Institut für Theoretische Physik

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

Quark susceptibilities in a generalized quasiparticle model

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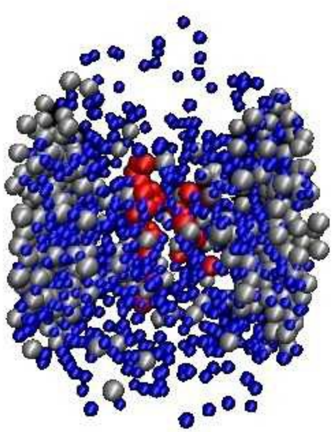
Wolfgang Cassing

FAIRNESS

Sitges, 01.06.2017



HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

H-QM | Helmholtz Research School Quark Matter Studies

PHSD

Overview

- QCD equation of state
- Thermodynamics in the quasiparticle limit
- Generalized quasiparticle model
- Extension to finite chemical potential
- Transport coefficients

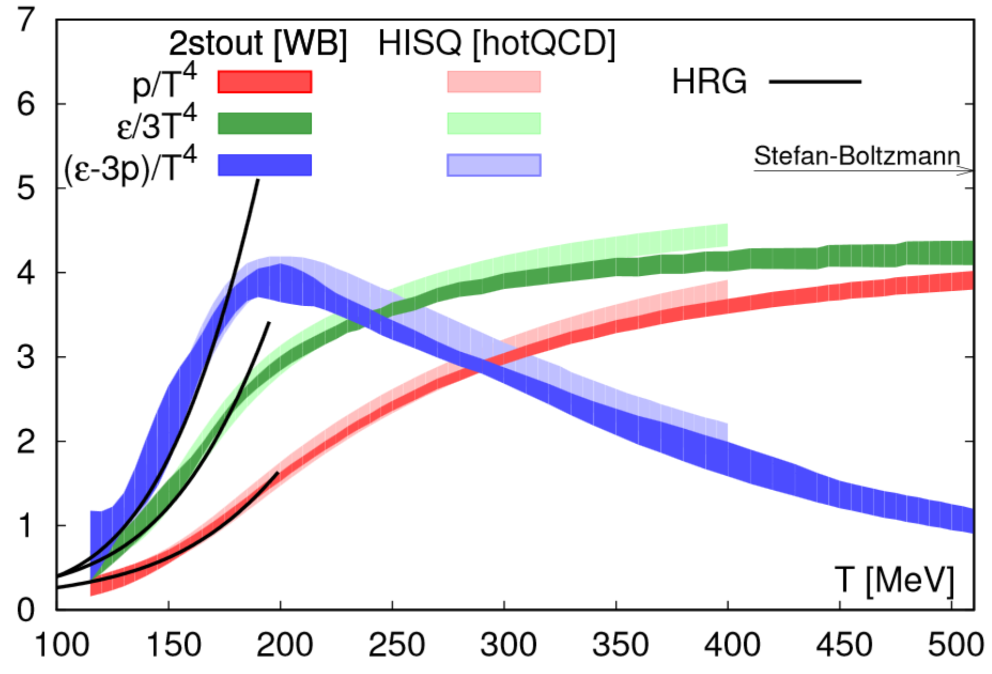
Lattice QCD

- **Different lattice EoS's have converged.**

Open problems:

- **No simulations for finite μ.**
- **No calculations out of equilibrium.**

Use effective models!



Wuppertal-Budapest: Phys. Lett. B 370 (2014) 99-104
HotQCD: Phys. Rev. D 90, 094503

Quasiparticle thermodynamics 4

- Idea: treat partons as dynamical quasiparticles.

Propagator with effective mass M and width γ:

$$G(\omega,\mathbf{p}) = \frac{-1}{\omega^2 - \mathbf{p}^2 - M^2 + 2i\gamma\omega} = \frac{-1}{\omega^2 - \mathbf{p}^2 - \Sigma}$$

$$A(\omega,\mathbf{p}) = \frac{2\gamma\omega}{(\omega^2 - \mathbf{p}^2 - M^2)^2 + 4\gamma^2\omega^2}$$

- Grand canonical potential in propagator representation:

$$\beta\Omega[D,S] = \frac{1}{2}\mathrm{Tr}[\ln D^{-1} - \Pi D] - \mathrm{Tr}[\ln S^{-1} + \Sigma S] + \Phi[D,S]$$

with selfenergies $\frac{\delta\Phi}{\delta D} = \frac{1}{2}\Pi \qquad \frac{\delta\Phi}{\delta S} = -\Sigma$

$\Phi[D,S]$ has no contribution to entropy or density.

J.P. Blaizot, E. Iancu and A. Rebhan, Phys. Rev. D 63 (2001) 065003

Quasiparticle thermodynamics 5

$$s = -\frac{1}{V}\frac{\partial \Omega}{\partial T} \qquad n = -\frac{1}{V}\frac{\partial \Omega}{\partial \mu}$$

- **Entropy and density for a given propagator D:**

$$S/V = -d \int \frac{d^3k}{(2\pi)^3}\frac{d\omega}{2\pi}\frac{\partial n_{B/F}}{\partial T}\left(\mathrm{Im}\left(\ln D^{-1}\right) - \mathrm{Re}\,(D)\,\mathrm{Im}\,(\Pi)\right)$$

$$N/V = -d \int \frac{d^3k}{(2\pi)^3}\frac{d\omega}{2\pi}\frac{\partial n_{B/F}}{\partial \mu}\left(\mathrm{Im}\left(\ln D^{-1}\right) - \mathrm{Re}\,(D)\,\mathrm{Im}\,(\Pi)\right)$$

In the on-shell limit $\gamma \to 0$ they reduce to the noninteracting entropy and density.

J.P. Blaizot, E. Iancu and A. Rebhan, Phys. Rev. D 63 (2001) 065003

Quasiparticle thermodynamics 5

$$s = -\frac{1}{V}\frac{\partial\Omega}{\partial T} \qquad n = -\frac{1}{V}\frac{\partial\Omega}{\partial\mu}$$

- **Different contributions decouple in the entropy.**

Divide the entropy in a pole- and a damping-term:

$$s^{(0)} = \frac{1}{2\pi^2}\int_0^\infty dk\ k^2 \left(\frac{\omega_k - \mu}{T} n_{B/F}(\omega_k) - S\ln\left(1 - Se^{-(\omega_k-\mu)/T}\right)\right)$$

$$\Delta s = \int_{d^4k} \frac{\partial n_{B/F}(\omega)}{\partial T}\left(2\gamma\omega\frac{\omega^2 - \mathbf{p}^2 - M^2}{(\omega^2 - \mathbf{p}^2 - M^2)^2 + 4\gamma^2\omega^2} - \arctan\left(\frac{2\gamma\omega}{\omega^2 - \mathbf{p}^2 - M^2}\right)\right)$$

The QCD-EoS follows from the entropy via grandcanonical thermodynamic relations:

$$\epsilon = sT - P + \mu n$$

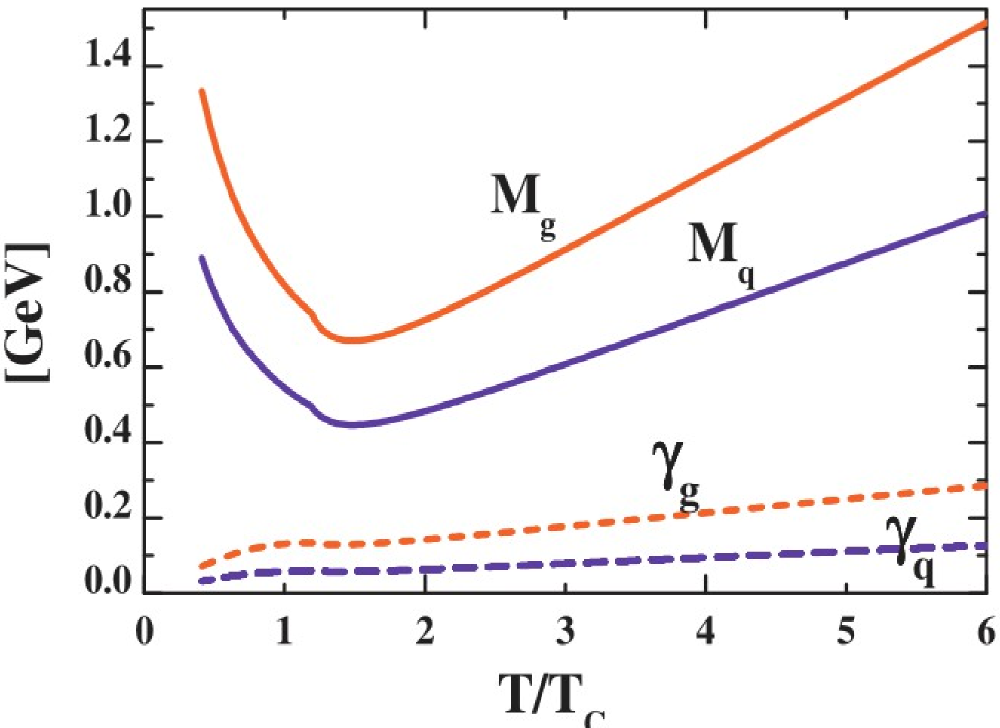
Effective mass and width

$$M_g^2 = \frac{g^2}{6}\left(\left(N_c + \frac{1}{2}N_f\right)T^2 + \frac{N_c}{2}\sum_q \frac{\mu_q^2}{\pi^2}\right)$$

$$\gamma_g = \frac{1}{3}N_c \frac{g^2 T}{8\pi}\ln\left(1 + \frac{2c}{g^2}\right)$$

$$M_{q,\bar{q}}^2 = \frac{N_c^2 - 1}{8N_c} g^2 \left(T^2 + \frac{\mu_q^2}{\pi^2}\right)$$

$$\gamma_{q,\bar{q}} = \frac{1}{3}\frac{N_c^2 - 1}{2N_c}\frac{g^2 T}{8\pi}\ln\left(1 + \frac{2c}{g^2}\right)$$



DQPM

Masses motivated by HTL

Width fixed by correlators

$$M \sim gT \qquad \gamma \sim g^2 T$$

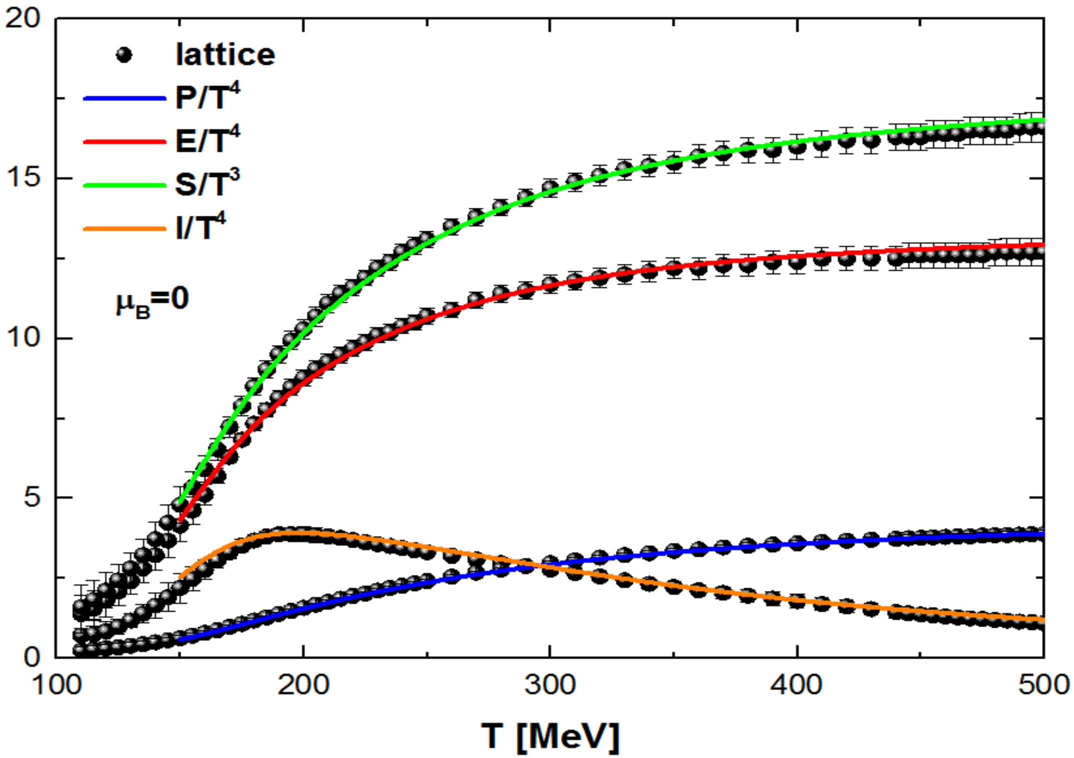
A. Peshier, W. Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

Effective coupling

- **1) Parametrisation of the effective coupling:**

$$g^2(T,T_c) = \frac{48\pi^2}{(11N_c - 2N_f)\ln(\lambda^2((T-T_s)/T_c)^2)}$$

- **2) or fit explicitly to the lattice EoS:** $g^2(S/S_{SB})$:



$$P(T) = \int_0^T S(T')dT'$$

$$E = TS - P + \mu N$$

Lattice QCD at finite μ

- **Sign problem prevents simulations for finite μ.**

$$P(T,\{\mu_i\}) = \frac{T}{V}\ln Z(T,\{\mu_i\})$$

- **Pressure is obtained via Taylor expansion:**

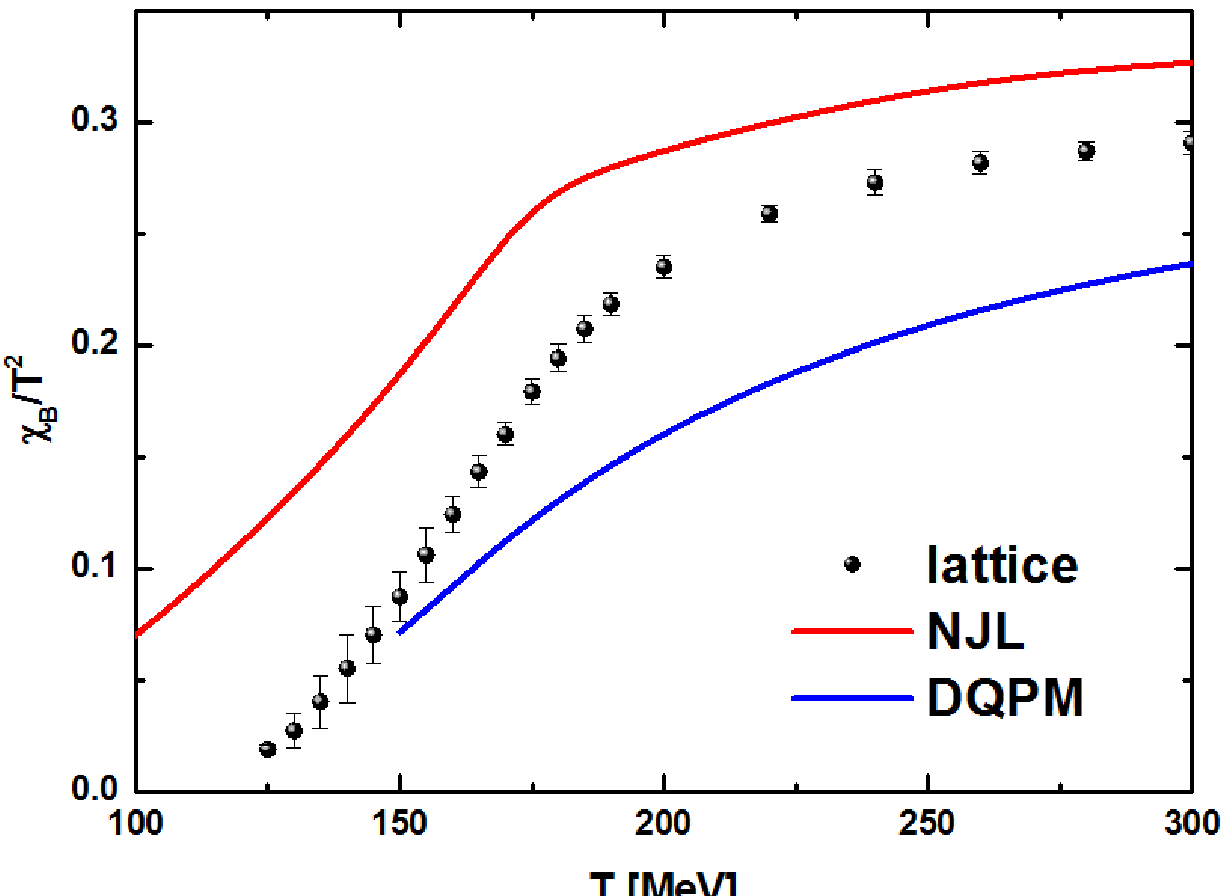
$$\frac{P(T,\{\mu_i\})}{T^4} = \frac{P(T,\{0\})}{T^4} + \frac{1}{2}\sum_{i,j}\frac{\mu_i\mu_j}{T^2}\chi_2^{ij}$$

$$\chi_2^{ij} = \frac{T}{V}\frac{1}{T^2}\frac{\partial^2 \ln Z}{\partial\mu_i\partial\mu_j}\bigg|_{\mu_i=\mu_j=0}$$

Lattice EoS at finite μ is controlled by the susceptibilities χ.

Susceptibilities

- First glimpse on finite chemical potentials.
- Contains only informations from quarks.



$$\chi_L = \frac{T}{V} \frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_L^2}\bigg|_{\mu_L=0}$$

DQPM quarks appear too heavy!

NJL quarks seem too light!

Lattice: S. Borsanyi, *et al.*, JHEP 1208 (2012) 053

„Non-perturbative“ QCD

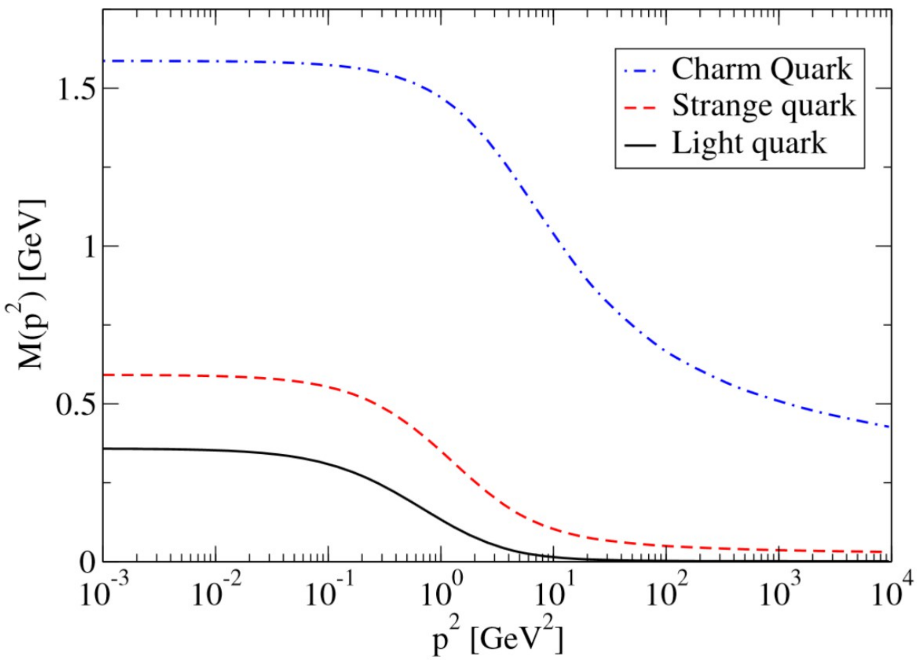
- **Heavy partons in the perturbative regime.**
- **The quark masses have to drop for higher energies to reach the perturbative limit!**

We introduce a correction factor to model this behavior:

$$h(\Lambda, p) = \frac{1}{\sqrt{1 + \Lambda \cdot p^2 \cdot (T_c/T)^2}}$$

$$M \rightarrow M(p) = M \cdot h(\Lambda, p)$$

$$\gamma \rightarrow \gamma(p) = \gamma \cdot h(\Lambda, p)$$



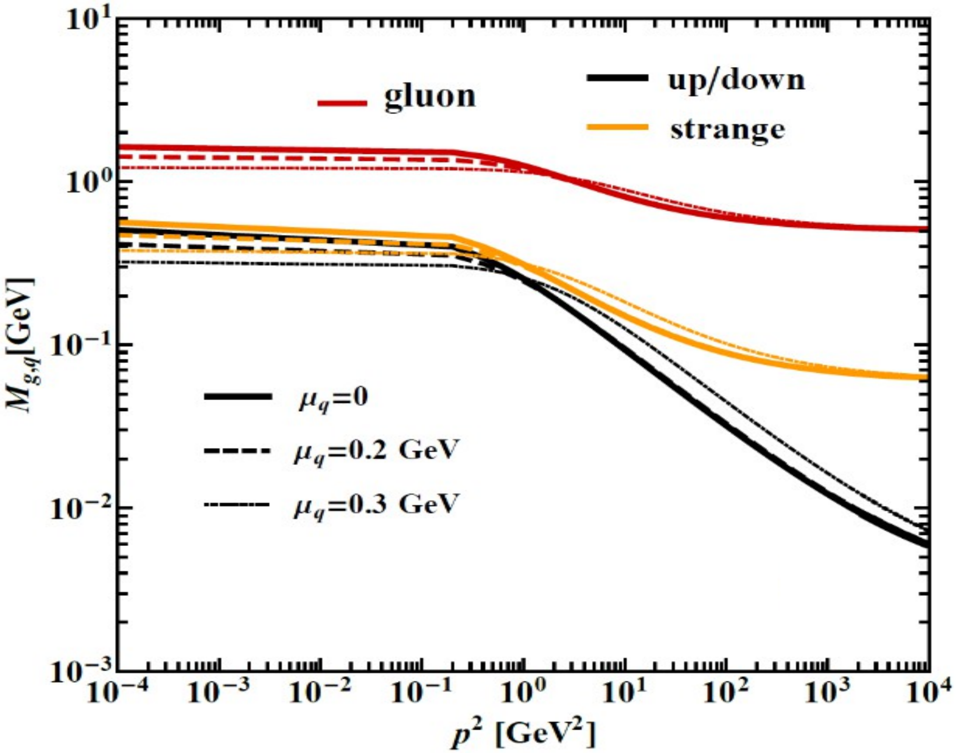
C. S. Fischer, *et al.*, Phys. Rev. D 90 (2014) 3, 034022

DQPM*

- This introduces a momentum dependence into the selfenergies:

$$\Sigma \to \Sigma(p)$$

- Propagator remains analytic in the upper half plane.

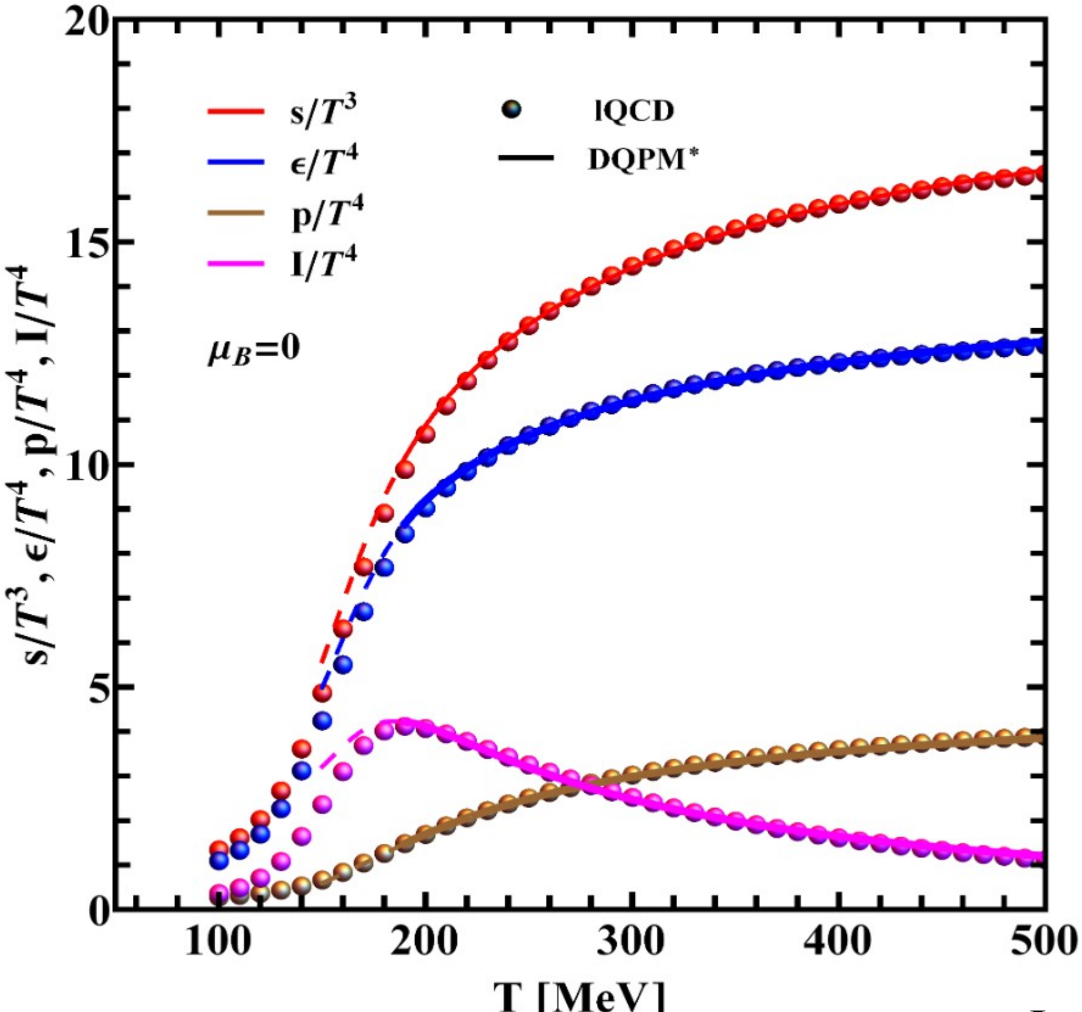


Calculate the entropy and particle density with the mom. dep. selfenergies.

This defines the generalized quasiparticle model DQPM*.

DQPM* EoS

- **EoS by thermodynamic relations:**



$$P(T) = \int_0^T S(T')dT'$$

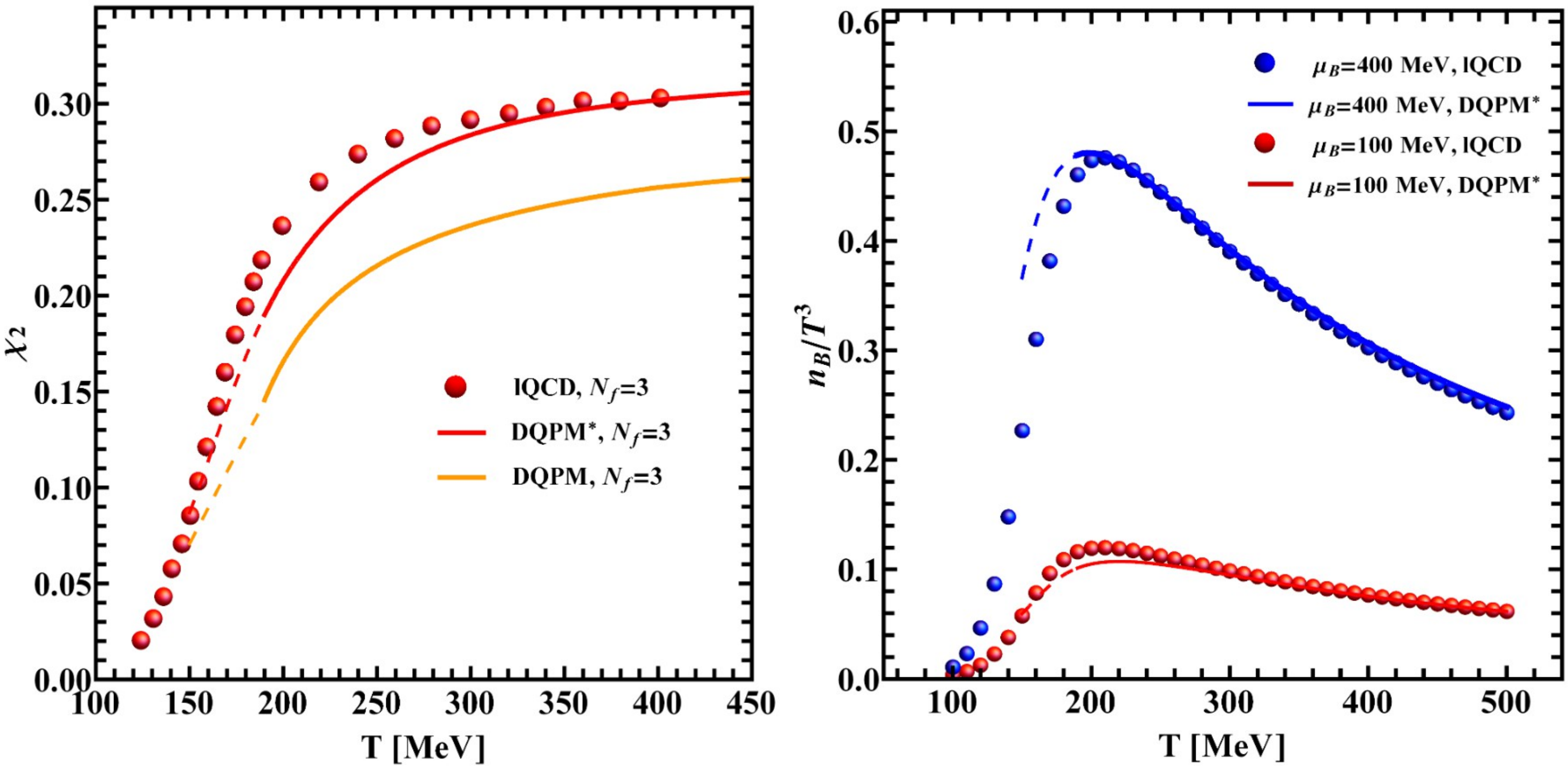
$$E = TS - P + \mu N$$

Momentum dependent DQPM* reproduces the EoS at T > 170 MeV.

Phys. Rev. C93 (2016) no. 4, 044914
Int. J. Mod. Phys. E25 (2016) no. 07, 1642003

DQPM* susceptibility

- Mom. dep. DQPM* reproduces the EoS at T>170 MeV.



- Momentum dependence drasticaly improves the susceptibility.

Phys. Rev. C93 (2016) no. 4, 044914
Int. J. Mod. Phys. E25 (2016) no. 07, 1642003

Extension to finite μ

- Entropy density and particle density are both derived from the same potential.

- They have to fulfill the Maxwell relation:

$$\left.\frac{\partial s}{\partial \mu_B}\right|_T = \left.\frac{\partial n_B}{\partial T}\right|_{\mu_B}$$

- This leads to a differential equation for the coupling g^2:

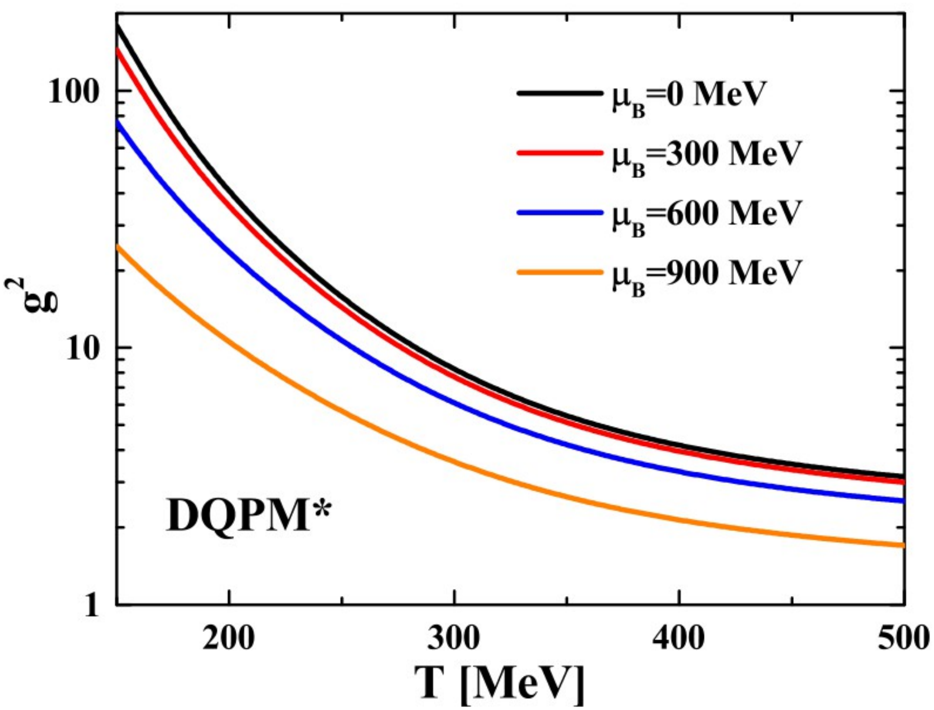
$$a_T \frac{\partial g^2}{\partial T} + a_\mu \frac{\partial g^2}{\partial \mu_B} = a_0$$

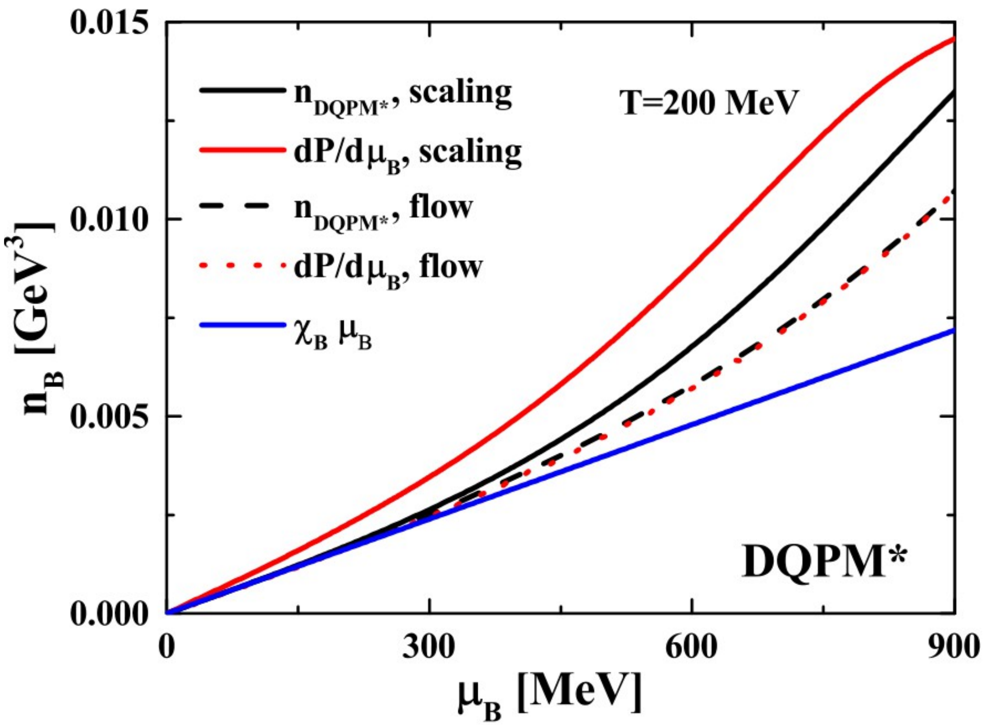
- We use $g^2(T,0)$ as initial condition for the equation.

DQPM* at finite μ

- Maxwell relation leads to a differential equation for the effective coupling, that ensures thermodynamic consistency:

$$\oint dP = 0$$

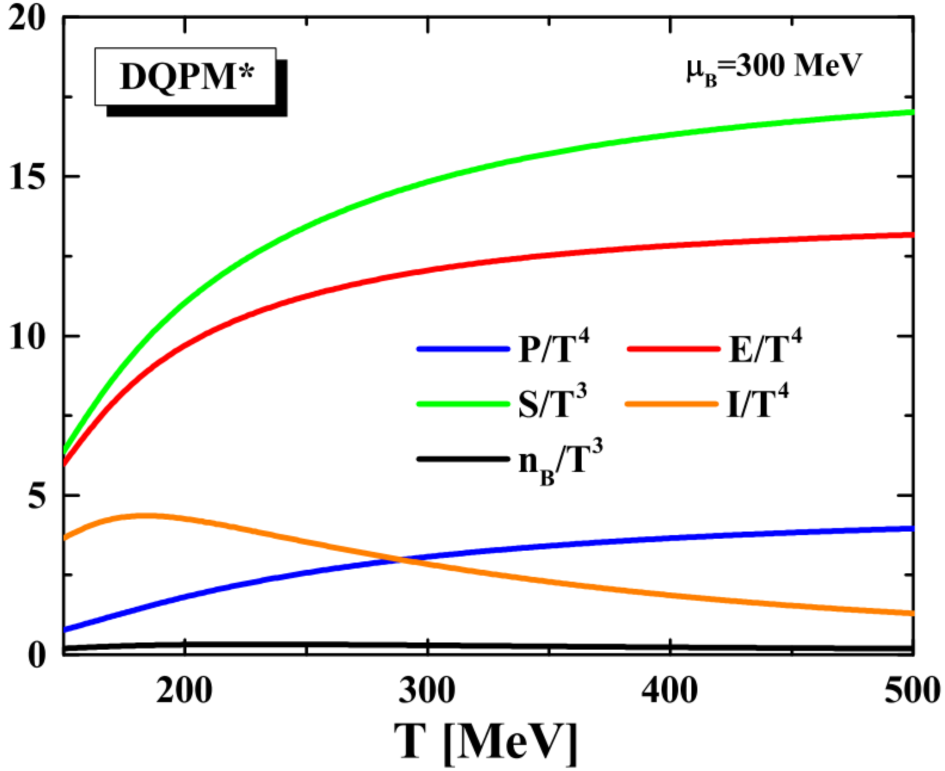


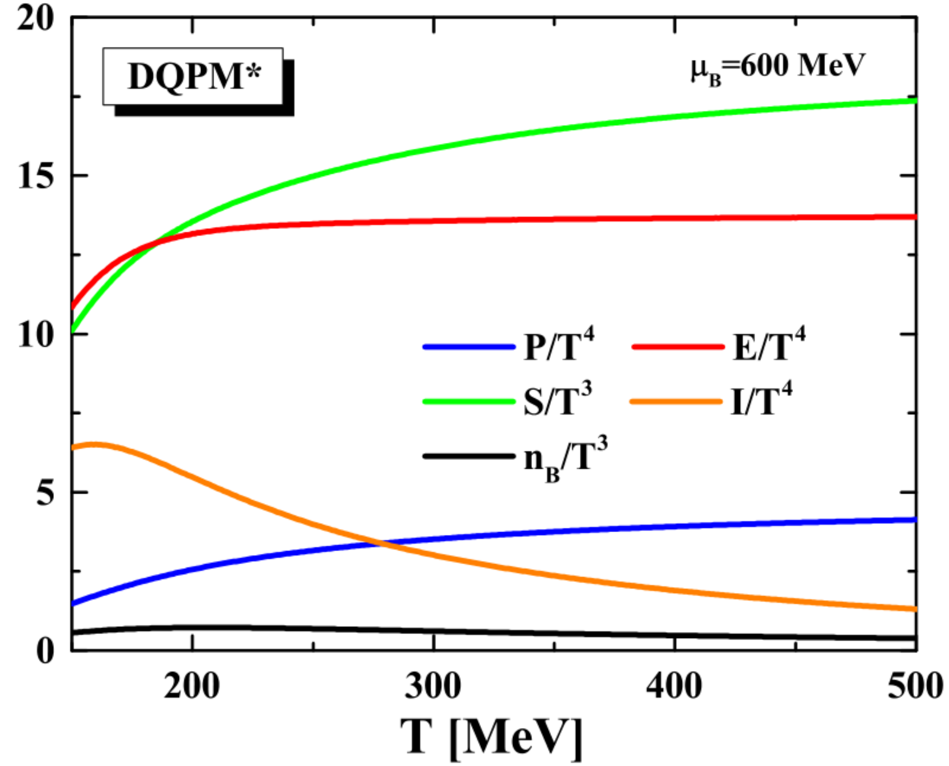


EoS at finite μ

- The effective coupling defines the EoS at arbitrary chemical potential:

$$P(T,\mu_B) = P(T,0) + \int_0^{\mu_B} n_B(T,\mu)\, d\mu$$



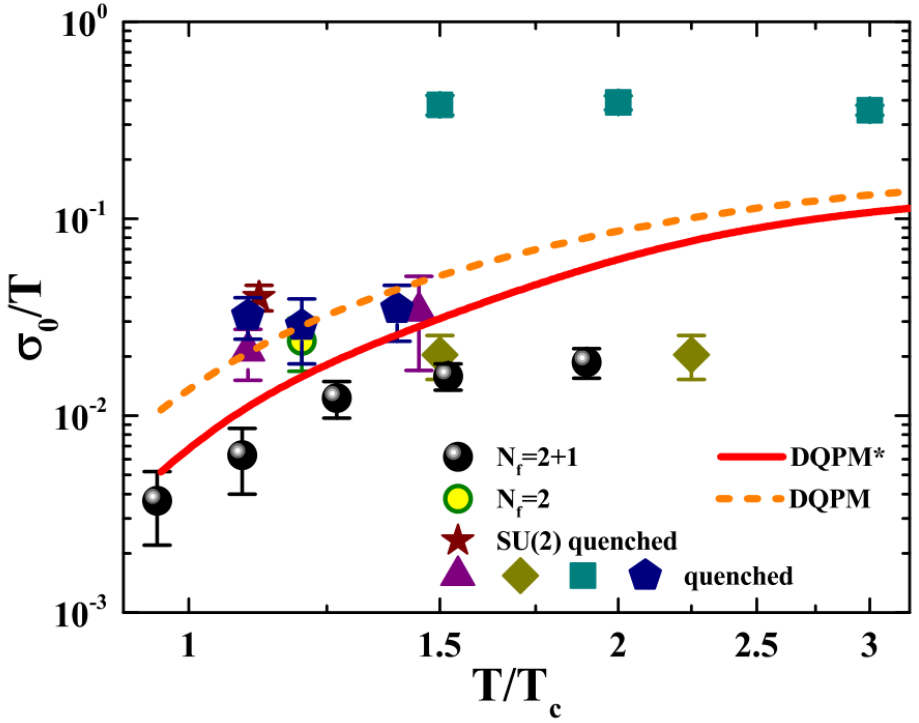


Transport coefficients

- The width so far is not well fixed by the EoS.
- Use transport coefficients

Electric conductivity in relaxation time approach:

$$\sigma_e(T,\mu_q) = \sum_{f,\bar{f}}^{u,d,s} \frac{e_f^2\; n_f^{\rm off}(T,\mu_q)}{\bar{\omega}_f(T,\mu_q)\;\underline{\bar{\gamma}_f(T,\mu_q)}}$$



Conductivity probes only the quark width γ_f.

Viscosity

- Viscosities probe the whole system, quarks and gluons!
- Separate contributions in RTA for quarks and gluons:

$$\eta = \eta_{\text{gluon}} + \eta_{\text{quarks}}$$

$$\zeta = \zeta_{\text{gluon}} + \zeta_{\text{quarks}}$$

- Contributions are proportional to the relaxation times.

$$\eta_{\text{gluon}} \sim \tau_g$$

$$\zeta_{\text{gluon}} \sim \tau_g$$

DQPM:

$$\tau_i = \frac{\hbar c}{\gamma_i}$$

Viscosity

- **Bulk visc. depends directly on the EoS.**

Speed of sound and mean field contribution

Relaxation times

$$\eta = \frac{1}{15T} \sum_{g,u,d,s} d_i \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \omega A_i(\omega, p) f_i(\omega) \Theta(\omega^2 - p^2) \boxed{\tau_i} \frac{p^4}{\omega^2}$$

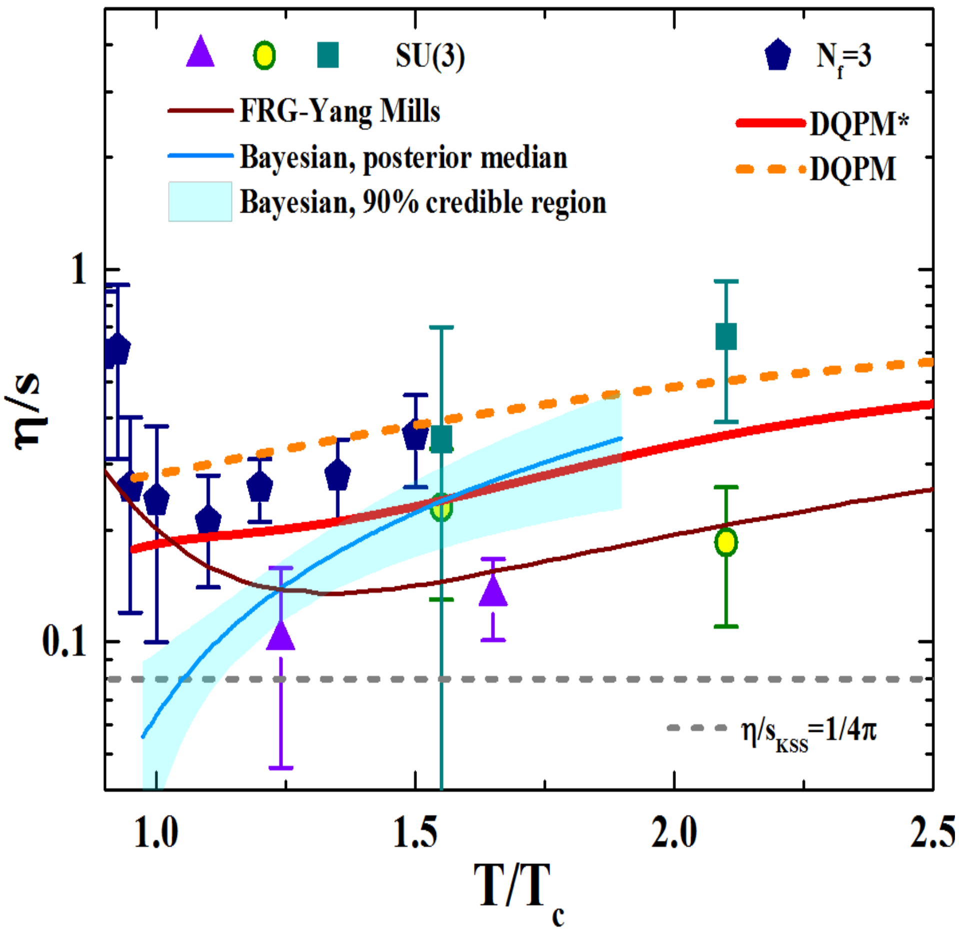
$$\zeta = \frac{1}{9T} \sum_{g,u,d,s} d_i \int \frac{d^3p}{(2\pi)^3} \int \frac{d\omega}{2\pi} \omega A_i(\omega, p) f_i(\omega) \Theta(\omega^2 - p^2) \boxed{\tau_i}$$

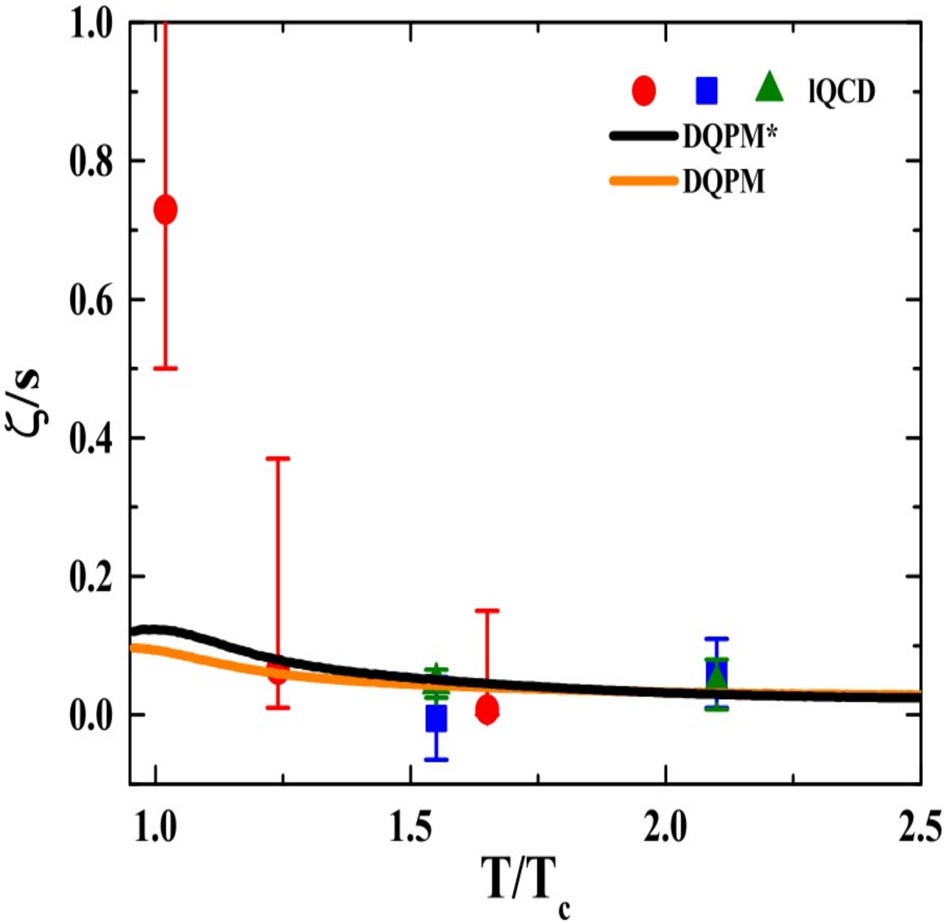
$$\boxed{\times \left[p^2 - 3c_s^2 \left(\omega^2 - T^2 \frac{dM_i^2}{dT^2} \right) \right]^2}$$

P. Chakraborty, J. Kapusta, Phys.Rev. C83 (2011) 014906

Transport coefficients

- Transport coefficients are sensitive to the width.
- Matching to lattice justifies functional form:





Transport coeff. at finite μ

- **All functions show the same behavior for small μ_B:**

$$\frac{X(T,\mu_B)}{X(T,0)} \approx 1 + c_X \left(\frac{\mu_B}{T}\right)^2$$

- **Comparison of c_X reveals scaling of the coefficients.**

We compare the transport coefficents with the quasiparticle densities:

$$n_i = \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \omega A_i(\omega,p) f_i(\omega) \Theta(\omega^2 - p^2)$$

Transportcoeff. at finite μ

- We fit the c-factor for $\mu_B < 300$ MeV:

$$\eta : c = 0.032 - 0.038 \qquad n_{g+q+\bar{q}} : c = 0.029 - 0.037$$

$$\sigma_0 : c = 0.050 - 0.057 \qquad n_{l+\bar{l}} : \quad c = 0.051 - 0.055$$

Conductivity scales with the light quarks.

Shear viscosity with the quasi particle density.

No clear scaling for bulk viscosity:

$$c_\zeta(T = 200\ \text{MeV}) \approx 0 \qquad c_\zeta(T = 500\ \text{MeV}) \approx 0.045$$

Summary

- **QCD EoS is well known from lQCD and can be reproduced by QP models with an effective mass.**

- **Susceptibilities challenge quasiparticle models**
- **Mom. dep. Selfenergies reproduce EoS + χ_B**
- **Extension to finite μ_B by Maxwell relations**
- **Width is controlled by transport coefficients**

<u>DQPM* is in line with lQCD EoS and correlators.</u>