Calculating hadron properties from dynamical hadronization within the FRG

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- Brief introduction to the FRG
- Dynamical Hadronization
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Motivation

Observable properties of hadrons are difficult to extract from QCD’s degrees of freedom:

- Need theoretical assumptions:

- Bound states and QCD (at hadronic energies) are not perturbative.

Many approaches and models are built to solve these problems.

- Lattice QCD.

- Functional methods.

  - **Dyson-Schwinger equations (DSE).**

  - **Bethe-Salpeter equations (BSE).**
Both equations need to be truncated to be numerically solved.

- **Rainbow-Ladder truncation:**

\[
\begin{align*}
\quad & -1 = \quad -1 + \quad \Downarrow \\
\quad & -1 = \quad -1 + \alpha(k^2)
\end{align*}
\]

- **Bethe-Salpeter equation:**

\[
\psi = \begin{pmatrix} K \end{pmatrix} \psi \implies \psi = \begin{pmatrix} \psi \end{pmatrix}
\]
Results for the pion and other ground state mesons are well understood. However:

- The solution relies strongly on the truncation.
- For more complex systems:
  - Other terms appear in the DSE.
  - Rainbow-Ladder truncation is not good enough.
  - New technical issues appear.

Work in a different approach:

- Use of the **Functional Renormalization Group (FRG)** to find properties of mesons.
- The FRG approach is consistent with BSE.
Introduction to the FRG

The properties of the scale-dependent effective action lead to the **1-loop** integral-differential equation:

\[
\partial_t \Gamma_k = \frac{1}{2} \text{Tr} \left( \partial_t R_k \left[ \Gamma_k^{(2)} + R_k \right]^{-1} \right)
\]

**Wetterich’s Flow Equation**

with

\[
t = \ln \left( \frac{k}{\Lambda_{\text{UV}}} \right) \quad \partial_t = k \frac{d}{dk}
\]

The properties of this exact flow equation are very convenient for physical calculations since it is an Euclidean **1-loop** integral-differential equation.
Initial and final conditions are fixed in theory space:

The choice of the regulator is not unique.
Wetterich’s equation can be solved applying vertex expansion:

\[
\Gamma_k[\phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \int_{p_1...p_n} \Gamma_k^{(n)}(p_1, \ldots p_n) \phi(p_1) \ldots \phi(p_n)
\]

Applying \(n\)–derivatives and averaging the fields one obtains the flow of the momentum dependent vertex functions.

**Remark:** a truncation/approximation is needed.
Dynamical Hadronization

Macroscopic QCD degrees of freedom are mesons and baryons. Introduced in the effective action through 4-Fermi Hubbard-Stratonovich transf.: 

\[ \lambda \xrightarrow{\text{H-S transformation}} h_{\phi} \]

- **Problem**: 4-Fermi interaction flow non-zero, H-S transformation must be applied in every RG-step \( \xrightarrow{\text{Solved by Dynamical Hadronization}} \).
Introduction of scale dependent bosonic field:

\[ \partial_t \phi_k(p) = \partial_t A_k(p)(\bar{\psi}_\tau \psi)(p) + \partial_t B_k(p)\phi_k(p) \]

with \( \partial_t A_k \) and \( \partial_t B_k \) defined such that 4-Fermi flow is cancelled:

- This generalizes Hubbard-Stratonovich transf. for every RG-step.
- Green’s functions computed with meson exchange diagrams.
Analytical Continuation

Search for bound state properties through real-time Green’s functions: analytical continuation must be performed.

Since the inverse propagator of 2-point function is proportional to $(p^2 + M^2)$ in Euclidean Space, goal is to continue to purely imaginary $p_0$.

- **Extrapolation** by fitting Euclidean momenta $p^2$ data to a parametrized function and evaluating it at Minkowski momenta $-p^2$.

- **Direct calculation** using the properties of the regulators.
Analytical continuation by extrapolation:

- Padé approximant:

\[ R^{(m,n)}(x) = \frac{\sum_{i=0}^{m} c_i x^i}{1 + \sum_{j=1}^{n} d_j x^j} \]

- Schlessinger point method:

\[ C(x) = \frac{F(x_1)}{1 + \frac{z_1(x-x_1)}{1 + \frac{z_2(x-x_2)}{\vdots + \frac{z_M(x-x_M)}}}} \]
Analytical continuation by direct calculation:


- Successfully performed with a 4D modified regulator for zero temperature $O(N)$ model [J. M. Pawlowski and N. Strodthoff, arXiv:1508.01160v3].

$$R_k;\Delta m_r^2(p^2) = \left( \Delta \Gamma_k^{(2)}(p^2)|_{\phi=\phi_0} + \Delta m_r^2 \right) r \left( \frac{p^2 + \Delta m_r^2}{k^2} \right)$$
- Modified regulator moves poles out the integrating region $\implies$ physical implications induced.

\[ x = \frac{(p^2_0 + \vec{p}^2)}{k^2} \text{ for values of } M^2 = \left( m^2 - \Delta m^2_r \right)/k^2. \]

[J. M. Pawlowski and N. Strodthoff, arXiv:1508.01160v3]
Preliminary results in the Quark-Meson model

Why using QM as starting point?

A.Cyrol et al, arXiv:1605.01856

Low-energy effective theory with gluons decoupled.
Free massless quark effective action + 4-Fermi interaction:

\[
\Gamma_k[\bar{\psi}, \psi] = \int \frac{d^4p}{(2\pi)^4} Z_{k,\psi} \bar{\psi}_a i\gamma^\mu \psi_a^A + \Gamma^{4-int}_k[\bar{\psi}, \psi]
\]

Applying Hubbard-Stratonovich transformation introducing \(\sigma\) and \(\vec{\pi}\) fields:

\[
\Gamma_k[\bar{\psi}, \psi, \sigma, \vec{\pi}] = \int_p \left\{ Z_{\psi,k} \bar{\psi}_a i\gamma^\mu \psi_a^A + \\
+ \frac{1}{2} \left( Z_{k,\phi} p^2 + m^2_{k,\phi} \right) (\sigma^2 + \pi^z \pi_z) - c \sigma + \\
+ \int_q h_k \bar{\psi}_a \left( \frac{\sigma}{2} \delta_{ab} + i \gamma_5 (\tau_z)_{ab} \pi^z \right) \psi_b^A \right\}
\]
Local potential approximation (LPA)

Wave-function renormalizations $Z_{k,i} \to 1$ and non-kinetic bosonic terms rewritten in a $O(N)$ potential:

$$V_k(\rho) = \sum_{n=0}^{\infty} \frac{V_k^{(n)}}{n!} (\rho - \rho_0)^n$$

with $\rho = \frac{1}{2} (\sigma^2 + \pi_z \pi^z)$ and $\rho_0$ constant expansion point related to $\langle \sigma \rangle \neq 0$.

Infinite set of flow equations for $V_k^{(n)}$, but result unchanged for $n > 6$. 
<table>
<thead>
<tr>
<th>Quark Mass $m_\psi$ (MeV)</th>
<th>$p^2$ (MeV$^2$)</th>
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<tr>
<td>284</td>
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</tbody>
</table>

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Calculating hadron properties within the FRG
Motivation Introduction to the FRG Dynamical Hadronization Analytical Continuation Preliminary results Summary

Inverse Pion propagator (MeV$^2$)

Pion Propagator (MeV$^2$)

Sigma Inverse Propagator (MeV$^2$)

Sigma Propagator (MeV$^2$)
Applying analytical continuation using the Padé method with numerical noise, common poles appear for different orders at:

- Pole \( m_\pi^2 \approx 120 \text{ MeV} \)
- Pole \( m_\sigma^2 \approx 400 \text{ MeV} \)

Results agree with Schlessinger point method.

LPA values for mesonic pole masses differ from curvature masses and quark mass does not behave as expected. **Different approximations need to be used.**
**Current status:**

Quark-Meson model in a different approximation (LPA’), where:

- $Z_{k,i} \neq 1$, therefore anomalous dimensions appear.
- A potential $V_k(\rho)$ is still used.
- The expansion point at $\langle \sigma \rangle$ is no longer constant.

Once results are obtained in the LPA’, the gauge bosons will be introduced and start working with QCD effective action.
The FRG provides an alternative procedure to the BSE/Faddeev equation to obtain resonance masses and decay widths.

Analytical continuation of the 4-fermi interaction to timelike momenta are to be performed in the Quark-Meson model.

Big numerical effort and tools are needed to obtain accurate results.

Provided that results in the FRG are compatible with DSE/BSE, the viability of this method should be analysed.

THANK YOU FOR YOUR ATTENTION