Magneto-hydrodynamical simulations of Heavy Ion Collisions with ECHO-QGP

5th FAIRNESS workshop

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Outline

Outline of the talk:

- Motivations
- Estimates of the magnitude of the magnetic field in H.I.C.
- Implementation of ideal magnetohydrodynamics in ECHO-QGP
- Applications to H.I.C.
- Discussions and conclusions
Strong magnetic fields may produce many interesting effects:

- **The Chiral Magnetic Effect**

- **Pressure anisotropy in QGP**

- **A shift in meson masses**

- **Mass shifts in quarkonia states**
  Suzuki and Yoshida - arXiv: 1601.02178

- **Shift of the Critical Temperature**

- **Influence on the elliptic flow**
  Pang, Endrödi and Petersen - arXiv: 1602.06176v1

- **Influence on directed flow**
Possible time evolution of $|\vec{B}|$ in HIC

The medium plays a crucial role:
Blue line: $\sigma = 0. \text{ fm}^{-1}$
Red line: $\sigma = 0.023 \text{ fm}^{-1}$

Event by event estimates by Roy and Pu

\[ \sigma(x, y, \vec{b}) = \frac{B^2(x, y, \vec{b})}{2\varepsilon(x, y, \vec{b})} \], Au-Au collision at \( \sqrt{s_{NN}} = 200 \) GeV, Glauber-M.C.

Our initial conditions: basic formula for point charge

For an observer at \( \mathbf{r} = z \hat{z} + \mathbf{b} \), \((\mathbf{b} \cdot \hat{z} = 0)\), if \( \gamma = 1/\sqrt{1 - v^2} \gg 1 \):

\[
H(t, b r) = H(t, r) \hat{\phi} = \\
\frac{e}{2\pi \sigma} \hat{\phi} \int_{0}^{\infty} \frac{J_1(k_{\perp} b) k_{\perp}^2}{\sqrt{1 + \frac{4k_{\perp}^2}{\gamma^2 \sigma^2}}} \exp \left\{ \frac{1}{2} \sigma \gamma^2 x_- \left( 1 - \sqrt{1 + \frac{4k_{\perp}^2}{\gamma^2 \sigma^2}} \right) \right\} dk_{\perp}
\]

where \( x_- = t - z/v \) and \( \hat{\phi} \) is the unit vector of the angular polar coordinates in the transverse plane \( x, y \).

Electrical conductivity \( \sigma \) is constant. Ohm law simply: \( \mathbf{J} = \sigma \mathbf{E} \).

We model the nuclei as uniformly charged spheres which freely propagate into a medium, before and after the collision.
More recent equations which are used at present


\[
B_\phi (t, x) = \frac{Q}{4\pi} \cdot \frac{v\gamma x_T}{\Delta^{3/2}} \left( 1 + \frac{\sigma v\gamma}{2} \sqrt{\Delta} \right) e^A,
\]

\[
B_r (t, x) = -\sigma\chi \frac{Q}{8\pi} \cdot \frac{v\gamma^2 x_T}{\Delta^{3/2}} \cdot \left[ \gamma (vt - z) + A\sqrt{\Delta} \right] e^A,
\]

\[
B_z (t, x) = \sigma\chi \frac{Q}{8\pi} \cdot \frac{v\gamma}{\Delta^{3/2}} \cdot \left[ \gamma^2 (vt - z)^2 \left( 1 + \frac{\sigma v\gamma}{2} \sqrt{\Delta} \right) + \Delta \left( 1 - \frac{\sigma v\gamma}{2} \sqrt{\Delta} \right) \right] e^A
\]

where: \( \sigma \) is the electric conductivity, \( \sigma\chi \) the chiral magnetic conductivity, \( \Delta \equiv \gamma^2 (vt - z)^2 + x_T^2, \) \( A \equiv (\sigma v\gamma/2)[\gamma (vt - z) - \sqrt{\Delta}] \)
Collision at $\sqrt{s_{\text{NN}}} = 5.5$ TeV, $b=7$ fm:

Magnetic permeability $\mu \sim 1$

Black line: $e = 1 \text{GeV}/\text{fm}^3$, gray line: $e = 150 \text{MeV}/\text{fm}^3 \sim 140 \text{MeV}$. 
How do magnetic fields compare with thermal pressure?

\[
\log_{10} \beta^{-1}, \text{ where } \beta = 2 \frac{p}{B^2},
\]

at \( \sqrt{s_{NN}} = 5.5 \text{ TeV}, b=7 \text{ fm} \)

\[
\log_{10} \beta^{-1}, \text{ where } \beta = 2 \frac{p}{B^2}, \text{ at } \sqrt{s_{NN}} = 200 \text{ GeV}
\]

What is ECHO-QGP

ECHO-QGP derives from the Eulerian Conservative High-Order astrophysical code for general relativistic magnetohydrodynamics, developed by L. Del Zanna.

(Del Zanna, Zanotti, Bucciantini, and Londrillo, A&A 473 (2007))

A collaboration lead by F. Becattini adapted ECHO-QGP to run second order dissipative hydrodynamical simulations of heavy ion collisions, including the computation of particle spectra following the Cooper-Frye prescription.


Floerchinger, Wiedemann, Beraudo, Del Zanna, Inghirami, Rolando, PLB 735 (2014)


Website: http://theory.fi.infn.it/echoqgp/
The fundamental equations

- Energy and momentum conservation: \( d_\mu T_{\mu\nu} = 0 \)
- Baryonic number conservation: \( d_\mu N^\mu = 0 \)
- Second law of thermodynamics: \( d_\mu s^\mu \geq 0 \)
- Maxwell equations: \( d_\mu F^{\mu\nu} = -J^\nu \quad (d_\mu J^\mu = 0) \quad d_\mu F^{*\mu\nu} = 0 \)

The fundamental assumptions

- We neglect all dissipative effects
- We neglect polarization and magnetization effects
- We assume infinite electrical conductivity
- We assume local thermal equilibrium
The ideal RHMD energy-momentum tensor

Polarization and magnetization neglected

\[ T^\mu_\nu_f = F^\mu_\lambda F^\nu_\lambda - \frac{1}{4} (F^\lambda_\kappa F^\kappa_\lambda) g^\mu_\nu \]
from Maxwell equations: \( d_\mu T^\mu_\nu_f = J_\mu F^\mu_\nu \)

Dissipative effects neglected:

Eckart frame = Landau frame \( \Rightarrow \) single fluid \( u^\mu \) (\( u_\mu u^\mu = -1 \))

Infinite electrical conductivity

Ohm’s law: \( J^\mu = \rho_e u^\mu + j^\mu ; \quad j^\mu = \sigma^{\mu\nu} e_\nu \Rightarrow e^\mu = 0 \)

Energy-momentum tensor \( T^{\mu\nu} \)

\[ T^{\mu\nu} = T^{\mu\nu}_m + T^{\mu\nu}_f \]
Matter: \( T^{\mu\nu}_m = (e + p) u^\mu u^\nu + pg^{\mu\nu} \)
Electromagnetic field: \( T^{\mu\nu}_f = b^2 u^\mu u^\nu + \frac{1}{2} b^2 g^{\mu\nu} - b^\mu b^\nu \)
The energy momentum tensor components

Lorentz transformations from the laboratory to the comoving frame:
\[ e^\mu = (\gamma v_k E^k, \gamma E^i + \gamma \varepsilon^{ijk} v_j B_k) \]
\[ b^\mu = (\gamma v_k B^k, \gamma B^i - \gamma \varepsilon^{ijk} v_j E_k) \]
where:
\[ \varepsilon^{ijk} \text{ is the Levi-Civita pseudo-tensor of the spatial three-metric} \]
\[ \gamma = \text{Lorentz factor, } g_{ij} = \text{diag}(1, 1, 1) \text{ or } g_{ij} = \text{diag}(1, 1, \tau^2) \]
e and \( p \) are measured in the comoving fluid frame,
\( \vec{E} \) and \( \vec{B} \) are measured in the laboratory frame

Components of the energy-momentum tensor

Energy density \( \mathcal{E} \equiv -T^0_0 = (e + p)\gamma^2 - p + \frac{1}{2}(E_k E^k + B_k B^k) \)
Momentum density \( S_i \equiv T^0_i = (e + p)\gamma^2 v_i + \varepsilon_{ijk} E^j B^k \)
Stresses \( T^i_j = (e + p)\gamma^2 v^i v_j + (p + \frac{1}{2}(E_k E^k + B_k B^k))\delta^i_j - E^i E_j - B^i B_j \)
The evolution equations

Ideal Ohm's law in the laboratory frame

\[ e^\mu = 0 \Rightarrow E_i = -\varepsilon_{ijk}v^j B^k \]

The evolution equations in conservative form

\[ \partial_0 U + \partial_i F^i = S \]

where

\[ U = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n \\ S_j \equiv T_j^0 \\ \mathcal{E} \equiv -\varepsilon_0^0 \\ B^j \end{pmatrix}, \quad F^i = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n v^i \\ T_j^i \\ S^i \equiv -T_0^i \\ v^i B^j - B^j v^i \end{pmatrix}, \quad S = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2} T^{ik} \partial_j g_{ik} \\ -\frac{1}{2} T^{ik} \partial_0 g_{ik} \\ 0 \end{pmatrix} \]
Example of 2D+1 simulation: initial conditions

2D+1 simulation in Milne coordinates
Au+Au collision at 200 GeV $\sqrt{s_{NN}}$
Geometrical Glauber initial conditions.
b=10 fm, $\tau_0=0.4$ fm/c, EoS=$p=e/3$
Electromagnetic field computed with Tuchin’s model.
Electrical conductivity of the medium ($\tau \leq \tau_0$): $\sigma = 5.8$ MeV, constant.
Electrical conductivity of the QGP ($\tau > \tau_0$): $\sigma = \infty$. 
Motivations

Time evolution of the magnetic field at the center of the grid

![Graph showing the time evolution of the magnetic field](image)

**Magnetic field (lab frame) at the center of the grid.**
The $p_T$ spectrum and the elliptic flow of charged pions

Transverse momentum distribution of $\pi^+$, computed with the Cooper-Frye formula.

$v_2$ of $\pi^+$, computed with the Cooper-Frye formula.
Assuming that magnetic fields may be larger than in our estimates, what is their effect on $v_2$? What could be their impact on the shear viscosity?
The electrical conductivity of the Quark Gluon Plasma

\[ C_{em} = e^2 \sum_f q_f^2 \]

(Plot from: Aarts et al, JHEP 1502 (2015) 186)

Electrical conductivity is finite and temperature dependent!

(See also: Greif, Bouras, Xu and Greiner, Phys. Rev. D 90, 094014 (2014))

And it is not isotropic!

(See: Hattori and Satow, Phys. Rev. D 94, 114032)
Beyond flows: other RMHD applications

**Λ - Λ polarization**

\[ \Lambda \text{ this study} \]
\[ \bar{\Lambda} \text{ this study} \]
\[ \Lambda \text{ PRC76 024915 (2007)} \]
\[ \bar{\Lambda} \text{ PRC76 024915 (2007)} \]

**Chiral currents**

\[ \partial_\mu T^{\mu\nu} = e F^{\nu\lambda} j_\lambda \]
\[ \partial_\mu j^\mu = 0 \]
\[ \partial_\mu j_5^\mu = C E^\mu B_\mu \]

where \( C \) is the anomaly coefficient and the electric and the axial vector currents are:

\[ j^\mu = n u^\mu + \frac{C \mu_5}{e} \left( 1 - \frac{\mu_5 n_5}{\varepsilon + p} \right) \]
\[ j_5^\mu = n_5 u^\mu + \frac{C \mu}{e} \left( 1 - \frac{\mu n}{\varepsilon + p} \right) \]

(For details, see: Hongo, Hirono and Hirano, arXiv:1309.2823)

Ongoing collaboration with Y.Hirono, M.Mace and D.Kharzeev to couple ECHO-QGP with their anomalous code.

STARS collaboration, arXiv:1701.06657
For theory, see: Becattini, Karpenko et al, Phys. Rev. C 95, 054902 (2017)
From a technical point of view, it is possible to study magnetic fields also at FAIR energies, but:

- how good is the hydro model at such low energies for peripheral collisions?
- we start hydro at ≈ 3 fm/c and we run it only for a short time: how large is the impact of this phase compared to the pre-equilibrium and to the hadronic phases?
Conclusions and future perspectives

- Magnetic fields may produce some relevant effects on several observable quantities
- It is uncertain whether they are strong and persistent enough to play a role in HIC at LHC and RHIC energies
- Magnetic fields effects might be detectable at low RHIC BES and FAIR energies, but probably RMHD is not the right tool to investigate them
- However, ECHO-QGP may help to study the influence and the evolution of magnetic fields in the QGP phase
- Future work will involve full 3D+1 simulations with different sets of initial conditions

Thank you!
## Conversions

### Constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.0072973525664</td>
</tr>
<tr>
<td>$m_{\pi^0}$</td>
<td>139.57018 MeV</td>
</tr>
<tr>
<td>$m_{\pi^0}^2$</td>
<td>0.01932 MeV$^2$</td>
</tr>
<tr>
<td>$4\pi\alpha = e^2$</td>
<td>$e = \sqrt{4\pi\alpha} = 0.30282212$</td>
</tr>
<tr>
<td>$\hbar c$</td>
<td>0.197326 GeVfm</td>
</tr>
<tr>
<td>$(\hbar c)^{3/2}$</td>
<td>0.087655 (GeVfm)$^{3/2}$</td>
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