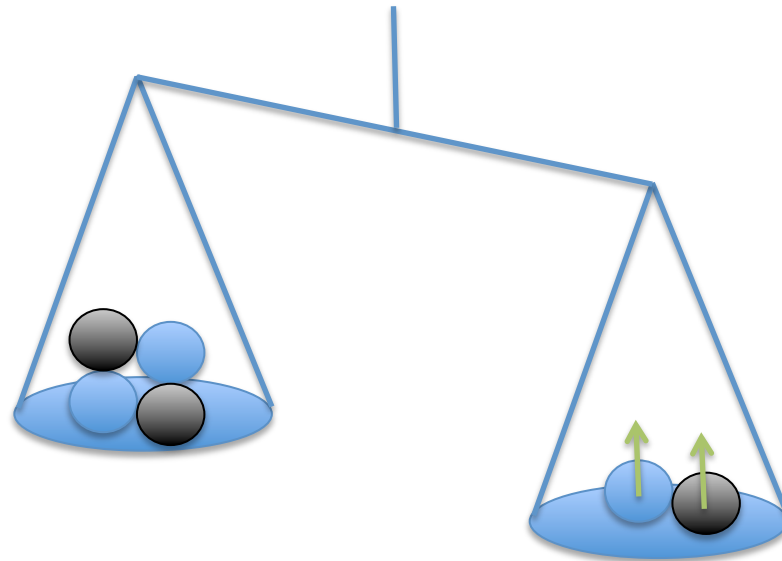


(Light) tetraquarks in a Dyson-Schwinger, Bethe-Salpeter approach

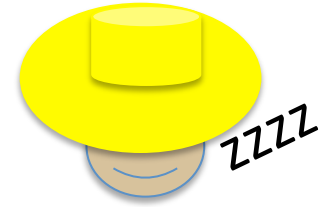


P. C.Wallbott, G. Eichmann, C. S. Fischer, W. Heupel

Contents

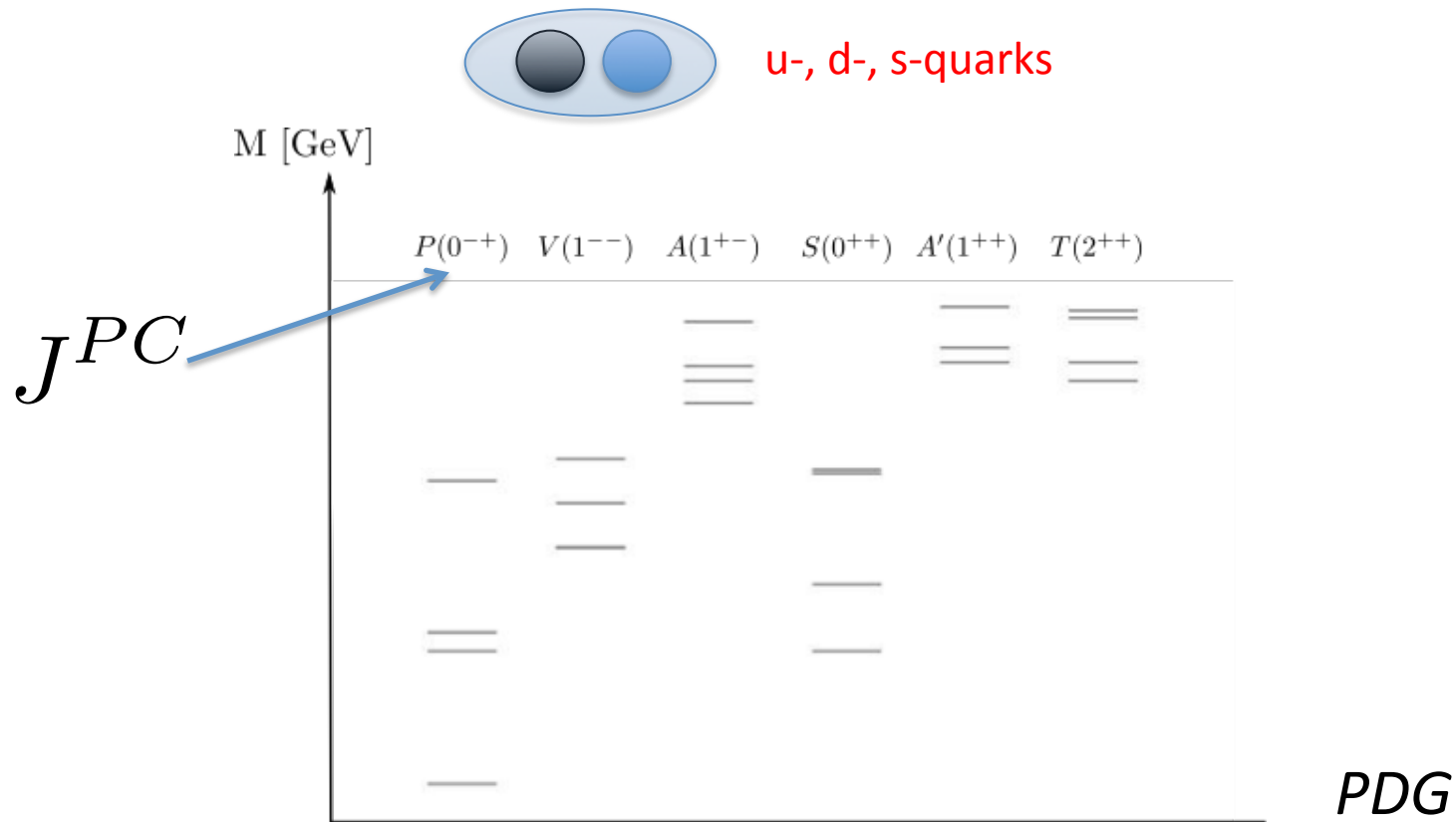
- Physics: The scalar mesons
- Calculations: How the method works
- Outlook

Contents

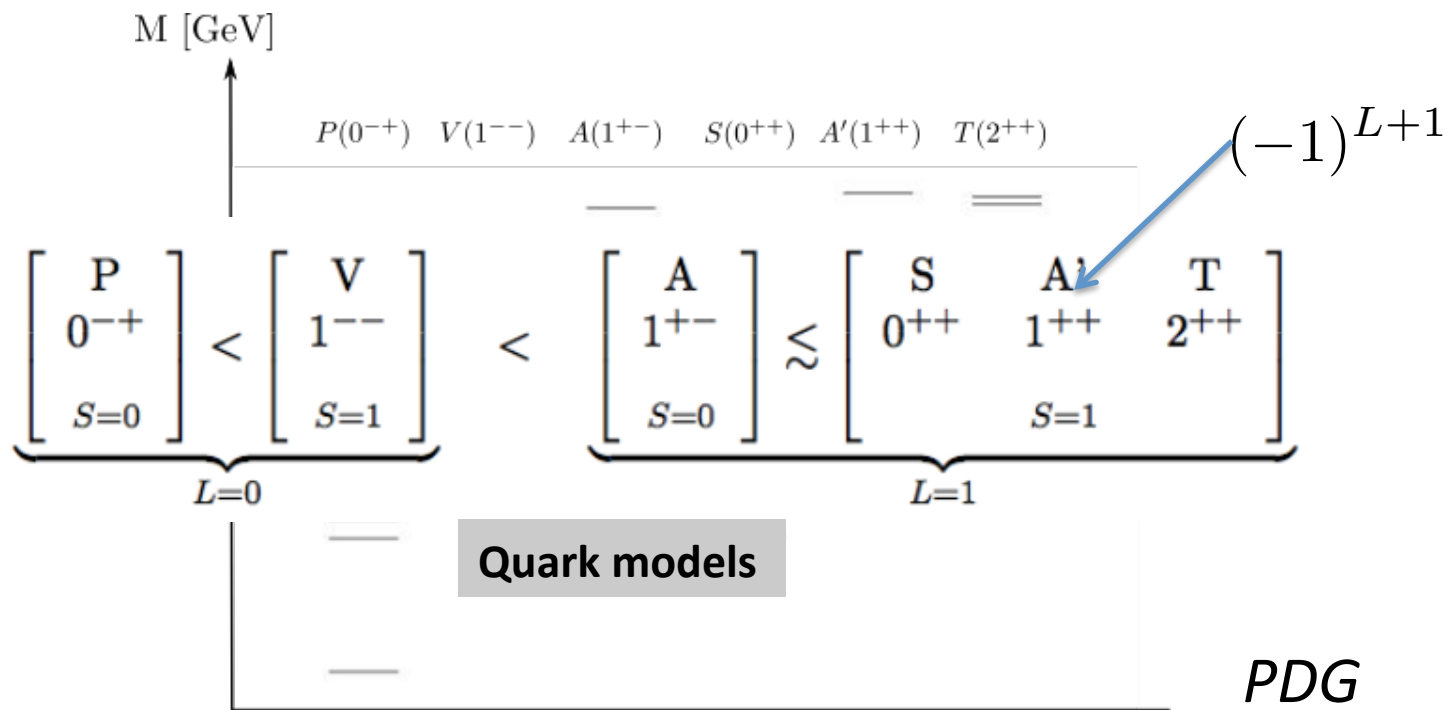


- Physics: The scalar mesons
- Calculations: How the method works
- Outlook

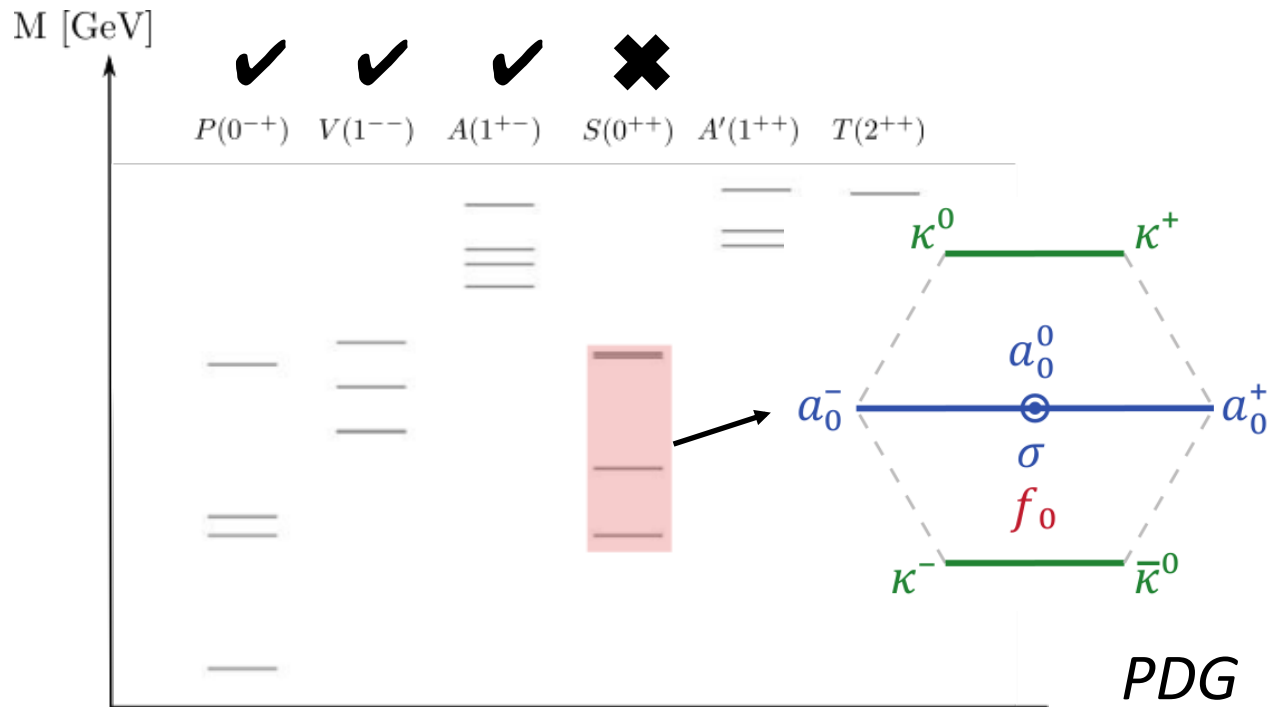
Light mesons: Multiplet order



Multiplet order



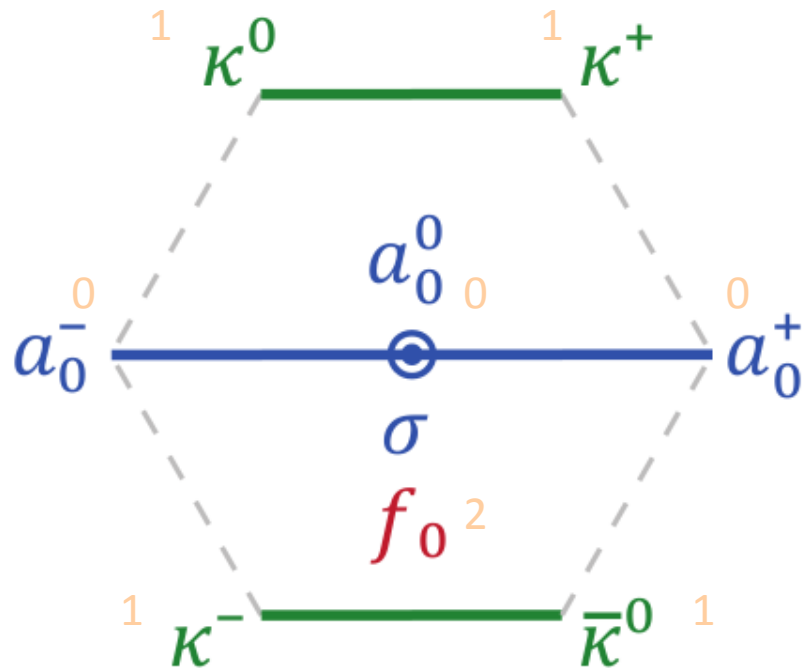
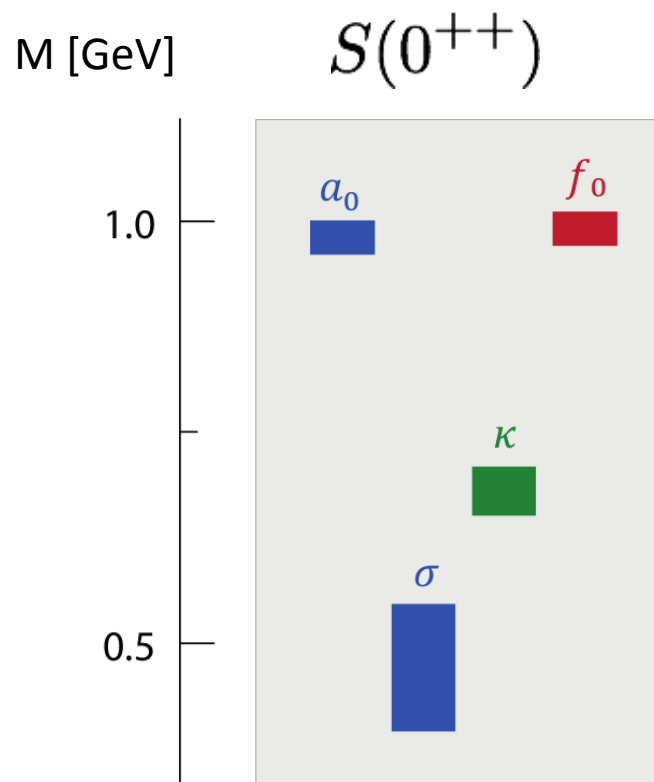
Multiplet order



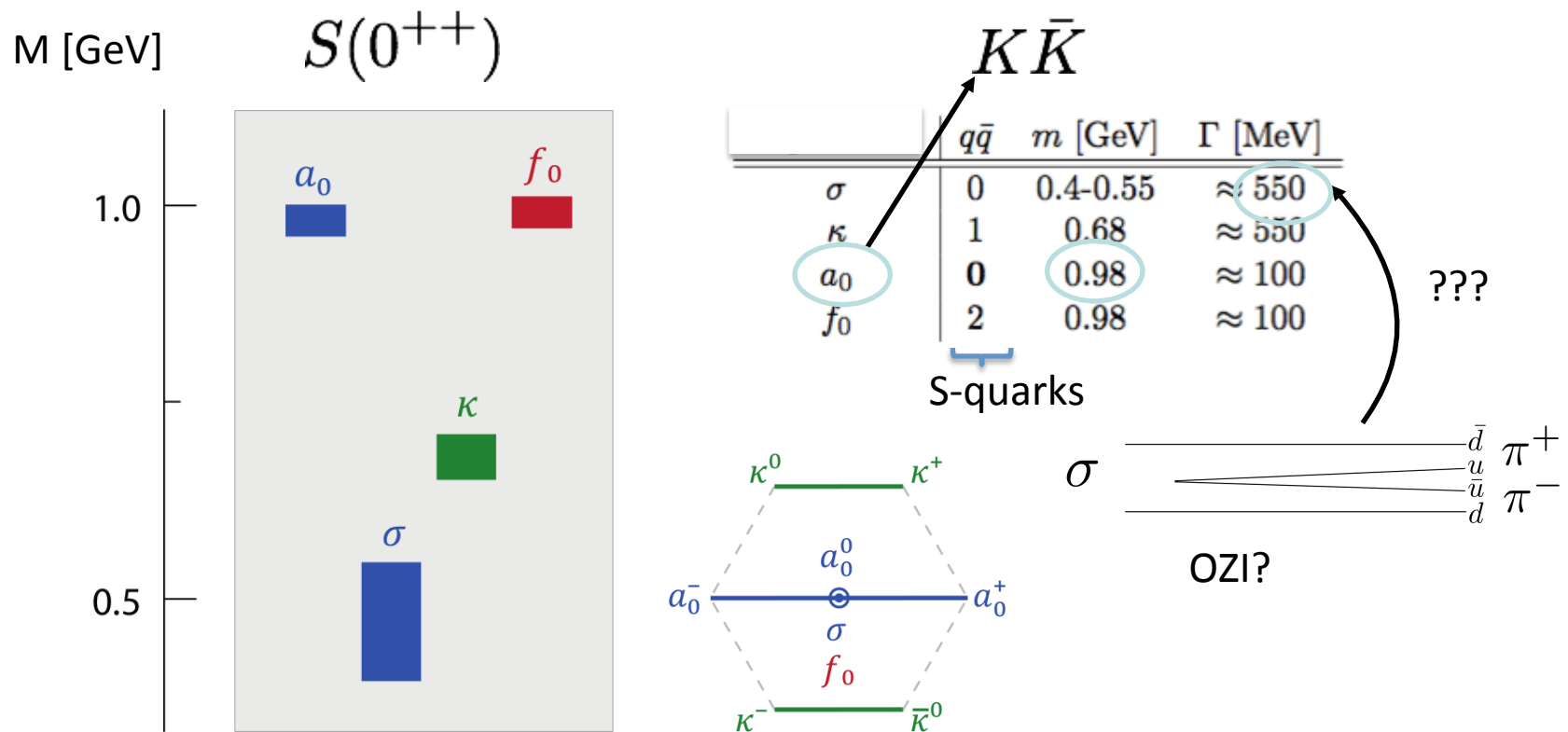
PDG

$$\underbrace{\begin{bmatrix} P \\ 0^{-+} \\ S=0 \end{bmatrix}}_{L=0} < \underbrace{\begin{bmatrix} V \\ 1^{--} \\ S=1 \end{bmatrix}}_{L=1} < \underbrace{\begin{bmatrix} A \\ 1^{+-} \\ S=0 \end{bmatrix}}_{L=1} \gtrsim \underbrace{\begin{bmatrix} S & A' & T \\ 0^{++} & 1^{++} & 2^{++} \\ & S=1 & \end{bmatrix}}_{L=1}$$

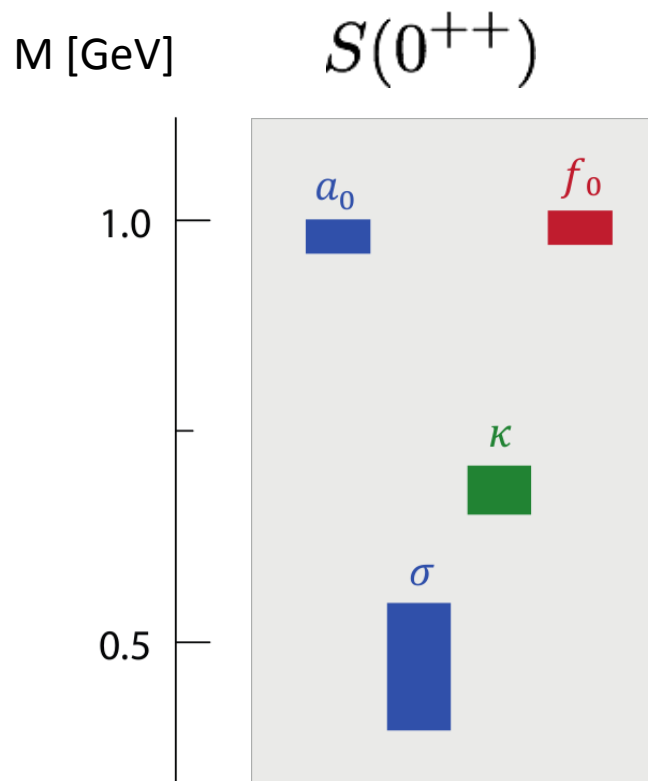
Masses ...



... and widths

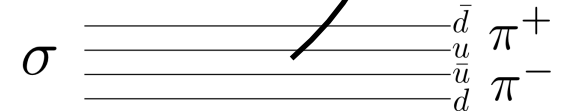
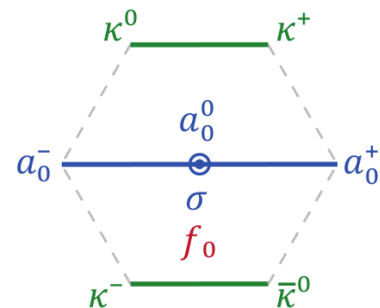


Are they tetraquarks?



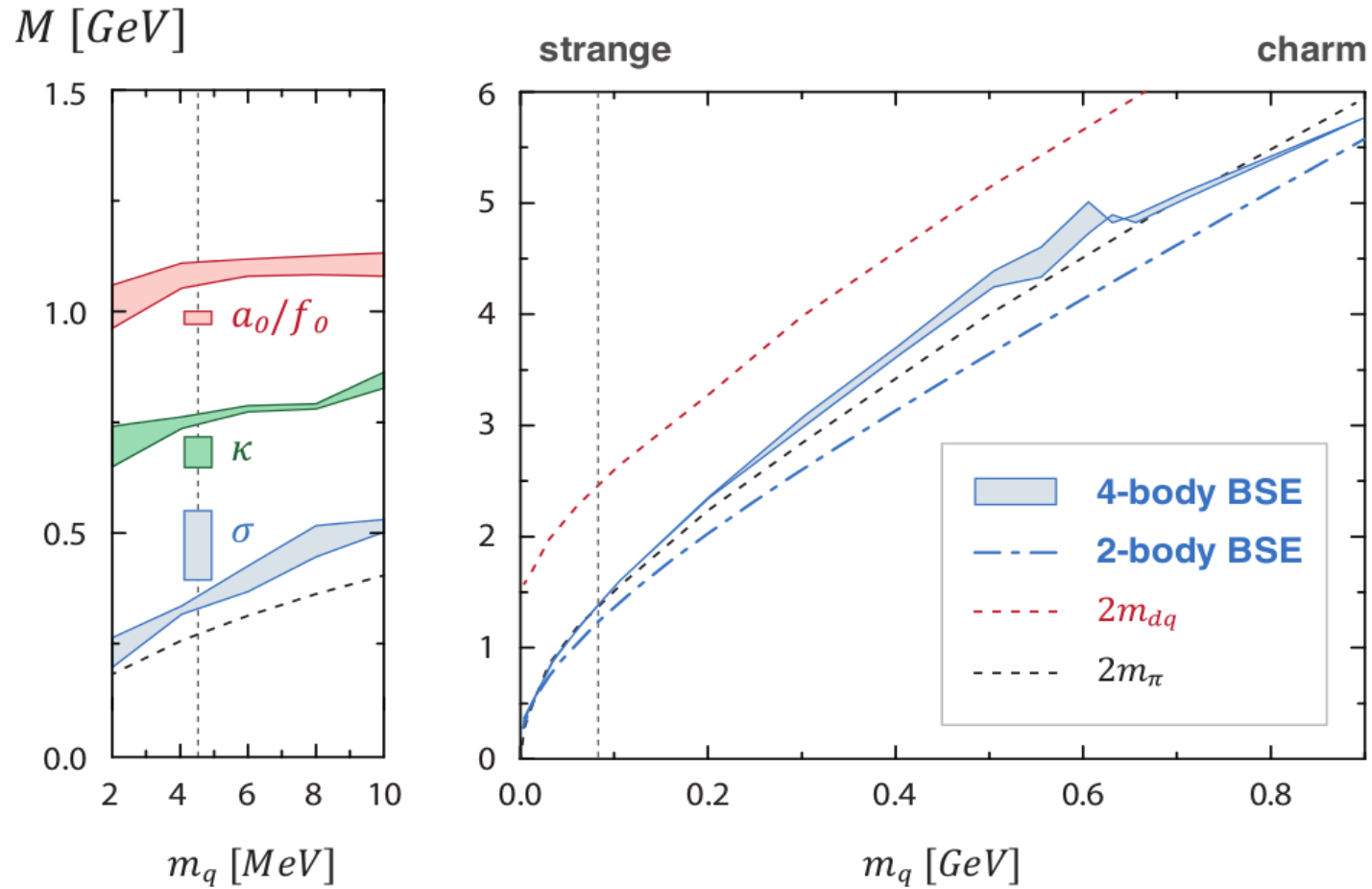
	$q\bar{q}$	$qq\bar{q}\bar{q}$	m [GeV]	Γ [MeV]
σ	0	0	0.4-0.55	≈ 550
κ	1	1	0.68	≈ 550
a_0	0	2	0.98 ✓	≈ 100
f_0	2	2	0.98	≈ 100

S-quarks



Jaffe, RJ: *Physical Review D*, 15(1):267, 1977

Results in BSE/DSE



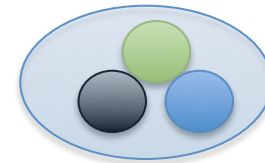
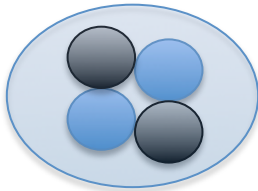
Eichmann, Fischer, Heupel Phys. Lett. B753:282-287, 2016

Calculation roadmap

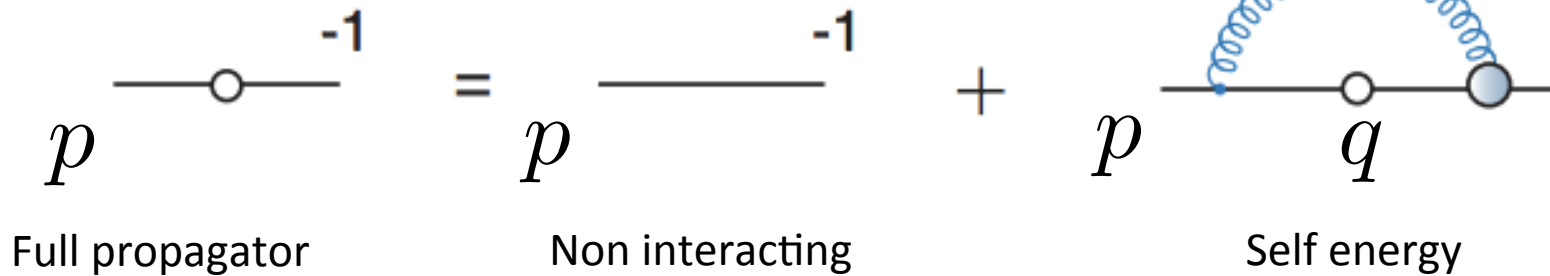
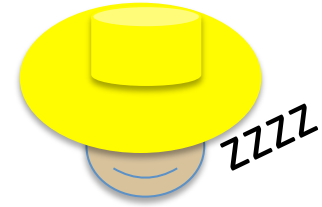
- Quark propagator (DSE)



- Bound states (BSE) for mesons, baryons, tetraquarks...

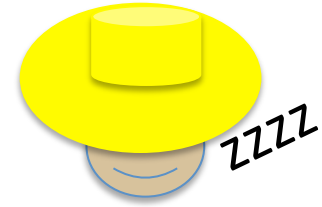


Quark DSE



$$S^{-1}(p) = S_0^{-1}(p) + \int_q D^{\mu\nu}(k) \gamma^\mu S(q) \Gamma^\nu(p, q)$$

Quark DSE



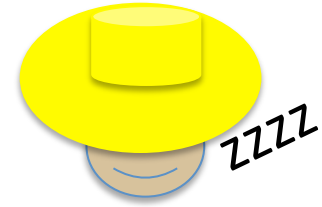
$$D^{\mu\nu} = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) \frac{Z(k^2)}{k^2} \Gamma^\mu(q, k)$$

$$S^{-1}(p) = -ip^\mu \gamma^\mu A(p^2) + B(p^2)$$

$$S_0^{-1}(p) = -ip^\mu \gamma^\mu + m$$

$$S^{-1}(p) = S_0^{-1}(p) + \int_q D^{\mu\nu}(k) \gamma^\mu S(q) \Gamma^\nu(p, q)$$

Quark DSE



$$D^{\mu\nu} = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) \frac{Z(k^2)}{k^2} \Gamma^\mu(q, k) \propto \gamma^\mu \alpha_{eff}(k^2)$$

$$S^{-1}(p) = -ip^\mu \gamma^\mu \overset{??}{A}(p^2) + \overset{??}{B}(p^2)$$

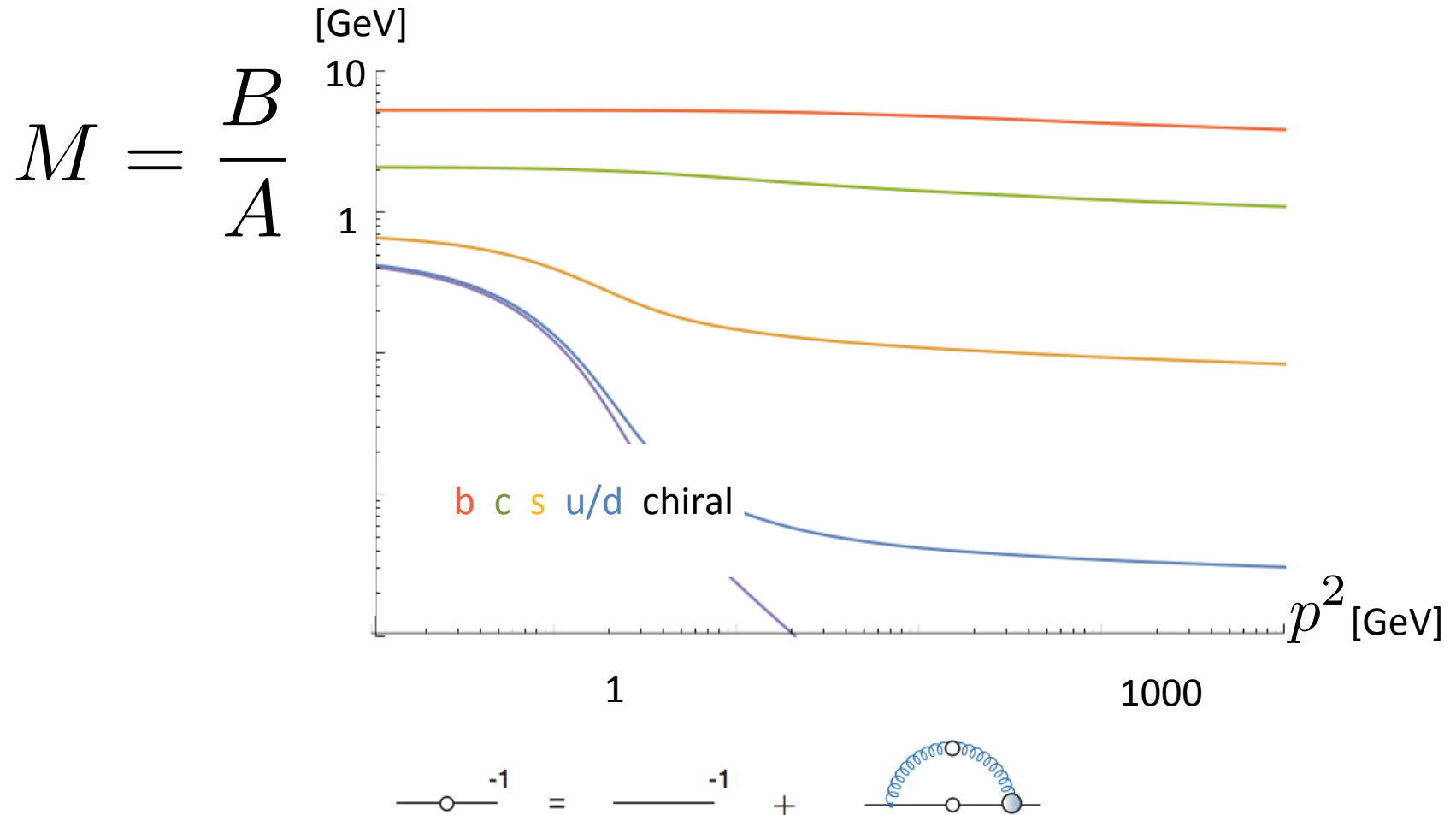
$$S_0^{-1}(p) = -ip^\mu \gamma^\mu + m$$

Diagram illustrating the Dyson-Schwinger equation for the quark propagator. The equation shows the full propagator $S^{-1}(p)$ as the sum of the free propagator $S_0^{-1}(p)$ and a loop diagram. The loop diagram consists of a quark line (solid line) and a gluon line (curly line). The vertices are represented by circles. The diagram is labeled with -1 for the free propagator and $S(q)$ for the full propagator in the loop.

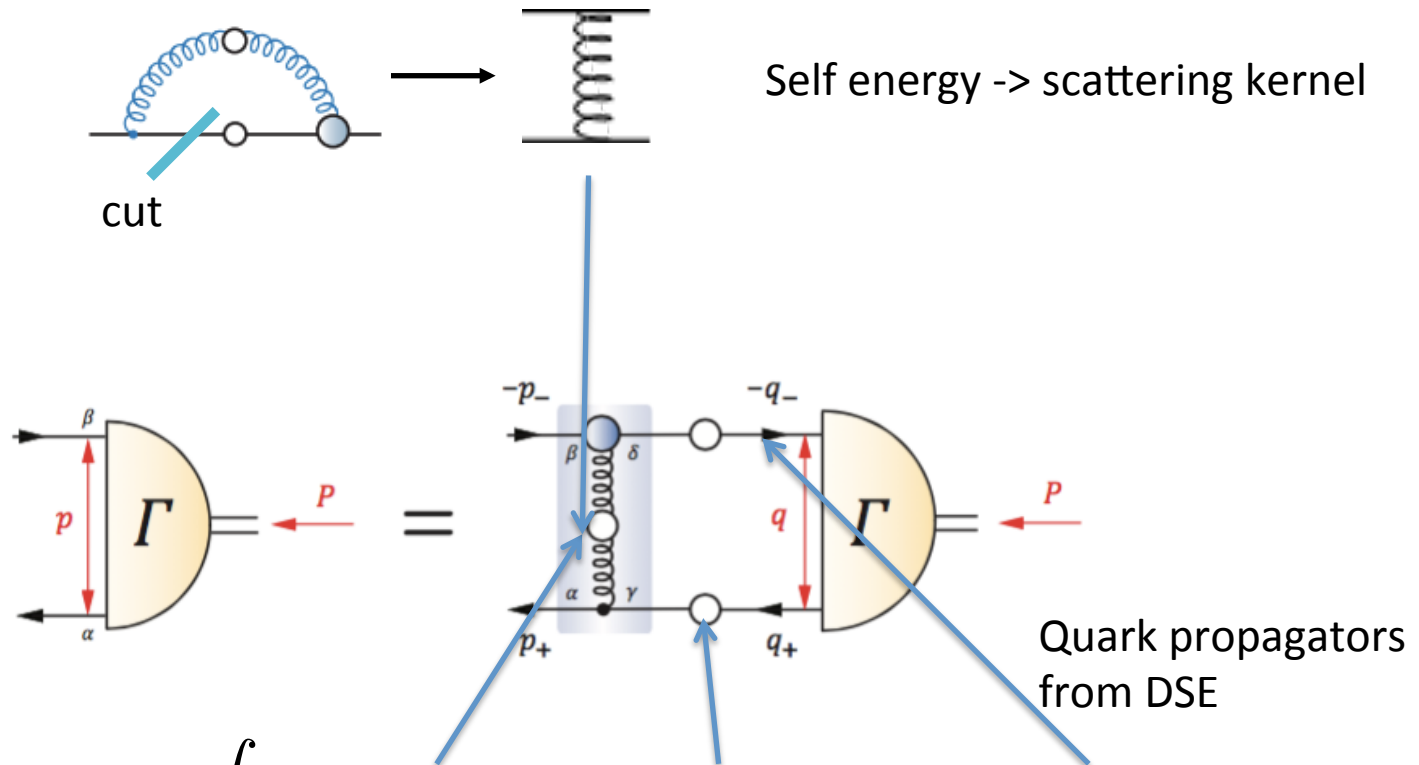
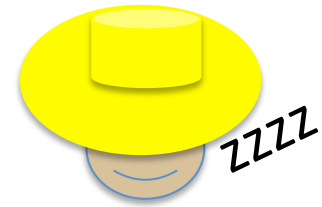
✓ Rainbow-Ladder
+ Maris-Tandy

In RL + MT only A,B unknown
-> solve for A,B
-> quark propagator

Quark DSE, results:



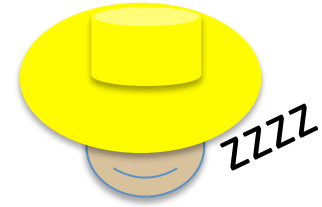
Meson BSE setup



$$\Gamma_{ab}(p, P) = \int_q K_{ad,cb}(p, q, P) [S(q_+) \Gamma(q, P) S(-q_-)]_{cd}$$

$$\Gamma = K \cdot \Gamma$$

Meson BSE solve it



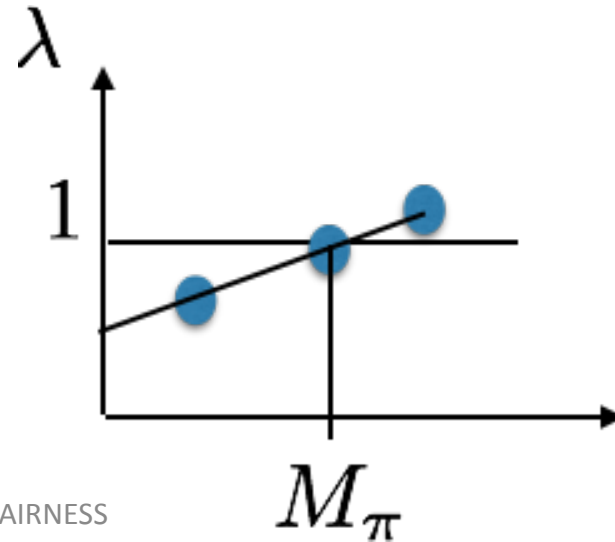
$$\Gamma_{ab}(p, P) = \int_q K_{ad,cb}(p, q, P) [S(q_+) \Gamma(q, P) S(-q_-)]_{cd}$$

$$\Gamma = \sum_{i=1}^4 f_i(\Omega) \tau_i \otimes color \otimes flavor$$

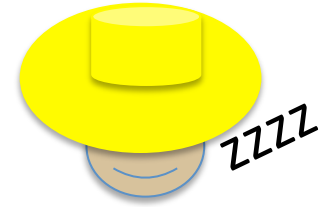
$\Omega(p^2, p \cdot P)$

$$\lambda(M)$$

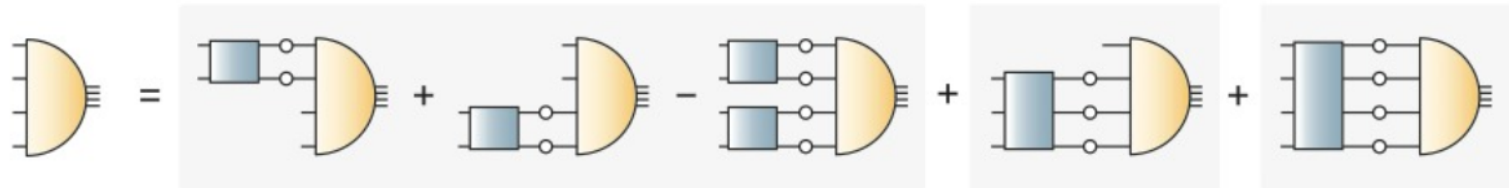
$$\Gamma = K \cdot \Gamma$$



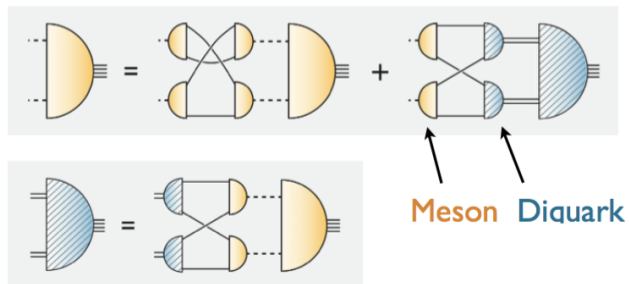
Tetraquark BSE(s)



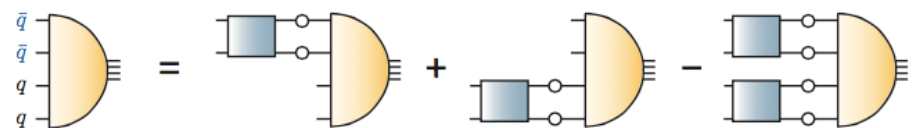
A. Khvedelidze, A. Kvinikhidze, *Theor. Math. Phys.* 90, 62 (1992)



2-body approximation



full equation

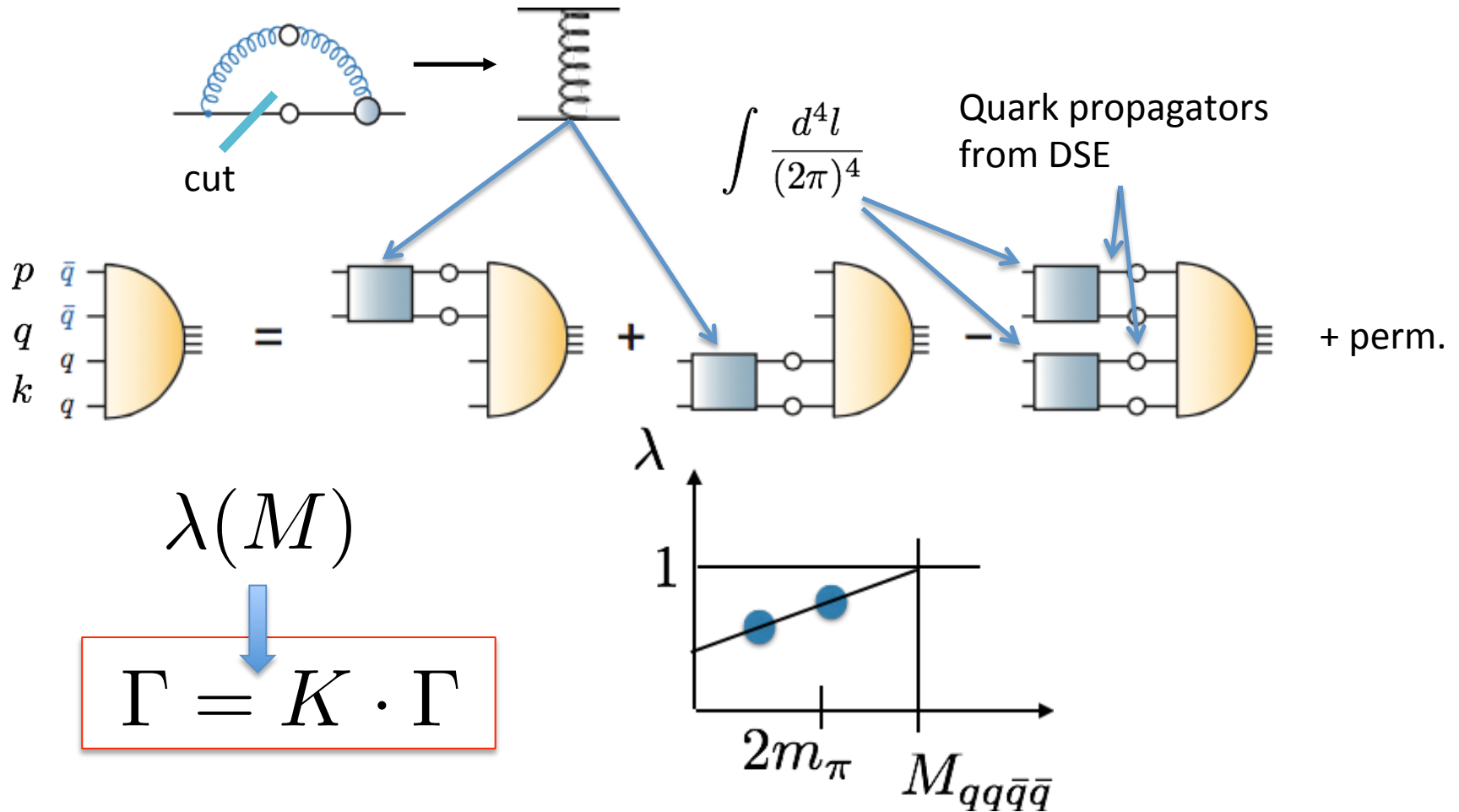


Eichmann, Fischer, Heupel
Phys. Lett. B 753:282-287, 2016

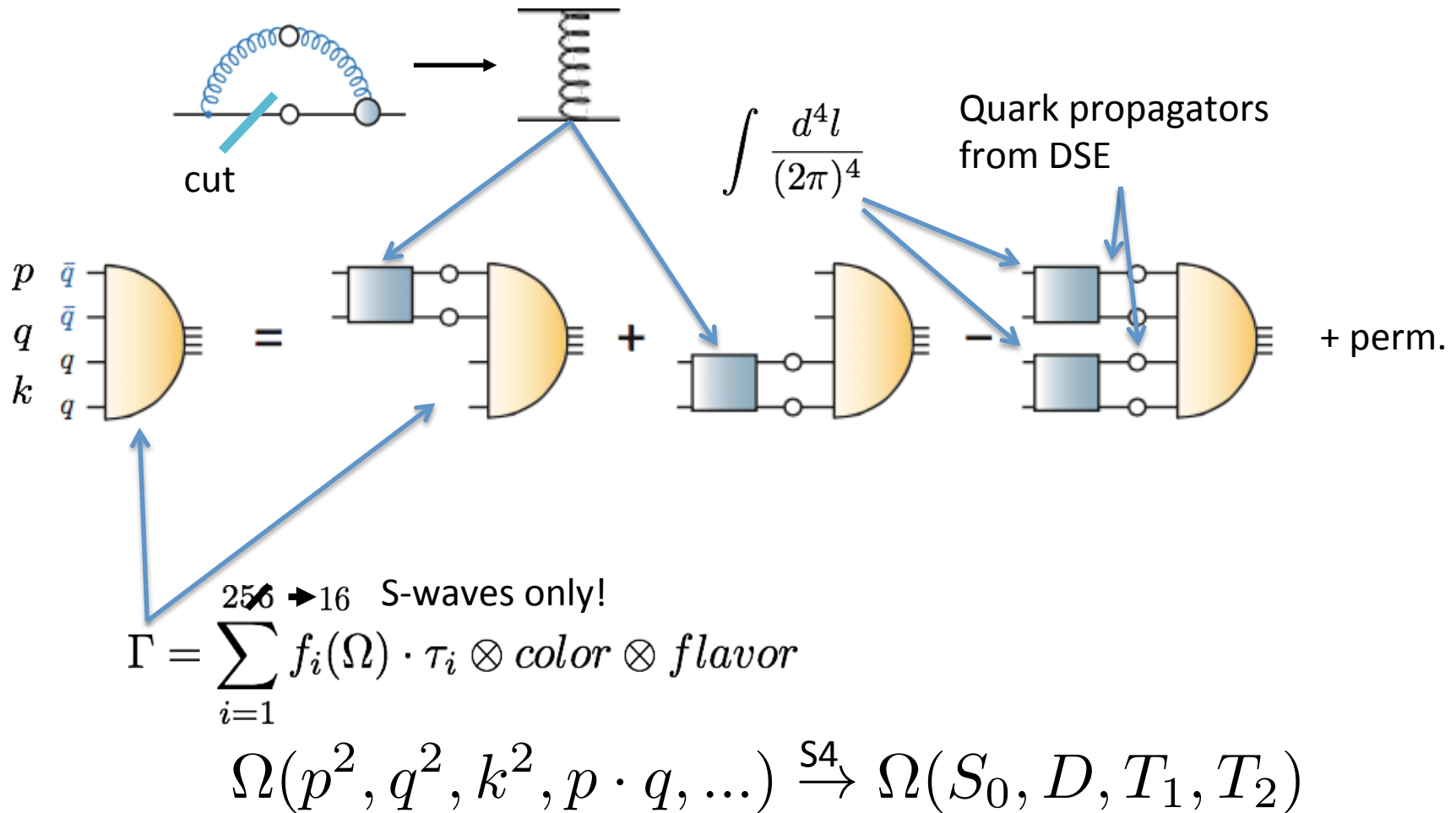
Heupel, Eichmann Fischer
Phys. Lett. B 718:545-549, 2012

Wallbott, tetraquarks in a DSE, BSE approach

Tetraquark BSE, setup



Tetraquark BSE, setup



Importance of multiplets

$$m_\sigma$$

$$f_i(S_0, \not{D}, \not{T}_1, \not{T}_2) \rightarrow 1.4 \text{ GeV}$$

$$f_i(S_0, \not{D}, \not{T}_1, T_2) \rightarrow 1.4 \text{ GeV}$$

$$f_i(S_0, \not{D}, T_1, \not{T}_2) \rightarrow 1.1 \text{ GeV}$$

$$f_i(S_0, D, T_1, \not{T}_2) \rightarrow \mathbf{0.35 \text{ GeV}}$$

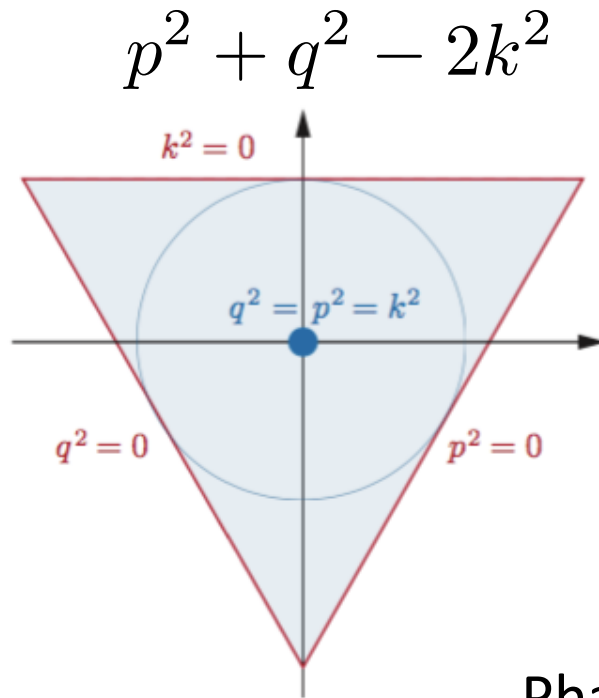
$$S_0 = \frac{p^2 + q^2 + k^2}{4} \quad T_i = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

$$D = \frac{1}{4S_0} \left(\sqrt{3}q^2 - p^2 \right)$$

$\not{}$:= set to
constant value

D important, T not!

Why is D important?



$$D = \frac{1}{4S_0} \begin{pmatrix} \sqrt{3}q^2 - p^2 \\ p^2 + q^2 - 2k^2 \end{pmatrix}$$

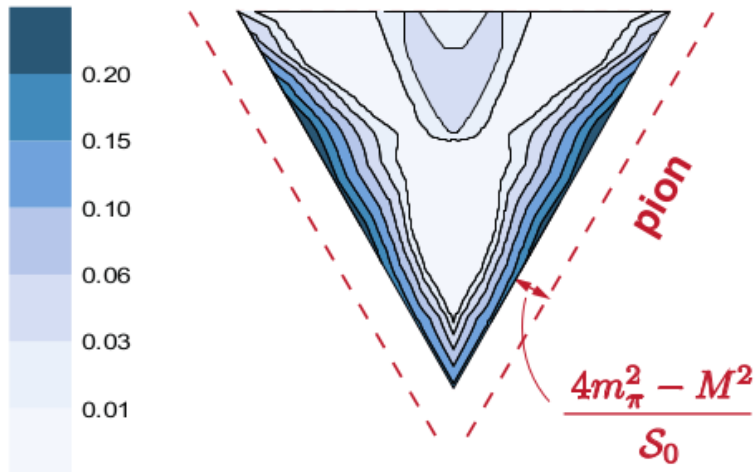
$$\sqrt{3}q^2 - p^2$$

Phase space restricted to triangle!

Poles!

$$f_1(\sqrt{3}q^2 - p^2, p^2 + q^2 - 2k^2) = f_1(D)$$

diquark



$$\Gamma = \sum_{i=1}^4 f_i(\Omega) \tau_i \otimes color \otimes flavor$$

$$\tau_1 = \gamma_5 \otimes \gamma_5$$

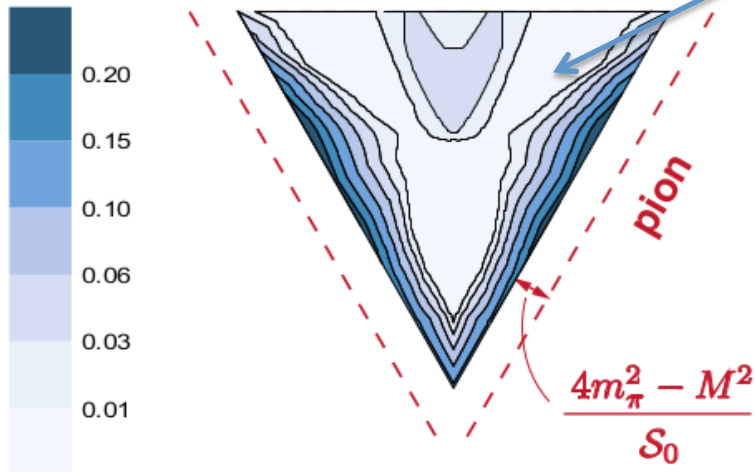
Poles!

$$f_1(\sqrt{3}q^2 - p^2, p^2 + q^2 - 2k^2) = f_1(D)$$

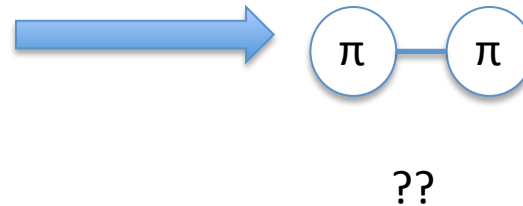
diquark

$$f_1 = \frac{1}{m_\pi^2 + (p1 + p3)^2} \cdot \frac{1}{m_\pi^2 + (p2 + p4)^2}$$

Exhibits poles!

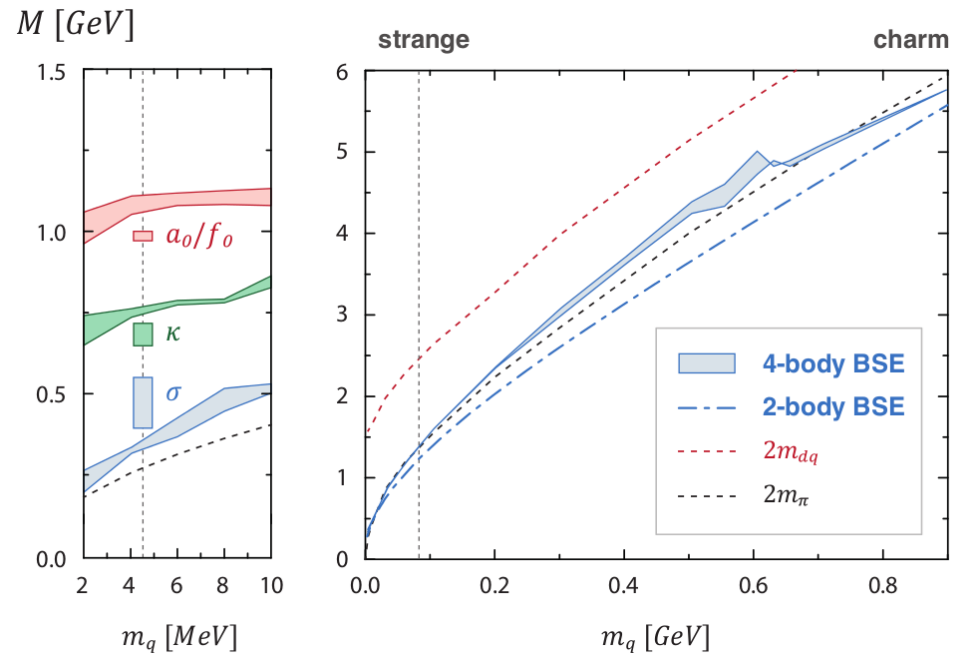


Moving poles!



Summary of results

- Light scalars, because **dynamically generated** meson poles dominate
- Tetraquark is resonance above two pion threshold

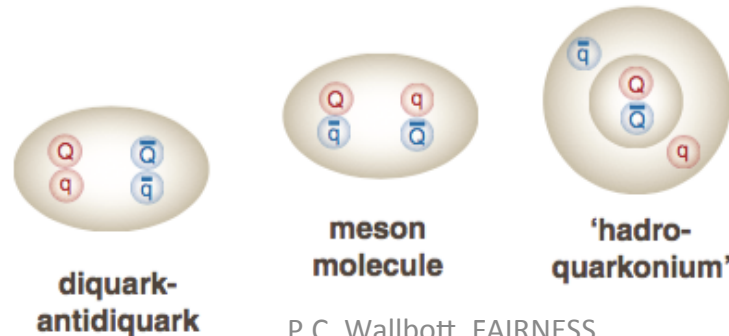


Outlook

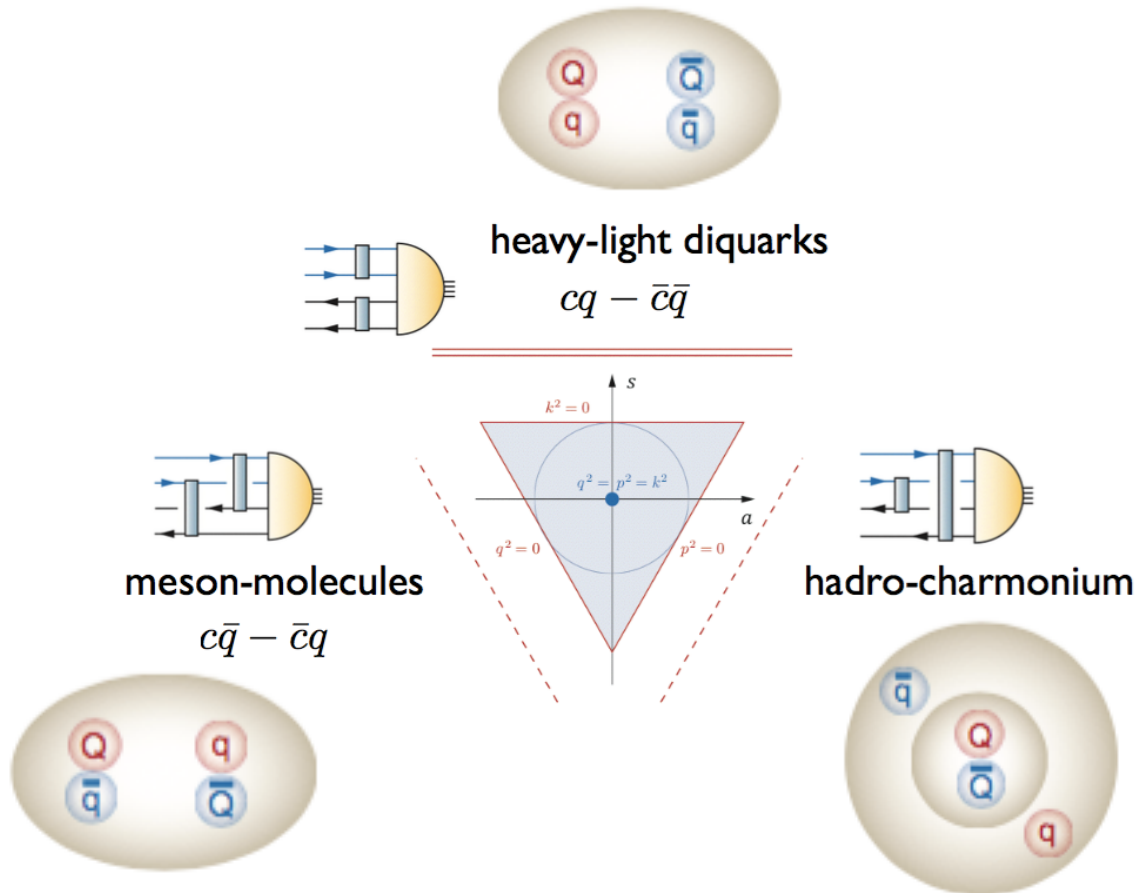
- Technical improvements
 - Analysis of higher partial waves
 - More systematic studies of results
 - Rigorous calculation of decay properties

$$\sigma \rightarrow \pi\pi$$

- Establish method for heavy-light systems

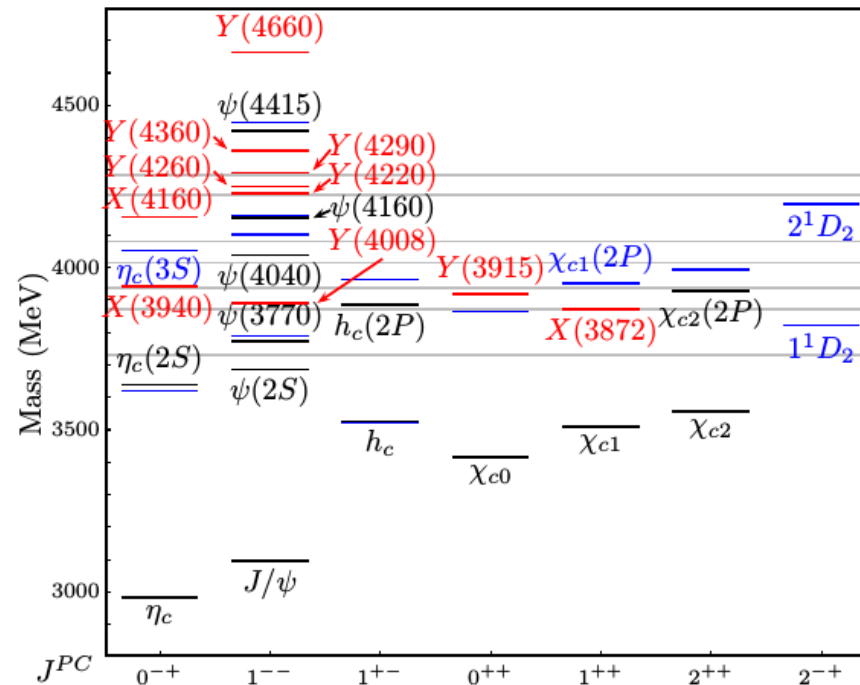


Outlook



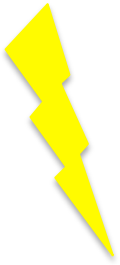
Outlook

- Establish method for heavy-light systems
- And other quantum numbers

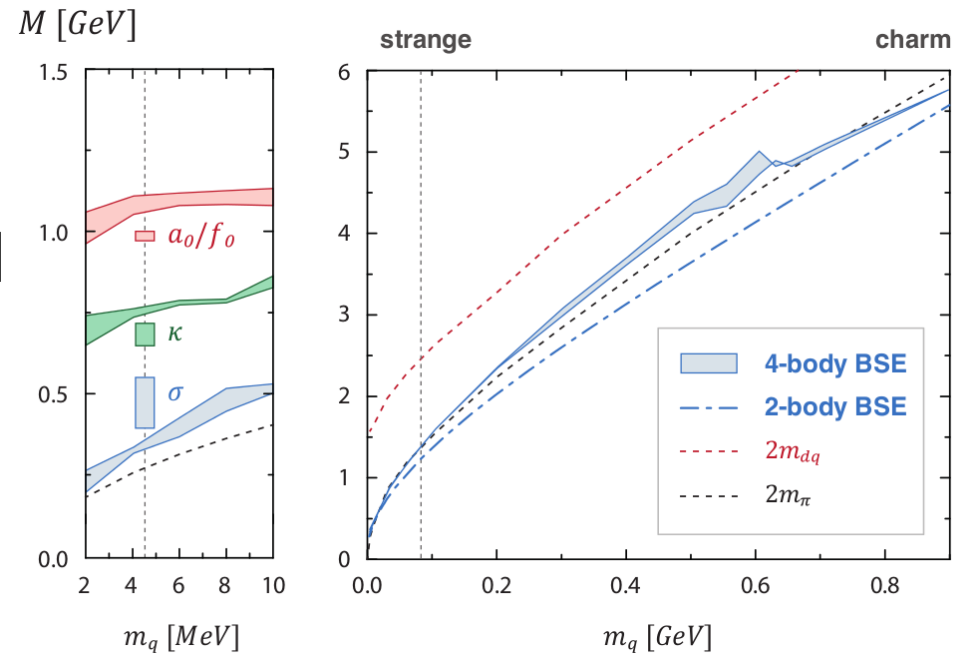


Esposito et al., I Journal Mod Phys A 30, 2014

Outlook (approach)

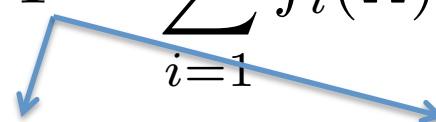


Hard calculation, limited to (almost) equal quark masses



Outlook (approach)

“Physical construction” + poles

$$\Gamma = \sum_{i=1}^4 f_i(\Omega) \tau_i \otimes color \otimes flavor$$


$$\Gamma_{q\bar{q}q\bar{q}} = \Gamma_{\pi} \otimes \Gamma_{\pi} + \Gamma_{\rho} \otimes \Gamma_{\rho} + \dots$$

PW, Tetraquarks in a DSE/BSE approach


$$1^- \otimes 1^- = 0^+$$

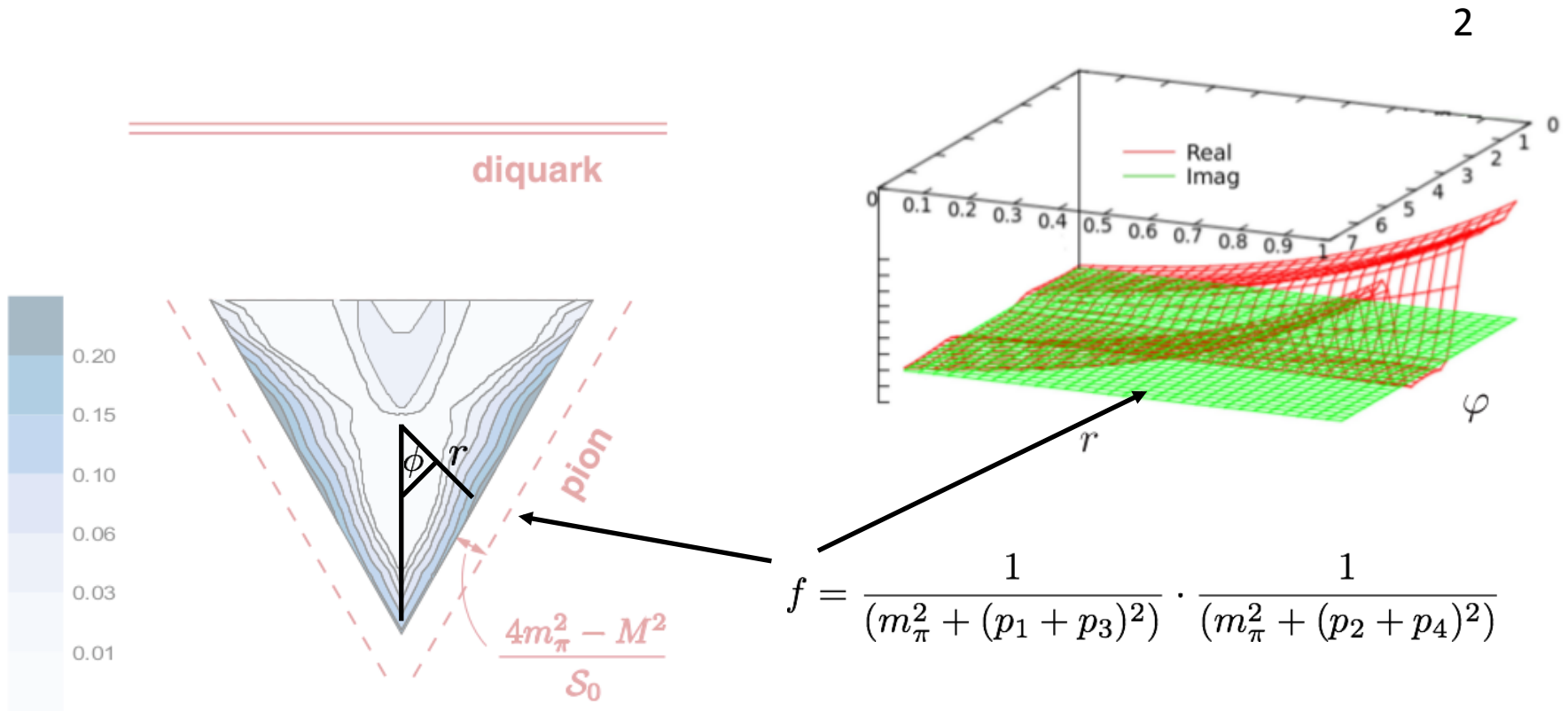
Now back to s-wave truncated amplitude

Summary

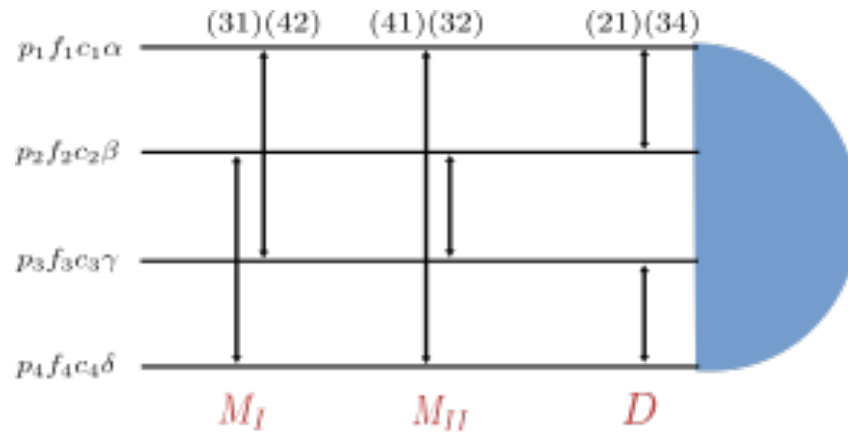
- Strength:
 - Once truncation is set, no further input necessary
 - System chooses configuration dynamically (π - π)
 - Growing computation power -> full solution possible
- Challenges:
 - heavy-light systems
 - Decays (extrapolation)
 - Mixing
 - Redo lots of coding

Also interesting

Why D? Poles!



Tetraquarks in a DSE BSE approach



Wallbott, tetraquarks in a BSE/DSE approach

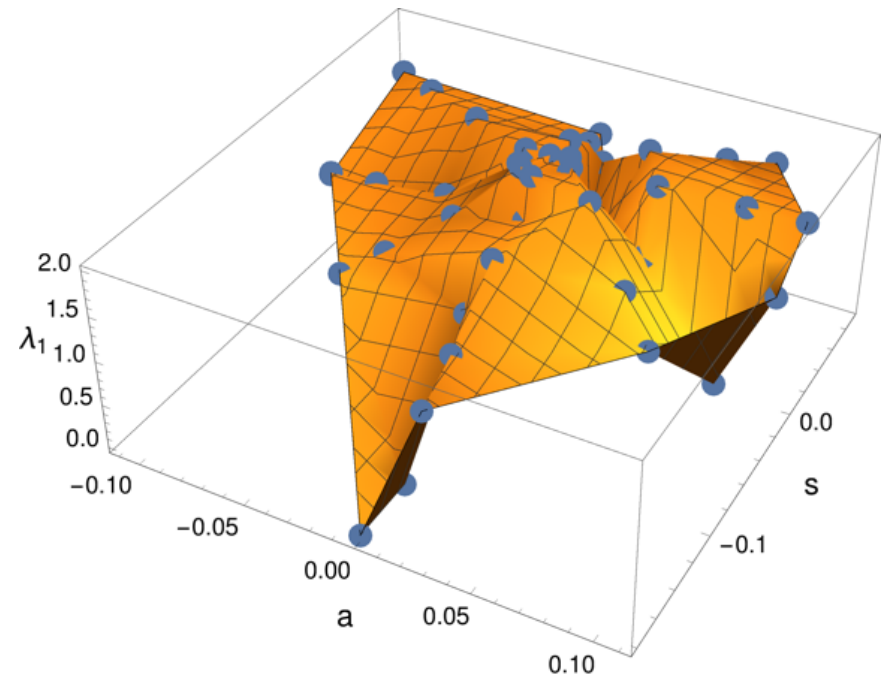
$$\Gamma = \gamma^5 \otimes \gamma^5 |_{M_1} \otimes color \otimes flavor \cdot f_1(\Omega)$$

$$\gamma^5 \otimes \gamma^5 |_{M_2} \otimes color \otimes flavor \cdot f_2(\Omega)$$

$$f = \frac{1}{(m_\pi^2 + (p_1 + p_3)^2)} \cdot \frac{1}{(m_\pi^2 + (p_2 + p_4)^2)}$$

Tetraquarks in a DSE BSE approach

- still strong doublet D dependence of results
- Problem: Symmetries!

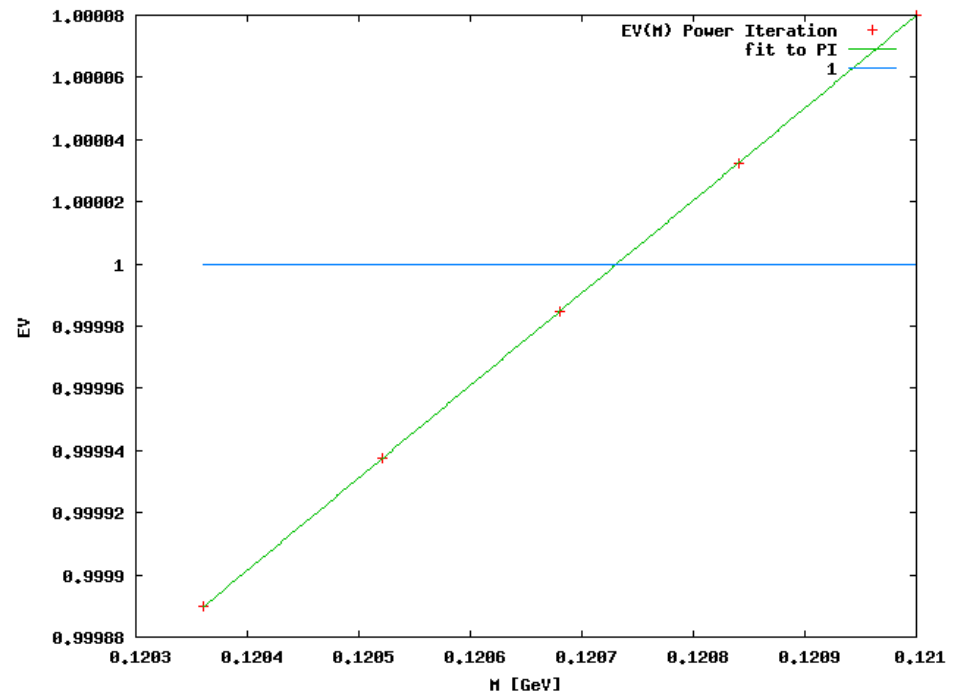


Wallbott, tetraquarks in a BSE/DSE approach

BSE eigenvalue/ extrapolation

$$K\Psi = \lambda(P^2)\Psi$$

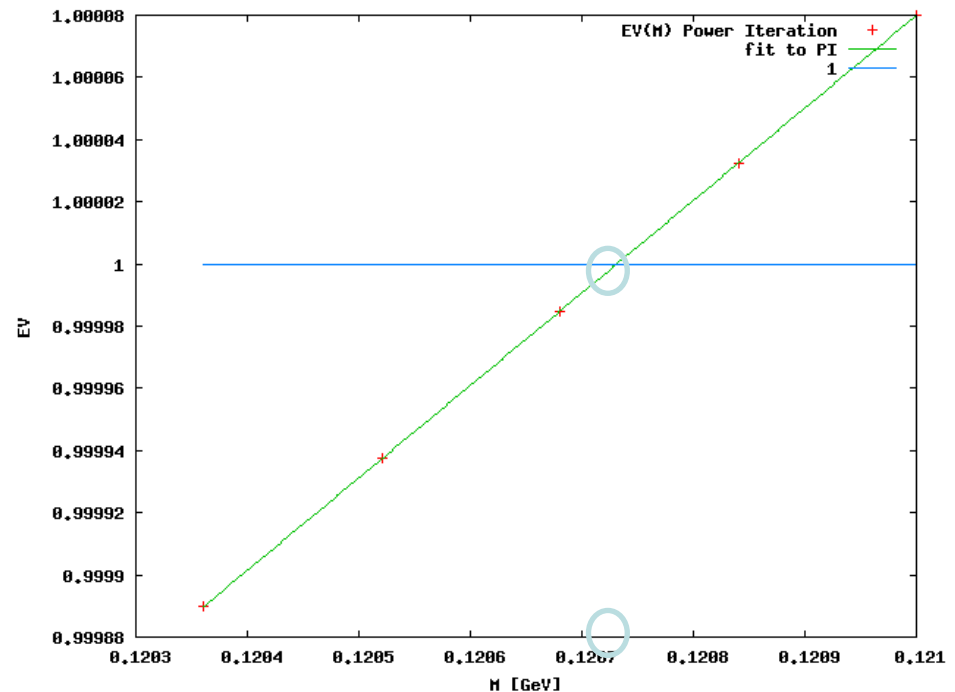
$$\lambda(P^2 = M_{physical}^2) = 1$$



BSE eigenvalue/ extrapolation

$$K\Psi = \lambda(P^2)\Psi$$

$$\lambda(P^2 = M_{physical}^2) = 1$$



BSE eigenvalue/ extrapolation

$$K\Psi = \lambda(P^2)\Psi$$

$$\lambda(P^2 = M_{physical}^2) = 1$$

