Motivation BH-Model Susceptibilities $\mu_B \neq 0$ Conclusions

Susceptibilities from a black hole engineered EoS with a critical point

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Lattice QCD

Perform calculations at $\mu_B = 0$, and extrapolate via Taylor expansion to finite $\mu_B$

Black Hole Engineering

Based on Lattice data at $\mu_B = 0$, allows us to calculate observables at finite density.

Susceptibilities of Conserved Charges

Are sensitive to the critical point and can be measured
Lattice QCD

QCD on a discretized lattice

- Study QCD from first principles in the non-perturbative region (Performs path integral using Monte-Carlo technique).
- Calculate equilibrium properties at $\mu_B = 0$ or at imaginary-$\mu_B$ (sign problem!). Calculations can be extrapolated to a small regime of $\mu_B$.
- Has technical difficulties to compute transport properties.
Black Hole Engineering

Holography (gauge/string duality at Strong Coupling)

<table>
<thead>
<tr>
<th>Quantum Field Theory in 4-dimensions</th>
<th>Classical Gravity in at least 5-dimensions</th>
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- Coupling $> > 1$ in QFT $\rightarrow$ vanishing string coupling
- $(T, \mu_B)$ in QFT $\rightarrow$ black hole solution
- Holography $\rightarrow$ Near Perfect fluidity

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Holographic Model

Non-conformal holographic gravity
dual in 5 dimensions

$$S = \frac{1}{16\pi G_5} \int d^5 x \sqrt{-g} \left[ R - \frac{1}{2} (\partial_M \phi)^2 - V(\phi) - \frac{1}{4} f(\phi) F_{MN}^2 \right]$$

Input parameters are fixed by lattice QCD results at $\mu_B = 0$

Finite $T$ and $\mu_B \to$ Predictions

Black Hole Model

- Input parameters are fixed by Lattice data at \((T, \mu_B = 0)\)
- Non-conformal Equation of State
  - at finite \(T\) and finite \(\mu_B\)
  - with a critical end point
  - agrees with lattice data at small \(\mu_B\)
- Near perfect fluidity
  - Ability to compute transport coefficients near the crossover and at large \(\mu_B\)
Model Predictions at $\mu_B = 0$

A system in thermal equilibrium is characterized by

\[ Z = \text{Tr} \left[ -\frac{H - \sum_i \mu_i Q_i}{T} \right] \]

The Pressure

\[ P = \frac{T}{V} \ln Z \]

The Baryonic Susceptibilities \( \chi^B_n \) are defined as

\[ \chi^B_n(T, \mu_B) = \frac{\partial^n}{\partial (\mu_B/T)^n} \left( \frac{P}{T^4} \right) \]
The susceptibilities $\chi_n = \chi^B_n(T, \mu_B)$ are related directly to the moments of the distribution.

The volume-independent ratios are useful quantities to compare to experimental data.

- mean: $M = \chi_1$
- variance: $\sigma^2 = \chi_2$
- skewness: $S = \chi_3/\chi_2^{3/2}$
- kurtosis: $\kappa = \chi_4/\chi_2^2$

$M/\sigma^2 = \chi_1/\chi_2$

$S\sigma = \chi_3/\chi_2$

$\kappa\sigma^2 = \chi_4/\chi_2$

$S\sigma^3/M = \chi_3/\chi_1$


Chemical freeze-out: all inelastic interactions cease. The chemical composition of the system is fixed

Kinetic freeze-out: all elastic interactions cease: the spectra of the particles are fixed

We want to study the chemical freeze-out

Observables: susceptibilities of conserved charges
- They are fixed at the freeze-out
- They can be measured and calculated
- They are sensitive to the critical point
The probability distribution for the order parameter

$$P[\sigma] \sim \exp \{-\Omega[\sigma]/T\}$$

$$\Omega = \int d^3x \left[ \frac{1}{2} (\nabla \sigma)^2 + \frac{m_\sigma^2}{2} \sigma^2 + \frac{\lambda_3}{3} \sigma^3 + \cdots \right]$$

The correlation length ($\xi = m_\sigma^{-1}$)

$$\xi \sim |T - T_c|^{-\nu} \text{ where } \nu > 0$$

$$\chi_2 = V T \xi^2$$
$$\chi_3 = 2 V T^{3/2} \hat{\lambda}_3 \xi^{9/2}$$
$$\chi_4 = 6 V T^2 [2 \hat{\lambda}_3^2 - \hat{\lambda}_4] \xi^7$$


Taylor expansion of observables in terms of susceptibilities

\[ \chi_n = \chi_n^B(T, \mu_B = 0) \]

- **Pressure**

\[ \frac{p(T, \mu_B) - p(T, \mu_B = 0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n)!} \left( \frac{\mu_B}{T} \right)^{2n} \]

- **Baryonic density**

\[ \frac{\rho_B(T, \mu_B)}{T^3} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n-1)!} \left( \frac{\mu_B}{T} \right)^{2n-1} \]

- **Susceptibilities \( \chi_2 \) and \( \chi_4 \)**

\[ \chi_2(T, \mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+2}}{(2n)!} \left( \frac{\mu_B}{T} \right)^{2n} \]

\[ \chi_4(T, \mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+4}}{(2n)!} \left( \frac{\mu_B}{T} \right)^{2n} \]
Reconstruction of thermodynamic quantities at different values of $\mu_B/T$ via Taylor series from calculations at $\mu_B = 0$. 

The black hole model contains a critical end point at

- \( \mu_B = 723 \pm 36 \text{ MeV} \)
- \( T = 89 \pm 11 \text{ MeV} \)

R. Critelli, I. P. et al., to appear.
Connection to Experiment

- We compare the baryonic BH susceptibilities ratios with the net-proton moments measured at STAR.
- Freeze-out parameters are extracted by fitting the experimental values for $\chi_1/\chi_2$ and $\chi_3/\chi_2$.
- $\chi_4/\chi_2$ predicted at the minimum in speed of sound.

Freeze out parameters from the Black Hole model

Trajectories in the \([ T − \mu ]\) plane that satisfy the experimental values

\[
\sqrt{s} = 27 \text{ GeV} \quad \text{and} \quad \sqrt{s} = 39 \text{ GeV}
\]

Freeze out points \([ T − \mu_B ]\) are extracted from the line made by the closest points between \(\chi_1/\chi_2\) and \(\chi_3/\chi_2\)
Freeze-out Line

Motivation

$\mu_B \neq 0$

Conclusions

Lattice QCD

BH Model

$\mu_B$ [MeV]

$T$ [MeV]

$\chi^2$ inflection line

Speed of sound

CEP

[WB:] R. Bellwied et. al.,

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We estimate a collision energy needed to hit the CEP

\[ \sqrt{s} = 2.5 - 4.1 \text{ GeV} \]

The collision energy is reachable by the next generation of colliders

The holographic Black Hole Model

- Reproduces lattice data at $\mu_B = 0$
- Contains a critical end point at $\mu_B = 723 \pm 36$ MeV and $T = 89 \pm 11$ MeV
- Allows us to compute baryonic susceptibilities, and extract freeze-out parameters
- Estimates that the collision energy needed to hit the CEP should be $\sqrt{s} = 2.5 - 4.1$ GeV