

Heavy quark potentials in effective string theory

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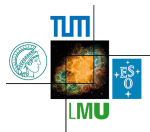
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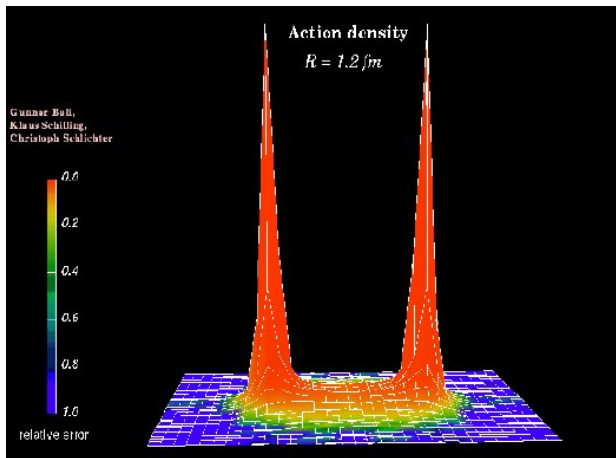


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Confinement in QCD [Wilson, PRD (1974)]

- Definition and characteristics
 - No single color charged object isolated at low-energy (i.e., $E \leq \Lambda_{QCD}$)
 - Detect only hadrons in the form of jets
 - Perturbative approach breaks down at low-energy ($\Lambda_{QCD} \sim 200MeV$)
- Any (analytical) solution?
 - Not in the form of perturbative method
 - Phenomenologically treat hadrons as DOFs (e.g., χ PT, CQM, etc)
 - Non-perturbative approach: Lattice Gauge Theory (Lattice QCD)
- Another approach:
QCD flux tube model for heavy meson [Nambu, Phys. Lett. B (1979)]



R : distance between $Q\bar{Q}$ rescaled by lattice spacing

A (brief) historical development in QCD flux tube model

- Nambu's work [Nambu, Phys. Lett. B (1979)]
- Kogut and Parisi's work [Kogut, Parisi, PRL (1981)]
 - Spin-spin potential between widely separated heavy quark-antiquark
- Progress during the last decades
 - Wilson loop to the heavy $Q\bar{Q}$ potentials [Brambilla, Pineda, Soto, Vairo, PRD (2000)]
 - Wilson loop to the string picture and the heavy $Q\bar{Q}$ potentials (partially complete) [Perez-Nadal, Soto, PRD (2009)]
 - Wilson loop to the string picture and the heavy $Q\bar{Q}$ potentials up to leading order [Brambilla, Groher, Martinez, Vairo PRD (2014)]
- Recent work (N. Brambilla, SH, A. Vairo)
 - Full EFT systematics for the NLO contribution to potentials
 - Constraints to (dimensionful) parameters in the string picture via Poincaré invariance in QCD

Brief guide towards QCD potentials in EFT framework

- 1 Effective description of heavy quarks at low-energy (NRQCD, pNRQCD)
- 2 Heavy quark potentials
- 3 Effective string theory (EST)
- 4 NLO to the heavy quark potentials in EST
- 5 Summary and outlook

Non-relativistic description of QCD for the heavy quark/antiquark

[Caswell, Lepage, Phys. Lett. B (1986), Bodwin, Braaten, Lepage PRD (1995)]

- Hierarchy of scales for the heavy quark/antiquark system
 - $M \gg Mv, \Lambda_{QCD}$ (M : mass, v : relative velocity between quarks)
 - Integrate out M from QCD
 - Lagrangian organized by $1/M$ expansion
- Degrees of freedom
 - Pauli spinors for heavy quark and antiquark fields
 - Dirac spinors for light quark and antiquark fields
 - Soft gluons for $SU(3)$ gauge symmetry

Potential NRQCD: effective theory for heavy quarkonium

[Brambilla, Pineda, Soto, Vairo, Nucl. Phys. B (2000)]

- Hierarchy of scales for the heavy quark-antiquark bound states
 - $M \gg Mv \gg Mv^2$
 - Integrate out momentum transfer $\sim 1/r$ from NRQCD
 - Lagrangian organized by multipole expansion
- Degrees of freedom
 - Color singlet $S(t, \mathbf{r}, \mathbf{R})$ and octet $O^a(t, \mathbf{r}, \mathbf{R})$
 - Dirac spinors for light quarks and antiquarks
 - Ultra-soft gluons $A_\mu^a(t, \mathbf{R})$ for $SU(3)$ gauge symmetry
- Lagrangian: appearance of potentials

$$\begin{aligned} \mathcal{L}_{\text{pNRQCD}} = & \int d^3r \left\{ \text{Tr} [S^\dagger (i\partial_0 - V_s(r) + \dots) S + O^\dagger (iD_0 - V_o(r) + \dots) O] \right. \\ & \left. + g V_A(r) \text{Tr} [O^\dagger \mathbf{r} \cdot \mathbf{E} S + S^\dagger \mathbf{r} \cdot \mathbf{E} O] + g \frac{V_B(r)}{2} \text{Tr} [O^\dagger \mathbf{r} \cdot \mathbf{E} O + O^\dagger O \mathbf{r} \cdot \mathbf{E}] \right\} - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu, a} \end{aligned}$$

Relativistic corrections to the static potentials in pNRQCD

- Heuristically: $V = V^{(0)} + V^{(1/M)} + V^{(1/M^2)} + \dots$
- Static singlet potential up to $1/M^2$

$$\begin{aligned}
 v(r) = & V^{(0)}(r) + \frac{2}{M} V^{(1,0)}(r) + \frac{1}{M^2} \left\{ \left[2 \frac{V_{L^2}^{(2,0)}(r)}{r^2} + \frac{V_{L^2}^{(1,1)}(r)}{r^2} \right] L^2 \right. \\
 & + \left[V_{LS}^{(2,0)}(r) + V_{L_2 S_1}^{(1,1)}(r) \right] \mathbf{L} \cdot \mathbf{S} + V_{S^2}^{(1,1)}(r) \left(\frac{S^2}{2} - \frac{3}{4} \right) + V_{S_{12}}^{(1,1)}(r) S_{12}(\hat{\mathbf{r}}) \\
 & \left. + \left[2V_{p^2}^{(2,0)}(r) + V_{p^2}^{(1,1)}(r) \right] \mathbf{p}^2 + 2V_r^{(2,0)}(r) + V_r^{(1,1)}(r) \right\}
 \end{aligned}$$

- Notations

- (a, b) : a the order of $1/M_1$ and b the order of $1/M_2$ expansion
- $1/M_1 = 1/M_2$
- $S_i = \frac{\sigma^i}{2}$ ($i \in \{1, 2\}$): spin of the heavy quark/antiquark
- V s: Wilson coefficients to be matched to NRQCD

NRQCD and pNRQCD matching [Brambilla, Pineda, Soto, Vairo, PRD (2000)]

- Static potential:

$$V^{(0)}(r) = \lim_{T \rightarrow \infty} \frac{i}{T} \ln \langle W_{\square} \rangle$$

- Matching $Q\bar{Q}$ correlator to singlet propagator
- Wilson loop: $W_{\square} \equiv P \exp \left\{ -ig \oint_{r \times T} dz^{\mu} A_{\mu}(z) \right\}$
- $\langle \dots \rangle$: average value over Yang-Mills action
- 1st order relativistic correction:

$$V^{(1,0)}(r) = -\frac{1}{2} \lim_{T \rightarrow \infty} \int_0^T dt \langle \langle \mathbf{gE}_1(t) \cdot \mathbf{gE}_1(0) \rangle \rangle_c$$

- $\langle \dots \rangle \equiv \langle \dots W_{\square} \rangle / \langle W_{\square} \rangle$
- $\langle \langle O_1(t_1) O_2(t_2) \rangle \rangle_c \equiv \langle \langle O_1(t_1) O_2(t_2) \rangle \rangle - \langle \langle O_1(t_1) \rangle \rangle \langle \langle O_2(t_2) \rangle \rangle$, for $t_1 \geq t_2$
- $\mathbf{E}_{1,2}(t) \equiv \mathbf{E}(t, \pm r/2)$; heavy quark/antiquark located at $(0, 0, \pm r/2)$

• 2nd order corrections [Pineda, Vairo, PRD (2001)]

- ① Spin-independent part: $V_{\mathbf{p}^2}^{(2,0)}(r), V_{\mathbf{p}^2}^{(1,1)}(r), V_{L^2}^{(2,0)}(r), V_{L^2}^{(1,1)}(r), V_r^{(2,0)}(r), V_r^{(1,1)}(r)$

$$V_{\mathbf{p}^2}^{(2,0)}(r) = \frac{i}{2} \hat{r}^i \hat{r}^j \int_0^\infty dt t^2 \langle\langle g \mathbf{E}_1^i(t) g \mathbf{E}_1^j(0) \rangle\rangle_c$$

$$V_{\mathbf{p}^2}^{(1,1)}(r) = i \hat{r}^i \hat{r}^j \int_0^\infty dt t^2 \langle\langle g \mathbf{E}_1^i(t) g \mathbf{E}_2^j(0) \rangle\rangle_c$$

...

(1)

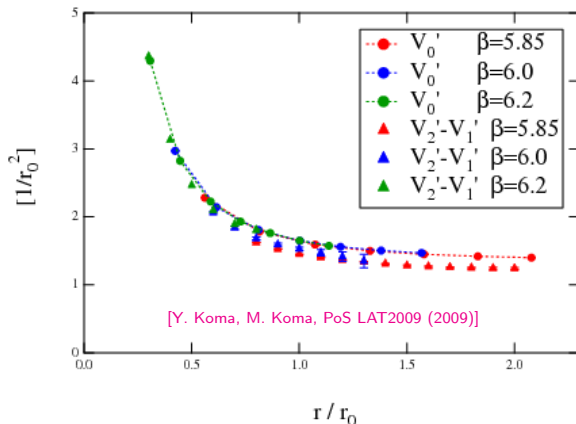
- ② Spin-dependent part: $V_{LS}^{(2,0)}(r), V_{L_2 S_1}^{(1,1)}, V_{S^2}^{(1,1)}, V_{S_{12}}^{(1,1)}$,

$$V_{LS}^{(2,0)}(r) = -\frac{c_F^{(1)}}{r^2} i \mathbf{r} \cdot \int_0^\infty dt t \langle\langle g \mathbf{B}_1(t) \times g \mathbf{E}_1(0) \rangle\rangle + \frac{c_S^{(1)}}{2r^2} \mathbf{r} \cdot (\nabla_r V^{(0)})$$

$$V_{L_2 S_1}^{(1,1)}(r) = -\frac{c_F^{(1)}}{r^2} i \mathbf{r} \cdot \int_0^\infty dt t \langle\langle g \mathbf{B}_1(t) \times g \mathbf{E}_2(0) \rangle\rangle$$

...

(2)



Confirmation of the Gromes relation: $\frac{1}{2r} \frac{dV^{(0)}}{dr} = V_{L_1 S_2}^{(1,1)} - V_{LS}^{(2,0)}$

- Fits nicely to Poincaré invariance [D. Gromes, Z. Phys. C (1984)]
- But 3- and 4-gauge field insertions **unknown** from lattice
- Need other ways to calculate V_r (contains 3 and 4 gauge fields insertions)

Explicit formulation of the potential

- We proceed with the **effective string theory** (EST) and compare the result to the lattice simulations

Comparison between two EFTs **below the confinement scale**

- Chiral perturbation theory (χ PT) for light quarks
 - DOF: light mesons (π, η, ρ , etc)
 - Expansion parameters: p/Λ_χ or m_π/Λ_χ ; $\Lambda_\chi \sim 1\text{GeV}$
- **Effective string theory** for heavy quarks
 - DOF: String coordinates instead of gluons
 - Expansion parameter: p/Λ_{QCD} ; $\Lambda_{QCD} \sim 200\text{MeV}$

Wilson loop-string partition function equivalence conjecture

[Lüscher, Nucl. Phys. B180, 317 (1981)]

$$\lim_{T \rightarrow \infty} \langle W_{\square} \rangle = Z \int \mathcal{D}\xi^1 \mathcal{D}\xi^2 e^{iS_{\text{String}}[\xi^1, \xi^2]}$$

- ξ^1, ξ^2 : string coordinates
- Z : normalization constant
- S_{String} : string action

- 1 Hierarchy: $r\Lambda_{QCD} \gg 1$, r distance between heavy quark-antiquark
- 2 Boundary conditions : $\xi^l(t, z) = 0$; $z = \pm r/2$
 - Transversal string vibrations: $\xi^l(t, z)$, for $l = x, y$ (or 1, 2)
- 3 Power counting: $\partial_a \sim 1/r$, $\xi^l \sim 1/\Lambda_{QCD}$ (i.e., $\partial_a \xi^l \sim (r\Lambda_{QCD})^{-1}$)
- 4 String action in 4D with static gauge (starting from Nambu-Goto action):

$$\begin{aligned} S &= -\sigma \int dt dz \sqrt{-\det(\eta_{ab} + \partial_a \xi^l \partial_b \xi^l)} \\ &= -\sigma \int dt dz \left(1 - \frac{1}{2} \partial_a \xi^l \partial^a \xi^l + \dots \right) \end{aligned}$$

- σ : string tension ($\sim \Lambda_{QCD}^2$), determined by lattice
- Truncation up to $(r\Lambda_{QCD})^{-2}$
- $a = z, t$
- Static potential: $V^{(0)} \approx \sigma r$ (leading order)

[Lüscher, K. Symanzik, P. Weisz, Nucl. Phys. B173, 365 (1980)]

- Correlator in Euclidean time ($t \rightarrow -it$):

$$G(t, t'; z, z') = \frac{\delta^{lm}}{4\pi\sigma} \ln \left(\frac{\cosh[(t - t')\pi/r] + \cos[(z + z')\pi/r]}{\cosh[(t - t')\pi/r] - \cos[(z - z')\pi/r]} \right) \quad (3)$$

- Mapping from gauge fields insertion into Wilson loop to EST:

[Kogut, Parisi, PRL (1981), Perez-Nadal, Soto, PRD (2009), Brambilla, Groher, Martinez, Vairo PRD (2014)]

- 1 Symmetries: rotations w.r.t. z-axis, reflection w.r.t. xz plane, CP , T
- 2 Match the mass dimensions
- 3 Truncate up to $(r\Lambda_{QCD})^{-2}$

$$\begin{aligned} \langle \dots \mathbf{E}_{1,2}^l(t) \dots \rangle &= \langle \dots \Lambda^2 \partial_z \xi^l(t, \pm r/2) \dots \rangle \\ \langle \dots \mathbf{E}_{1,2}^3(t) \dots \rangle &= \langle \dots \Lambda'^2 \dots \rangle \\ \langle \dots \mathbf{B}_{1,2}^l(t) \dots \rangle &= \langle \dots \pm \Lambda' \epsilon^{lm} \partial_t \partial_z \xi^m(t, \pm r/2) \dots \rangle \\ \langle \dots \mathbf{B}_{1,2}^3(t) \dots \rangle &= \langle \dots \pm \Lambda''' \epsilon^{lm} \partial_t \partial_z \xi^l(t, \pm r/2) \partial_z \xi^m(t, \pm r/2) \dots \rangle \end{aligned} \quad (4)$$

Potentials at leading order

[Perez-Nadal, Soto, PRD (2009), Brambilla, Groher, Martinez, Vairo PRD (2014)]

- Wilson loop expectation values by using Eq. (3) and (4):

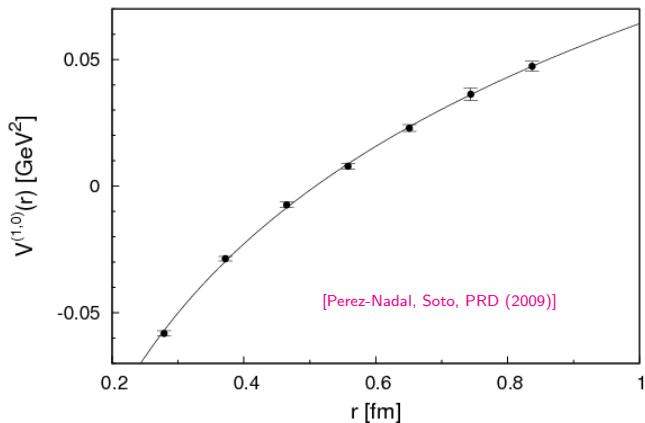
$$\begin{aligned} \langle\langle \mathbf{E}_1^i(-it) \mathbf{E}_1^j(0) \rangle\rangle_c &= \delta^{ij} \frac{\pi \Lambda^4}{4\sigma r^2} \sinh^{-2} \left(\frac{\pi t}{2r} \right), & \langle\langle \mathbf{E}_1^3(-it) \mathbf{E}_1^3(0) \rangle\rangle_c &= 0, \\ \langle\langle \mathbf{E}_1^i(-it) \mathbf{E}_2^j(0) \rangle\rangle_c &= -\delta^{ij} \frac{\pi \Lambda^4}{4\sigma r^2} \cosh^{-2} \left(\frac{\pi t}{2r} \right), & \langle\langle \mathbf{E}_1^3(-it) \mathbf{E}_2^3(0) \rangle\rangle_c &= 0, \end{aligned}$$

...

- Relativistic correction to static potential

$$V^{(1,0)}(r) = \frac{g^2 \Lambda^4}{2\pi\sigma} \ln(\sigma r^2) + \mu_1, \quad V_{\mathbf{p}^2}^{(2,0)}(r) = V_{\mathbf{p}^2}^{(1,1)}(r) = 0, \quad V_{\mathbf{L}^2}^{(2,0)}(r) = -V_{\mathbf{L}^2}^{(1,1)}(r) = -\frac{g^2 \Lambda^4 r}{6\sigma},$$

...



Confirmation to logarithmic growth: $V^{(1,0)}(r) \sim \ln(\sigma r^2)$

Emergence of parameters: $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \dots$

- μ_i : renormalization constants
- Exploit symmetries of the fundamental theory \rightarrow reduce parameters
- Poincaré invariance (see my talk last year)
 - Theory independent of the reference frame interaction observed
 - Spatial rotations, **boosts**, as well as spacetime translations
- Poincaré invariance in QCD constrains parameters

[Brambilla, Gromes, Vairo, Phys. Lett. B (2003), Berwein, Brambilla, SH, Vairo, TUM-EFT 74/15]:

- Gromes relation: $\frac{1}{2r} \frac{dV^{(0)}}{dr} + V_{LS}^{(2,0)} - V_{L_2 S_1}^{(1,1)} = 0 \rightarrow \mu_2 = \frac{\sigma}{2}$
- Momentum dependent potentials: $\frac{r}{2} \frac{dV^{(0)}}{dr} + 2V_{L_2}^{(2,0)} - V_{L_2}^{(1,1)} = 0 \rightarrow g\Lambda^2 = \sigma$
- $-\nabla_1 V^{(0)} = \langle\langle g\mathbf{E}_1 \rangle\rangle$ [Brambilla, Pineda, Soto, Vairo, PRD (2000)]: $g\Lambda'^2 = -\sigma$

- In CM, Hamiltonian of the system given, $H = \mathbf{p}^2/M + V$
 [Brambilla, Groher, Martinez, Vairo PRD (2014)]

$$\begin{aligned}
 V(r) = & V^{(0)}(r) + \frac{2}{M} V^{(1,0)}(r) + \frac{1}{M^2} \left\{ \left[2 \frac{V_{\mathbf{L}^2}^{(2,0)}(r)}{r^2} + \frac{V_{\mathbf{L}^2}(r)}{r^2} \right] \mathbf{L}^2 + \left[V_{\mathbf{L}\mathbf{S}}^{(2,0)}(r) + V_{\mathbf{L}_2\mathbf{S}_1}^{(1,1)}(r) \right] \mathbf{L} \cdot \mathbf{S} \right. \\
 & \left. + V_{\mathbf{S}^2}^{(1,1)}(r) \left(\frac{\mathbf{S}^2}{2} - \frac{3}{4} \right) + V_{\mathbf{S}_{12}}^{(1,1)}(r) \mathbf{S}_{12}(\hat{\mathbf{r}}) + 2V_r^{(2,0)}(r) + V_r^{(1,1)}(r) \right\} \\
 = & \sigma r + \frac{1}{M} \frac{\sigma}{\pi} \ln(\sigma r^2) + \frac{1}{M^2} \left(-\frac{\sigma}{6r} \mathbf{L}^2 - \frac{\sigma}{2r} \mathbf{L} \cdot \mathbf{S} - \frac{9\zeta_3 \sigma^2 r}{2\pi^3} \right) + \mathcal{O}(r^{-2})
 \end{aligned}$$

- Significance
 - Potential depends only on string tension and mass
 - Parameters determined by lattice simulations
- Towards NLO
 - Non-trivial impact of the sub-leading terms to the potentials
 - NLO in the gauge field insertions: $(r\Lambda_{QCD})^{-2}$ suppressed
 - Subleading contributions from the action might also be needed

Possible contributions at NLO:

1 Non-gaussian action

- Add $(\partial_a \xi^l \partial^a \xi^l)^2$, $(\partial_a \xi^l \partial_b \xi^l)(\partial^a \xi^m \partial^b \xi^m)$ [Lüscher, Weisz, JHEP (2004)]
- Solution via perturbative expansion
- **But** the Green's function is at $\mathcal{O}(\sigma^{-3} r^{-6})$, counted as NNLO

2 String Mapping

- Gaussian action
- Symmetries from $Q\bar{Q}$ bound state (CP , T , rotation, reflection):

$$\langle \dots E_{1,2}^l(t) \dots \rangle = \langle \dots \Lambda^2 \partial_z \xi^l(t, \pm r/2) + \bar{\Lambda}^2 \partial_z \xi^l(t, \pm r/2) (\partial_a \xi^m(t, \pm r/2))^2 \dots \rangle$$

$$\langle \dots B_{1,2}^l(t) \dots \rangle = \langle \dots \pm \Lambda'^2 \epsilon^{lm} \partial_t \partial_z \xi^m(t, \pm r/2) \pm \bar{\Lambda}'^2 \epsilon^{lm} \partial_t \partial_z \xi^m(t, \pm r/2) (\partial_a \xi^n(t, \pm r/2))^2 \dots \rangle$$

$$\langle \dots E_{1,2}^3(t) \dots \rangle = \langle \dots \Lambda''^2 + \bar{\Lambda}''^2 (\partial_a \xi^m(t, \pm r/2))^2 \dots \rangle$$

$$\langle \dots B_{1,2}^3(t) \dots \rangle = \langle \dots \pm \Lambda''' \epsilon^{lm} \partial_t \partial_z \xi^l(t, \pm r/2) \partial_z \xi^m(t, \pm r/2) \\ \pm \bar{\Lambda}''' \epsilon^{lm} \partial_t \partial_z \xi^l(t, \pm r/2) \partial_z \xi^m(t, \pm r/2) (\partial_a \xi^n(t, \pm r/2))^2 \dots \rangle$$

4-string field correlator

- Product of two Green's functions
- Three sources of divergence
 - $\partial_z \partial_{z'} G(t, t'; z, z')|_{t=t', z=z'=\pm r/2}$
 - $\partial_t \partial_z \partial_{z'} G(t, t'; z, z')|_{t=t', z=z'=\pm r/2}$
 - $\partial_t \partial_{t'} G(t, t'; z, z')|_{t=t', z=z'=\pm r/2}$
- Infinite sum over **vibrational modes** and integral over all **momentum space** in the Green's function, Eq. (3)

$$G(t, t'; z, z') = \frac{1}{\pi \sigma r} \sum_{n=0}^{\infty} \sin\left(\frac{n\pi}{r} z - \frac{n\pi}{2}\right) \sin\left(\frac{n\pi}{r} z' - \frac{n\pi}{2}\right) \int_{-\infty}^{\infty} dk \frac{e^{-ik(t-t')}}{k^2 + \left(\frac{n\pi}{r}\right)^2}$$

- **Zeta function** and **dimensional** regularization

- $\partial_z \partial_{z'} G|_{t=t', z=z'=\pm r/2} \propto \sum_n n = -1/12$
- $\partial_t \partial_z \partial_{z'} G|_{t=t', z=z'=\pm r/2} \propto \sum_n n^2 = 0$
- $\partial_t \partial_{t'} G|_{t=t', z=z'=\pm r/2} \propto 0 \cdot \int dy [y^2/(y^2 + 1)] = 0 \cdot (-2\pi)$

- **Poincaré invariance** → reduce the number of parameters

- Additional equality: $-4V_{p^2}^{(2,0)} + 2V_{p^2}^{(1,1)} - V^{(0)} + r \frac{dV^{(0)}}{dr} = 0$

- Exact result:

$$V_{p^2}^{(2,0)}(r)|_{NLO} = \left(\frac{1}{12\pi} + \frac{\pi}{36} \right) \frac{1}{r} - \frac{\mu}{4}, \quad V_{p^2}^{(1,1)}(r)|_{NLO} = \left(\frac{1}{6\pi} - \frac{\pi}{36} \right) \frac{1}{r},$$

$$V_{L^2}^{(2,0)}(r)|_{NLO} = \left(\frac{1}{18\pi} - \frac{\pi}{108} \right) \frac{1}{r}, \quad V_{L^2}^{(1,1)}(r)|_{NLO} = \left(\frac{1}{9\pi} + \frac{5\pi}{216} \right) \frac{1}{r},$$

...

- Full expression:

$$\begin{aligned}
 V(r) = & \left\{ \sigma + \left[-\frac{9\zeta_3}{2\pi^3} + \frac{a+b}{4\pi^3} + \left(\frac{\pi}{540} - \frac{1}{9\pi} \right) \left(\frac{1}{3} + \frac{1}{4\pi^2} \right)^2 \right] \frac{\sigma^2}{M^2} \right\} r^1 \\
 & + \frac{\sigma}{\pi M} \ln(\sigma r^2) + \left(\mu + \frac{2\mu'_1}{M} - \frac{\mu}{2M^2} p^2 + \frac{\mu_2}{M^2} \right) r^0 \\
 & + \left\{ -\frac{\pi}{12} + \left[\frac{1}{3\pi} + \frac{\pi}{36} \right] \frac{p^2}{M^2} + \left(-\frac{L^2}{6} - \frac{L \cdot S}{2} + \left[\frac{50}{27\pi^2} - \frac{2}{9\pi^2} - \frac{1}{12\pi^6} \right] \right) \frac{\sigma}{M^2} \right\} r^{-1} + \mathcal{O}(r^{-2})
 \end{aligned}
 \tag{5}$$

- ① Parameters and constants in Eq. (5)
 - $a \approx 7.08603$, $b \approx 1.26521$ (from V_r 's at NLO)
 - μ from $V^{(0)} = \sigma r + \mu - \frac{\pi}{12r}$
 - μ'_1 and μ_2 : renormalization parameters
- ② Comparison to lattice data [Y. Koma, M. Koma, H. Wittig, PoS LAT2007 (2007)]
 - Confirmation of $V_{p^2}^{(2,0)}$ and $V_{p^2}^{(1,1)}$ at long-distance (≥ 0.5 fm)
 - Confirmation of $V_{L^2}^{(2,0)} + V_{L^2}^{(1,1)}$ at long-distance (≥ 0.5 fm)

- Summary

- Introduction to low-energy EFTs for QCD (NRQCD, pNRQCD)
- Wilson loop and heavy quark potential
- Wilson loop and EST
- Heavy quark potential via EST at LO
- Correction via NLO mapping
- Heavy quark potential up to NLO in EST

- Outlook

- Calculation of heavy quarkonium mass (in progress)
- Application of EST to heavy hybrids and baryonic states



K. G. Wilson (1974)

Confinement of quarks

Phys. Rev. D 10, 2445



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Poincaré invariance constrains on NRQCD and potential NRQCD

Phys. Lett. B 576, 314 - 327



M. Berwein, N. Brambilla, S. Hwang, A. Vairo (2016)

Poincaré invariance in NRQCD and pNRQCD


To be submitted to *PRD* (2016), TUM-EFT 74/15





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Poincaré invariance and the heavy-quark potential


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-  N. Brambilla, A. Pineda, J. Soto, and A. Vairo (2000)
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Phys. Lett. B 80 (1979), 372

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Z. Phys. C 26 (1984), 401



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A. Pineda, A. Vairo (2001)

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G. Perez-Nadal, J. Soto (2009)

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PRD 79 (2009), 114002



N. Brambilla, M. Groher, H.E. Martinez, A. Vairo (2014)

Effective string theory and the long-range relativistic corrections to the quark-antiquark potential

PRD 90 (2014), 114032



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