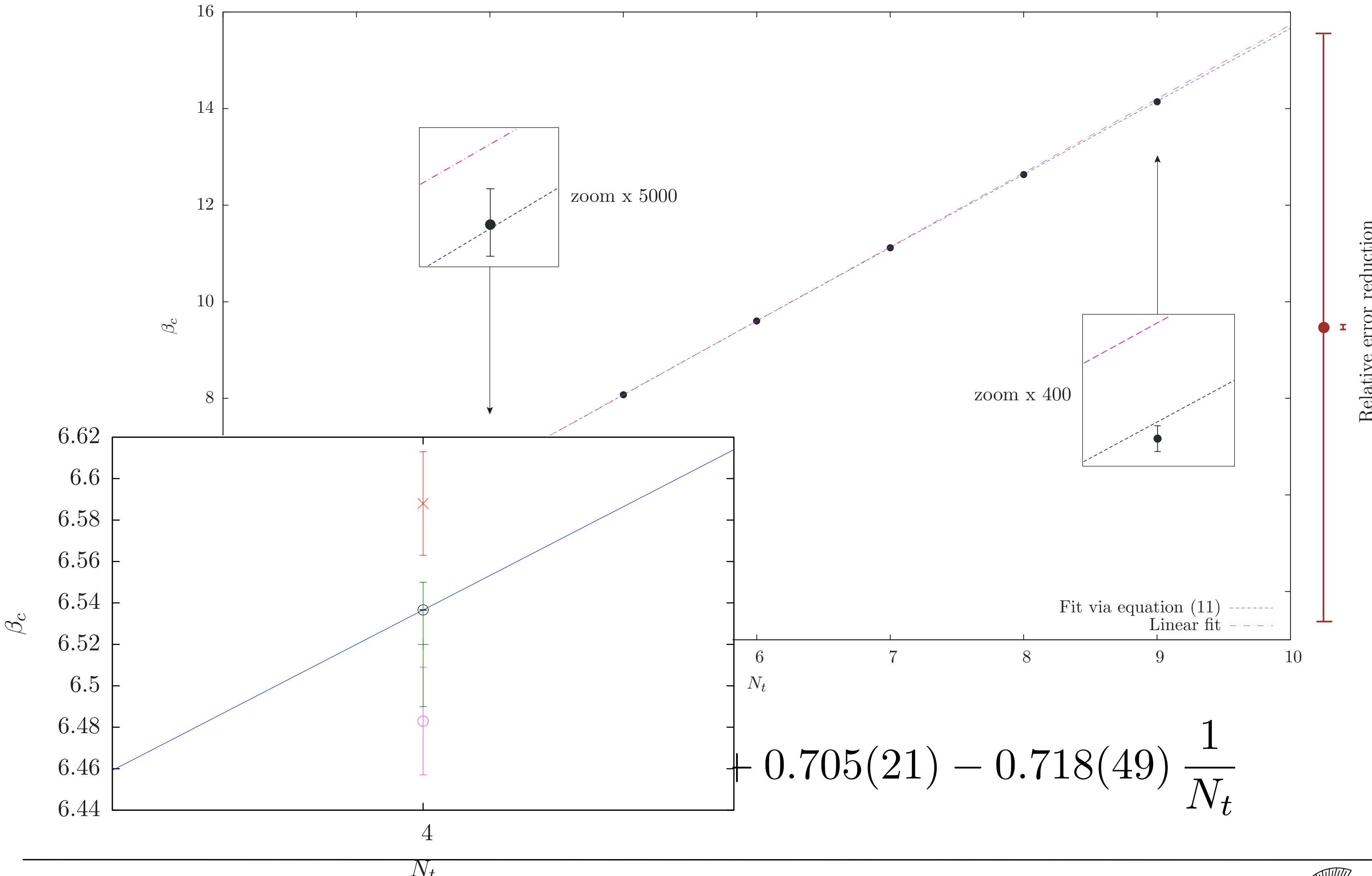


$$\beta_c(N_t) = 1.5028(21)\, N_t + 0.705(21) - 0.718(49)\, \frac{1}{N_t}$$

Critical Couplings



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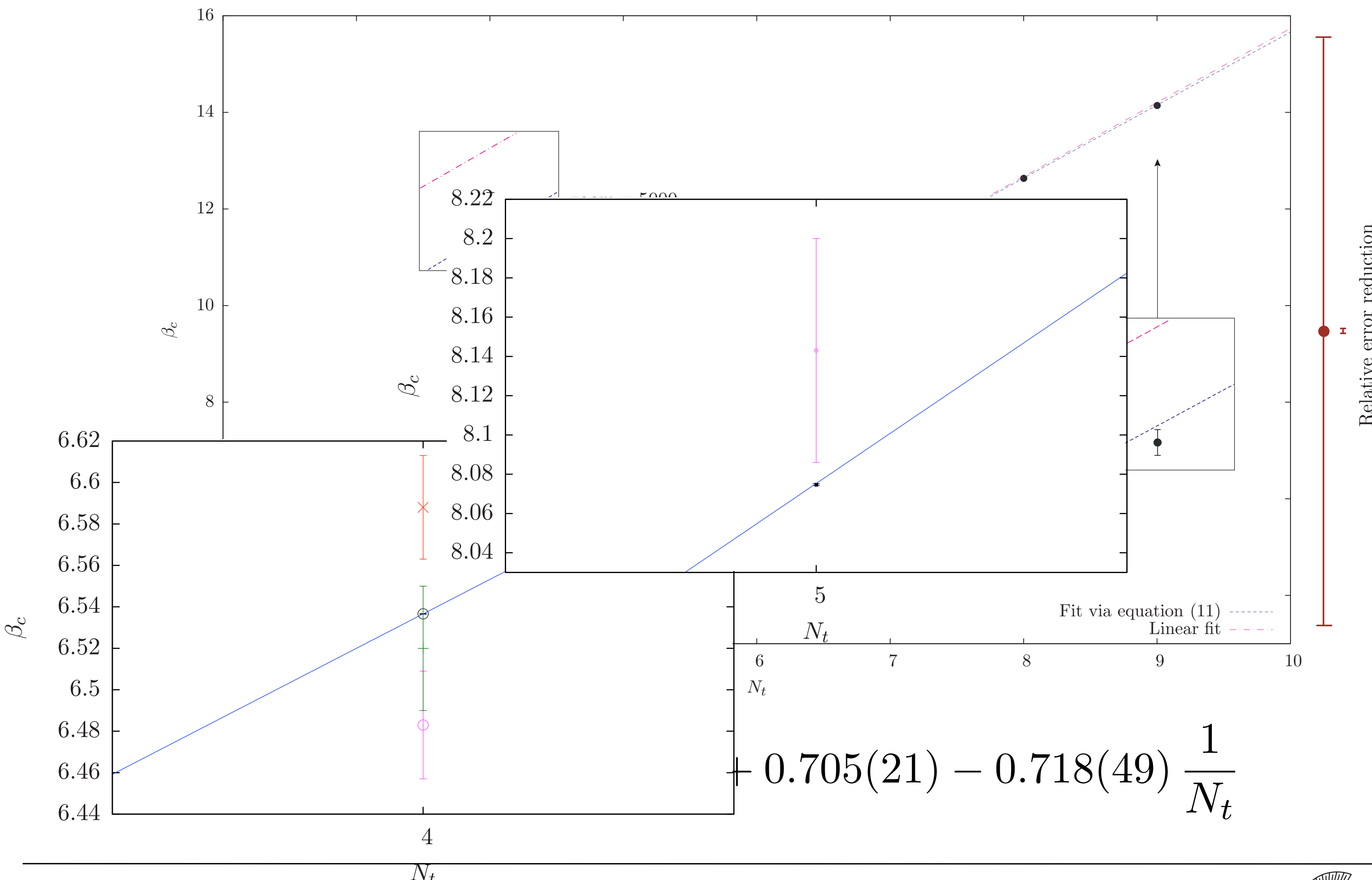

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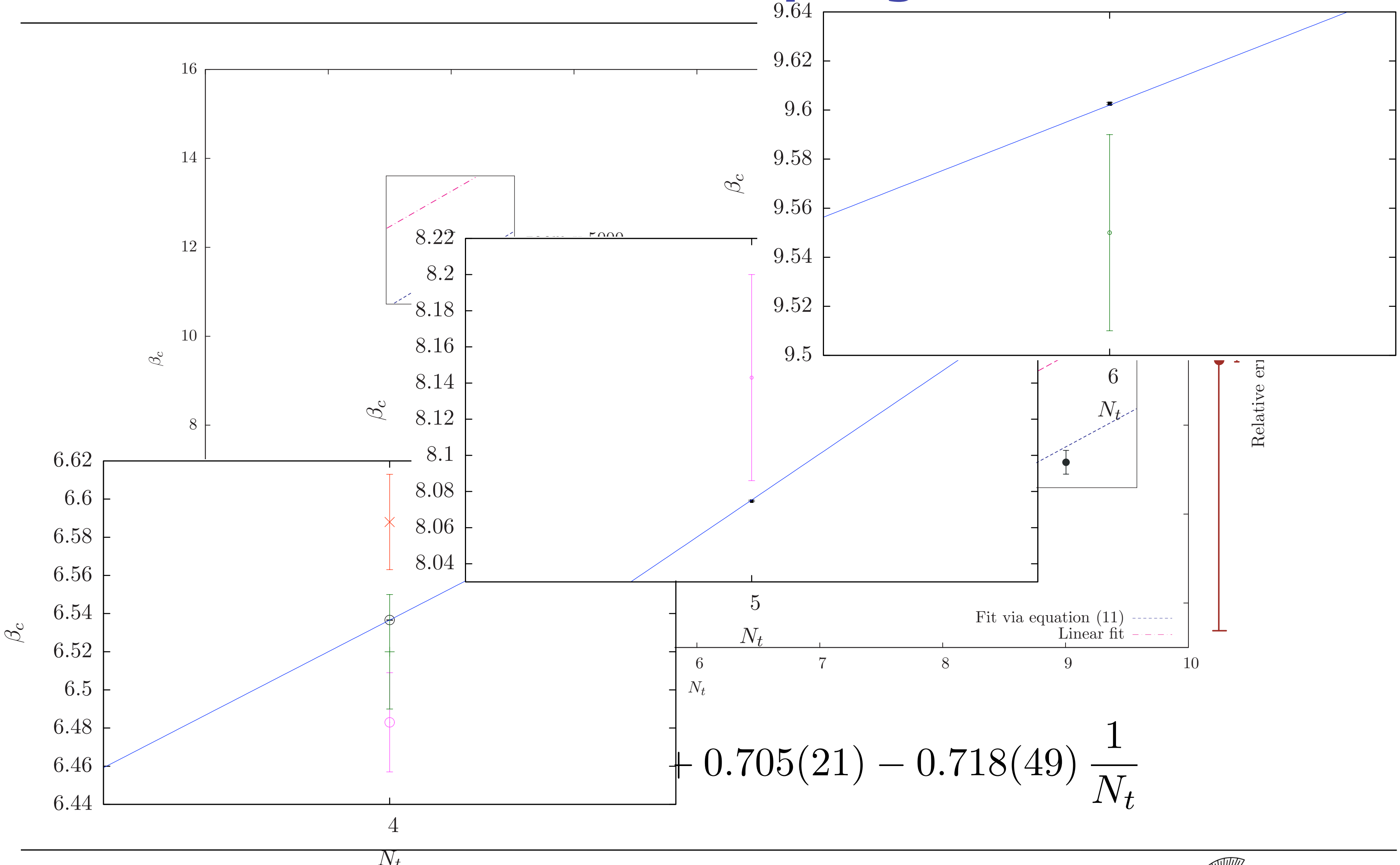

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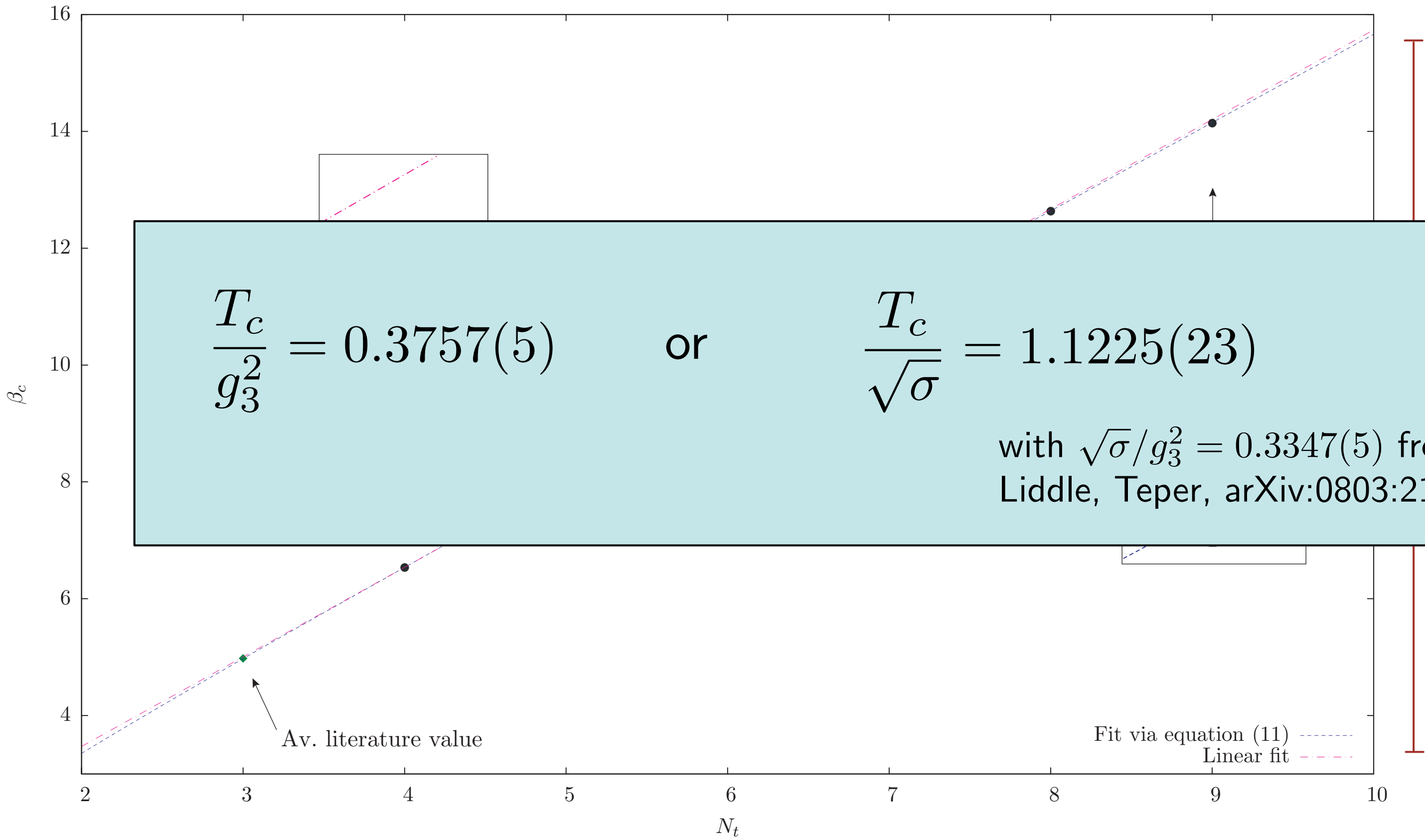


$$\frac{T_c}{g_3^2} = 0.3757(5) \qquad \text{or} \qquad \frac{T_c}{\sqrt{\sigma}} = 1.1225(23)$$

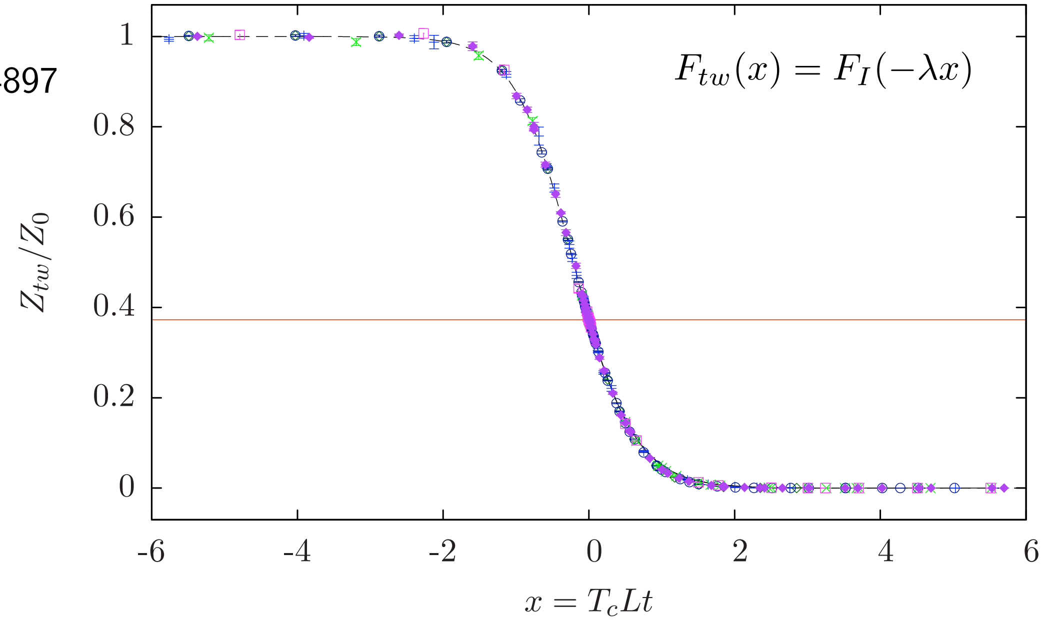
with $\sqrt{\sigma}/g_3^2 = 0.3347(5)$ from
Liddle, Teper, arXiv:0803:2128

$$\beta_c(N_t) = 1.5028(21)\, N_t + 0.705(21) - 0.718(49)\, \frac{1}{N_t}$$

Center Vortex vs Elecrtic Flux Free Energies

$F_I(x)$: Interface free energy in Ising model,
Wu et al., J. Phys. A: Math. Gen. 32 (1999) 4897

Center vortex: $F_{tw} = -\ln(Z_{tw}/Z\ )$



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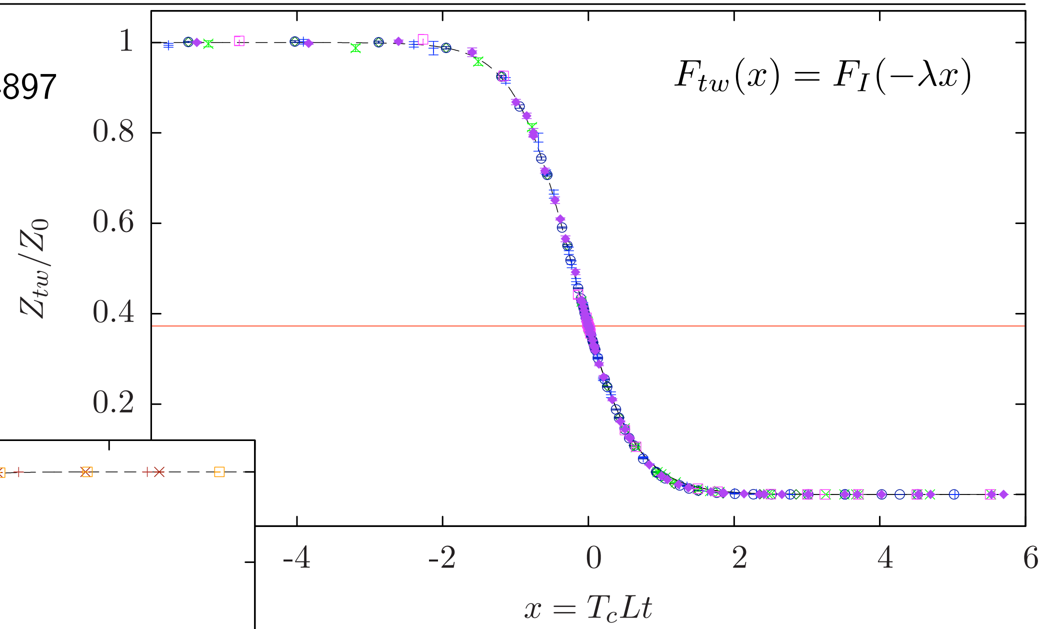

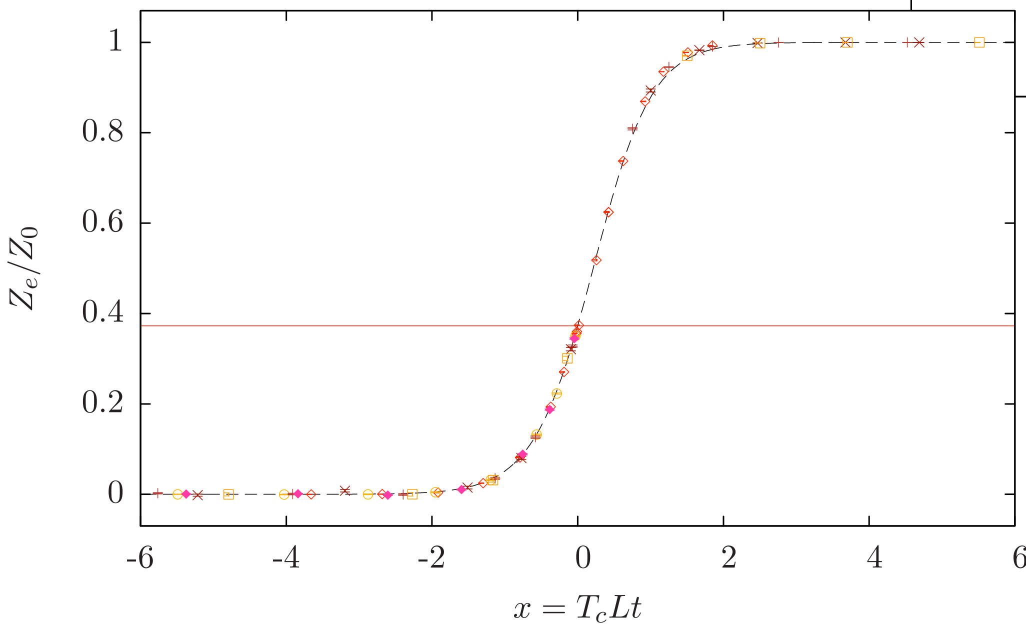

From 2 dim. Z_2 Fourier Transform
(also of Ising partition functions):

Electric flux (static charge):

$F_q = -\ln(Z_e/Z_0)$

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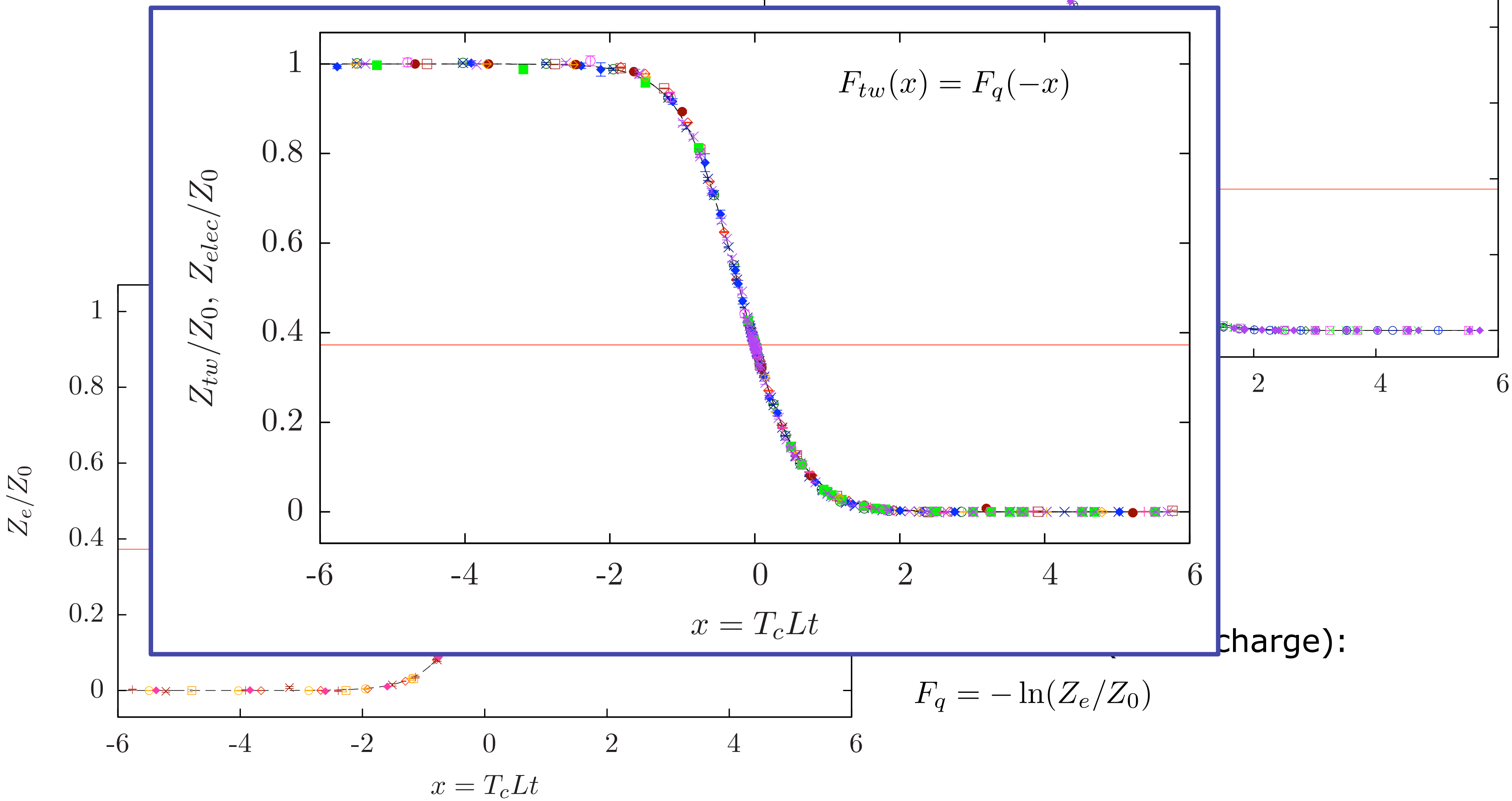

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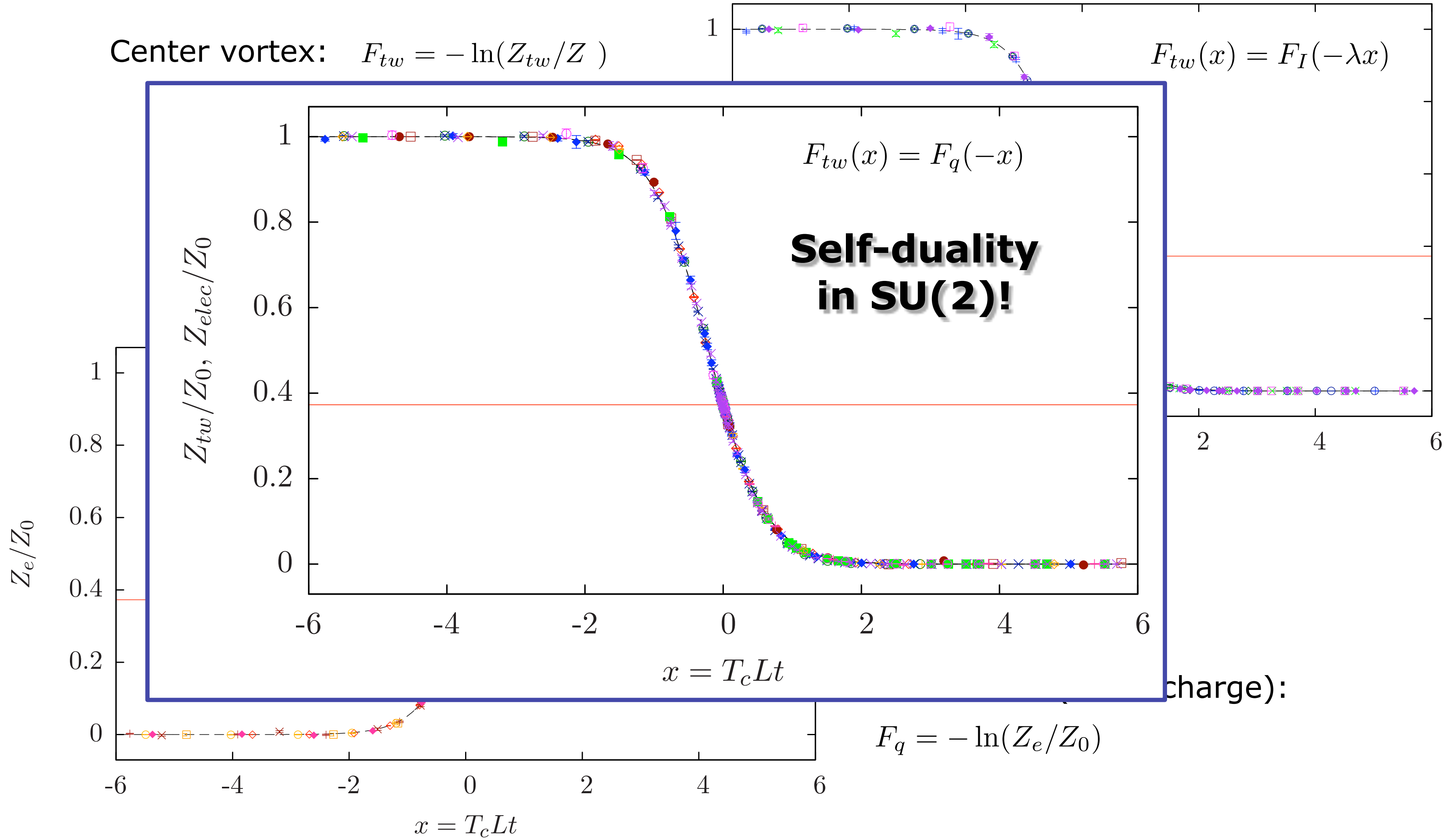

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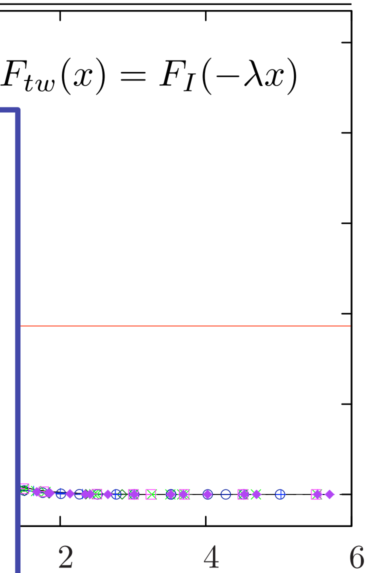

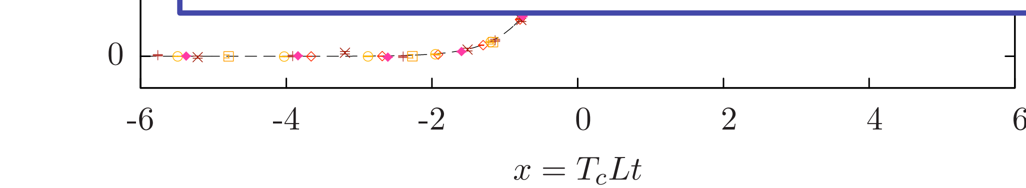

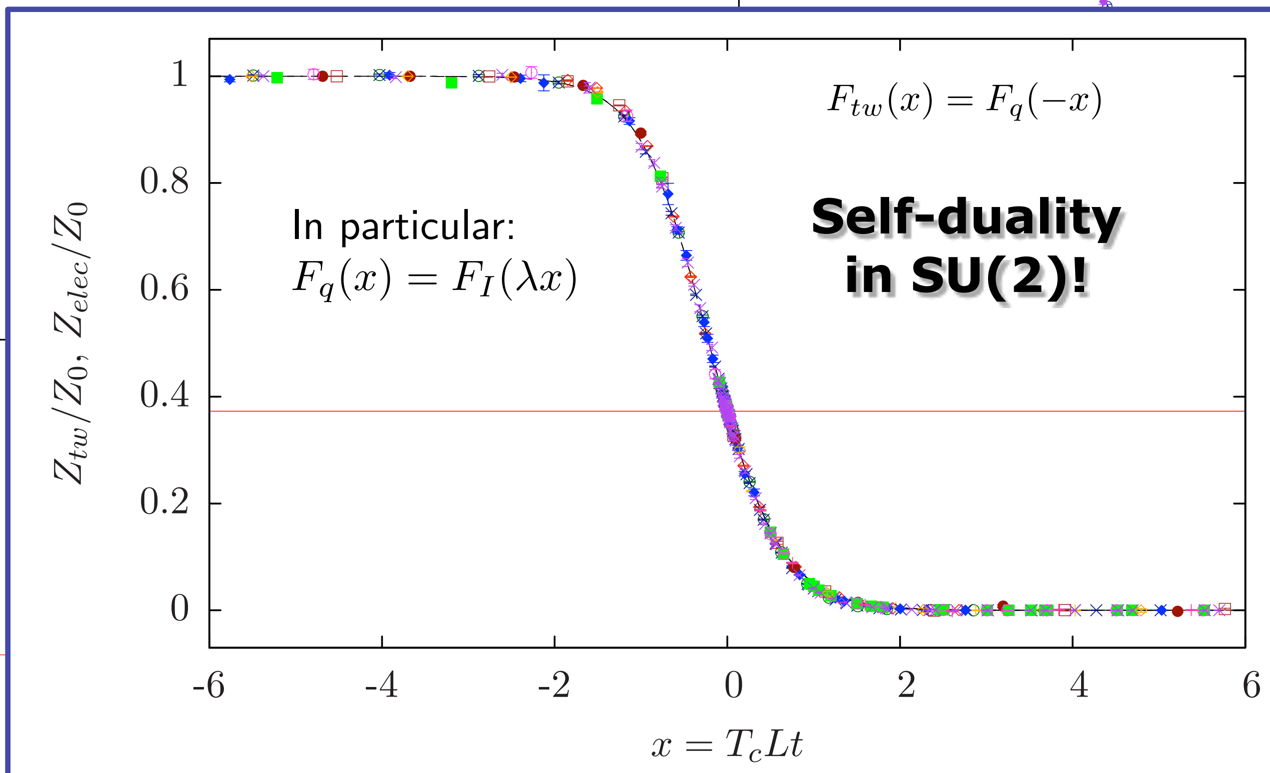

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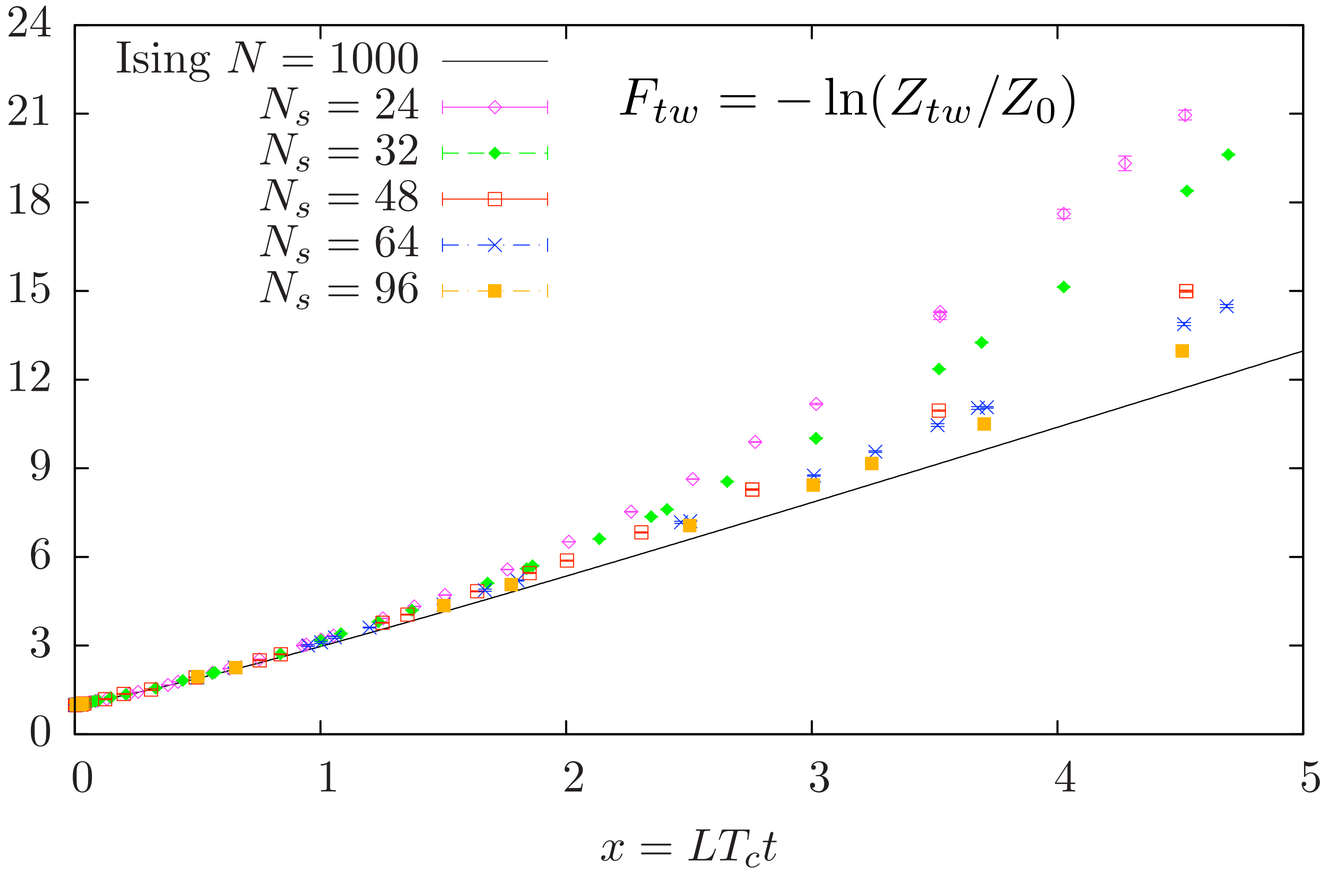
Vortex Free Energy

$T > T_c, \; N_t = 4$



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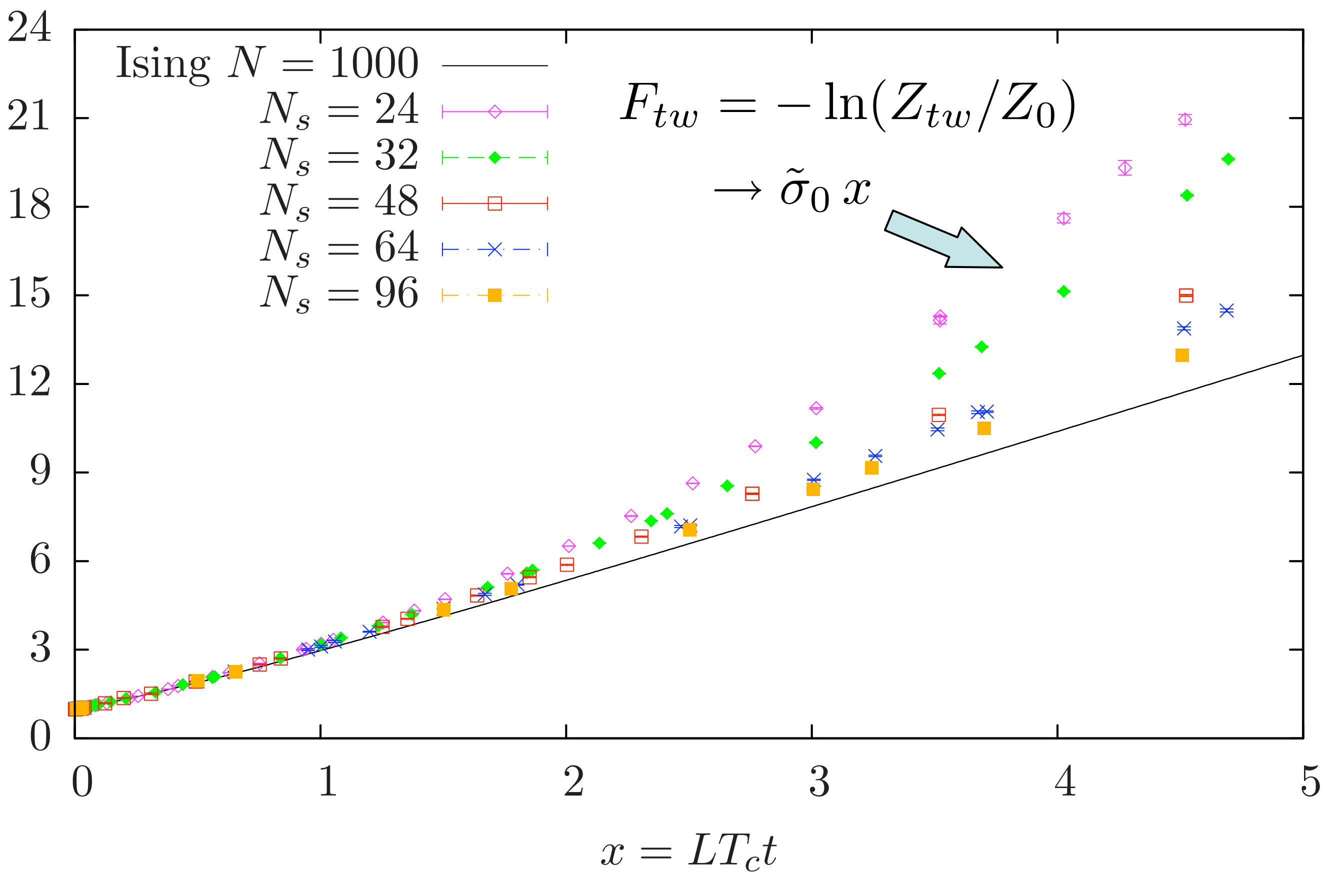

Finite-Size Scaling

- Exact results from the 2D Ising model with Interfaces:

2D Ising	2+1 D SU(2)
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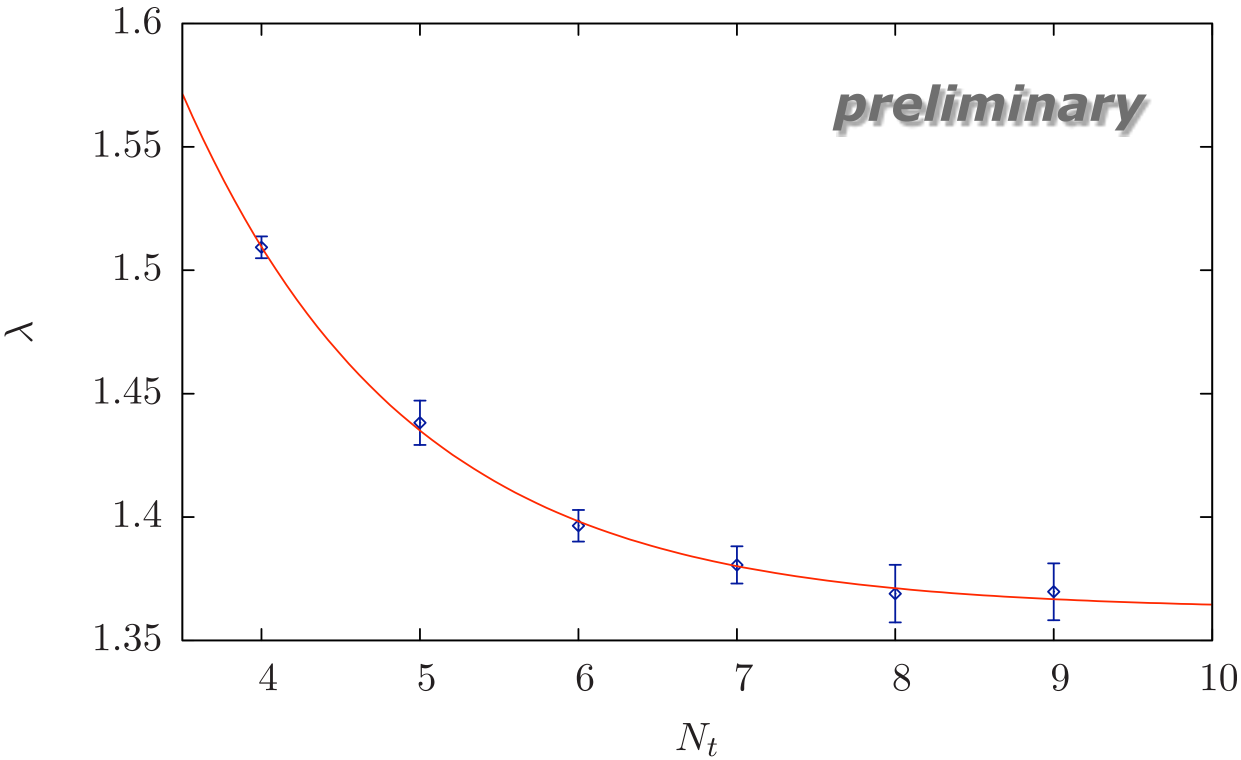
Continuum Limit

Temperature $T = \dfrac{1}{aN_t}$



$$x_{\text{Ising}} = -\lambda\, x$$

$N_t \to \infty$:

$$\lambda(N_t) \to 1.362(3)$$

$$\therefore \quad \tilde{\sigma} = \lambda T_c 2 \ln(1+\sqrt{2})\, t\,, \;\; t > 0$$

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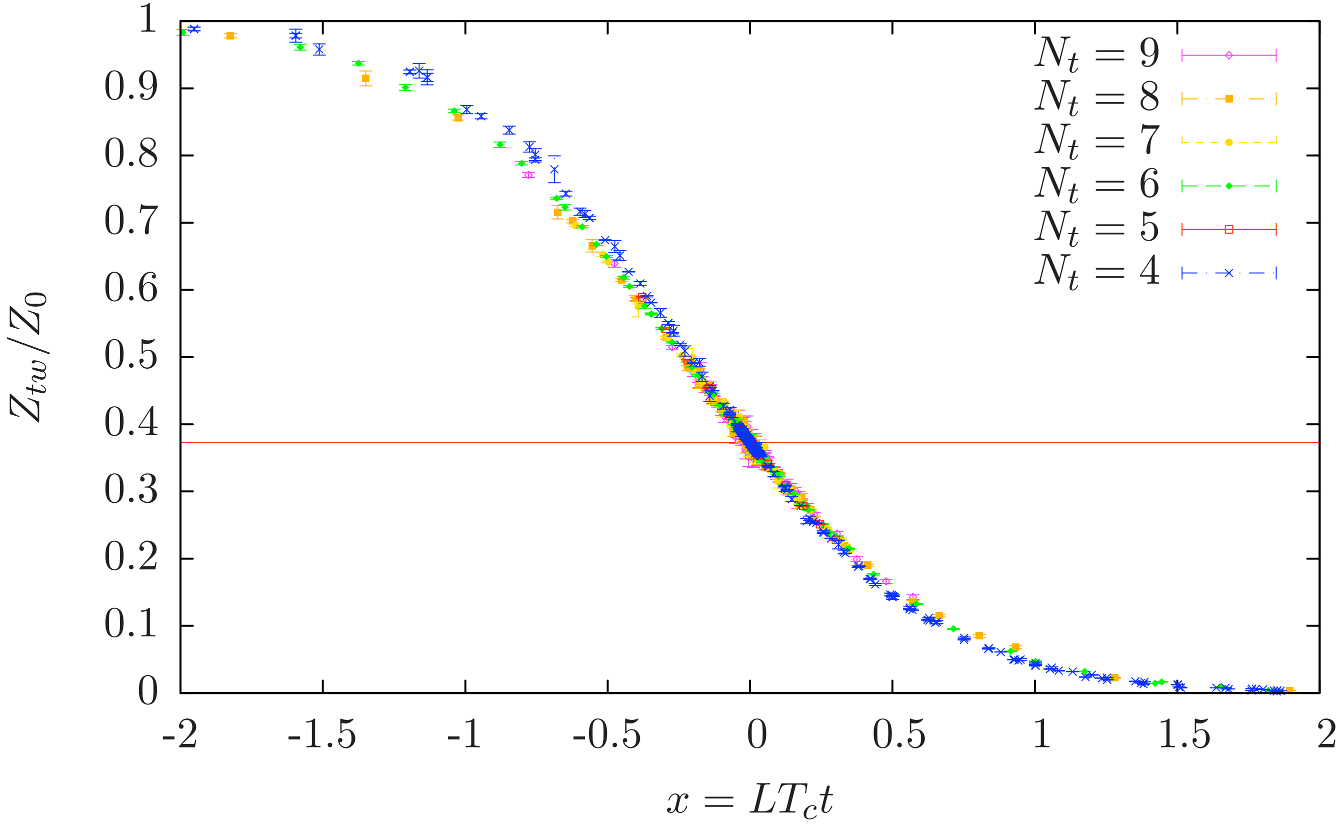
N_t - Scaling

- Compare different N_t lattices:



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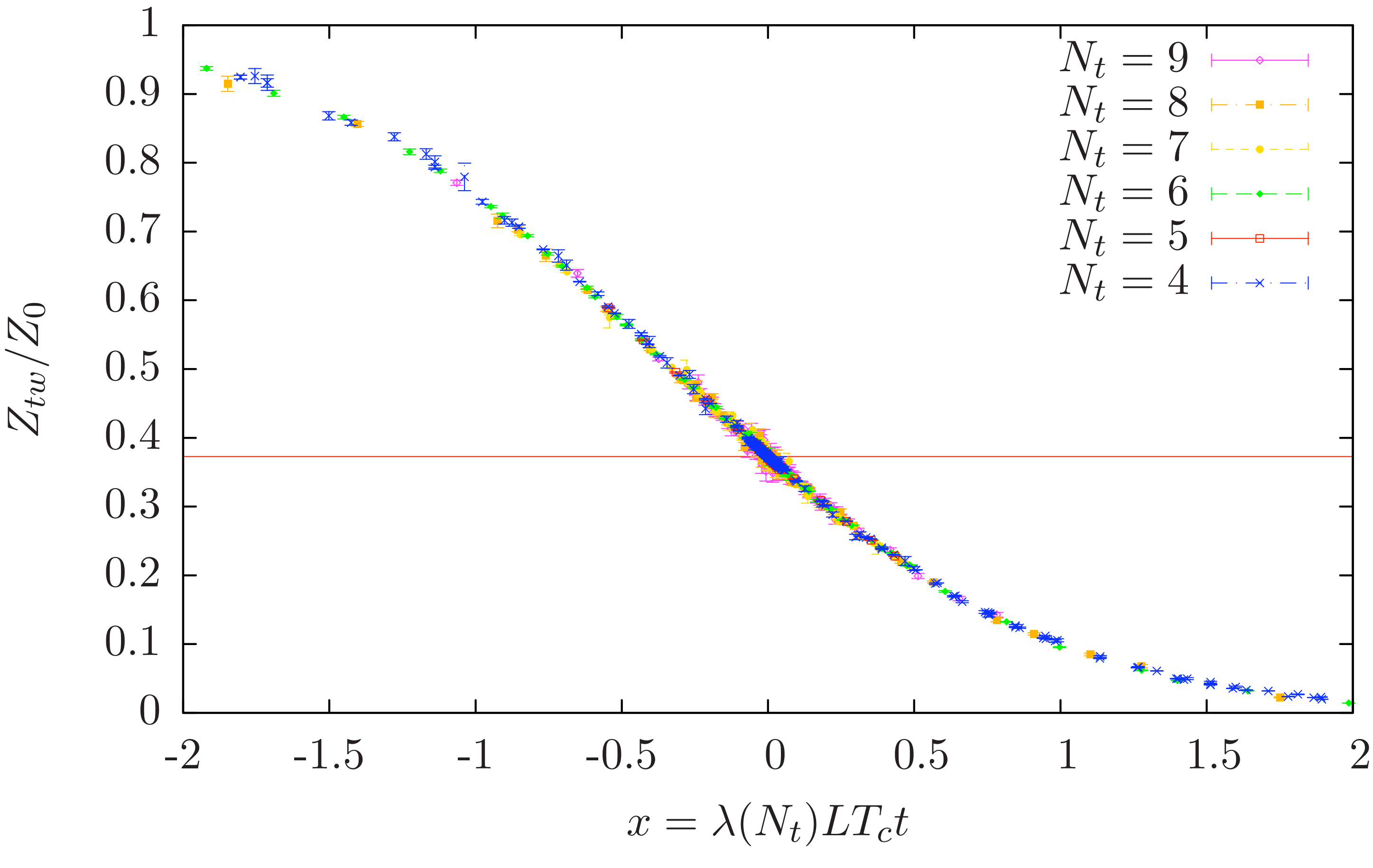
N_t - Scaling

- Rescale: $x \rightarrow \lambda(N_t)\, x$



Conclusions

- Exact results from 2D Ising model for 2+1 D SU(2):
  - precision determination of critical coupling β_c and temperature T_c/g_3^2
  - determine how temperature varies with lattice coupling β around T_c (at fixed N_t)
  - one parameter fits to vortex free energies around T_c, $F_{tw}(x) = F_I(-\lambda x)$ with $\lambda \to 1.362(3)$ for $N_t \to \infty$
  - finite size scaling analysis (including diff. N_t-lattices):

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Thank You!