# Determination of electric dipole transitions in heavy quarkonia using potential nonrelativistic QCD

**Jorge Segovia**, Sebastian Steinbeißer and Antonio Vairo

Physik-Department-T30f

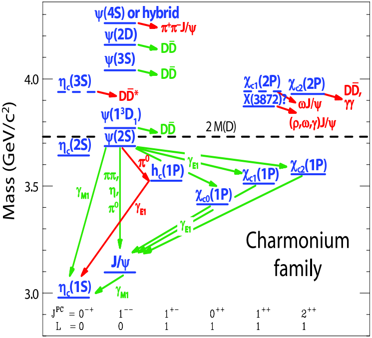
Technische Universität München

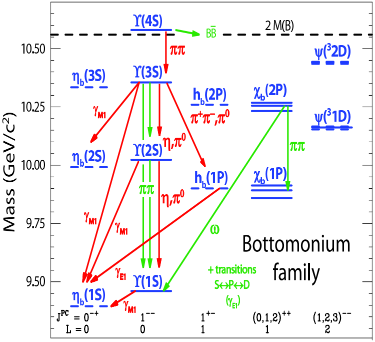
T30f
Theoretische Teilchen- und Kernphysik

Unterstützt von / Supported by

Alexander von Humboldt
Stiftung / Foundation

*The Charmonium and bottomonium systems were discovered in the 1970s*
*Experimentally clear spectrum of narrow states below the open-flavor threshold*




E. Eichten *et al.*, Rev. Mod. Phys. 80 (2008) 1161.

- Heavy quarkonia are bound states made of a heavy quark and its antiquark ($c\bar{c}$ charmonium and $b\bar{b}$ bottomonium).
- They can be classified in terms of the quantum numbers of a nonrelativistic bound state → Reminds positronium [$(e^+e^-)$-bound state] in QED.
- Heavy quarkonium is a very well established multiscale system which can serve as an ideal laboratory for testing all regimes of QCD.

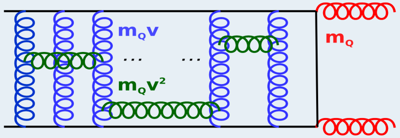
☞ Heavy quarkonium is a nonrelativistic system:

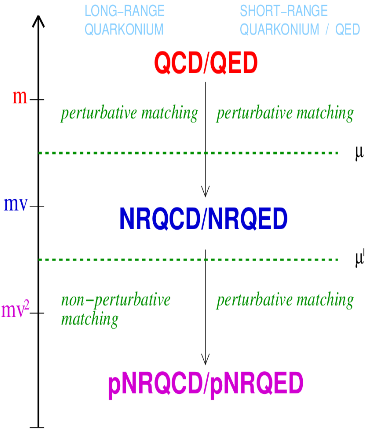
$$v_c \sim 0.55, \quad v_b \sim 0.32 \quad (v_{\rm light} = 1.0)$$

☞ Heavy quarkonium is a multiscale system:

$$M \gg p \sim 1/r \sim Mv \gg E \sim Mv^2$$

☞ Scales are entangled in full QCD




☞ Systematic expansions in the small heavy-quark velocity $v$ may be implemented at the Lagrangian level by constructing suitable effective field theories (EFTs):

- Expanding QCD in $p/M$, $E/M$ leads to **NRQCD.**
  → G.T. Bodwin, E. Braaten and G.P. Lepage, Phys. Rev. D51 (1995) 1125.
- Expanding NRQCD in $E/p$ leads to **pNRQCD.**
  → N. Brambilla, A. Pineda, J. Soto and A. Vairo, Nucl. Phys. B566 (2000) 275.

# There is another scale in QCD: $\Lambda_{\rm QCD}$

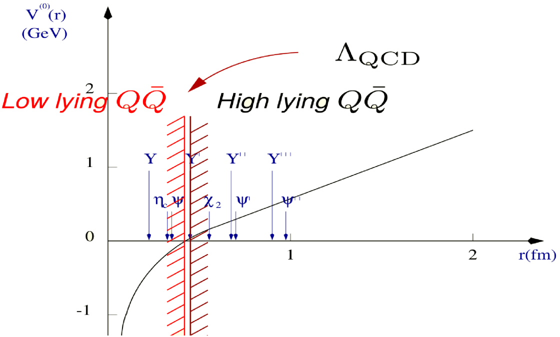
- The matching of QCD to NRQCD

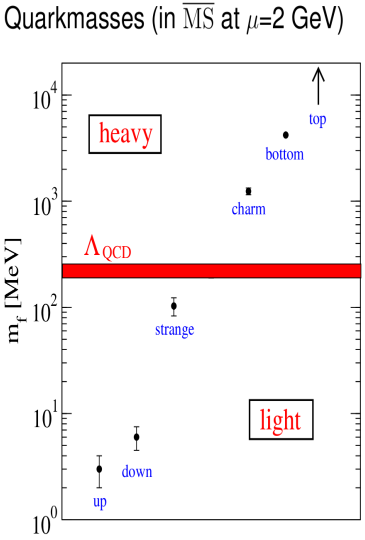
$M \gg \Lambda_{\rm QCD}$ $\rightarrow$ Perturbative matching.

- The matching of NRQCD to pNRQCD

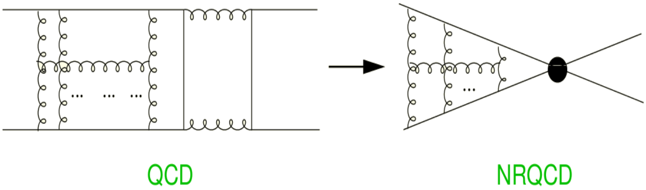
$p \sim 1/r \gg \Lambda_{\rm QCD}$ $\rightarrow$ Weak coupling regime. Perturbative matching.

$p \sim 1/r \gtrsim \Lambda_{\rm QCD}$ $\rightarrow$ Strong coupling regime. Nonperturbative matching.



☞ Physics at the scale $M$: Quarkonium annihilation and production.



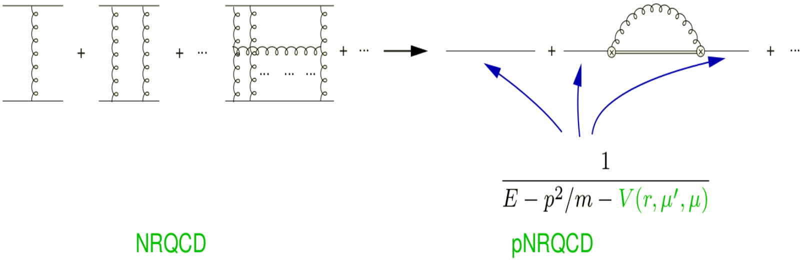

$\times c(\alpha_s(M), \mu)$

☞ The effective Lagrangian is organized as an expansion in $1/M$ and $\alpha_s(M)$:

$$\mathcal{L}_{\rm NRQCD} = \sum_n \frac{c_n(\alpha_s(M), \mu)}{M^n} \times \mathcal{O}_n(\mu, Mv, Mv^2, \ldots)$$

- $\mathcal{L}_{\rm NRQCD}$ is made of all low-energy operators $\mathcal{O}_n$ that may be built from the effective degrees of freedom and are consistent with the symmetries of $\mathcal{L}_{\rm QCD}$.
- The Wilson coefficients $c_n$ encode the high energy physics. They are calculated by imposing that $\mathcal{L}_{\rm NRQCD}$ and $\mathcal{L}_{\rm QCD}$ describe the same physics at $\mu = M$.

☞ Physics at the scale $Mv$: Quarkonium formation.




☞ The effective Lagrangian is organized as an expansion in $1/M$, $\alpha_s(M)$ and $1/p \sim r$:

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r \sum_n \sum_k \frac{c_n(\alpha_s(M),\mu)}{M^n} \times V_{n,k}(r,\mu',\mu) r^k \times \mathcal{O}_k(\mu', Mv^2, \ldots)$$

where a multipole expansion of the gluon field has been performed.

☞ The Wilson coefficients of pNRQCD depends on the distance $r$ (and scales $\mu, \mu'$):

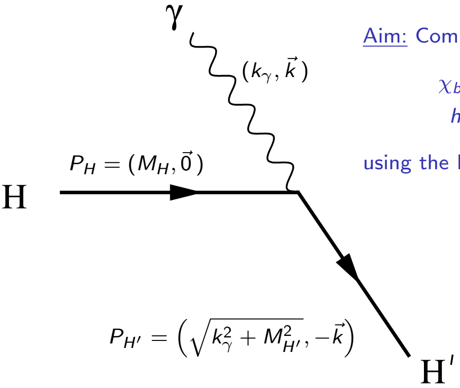
- $V_{n,0}$ are the potentials in the Schrödinger equation.
- $V_{n,k\neq0}$ are the couplings with the low-energy degrees of freedom, which provide corrections to the potential picture.

In summary...

- Provides a QM description from FT: the matching coefficients are the interaction potentials and the leading order dynamical equation is of the Schrödinger type.
- The degrees of freedom in pNRQCD (at weak coupling) are color singlet and octet fields and ultrasoft gluon fields.
- Account for non-potential terms as well. Singlet to Octet transitions via ultrasoft gluons provide loop corrections to the leading potential picture.
- The Quantum Mechanical divergences are canceled by the NRQCD matching coefficients.
- Poincaré invariance is realized via exact relations between different matching coefficients.

**Potential nonrelativistic QCD is the state-of-the-art tool for addressing Quarkonium bound state properties**

- Conventional meson spectrum: higher order perturbative corrections in $v$ and $\alpha_s$.
- Inclusive and semi-inclusive decays, E1 and M1 transitions, EM line-shapes.
- Doubly- and triply-heavy baryons.
- Precise extraction of Standard Model parameters: $m_c$, $m_b$, $\alpha_s$, ...
- Exotic states such as gluelumps and hybrids.
- Properties of Quarkonium systems at finite temperature.




Aim: Compute the electric dipole (E1) transitions:

$$\chi_{bJ}(1P) \to \gamma\Upsilon(1S) \text{ with } J = 0, 1, 2$$
$$h_b(1P) \to \gamma\eta_b(1S)$$

using the EFT called potential nonrelativistic QCD.

- Electromagnetic transitions are often significant decay modes of heavy quarkonium states that are below the open-flavour threshold.
- They can be classified in a series of electric and magnetic multipoles. The electric dipole (E1) and the magnetic dipole (M1) are the most important ones.
- Large set of accurate experimental data taken by B-factories, $\tau$-charm facilities and proton-proton colliders ask for a systematic and model-independent analysis.

☞ The LO decay width, which scales as $\sim k_\gamma^3/(mv)^2$, is

$$\Gamma^{(0)}_{\rm E1} = \frac{4}{9}\,\alpha_{\rm em}\,e_Q^2\,k_\gamma^3\left[I_3^{(0)}(n1\to n'0)\right]^2$$

☞ A probe of the internal structure of hadrons:

$$I_N^{(k)}(n\ell\to n'\ell') = \int_0^\infty dr\, r^N R^*_{n'\ell'}(r)\left[\frac{d^k}{dr^k}R_{n\ell}(r)\right]$$

☞ Up to order $k_\gamma^3/m^2$, the expressions we use for the decay rates under study are:

$$\Gamma(n\,^3P_J\to n'\,^3S_1+\gamma) = \Gamma^{(0)}_{E1}\left\{1+R^{S=1}(J)-\frac{k_\gamma}{6m}-\frac{k_\gamma^2}{60}\frac{I_5^{(0)}(n1\to n'0)}{I_3^{(0)}(n1\to n'0)}\right.$$
$$\left.+\left[\frac{J(J+1)}{2}-2\right]\left[-\left(1+\kappa_Q^{\rm em}\right)\frac{k_\gamma}{2m}+\frac{1}{m^2}\left(1+2\kappa_Q^{\rm em}\right)\frac{I_2^{(1)}(n1\to n'0)+2I_1^{(0)}(n1\to n'0)}{I_3^{(0)}(n1\to n'0)}\right]\right\}$$

$$\Gamma(n^1P_1\to n'\,^1S_0+\gamma) = \Gamma^{(0)}_{E1}\left\{1+R^{S=0}-\frac{k_\gamma}{6m}-\frac{k_\gamma^2}{60}\frac{I_5^{(0)}(n1\to n'0)}{I_3^{(0)}(n1\to n'0)}\right\}$$

- $R^{S=1}(J)$ and $R^{S=0}$ include the initial and final state corrections due to higher order potentials and higher order Fock states.
- The remaining corrections within the brackets are the result of taking into account $\mathcal{O}(v^2)$-suppressed electromagnetic interaction terms in the Lagrangian.
- The terms proportional to the anomalous magnetic moment, $\kappa_Q^{\rm em}$, go beyond our accuracy and are therefore not considered in the numerical analysis.

# Coulomb-like potential in the LO Hamiltonian

☞ The states are solutions of the Schrödinger equation:

$$H^{(0)}\psi^{(0)}_{n\ell m}(\vec{r}) = E^{(0)}_n \psi^{(0)}_{n\ell m}(\vec{r})$$

☞ Only the Coulomb-like term of the static potential is exactly included in the LO Hamiltonian ($C_F = 4/3$):

$$H^{(0)} = -\frac{\vec{\nabla}^2}{2m_r} + V^{(0)}_s(r) = -\frac{\vec{\nabla}^2}{2m_r} - \frac{C_F\alpha_s}{r}$$

☞ Therefore, $\psi^{(0)}_{n\ell m}(\vec{r})$ and $E^{(0)}_n$ can be written in the hydrogen-like form:

$$\psi^{(0)}_{n\ell m}(\vec{r}) = R_{n\ell}(r) Y_{\ell m}(\Omega_r) = N_{n\ell}\, \rho_n^\ell\, e^{-\frac{\rho_n}{2}}\, L^{2\ell+1}_{n-\ell-1}(\rho_n) Y_{\ell m}(\Omega_r)$$

$$E^{(0)}_n = -\frac{m_r C_F^2 \alpha_s^2}{2n^2}$$

$$\rho_n = 2r/na\,, \qquad a = 1/m_r C_F \alpha_s\,, \qquad N_{n\ell} = \sqrt{\left(\frac{2}{na}\right)^3 \frac{(n-\ell-1)!}{2n[(n+\ell)!]}}\,.$$

*These states are not eigenstates of the complete Hamiltonian due to higher order potentials and the presence of ultra-soft gluons that lead to singlet-to-octet transitions*

$\Downarrow$

**One has to consider corrections to the wave function which can contribute to the decay rate at the required order of precision**

☞ To account for $\mathcal{O}(v^2)$-corrections to the decay width, we need to consider the complete Hamiltonian:

$$H = -\frac{\vec{\nabla}^2}{2m_r} + V_s(r) + \delta H$$

☞ The static potential is:

$$V_s(r) = V_s^{(0)}(r)\left[1 + \sum_{n=1}^{\infty}\left(\frac{\alpha_s}{4\pi}\right)^n a_n(\nu, r)\right]$$

☞ The known $\mathcal{O}(\alpha_s^n)$ radiative corrections to the LO static potential are:

$$a_1(\nu, r) = a_1 + 2\beta_0 \ln\left(\nu e^{\gamma_E} r\right)$$

$$a_2(\nu, r) = a_2 + \frac{\pi^2}{3}\beta_0^2 + (4a_1\beta_0 + 2\beta_1)\ln\left(\nu e^{\gamma_E} r\right) + 4\beta_0^2 \ln^2\left(\nu e^{\gamma_E} r\right)$$

$$\begin{aligned} a_3(\nu, r) = {} & a_3 + a_1\beta_0^2\pi^2 + \frac{5\pi^2}{6}\beta_0\beta_1 + 16\zeta_3\beta_0^3 \\ & + \left(2\pi^2\beta_0^3 + 6a_2\beta_0 + 4a_1\beta_1 + 2\beta_2 + \frac{16}{3}C_A^3\pi^2\right)\ln\left(\nu e^{\gamma_E} r\right) \\ & + \left(12a_1\beta_0^2 + 10\beta_0\beta_1\right)\ln^2\left(\nu e^{\gamma_E} r\right) + 8\beta_0^3\ln^3\left(\nu e^{\gamma_E} r\right) \\ & + \delta a_3^{us}(\nu, \nu_{us}) \,. \end{aligned}$$

☞ The term $\delta H$ encodes the relativistic corrections to the static potential and to the nonrelativistic kinetic operator:

$$\delta H = -\frac{\vec{\nabla}^4}{4m^3} + \frac{V^{(1)}}{m} + \frac{V^{(2)}_{\rm SI}}{m^2} + \frac{V^{(2)}_{\rm SD}}{m^2}$$

☞ At order $1/m^2$, we can split the contributions into spin-independent (SI) and spin-dependent (SD) terms:

$$V^{(2)}_{\rm SI}(r) = V^{(2)}_{r}(r) + \frac{1}{2}\{V^{(2)}_{p^2}(r), -\vec{\nabla}^2\} + V^{(2)}_{L^2}(r)\,\vec{L}^2$$
$$V^{(2)}_{\rm SD}(r) = V^{(2)}_{LS}(r)\,\vec{L}\cdot\vec{S} + V^{(2)}_{S^2}(r)\,\vec{S}^2 + V^{(2)}_{S_{12}}(r)\,S_{12}$$

☞ In the weak-coupling case, the above potentials read at leading (non-vanishing) order in perturbation theory:

$$V^{(1)}(r) = -\frac{C_F C_A \alpha_s^2}{2r^2}\,, \qquad V^{(2)}_{r}(r) = \pi C_F \alpha_s \delta^{(3)}(\vec{r})\,,$$
$$V^{(2)}_{p^2}(r) = -\frac{C_F\alpha_s}{r}\,, \qquad V^{(2)}_{L^2}(r) = \frac{C_F\alpha_s}{2r^3}\,,$$
$$V^{(2)}_{LS}(r) = \frac{3C_F\alpha_s}{2r^3}\,, \qquad V^{(2)}_{S^2}(r) = \frac{4\pi C_F\alpha_s}{3}\delta^{(3)}(\vec{r})\,,$$
$$V^{(2)}_{S_{12}}(r) = \frac{C_F\alpha_s}{4r^3}\,.$$

- First order correction to the wave function ($|n\ell\rangle^{(0)} \equiv |n\ell\rangle$ and $E_n^{(0)} \equiv E_n$):

$$|n\ell\rangle^{(1)} = \sum_{n'\neq n} \frac{\langle n'\ell|V|n\ell\rangle}{E_n - E_{n'}} |n'\ell\rangle$$

- Second order correction to the wave function:

$$|n\ell\rangle^{(2)} = \sum_{k_1\neq n} \left[ \sum_{k_2\neq n} \frac{\langle k_1\ell|V|k_2\ell\rangle\langle k_2\ell|V|n\ell\rangle}{(E_n - E_{k_1})(E_n - E_{k_2})} - \frac{\langle k_1\ell|V|n\ell\rangle\langle n\ell|V|n\ell\rangle}{(E_n - E_{k_1})^2} \right] |k_1\ell\rangle - \frac{1}{2} \sum_{k_2\neq n} \frac{|\langle k_2\ell|V|n\ell\rangle|^2}{(E_n - E_{k_2})^2} |n\ell\rangle$$

- Identification with the Coulomb Green function:

$$\sum_{n'} \frac{|n'\ell\rangle\langle n'\ell|}{E_n - E_{n'}} - \sum_{n'=n} \frac{|n'\ell\rangle\langle n'\ell|}{E_n - E_{n'}} = (-1) \times \lim_{E\to E_n} \left( \frac{1}{E - H} - \frac{P_{n\ell}}{E - E_n} \right)$$

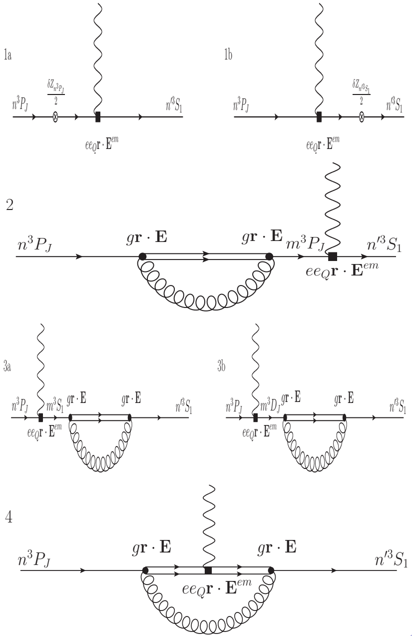
- Definition of the Coulomb Green function:

$$G(\vec{r}_1, \vec{r}_2; E) = \sum_{\ell=0}^{\infty} \frac{2\ell+1}{4\pi} P_\ell(\hat{r}_1 \cdot \hat{r}_2) G_\ell(r_1, r_2; E)$$

$$G_\ell(r_1, r_2; E) = \sum_{\nu=\ell+1}^{\infty} m_r a^2 \left( \frac{\nu^4}{\lambda} \right) \frac{R_{\nu\ell}(\rho_{\lambda,1}) R_{\nu\ell}(\rho_{\lambda,2})}{\nu - \lambda}$$

# Corrections due to higher order Fock states

- Diagrams 1a and 1b correspond to the renormalization of the initial and final wave functions.
- Diagrams 2, 3a and 3b account for the correction of the initial and final wave functions due to the presence of octet states.
- Diagram 4 represents an electric dipole transition mediated by the intermediate octet state.
- The first two diagrams contribute to relative order $\Lambda_{\rm QCD}^2/(mv)^2$ whereas the remaining ones scales as $\Lambda_{\rm QCD}^3/(m^3v^4)$.
- We do not consider these contributions herein because in the (strict) weak-coupling regime, $E \sim mv^2 \gg \Lambda_{\rm QCD}$, one can argue that they should be negligible.

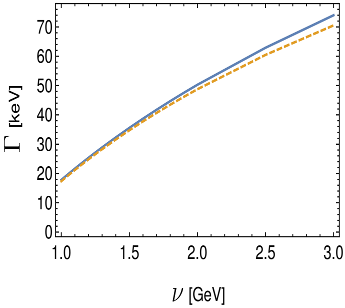



N. Brambilla *et al.*, Phys. Rev. D85 (2012) 094005.

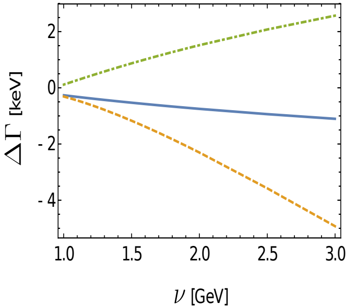
$$\Gamma_{n^3P_J \to n'^3S_1} = \Gamma^{(0)}_{E_1}\Bigg[1 + R^{S=1}(J) - \frac{k_\gamma}{6m} - \frac{k_\gamma^2}{60}\frac{I_5^{(0)}(n1 \to n'0)}{I_3^{(0)}(n1 \to n'0)}$$

$$+ \left(\frac{J(J+1)}{2} - 2\right)\left(-(1+\cancel{\kappa_Q^{\rm em}})\frac{k_\gamma}{2m} + \frac{1}{m^2}(1+\cancel{2\kappa_Q^{\rm em}})\frac{I_2^{(1)}(n1 \to n'0) + 2I_1^{(0)}(n1 \to n'0)}{I_3^{(0)}(n1 \to n'0)}\right)\Bigg]$$

Combined effect



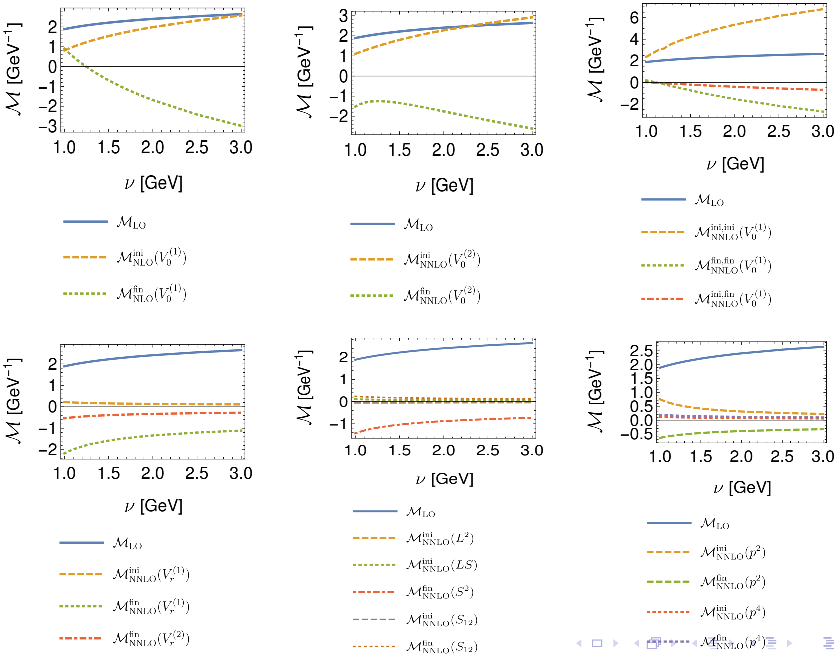

Individual contributions

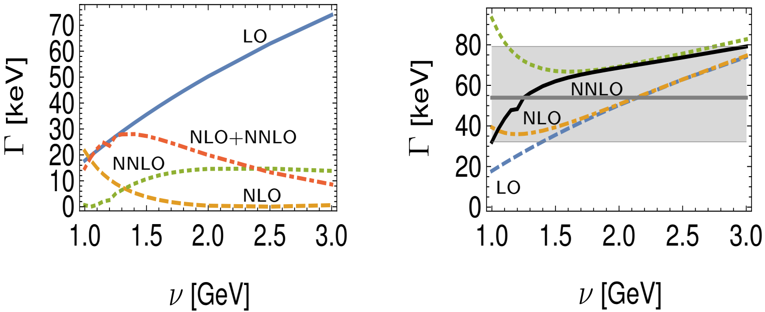



☞ Observations:

- The leading order decay width depends strongly on the typical scale $\nu$ of the heavy quarkonia involved in the reactions.
- The relativistic effects from higher order electromagnetic terms are small, as expected. This can be understood analyzing each contribution separately.

# Wave function corrections $[\chi_{b1}(1P) \to \gamma\Upsilon(1S)]$






☞ Observations:

- LO, NLO and NNLO contributions depend strongly on the renormalization scale $\nu$: running of $\alpha_s(\nu)$ and radiative terms ($\propto$ logs) of the static potential.
- Subleading contributions are of the same order of magnitude than the leading order term.
- The renormalization scale dependence of the decay width is slightly reduced as the NLO and NNLO corrections are included.
- Setting the terms proportional to $a_1(\nu, r)$ and $a_2(\nu, r)$ to zero, the decay width exhibits a different $\nu$-dependence (dotted green curve in the right panel).

# Static potential exactly included in the LO Hamiltonian

**Improving convergence:** *The perturbative expansion in pNRQCD can be rearranged in such a way that the static potential is exactly included in the LO Hamiltonian*

☞ The new expansion scheme was applied in:

- Y. Kiyo *et al.*, Nucl. Phys. B841 (2010) 231, for studying inclusive EM decays of heavy quarkonium.
- A. Pineda *et al.*, Phys. Rev. D87 (2013) 074024 for computing magnetic dipole transitions between low-lying heavy quarkonia.

☞ The exact treatment of the soft logarithms of the static potential made the factorization scale dependence much smaller.

☞ We proceed herein to apply the same formalism to the electric dipole transitions under study

$$H^{(0)}_{\rm new} = -\frac{\vec{\nabla}^2}{2m_r} + V_s^{(N)}(r)$$

where the static potential is again approximated by a polynomial of order $N+1$ in powers of $\alpha_s$

$$V_s^{(N)}(r) = -\frac{C_F\alpha_s(\nu)}{r}\left[1+\sum_{k=1}^{N}\left(\frac{\alpha_s}{4\pi}\right)^k a_k(\nu,r)\right]$$

☞ One first makes the substitution

$$(m, V_s(r)) = (m_X + \delta m_X, V_{s,X}(r) - 2\,\delta m_X)$$

where

$$\delta m_X^{(N)}(\nu_f) = \nu_f \sum_{k=0}^{N} \delta m_X^{(k)}\left(\frac{\nu_f}{\nu}\right) \alpha_s^{k+1}(\nu)$$

represents a residual mass that encodes the pole mass renormalon contribution, and $X$ stands for the specific renormalon subtraction scheme.

☞ We mainly use the RS' scheme:

$$\delta m_{\rm RS'}^{(0)} = 0$$

$$\delta m_{\rm RS'}^{(1)}\left(\frac{\nu_f}{\nu}\right) = N_m \frac{\beta_0}{2\pi} S(1,b)$$

$$\delta m_{\rm RS'}^{(2)}\left(\frac{\nu_f}{\nu}\right) = N_m \left(\frac{\beta_0}{2\pi}\right)\left[S(1,b)\frac{2d_0(\nu,\nu_f)}{\pi} + \left(\frac{\beta_0}{2\pi}\right) S(2,b)\right]$$

$$\delta m_{\rm RS'}^{(3)}\left(\frac{\nu_f}{\nu}\right) = N_m \left(\frac{\beta_0}{2\pi}\right)\left[S(1,b)\frac{3d_0^2(\nu,\nu_f) + 2d_1(\nu,\nu_f)}{\pi^2} + \left(\frac{\beta_0}{2\pi}\right) S(2,b)\frac{3d_0(\nu,\nu_f)}{\pi} + \left(\frac{\beta_0}{2\pi}\right)^2 S(3,b)\right]$$

where

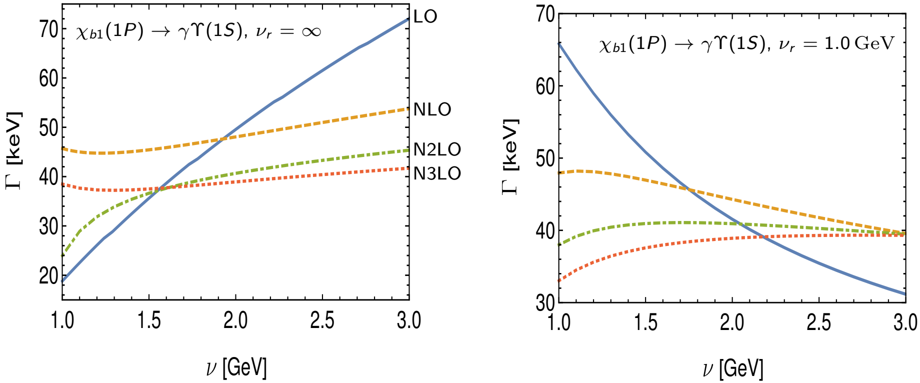
$$d_k(\nu,\nu_f) = \beta_k/2^{1+2k}\ln\frac{\nu}{\nu_f}\,, \qquad S(n,b) = \sum_{k=0}^{2} c_k \frac{\Gamma(n+1+b-k)}{\Gamma(1+b-k)}\,,$$

$b$, $c_0$, $c_1$ and $c_2$ are coefficients that only depend on the $\beta$'s and $N_m$ is a constant.

*The last improvement to the static potential that we consider herein is the absorption of the large logarithms into the running coupling constant.*

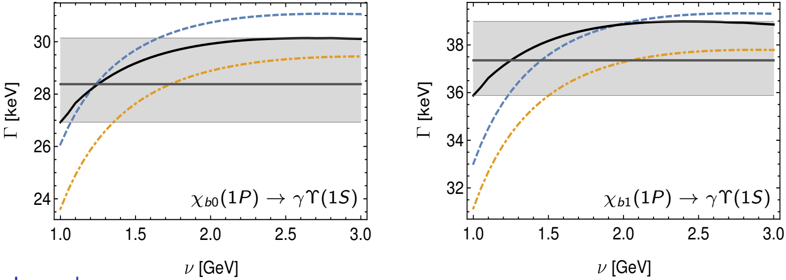
$$V_{s,\mathrm{RS}'}^{(N)}(\nu,\nu_f,\nu_r,r) = \begin{cases} V_s^{(N)} + 2\delta m_{\mathrm{RS}'}^{(N)}\big|_{\nu=\nu} \equiv \sum_{k=0}^{N} V_{s,\mathrm{RS}'}^{(k)} \alpha_s^{k+1}(\nu) & \text{if } r > \nu_r^{-1}\,, \\ V_s^{(N)} + 2\delta m_{\mathrm{RS}'}^{(N)}\big|_{\nu=1/r} \equiv \sum_{k=0}^{N} V_{s,\mathrm{RS}'}^{(k)} \alpha_s^{k+1}(1/r) & \text{if } r < \nu_r^{-1}\,, \end{cases}$$

- This has to be done carefully in order not to destroy the renormalon cancellation achieved order by order in $\alpha_s$.
- Large values of $\nu_f$ imply a large infrared cutoff. This makes our scheme become closer to a $\overline{\mathrm{MS}}$-like scheme. Such schemes still achieve renormalon cancellation but jeopardize the power counting.
- Low values of $\nu_f$ are preferred with the constraint that one should still obtain the renormalon cancellation.
- By also taking a low value of $\nu_r = \nu_f$ we find that the convergence is accelerated and the scale dependence is significantly reduced.




☞ Observations:

- One gets a strong dependence on the factorization scale when solving the Schrödinger equation with only the Coulomb-like term of the static potential.
- The $\nu$-scale dependence becomes mild as NLO, NNLO and NNNLO terms of the static potential are included in the Schrödinger equation.
- The convergence of the perturbative series has improved considerably with respect the purely analytical case. This feature is clearly seen at large values of $\nu$.
- The convergence is seen in the whole range of $\nu$ once the correct log-arithmetically modulated short distance behavior of the static potential is



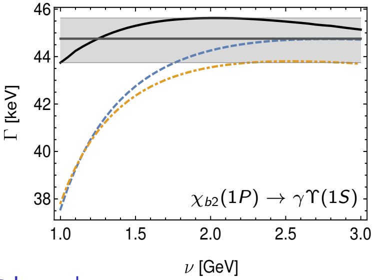

☞ Legend:

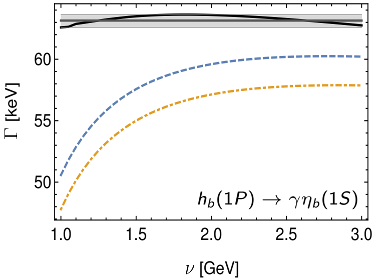
- *Dashed blue curve:* leading order non-relativistic decay rate.
- *Dot-dashed orange curve:* taking into account the relativistic contributions stemming from higher order electromagnetic operators.
- *Solid black:* including the relativistic corrections to the wave function of the initial and final states.

☞ Observations:

- The leading order decay width depends weakly on $\nu$, this feature is translated to the cases in which relativistic corrections are included.
- Both relativistic contributions to the leading order decay rate are much more under control than in the purely analytical case.
- Higher order EM operators tend to diminish the LO decay rate whereas the opposite effect is found when corrections to the wave functions are incorporated.

# Improved results (II)




☞ Legend:

- *Dashed blue curve:* leading order non-relativistic decay rate.
- *Dot-dashed orange curve:* taking into account the relativistic contributions stemming from higher order electromagnetic operators.
- *Solid black:* including the relativistic corrections to the wave function of the initial and final states.

☞ Observations:

- Similar conclusions to the ones already mentioned in the former slide apply here.
- The relativistic corrections stemming from higher order EM operators are clearly smaller whereas the effect due to corrected wave functions is 2-3 times bigger.
- The correction to the decay rate due to the modification of the initial and final state wave functions is much larger for the $h_b(1P) \to \gamma\Upsilon(1S)$ transition.

# Improved results (III)

☞ Final values for the decay widths:

$$\Gamma(\chi_{b0}(1P) \to \gamma\Upsilon(1S)) = 28^{+2}_{-1}\ \text{keV}\,,$$
$$\Gamma(\chi_{b1}(1P) \to \gamma\Upsilon(1S)) = 37^{+2}_{-1}\ \text{keV}\,,$$
$$\Gamma(\chi_{b2}(1P) \to \gamma\Upsilon(1S)) = 45^{+1}_{-1}\ \text{keV}\,,$$
$$\Gamma(h_b(1P) \to \gamma\eta_b(1S)) = 63^{+1}_{-1}\ \text{keV}\,.$$

☞ Comparison with several other theoretical approaches:

| Mode | LO | NNLO | CQM | R | GI | BT | LFQM | $SNR_{0/1}$ |
|---|---|---|---|---|---|---|---|---|
| $\chi_{b0}(1P) \to \gamma\Upsilon(1S)$ | 28.47 | 28.38 | 28.07 | 29.9 | 23.8 | 25.7 | - | - |
| $\chi_{b1}(1P) \to \gamma\Upsilon(1S)$ | 36.04 | 37.35 | 35.66 | 36.6 | 29.5 | 29.8 | - | - |
| $\chi_{b2}(1P) \to \gamma\Upsilon(1S)$ | 41.00 | 44.76 | 39.15 | 40.2 | 32.8 | 33.0 | - | - |
| $h_b(1P) \to \gamma\eta_b(1S)$ | 55.22 | 63.15 | 43.7 | 52.6 | 35.7 | - | 37.5 | 55.8/36.3 |

☞ Predicted total decay widths:

| Mode | $\mathcal{B} = \Gamma_i/\Gamma$ | $\Gamma_i$ | $\Gamma$ |
|---|---|---|---|
| $\chi_{b0}(1P) \to \gamma\Upsilon(1S)$ | $(1.76 \pm 0.35)\%$ | $28^{+2}_{-1}$ keV | $1.6^{+0.3}_{-0.3}$ MeV |
| $\chi_{b1}(1P) \to \gamma\Upsilon(1S)$ | $(33.9 \pm 2.2)\%$ | $37^{+2}_{-1}$ keV | $110^{+9}_{-8}$ keV |
| $\chi_{b2}(1P) \to \gamma\Upsilon(1S)$ | $(19.1 \pm 1.2)\%$ | $45^{+1}_{-1}$ keV | $234^{+15}_{-16}$ keV |
| $h_b(1P) \to \gamma\eta_b(1S)$ | $(52^{+6}_{-5})\%$ | $63^{+1}_{-1}$ keV | $121^{+14}_{-12}$ keV |

# Epilogue

*We have computed the electric dipole transitions $\chi_{bJ}(1P) \to \gamma\Upsilon(1S)$ with $J = 0,1,2$ and $h_b(1P) \to \gamma\eta_b(1S)$ within the weak coupling version of a low-energy effective field theory called potential non-relativistic QCD*

☞ Only the Coulomb term of the static potential is included in the LO Hamiltonian:

- The decay width reveals a severe dependence on the factorization scale $\nu$: running of $\alpha_s(\nu)$ and radiative terms ($\propto$ logs) of the static potential.
- Most of the corrections to the decay rate induced by $1/m$ and $1/m^2$ potentials are relatively small and behave well with respect the renormalization scale $\nu$.
- The general convergence of the perturbative series for all the studied electric dipole transitions is not as good as expected.

☞ The static potential is exactly included in the leading order Hamiltonian:

- The LO decay rate depends weakly on the factorization scale and this is translated to the result in which relativistic corrections are included.
- Relativistic corrections induced by higher order electromagnetic operators and wave functions are much more under control.
- Our results are in fair agreement with the ones reported by other theoretical approaches [$\Gamma(h_b(1P) \to \gamma\eta_b(1S))$ is slightly higher].
- Total decay widths of the $\chi_{bJ}(1P)$-family and the $h_b(1P)$ meson are predicted.