Critical scaling at the chiral phase transition

Jens A. Müller

TU-Darmstadt

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In collaboration with Christian S. Fischer

Outline

Chiral symmetry breaking in QCD

Massless QCD ($m_q = 0$)

classical symmetries

$SU_L(N_f) \times SU_R(N_f) \times U_A(1) \times U_B(1)$

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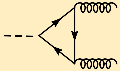
$$\downarrow$$

chiral anomaly



$$\partial_\mu J_5^\mu \propto F^a_{\mu\nu} \tilde{F}^{a,\mu\nu}$$

$$SU_L(N_f) \times SU_R(N_f) \times Z_{2N_f} \times U_B(1)$$

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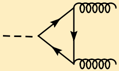

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$$\downarrow$$

spontaneous χSB

$$SU_{L+R}(N_f) \times Z_{2N_f} \times U_B(1)$$

$N_f^2 - 1$ Goldstone particles

Chiral symmetry restoration at $T \neq 0$

Massless QCD ($m_q = 0,\ N_f > 1$)

$$\begin{array}{ccc} \text{low temperature} & \overset{\text{phase transition}}{\longleftrightarrow} & \text{high temperature} \\ SU_{L+R}(N_f) & & SU_L(N_f) \times SU_R(N_f) \end{array}$$

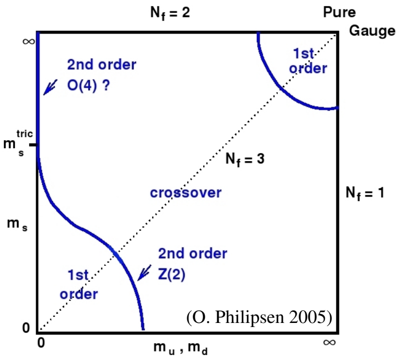
- Order parameters

$$\begin{array}{l} \langle \bar{q}_R q_L \rangle \neq 0,\, T < T_c \\ \langle \bar{q}_R q_L \rangle = 0,\, T > T_c \end{array} \quad \text{or} \quad \begin{array}{l} \mathrm{Tr}\, S^{-1} \neq 0,\, T < T_c \\ \mathrm{Tr}\, S^{-1} = 0,\, T > T_c \end{array}$$

- Transition depends on $\# N_f$ and m_q
- Phase structure obtained by universality argument (Pisarski & Wilczek '84)

Dependence on $\# N_f$ and m_q

$U_A(1)$ symmetry restored only above T_c



[F. R. Brown et al, PRL 65 (1990)]

Phase transition from microscopic theory

- Need to account for correct dof below and above the transition
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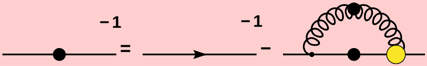
Applicability of DSE to phenomenology of chiral symmetry restoration

$m_q = 0$ and $N_f = 2 \rightarrow$ 2nd order PT $O(4)$ universality class

Dyson-Schwinger Equations

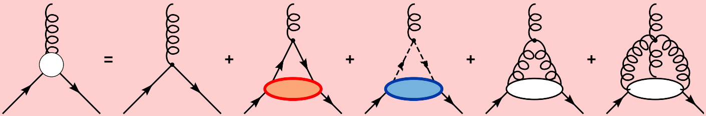
Examples:

Quark DSE



$$S^{-1}(p) = [S^{(0)}(p)]^{-1} - \Sigma(p)$$

DSE quark gluon vertex



Propagators in Matsubara formalism

- Quark propagator

$$S(p_{\omega_n}) = (i\,\vec{\gamma}\cdot\vec{p}\,A(p_{\omega_n}) + i\,\gamma_4\omega_n\,C(p_{\omega_n}) + B(p_{\omega_n}))^{-1}$$

$$p_{\omega_n} = (\omega_n, \vec{p}), \quad \omega_n = \pi T(2n+1)$$

- Landau gauge gluon propagator

$$D_{\mu\nu}(p_{\Omega_n}) = \Delta^T_{\mu\nu}(p_{\Omega_n})\frac{Z(p_{\Omega_n})}{p^2} + \Delta^L_{\mu\nu}(p_{\Omega_n})\frac{H(p_{\Omega_n})}{p^2}$$

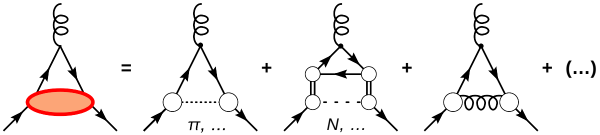
$$p_{\Omega_n} = (\Omega_n, \vec{p}), \quad \Omega_n = \pi T 2n$$

$\Delta^T_{\mu\nu}$, $\Delta^L_{\mu\nu}$ are transverse and longitudinal projectors wrt the heat bath

- Numerical results presented here obtained for $Z(p_{\Omega_n}) = H(p_{\Omega_n})$
- But also qualitative agreement for $Z(p_{\Omega_n}) \neq H(p_{\Omega_n})$

Truncation

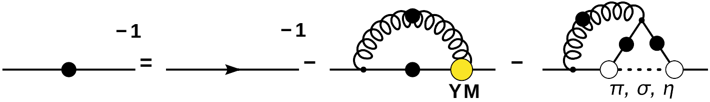
Connected quark-antiquark scattering amplitude



Separate resonant mesonic contributions

$$K^{rs}_{tu}(q,p,P) = \bar{\Gamma}^i_M(q,P)|_{rs}\ \frac{1}{\Omega_n^2 + u^2\vec{p}^2 + m^2}\ \Gamma^i_M(p,P)|_{tu} + R^{rs}_{tu}(q,p,P)$$

This allows to take into account mesonic effects



C. Fischer, D. Nickel, J. Wambach, PRD 76 (2007) 094009

C. Fischer, D. Nickel, R. Williams, arXiv:0807.3486, [hep-ph]

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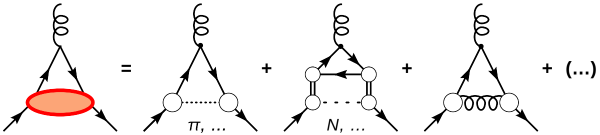

Modified dispersion relation

Pisarski & Tytgat PRD 54 R2989 (1996), Son & Stephanov PRL 88 (2002)

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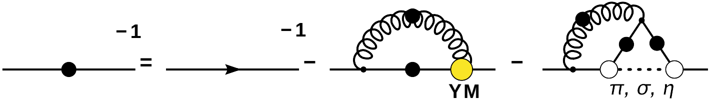

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- Pion/Sigma effects approximated wo solving for the Bethe-Salpeter amplitude
  - from axWTI BSA for low pion momenta ($P \to 0$)

$$\Gamma_\pi^i(p, P \to 0) = i\,\gamma_5\,\tau^i\,\frac{B(p)}{f_t}$$

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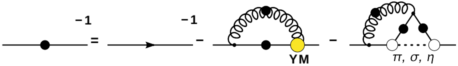
Order parameter $\mathrm{Tr}\, S^{-1} = 4\, B \Longrightarrow B \sim t^x, \quad t = (1 - \frac{T}{T_c})$

Projection on B:



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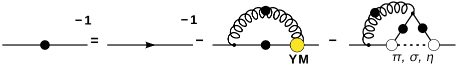

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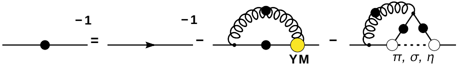

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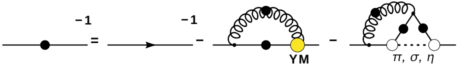

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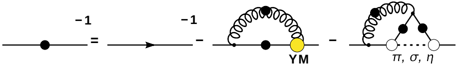

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$$B \sim \Sigma^B_{\pi,\sigma} \sim \frac{B^3}{u^2}\left(+ \frac{B^5}{u^2}\right) \xrightarrow{\text{consistent}} B \sim t^{\nu/2}$$

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- Known $\langle\bar{q}q\rangle \sim t^{\beta}, \quad u^2 \sim f_s^2 \sim t^{\nu}, \quad \beta = \nu/2$
  ($d = 3$ and wo anomalous dim. η)

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- Ansatz $u \sim t^{\nu/2} \longrightarrow B \sim t^{\nu/2}$

$$B \sim t^{\nu/2} \Rightarrow \begin{cases} \longrightarrow \langle \bar{q}q \rangle = Z_2 \, \mathrm{Tr}\, S \sim t^{\nu/2} \\ \xrightarrow{\text{Pagel-Stokar}} f_s^2 \sim t^{\nu} \end{cases}$$

simple scaling analysis!

closed equations in t

Can be confirmed numerically!

Numerical verification of scaling analysis (preliminary)

Truncation in numerical calculation

- only $\Omega_0^\pi = 0$ taken into account
- bare quark propagators in meson exchange contribution
- negelect momentum dependence in BSA $\Gamma_\pi(p, P) \to \frac{B(0)}{f_t}$

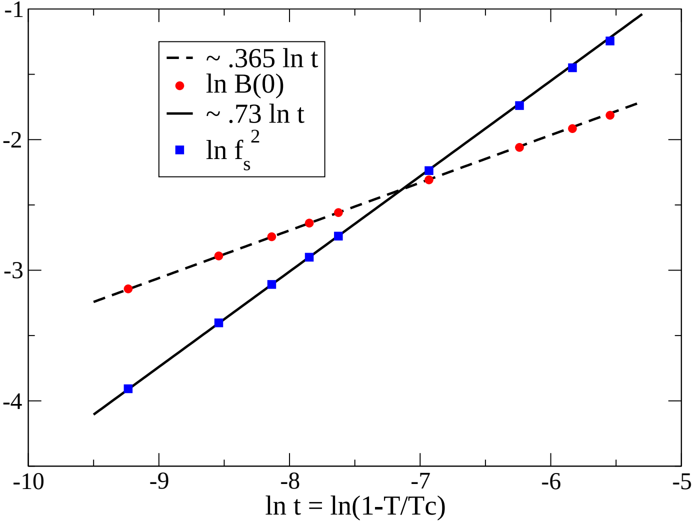
$\to$ meson exchange loop can be calculated analytical

- f_s not yet coupled back in dynamical system
- Assume $u \sim t^{\nu/2} \to$ check analysis by numerics

Preliminary! Not all components of meson exchange included in numerics

Numerical verification of scaling analysis (preliminary)

$u \sim t^{\nu/2}$ assumed, here with $\nu = .73$



Applicability of DSE to phenomenology of chiral symmetry restoration

- Truncation scheme

    Tractable meson back-reaction in SSB-phase $\longrightarrow$ accounts for important dof near PT

- Scaling analysis suggests that truncation scheme has essential elements to describe 2nd order phase transition

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- Check if complete system generates non mean-field critical scaling
- Maybe:
    - BSE at finite temperature