



Inverse magnetic catalysis in nonlocal chiral quark models

Valeria Paula Pagura

U. de Valencia - CSIC
Spain

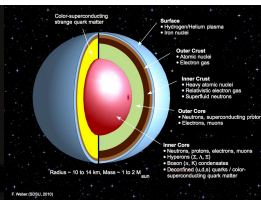
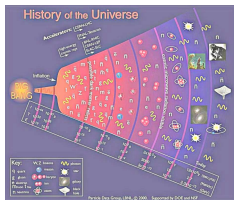
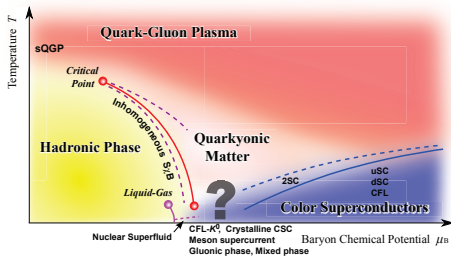
In collaboration with:

S.Noguera (Spain), D.Gómez Dumm and N.N.Scoccola (Argentina)

Phys. Rev. D **95**, 034013 (2017)

June, 2nd 2017

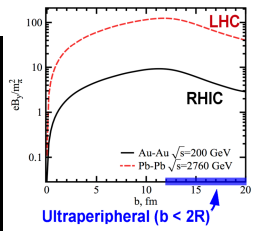
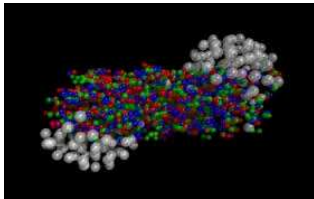
Motivation





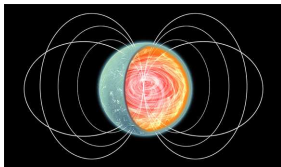
Motivation: why intense magnetic fields?

Non-central relativistic heavy ion collisions: $B \sim 10^{19}$ G



Magnetic field at $T = 0$
(A. Bzdak, V. Skokov (2012))

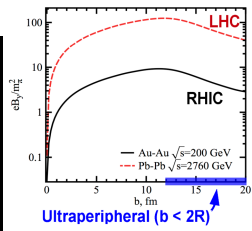
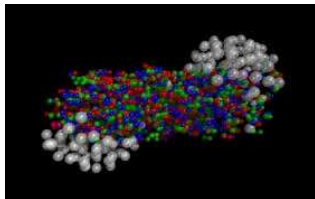
Magnetars: $B \sim 10^{13} - 10^{15}$ G (Duncan and Thompson (2012))





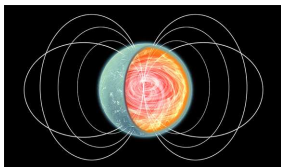
Motivation: why intense magnetic fields?

Non-central relativistic heavy ion collisions: $B \sim 10^{19}$ G



Magnetic field at $T = 0$
(A. Bzdak, V. Skokov (2012))

Magnetars: $B \sim 10^{13} - 10^{15}$ G (Duncan and Thompson (2012))



Including a non-zero uniform B

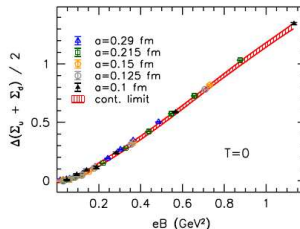
- How does QCD phase diagram looks like?
- Are there modifications in the nature of transitions?
- What is the fate of critical endpoints?



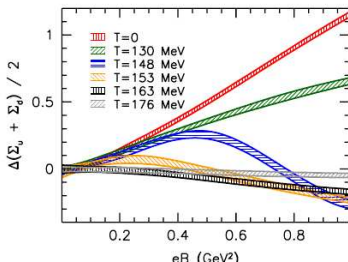
From LQCD calculations carried out with physical pion masses

(Bali et al JHEP **1202**, 044 (2012), Phys. Rev. D **86**, 071502 (2012))

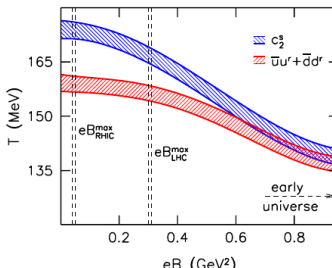
- Low temperature: **magnetic catalysis**
enhancement of the chiral condensate
- Close to the chiral restoration T_c :
inverse magnetic catalysis



nonmonotonic behavior of light quark condensates



decrease of the T_c with the magnetic field





Many scenarios have been considered to account for the IMC

- LQCD (Bali et al (2013); Bruckmann et al (2013))
- NJL (Kashiwa (2011); Menezes et al(2014); Krein et al (2014); Fayazbakhsh, Sadoogui (2014); Ferrer et al(2015); Mao(2016))
- Linear Sigma Model (Loewe et al (2014/15))
- PQM, QM models (Fraga et al (2014); Andersen, Naylor, Tranberg (2015))
- Holographic approach (Rougemont, Critelli, Noronha (2016))
- Other effective models (Fraga, Noronha, Palhares (2013); Chao, Chu, Huang (2013); Fukushima, Hidaka (2013); Mueller, Pawłowsky (2015))

However, the mechanism is not yet fully understood.



NJL Model

Model with chiral quark interaction (Nambu, Jona-Lasinio (1961))

Simplest version: **Scalar** and **pseudoscalar** interactions.

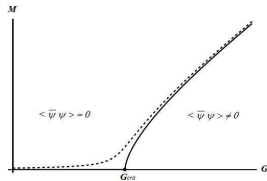
$$\mathcal{L}_{NJL} = \bar{\psi}(i \not{\partial} - m_c)\psi + \frac{G}{2} [(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2] \quad \psi^\dagger = (u, d)$$

In the mean field approximation (MFA):

$$(\bar{\psi}\mathcal{O}\psi)^2 \rightarrow 2\langle\bar{\psi}\mathcal{O}\psi\rangle\bar{\psi}\mathcal{O}\psi$$

Dynamical fermion mass generation:

$$M = m_c - 2G\langle\bar{\psi}\psi\rangle \quad G > G_{crit} \Rightarrow \langle\bar{\psi}\psi\rangle \neq 0$$



Parameters:

- Current quark mass m_c
- Coupling constant G (4 fermions)
- Cutoff Λ

Chosen to reproduce:

- $m_\pi = 139$ MeV
- $f_\pi = 92.4$ MeV
- Constituent mass
 $M_0 \sim 300 - 400$ MeV

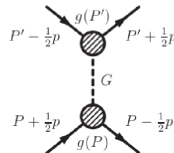


non-local NJL Model

$$S_E = \int d^4x \left\{ \bar{\psi}(x) (-i\not{D} + m_c) \psi(x) - \frac{G}{2} j_a(x) j_a(x) \right\}$$

$$j_a(x) = \int d^4z \mathcal{G}(z) \bar{\psi}(x + \frac{z}{2}) \Gamma_a \psi(x - \frac{z}{2})$$

$\Gamma_a = (\mathbf{1}, i\gamma_5 \vec{\tau})$, $\mathcal{G}(z)$ nonlocal form factor



- models effective OGE
- leads to momentum dependence in quark propagators
- finite to all orders in the loop expansion $\rightarrow$ no extra cutoffs
- properties of light mesons at zero and finite T and/or μ



Coupling to the magnetic field

$$\partial_\mu \rightarrow D_\mu \equiv \partial_\mu - i \hat{Q} \mathcal{A}_\mu(x), \quad \hat{Q} = \text{diag}(q_u, q_d), \quad \mathcal{A}_\mu = B x_1 \delta_{\mu 2}$$

The inclusion of gauge interactions implies a change in the nonlocal currents

$$\psi(x-z/2) \rightarrow \mathcal{W}(x, x-z/2) \psi(x-z/2) \quad \mathcal{W}(s, t) = \text{P exp} \left[-i \hat{Q} \int_s^t dr_\mu \mathcal{A}_\mu(r) \right]$$

We bosonize the action

$$S_{\text{bos}} = -\ln \det \mathcal{D}_{x,x'} + \frac{1}{2G} \int d^4x \left[\sigma(x) \sigma(x) + \vec{\pi}(x) \cdot \vec{\pi}(x) \right],$$

$$\begin{aligned} \mathcal{D}_{x,x'} &= \delta^{(4)}(x-x') \left(-i \not{D} + m_c \right) + \\ &\quad \mathcal{G}(x-x') \gamma_0 \mathcal{W}(x, \bar{x}) \gamma_0 \left[\sigma(\bar{x}) + i \gamma_5 \vec{\tau} \cdot \vec{\pi}(\bar{x}) \right] \mathcal{W}(\bar{x}, x') \end{aligned}$$

Mean Field Approximation, assuming σ has a nontrivial translational invariant

MF value $\bar{\sigma}, \bar{\pi}_i \rightarrow 0$



MFA action per unit volume

$$\frac{S_{\text{bos}}^{\text{MFA}}}{V^{(4)}} = \frac{\bar{\sigma}^2}{2G} - N_c \sum_{f=u,d} \frac{|q_f B|}{2\pi} \int \frac{d^2 p_{\parallel}}{(2\pi)^2} \left[\ln \left(p_{\parallel}^2 + M_{0,p_{\parallel}}^{\lambda_f, f^2} \right) + \sum_{k=1}^{\infty} \ln \Delta_{k,p_{\parallel}}^f \right]$$

Where

$$\Delta_{k,p_{\parallel}}^f = \left(2k|q_f B| + p_{\parallel}^2 + M_{k,p_{\parallel}}^{+,f} M_{k,p_{\parallel}}^{-,f} \right)^2 + p_{\parallel}^2 \left(M_{k,p_{\parallel}}^{+,f} - M_{k,p_{\parallel}}^{-,f} \right)^2$$

$$M_{k,p_{\parallel}}^{\lambda,f} = m_c + \bar{\sigma} g_{k,p_{\parallel}}^{\lambda,f}$$

$$g_{k,p_{\parallel}}^{\lambda,f} = \frac{4\pi}{|q_f B|} (-1)^{k\lambda} \int \frac{d^2 p_{\perp}}{(2\pi)^2} g(p_{\perp}^2 + p_{\parallel}^2) \exp(-p_{\perp}^2/|q_f B|) L_{k\lambda}(2p_{\perp}^2/|q_f B|)$$

$$p_{\parallel} = (p_3, p_4) \quad p_{\perp} = (p_1, p_2)$$

$$k_{\pm} = k - 1/2 \pm s_f/2 \quad s_f = \text{sign}(q_f B)$$

$$\lambda = +/-$$



minimizing the effective action $\rightarrow$ Gap equation

$$\frac{\bar{\sigma}}{G} = N_c \sum_{f=u,d} \frac{|q_f B|}{\pi} \sum_{k=0}^{\infty} \int \frac{d^2 p_{\parallel}}{(2\pi)^2} \sum_{\lambda=\pm} \hat{A}_{k,p_{\parallel}}^{\lambda,f} g_{k,p_{\parallel}}^{\lambda,f}$$

$$\hat{A}_{k,p_{\parallel}}^{\pm,f} = \frac{M_{k,p_{\parallel}}^{\mp,f} (2k|q_f B| + p_{\parallel}^2 + M_{k,p_{\parallel}}^{-,f} M_{k,p_{\parallel}}^{+,f}) + p_{\parallel}^2 (M_{k,p_{\parallel}}^{\pm,f} - M_{k,p_{\parallel}}^{\mp,f})}{\Delta_{k,p_{\parallel}}^f}$$

Quark condensate: $\langle \bar{q}_f q_f \rangle_B = \frac{\partial S_{bos}^{MFA}}{\partial m_c}$

To make contact with LQCD:

$$\Sigma_B^f = \frac{2m_c}{S^4} [\langle \bar{q}_f q_f \rangle_B^{\text{reg}} - \langle \bar{q}_f q_f \rangle_0^{\text{reg}}] + 1, \quad S = (135 \times 86)^{1/2} \text{MeV}$$

$$\Delta \Sigma_B^f = \Sigma_B^f - \Sigma_0^f \quad \Delta \bar{\Sigma}_B = (\Delta \Sigma_B^u + \Delta \Sigma_B^d)/2$$





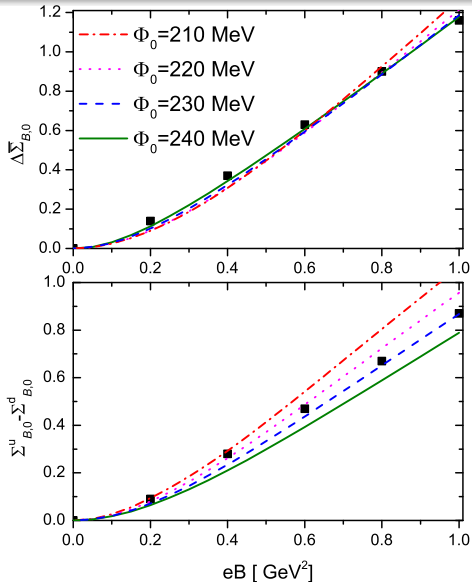
Condensates at T=0

Gaussian form factor

$$g(p^2) = \exp(-p^2/\Lambda^2)$$

$$\Phi_0 = -\langle \bar{q}_f q_f \rangle_{0,0}^{\text{reg}}$$

(Squared dots from Bali et al
Phys. Rev. D **86**, 071502
(2012))





Extension to finite temperature (Matsubara formalism)

$$\int \frac{d^2 p_{\parallel}}{(2\pi)^2} F(p_{\parallel}^2) \rightarrow T \sum_{n=-\infty}^{\infty} \int \frac{dp_3}{2\pi} F(p_{\parallel n}^2)$$

$$\bar{p}_n = (p_3, \omega_n) \quad \omega_n = (2n + 1)\pi T$$

Thermodynamical potential $\rightarrow$ Gap equation

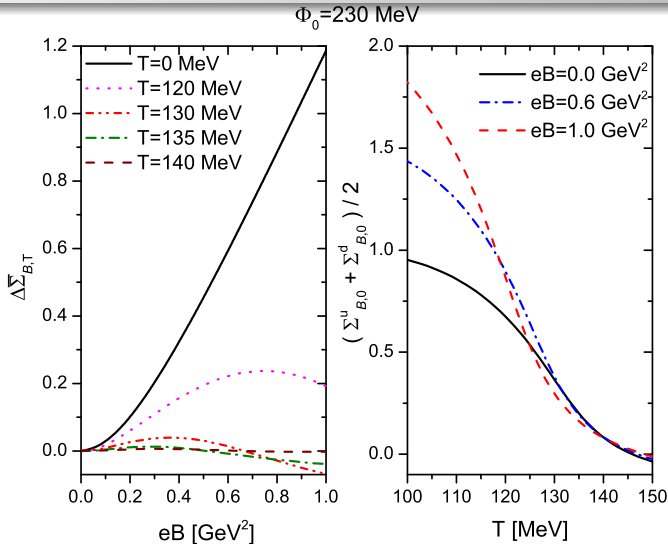
$$\Omega_{B,T}^{\text{MFA}} = \frac{\bar{\sigma}^2}{2G} - T N_c \sum_{n=-\infty}^{\infty} \sum_f \frac{|q_f B|}{2\pi} \int \frac{dp_3}{2\pi} \times$$

$$\left\{ \ln \left[p_{\parallel n}^2 + \left(M_{0,p_{\parallel n}}^{sf,f} \right)^2 \right] + \sum_{k=1}^{\infty} \ln \left[\Delta_{k,p_{\parallel n}}^f \right] \right\}$$

$$\Omega_{B,T}^{\text{MFA,reg}} = \Omega_{B,T}^{\text{MFA}} - \Omega_{B,T}^{\text{free}} + \Omega_{B,T}^{\text{free,reg}} \quad \frac{\partial \Omega_{B,T}^{\text{MFA,reg}}}{\partial \bar{\sigma}} = 0$$

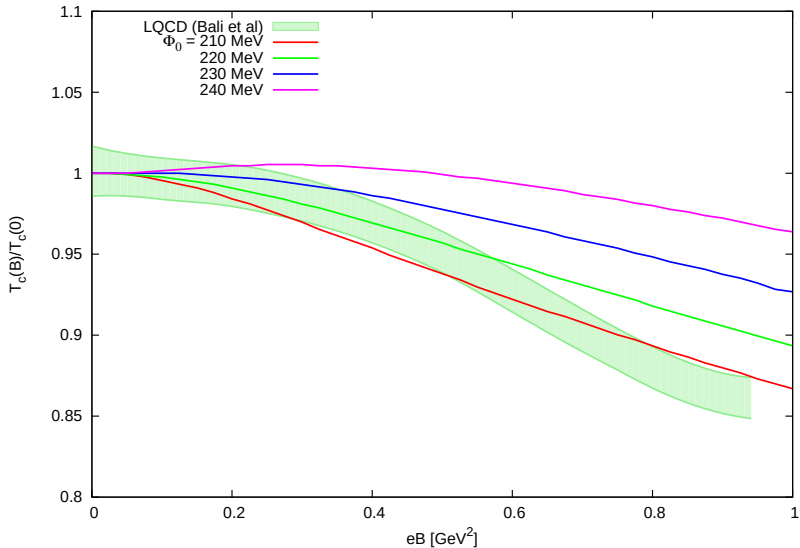


Condensates at finite T



IMC is naturally displayed!

$T_c(eB = 0) = 129.8 \text{ MeV}$ (higher if coupling to PL is included)



beyond $eB \simeq 0.4 \text{ GeV}^2 \rightarrow$ decrease of T_c

IMC is observed for all parametrizations!



Coupling to the Polyakov Loop

- Assuming quarks move on a constant colour background field:
 $\phi = ig \delta_{\mu 0} G_a^\mu \lambda^a / 2$
- Polyakov gauge $\phi = \phi_3 \lambda_3 + \phi_8 \lambda_8$
- $\Phi = \frac{1}{3} \text{Tr} \exp(i\phi/T)$ Order parameter for deconfinement

$$p_{\parallel n} \rightarrow p_{\parallel nc} = (p_3, [(2n+1)\pi T + \phi_c])$$

Effective gauge self-interactions through a PL potential $\mathcal{U}$

$$\Omega_{B,T}^{\text{MFA,reg}} \rightarrow \Omega_{B,T}^{\text{MFA,reg}} + \mathcal{U}(\Phi, T)$$

Gap equations

$$\frac{\partial \Omega_{B,T}^{\text{MFA,reg}}}{\partial \bar{\sigma}} = \frac{\partial \Omega_{B,T}^{\text{MFA,reg}}}{\partial \Phi} = 0$$





Polynomial potential (Ratti et al Phys. Rev. D **73**, 014019 (2006))

$$\frac{\mathcal{U}_{\text{poly}}(\Phi, T)}{T^4} = -\frac{b_2(T)}{2}\Phi^2 - \frac{b_3}{3}\Phi^3 + \frac{b_4}{4}\Phi^4$$

$$b_2(T) = a_0 + a_1 \left(\frac{T_0}{T}\right) + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3$$

Parameters fitted to reproduce pure gauge EoS and PL behavior

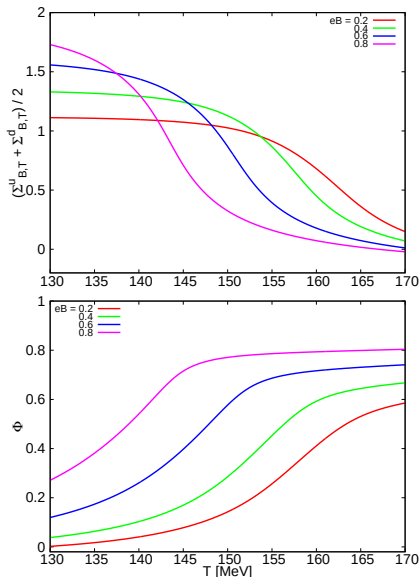
Improved Potential (Haas et al Phys. Rev. D **87**, 076004 (2013))

$$\frac{\mathcal{U}_{\text{glue}}(\Phi, t_{\text{glue}})}{T^4} = \frac{\mathcal{U}_{\text{YM}}[\Phi, t_{\text{YM}}(t_{\text{glue}})]}{T_{\text{YM}}^4}$$

$$t_{\text{YM}}(t_{\text{glue}}) = 0.57 t_{\text{glue}} = 0.57 \left(\frac{T - T_c^{\text{glue}}}{T_c^{\text{glue}}} \right)$$

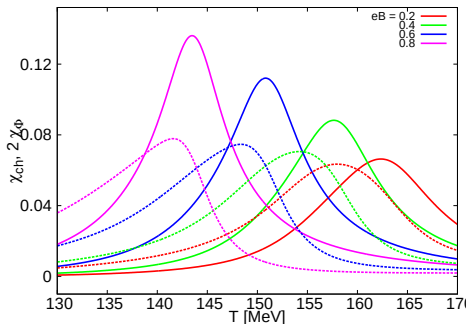


Condensates at finite T - nIPNJL

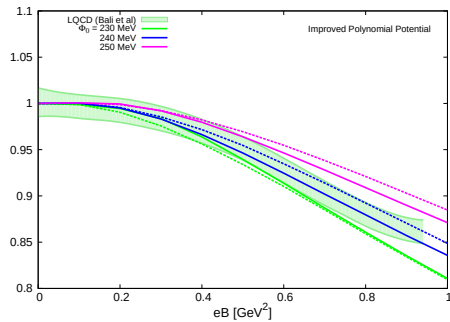
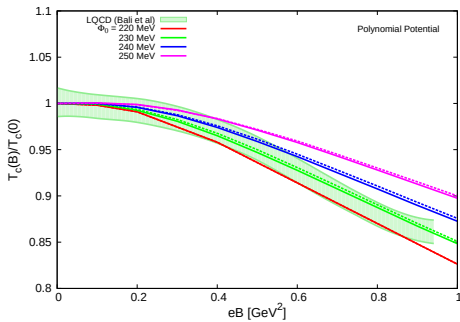


Improved Polynomial potential

$$\Phi_0 = 240 \text{ MeV}$$



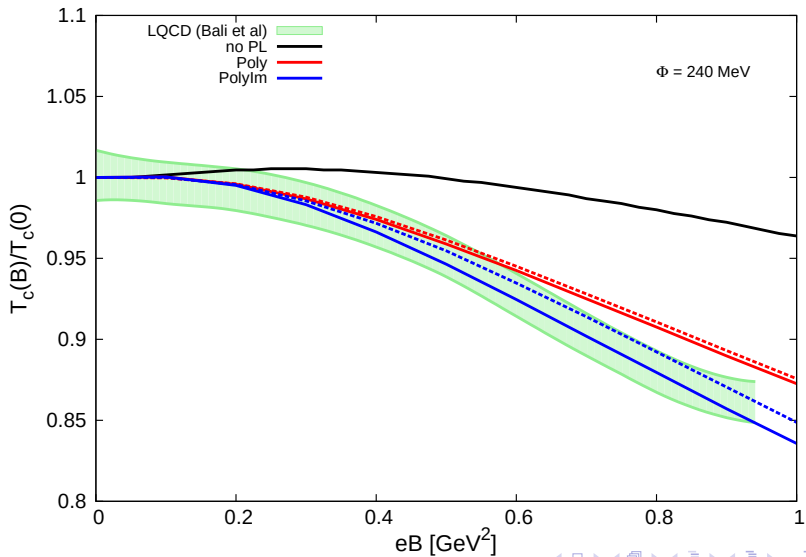
$$T_c(eB = 0) \sim 163 \text{ MeV}$$



IMC always displayed

Splitting of chiral restoration and deconfinement transitions (1 – 10 MeV)

$T_c(eB = 1) - T_c(eB = 0)$ agrees with LQCD estimates



Summary and outlook

In the context of a nonlocal extension of the nl(P)NJL model under an external homogeneous magnetic field...

- We observe at $T = 0$ the expected MC effect, with results in good quantitative agreement with LQCD.
- In contrast to what happens in the standard local NJL model, our results naturally lead to the IMC effect.
- When coupling to the PL is included ΔT_c observed from $eB = 0$ to 1 GeV^2 agrees with LQCD estimates

Work in progress...

- Extension to other form factors shape.
- Analysis with other PL thermodynamical potentials
- Inclusion of WFR.