Chiral and deconfinement transitions from Dyson-Schwinger equations

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C.F. and J. Mueller, arXiv:0908.0007 [hep-ph]

C.F., Phys. Rev. Lett. in press, arXiv:0904.2700 [hep-ph]

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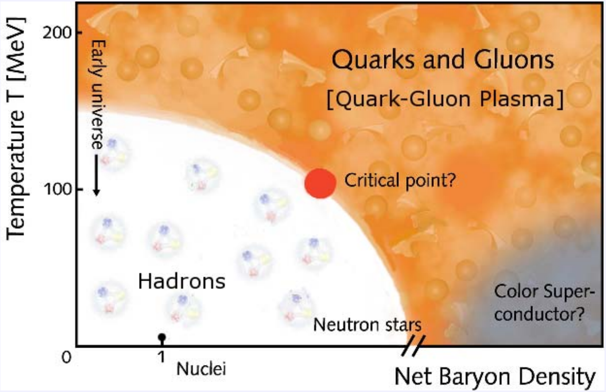
QCD phase transitions



- Chiral limit ($M_{weak} = 0$): order parameter chiral condensate
- Heavy quarks ($M_{weak} = \infty$): order parameter Polyakov-loop

Lattice QCD vs. DSE/FRG: Complementary!

- Lattice simulations
  - Ab initio
  - Gauge invariant

- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  - Space-Time-Continuum
  - Chiral symmetry: light quarks and mesons
  - Analytic solutions at small momenta
  - Chemical potential: no sign problem

QCD in covariant gauge

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c]\ \exp\left\{ -\int_0^{1/T} dt \int d^3x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Landau gauge ($\xi = 0$) propagators in momentum space, $q = (\vec{q}, \omega_q)$:

$$D^{\mathrm{Gluon}}_{\mu\nu}(q) = \frac{Z_T(q)}{q^2} P^T_{\mu\nu}(q) + \frac{Z_L(q)}{q^2} P^L_{\mu\nu}(q)$$

$$S^{\mathrm{Quark}}(q) = \frac{1}{-i\,\vec{\gamma}\vec{q}\,A(q) - i\,\gamma_4\omega_n\,C(q) + B(q)}$$

The Goal:

Gauge invariant information from gauge fixed functional approach

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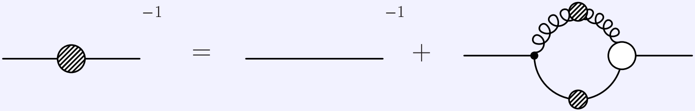
The ordinary chiral condensate



- Consider DSE in infinite volume/continuum

  C.F. and J.Mueller, arXiv:0908.0007 [hep-ph]

- Consider DSE on torus with $V = 1/T \times L^3$
  - spatial directions: periodic boundary conditions
  - temporal direction: antiperiodic boundary condition

  C.F.,PRL in press,arXiv:0904.2700 [hep-ph]

- Order parameter for chiral transition:

$$\langle\bar{\psi}\psi\rangle = Z_2 N_c \frac{T}{L^3} Tr_D \sum_{\vec{p},\omega_p} S(p_{\vec{p},\omega_p})$$

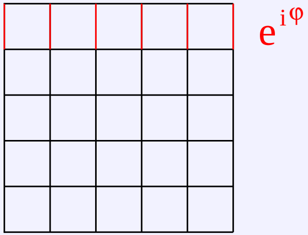
The dual condensate I

Consider general $U(1)$-valued boundary conditions in temporal direction for quark fields ψ:

$$\psi(\vec{x}, 1/T) = e^{i\varphi}\psi(\vec{x}, 0)$$

Matsubara frequencies: $\omega_p(n_t) = (2\pi T)(n_t + \varphi/2\pi)$

Lattice:



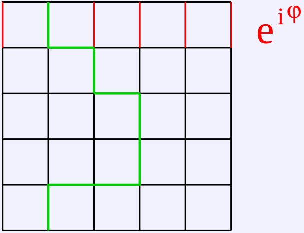
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007..

The dual condensate II

Relation of condensate to loops of link variables $U_\mu(x)$:

$$\langle\overline{\psi}\psi\rangle_\varphi = Tr\,[m + D_\varphi]^{-1} = \frac{1}{Vm}\sum_{l\in\mathcal{L}} \frac{e^{i\varphi n(l)}}{(2am)^{|l|}}\, Tr_c \prod_{(x,\mu)\in l} s(l)U_\mu(x)\,.$$

- geometric series of inverse staggered Dirac operator
- winding number $n(l)$ of loop l around temporal direction



E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

The dual condensate III

Then define dual condensate Σ_n:

$$\Sigma_n \quad = \quad -\int_0^{2\pi} \frac{d\varphi}{2\pi} \, e^{-i\varphi n} \, \langle \overline{\psi}\psi \rangle_\varphi$$

- $n = 1$ projects out loops with $n(l) = 1$: dressed Polyakov loop
- transforms under center transformation exactly like ordinary Polyakov loop
- Σ_1 is order parameter for center symmetry/deconfinement

  E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.
- Σ_1 is accessible with functional methods

  C.F., PRL in press, arXiv:0904.2700 [hep-ph]

The (dual) scalar quark dressing

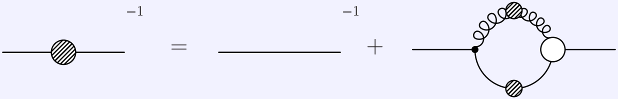
Inverse quark propagator:

$$S^{-1}(q) = -i\,\vec{\gamma}\vec{q}\,A(q) - i\,\gamma_4\omega_n\,C(q) + B(q)$$

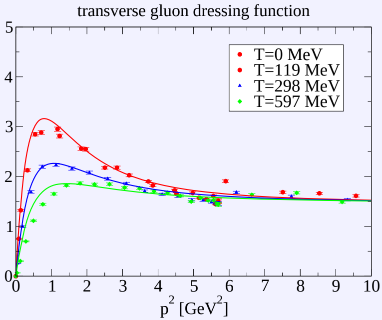
- Another order parameter for chiral symmetry breaking:
  Scalar quark dressing function $B(0, \pi T)$

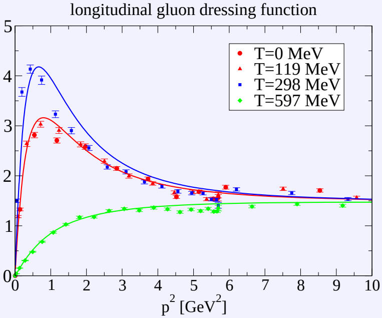
- Another order parameter for confinement/deconfinement:
  Dual scalar quark dressing:

$$\Sigma_B = -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi}\, B(0, \varphi T)\,,$$



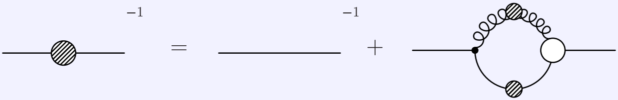
- *T*-dependent gluon propagator from lattice data





Cucchieri, Maas, Mendes, PRD75 (2007)

Input into quark-DSE



- T-dependent ansatz for quark-gluon vertex

$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3 \left(\delta_{4\nu}\gamma_4 \frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j \frac{A(k)+A(p)}{2} \right) \times \left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2+1]}{4\pi} \right)^{2\delta} \right).$$

- $T = 0$: Similar ansatz sucessfull in describing meson observables

CF and R. Williams, PRD **78**, 074006 (2008)

CF and R. Williams, Phys. Rev. Lett in press, arXiv:0905.2291 [hep-ph]

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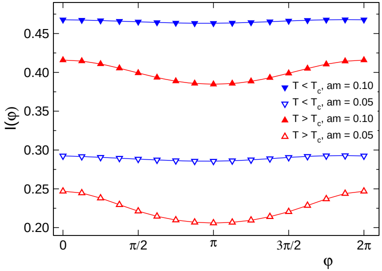
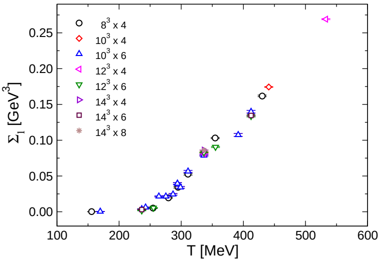
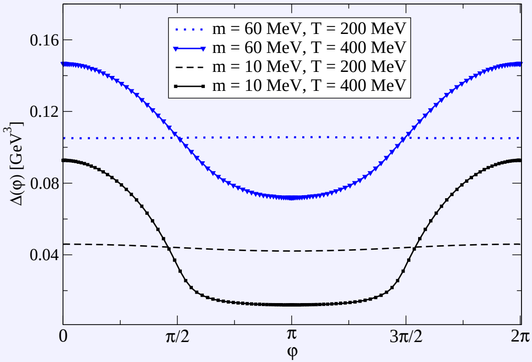




$$\Sigma_1 = -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi} \langle \overline{\psi}\psi \rangle_\varphi$$

E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

Angular dependence of condensate

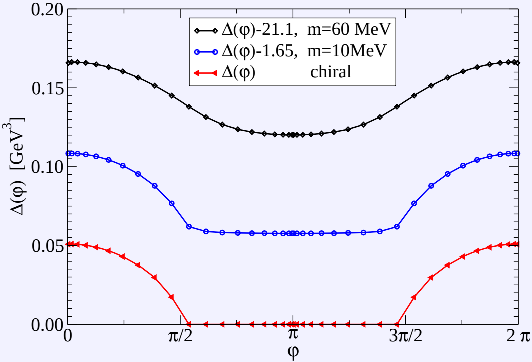


$$\Delta(\varphi) \equiv \langle \overline{\psi}\psi \rangle_{\varphi} = Tr\,[m + D_{\varphi}]^{-1} = \frac{1}{Vm} \sum_{l \in \mathcal{L}} \frac{e^{i\varphi n(l)}}{(2am)^{|l|}} Tr_c \prod_{(x,\mu)\in l} s(l) U_{\mu}(x)\,.$$

- Smaller mass: more contributions from loop with larger $n(l)$!

C.F., PRL in press, arXiv:0904.2700 [hep-ph]

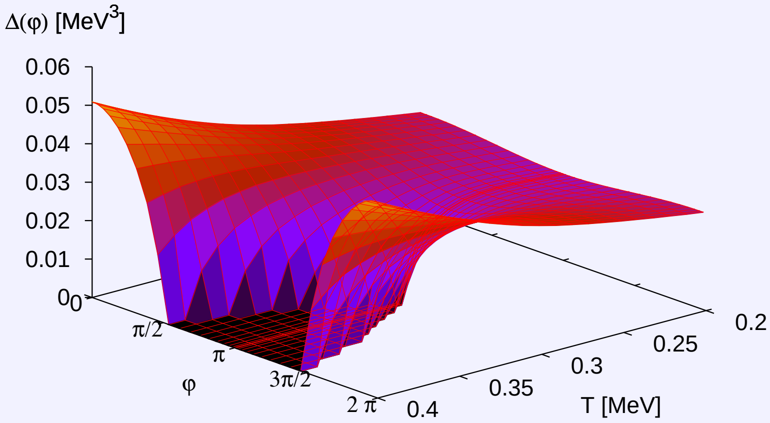
Angular dependence in chiral limit



- Chiral limit: need continuum DSEs
- Chiral limit: loop expansion breaks down
- Width of plateau is T-dependent!

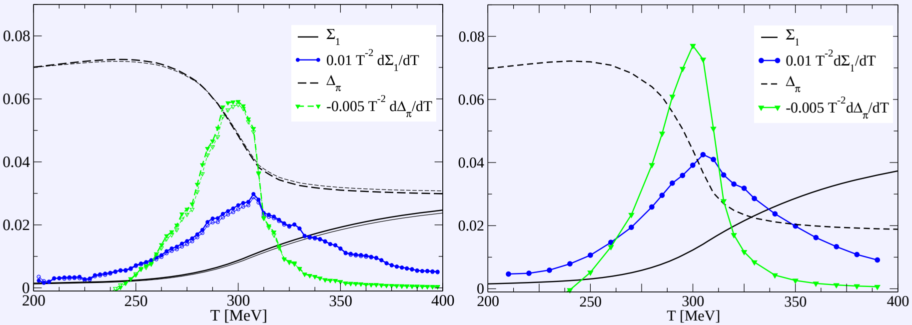
C.F. and Jens Mueller, arXiv:0908.0007 [hep-ph]

Angular and temperature dependence in chiral limit



- Width of plateau grows with T but saturates at $T \approx 2\,T_c$
- $\Delta(\varphi = 0)$ grows with T^2 for $T > 2\,T_c$

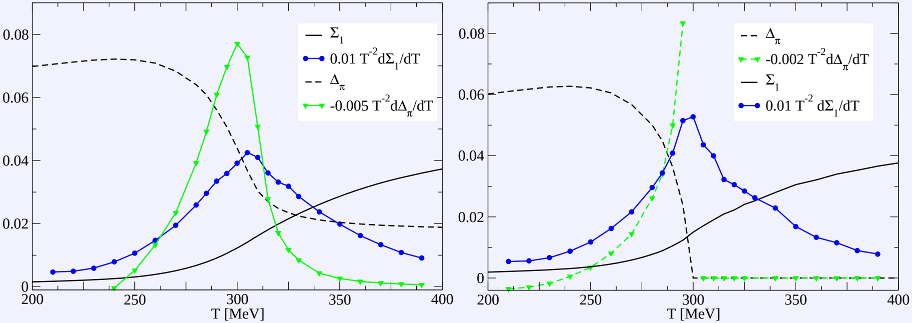
Transition temperatures for finite quark masses



τ	T_{χ_R/T^4}	T_{χ_R}	T_{deconf}
301(2)	304(1)	305(1)	308(2)

- $T_{chiral} \leq T_{deconf}$
- Systematic study of volume and discretisation effects possible.

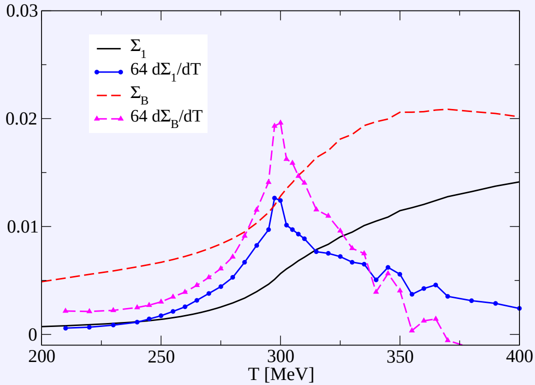
Transition temperatures in chiral limit



τ	T_{χ_R/T^4}	T_{χ_R}	T_{deconf}
298(1)	298(1)	298(1)	299(3)

- $T_{chiral} = T_{deconf}$
- Second order chiral phase transition

Dual scalar quark dressing Σ_B



- Transition temperatures of dual condensate and dual scalar quark dressing agree

$$\Sigma_B = -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi} B(0, \varphi T)\,,$$

Summary

Techniques:

- Dual condensate → deconfinement order parameter
- Dual scalar quark dressing → deconfinement order parameter
- Calculable with functional methods!

C.F., PRL in press, arXiv:0904.2700 [hep-ph]; C.F. and J. Mueller, arXiv:0908.0007 [hep-ph]

J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski,arXiv:0908.0008 [hep-ph]

J. Braun, H. Gies and J. M. Pawlowski, arXiv:0708.2413 [hep-th].

Results for DχSB and Deconfinement:

- (Slightly) Different transition temperatures at finite quark mass
- Same transition temperatures in chiral limit

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TECHNISCHE UNIVERSITÄT DARMSTADT

HELMHOLTZ GEMEINSCHAFT

GSI

HIC for FAIR

LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

EMMI

Helmholtz-Alliance: Extremes of density and temperature; cosmic matter in the laboratory

Gluon and Quark-Gluon-Vertex

Fit function for gluon:

$$Z_{T,L}(\vec{q},\omega_q,T) = \frac{q^2\Lambda^2}{(q^2+\Lambda^2)^2}\left\{\left(\frac{c}{q^2+\Lambda^2 a_{T,L}(T)}\right)^2 + \frac{q^2}{\Lambda^2}\left(\frac{\beta_0\alpha(\mu)\ln[q^2/\Lambda^2+1]}{4\pi}\right)^\gamma\right\}$$

Ansatz for Quark-Gluon-Vertex:

$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3\left(\delta_{4\nu}\gamma_4\frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j\frac{A(k)+A(p)}{2}\right)\times \times\left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2}\left(\frac{\beta_0\alpha(\mu)\ln[q^2/\Lambda^2+1]}{4\pi}\right)^{2\delta}\right).$$