Shear viscosity and entropy of a hadron gas

presented by Jean-Bernard Rose

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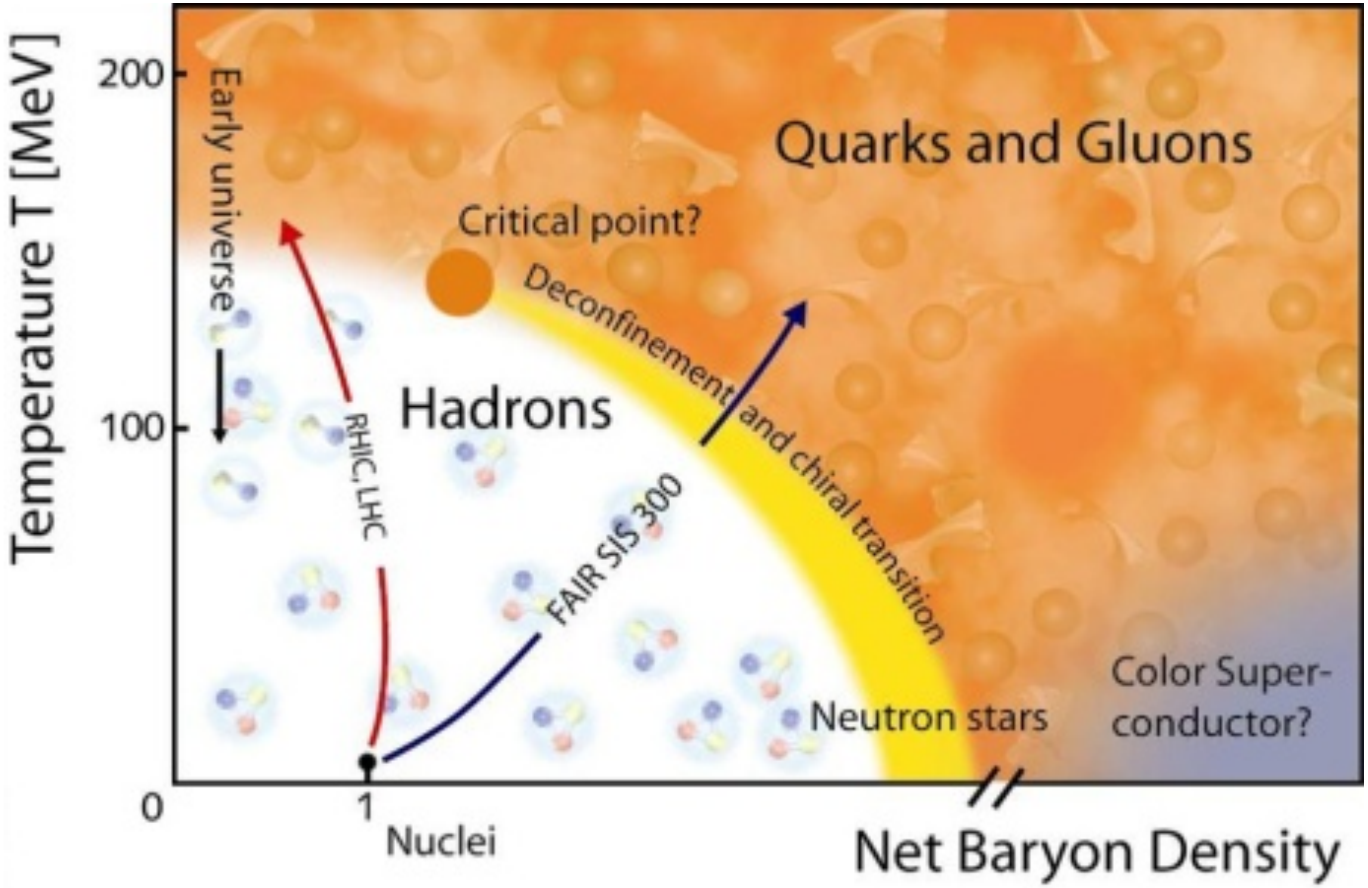
FAIRNESS Workshop, Sitges, Spain, June 1st, 2017

Outline

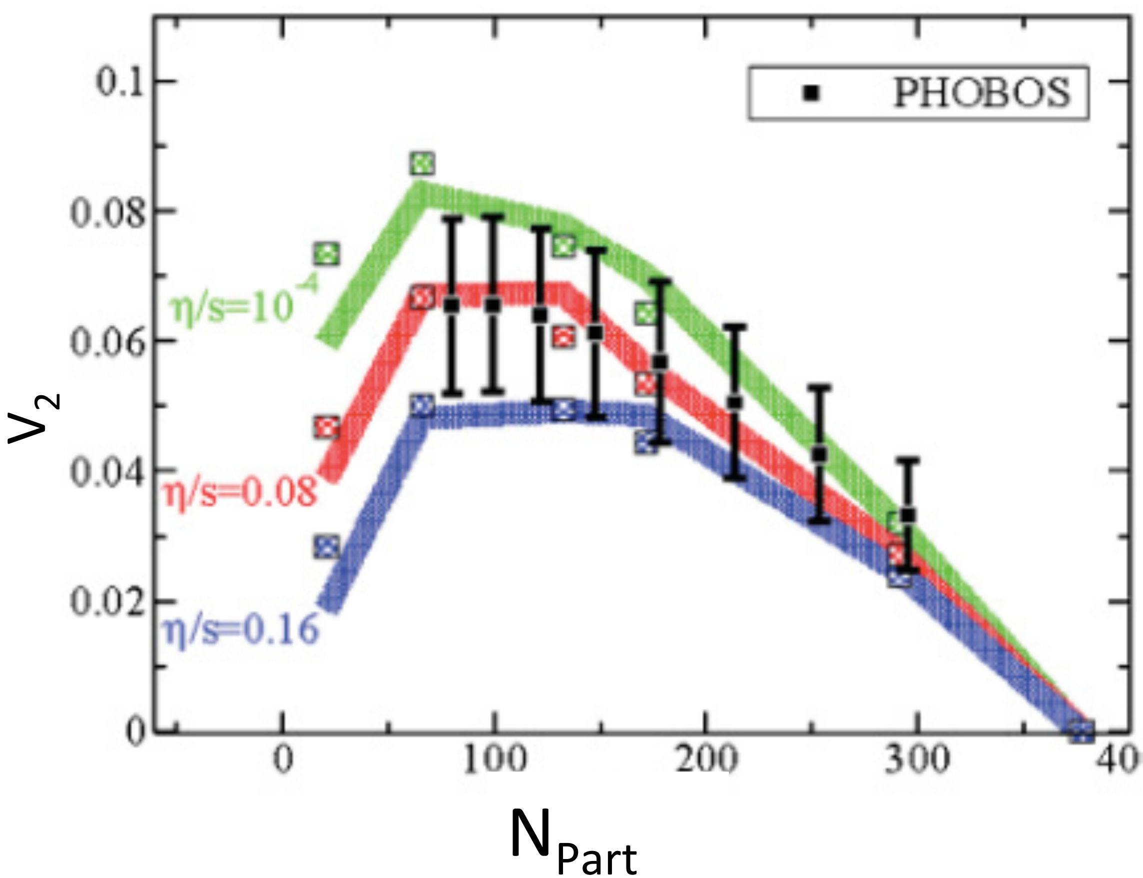
1. Intro: Viscosity of the hadron gas
2. Transport
    - SMASH
3. Viscosity considerations
    - Green-Kubo formalism
    - Test case: Pion gas
4. Entropy considerations
5. Results
    - Full hadron gas viscosity
6. Conclusion

What is the hadron gas?



Viscosity in heavy ion collisions

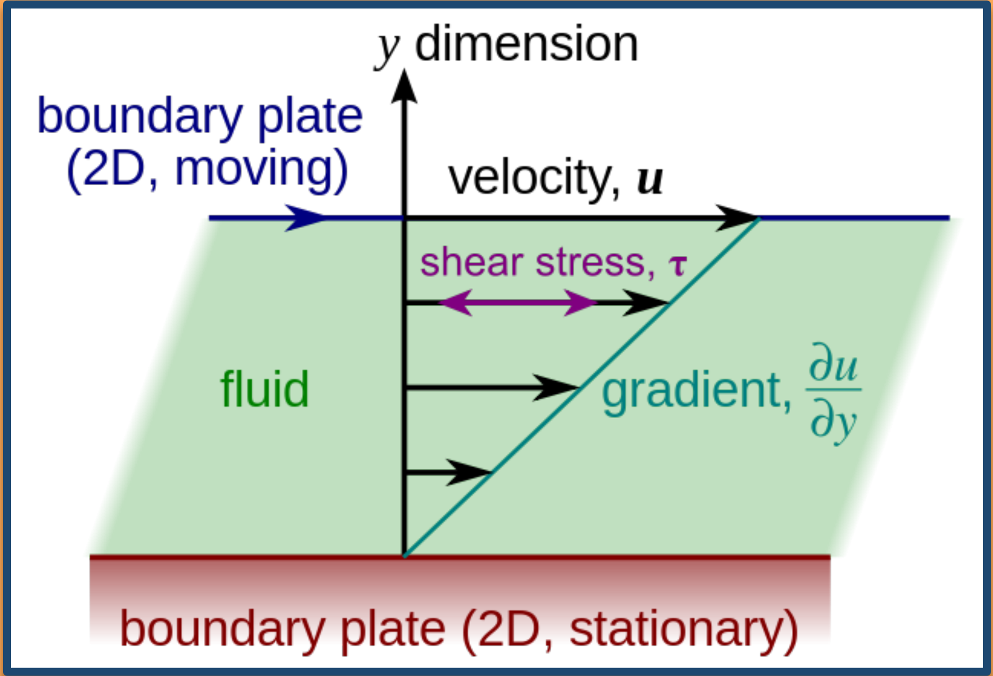
- RHIC and LHC measured large elliptic flow at the high energies corresponding to what is thought to be QGP
- Hydrodynamics relatively successful at explaining this with small *η/s*
- Still not clear what the behavior of η/s is at low energies (FAIR, late stage RHIC/LHC)



Luzum & Romatschke 10.1103/Phys. Rev. C 78.034915

What is viscosity?

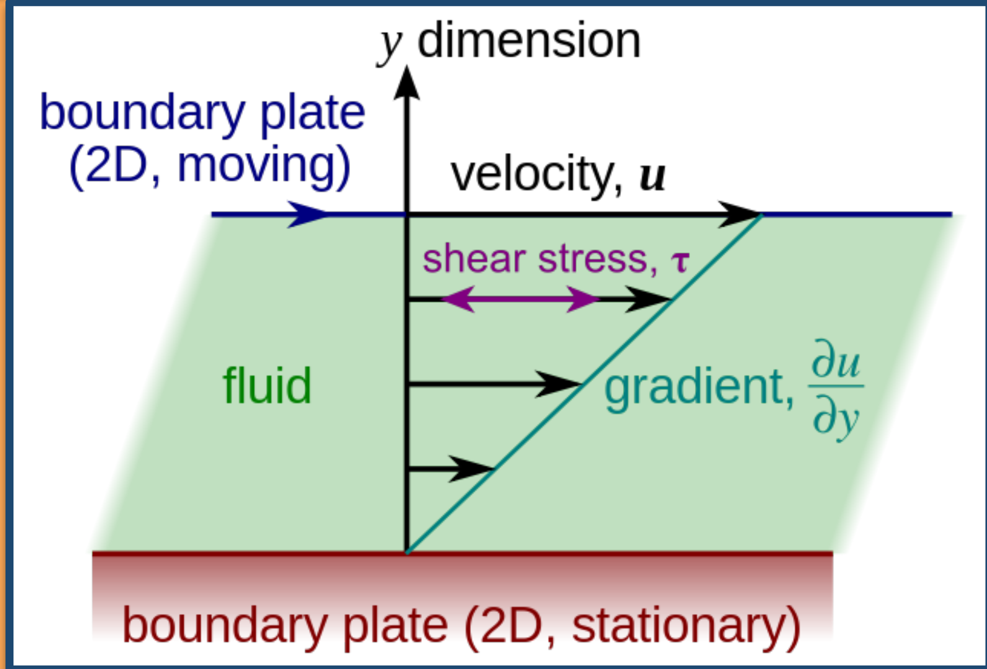
Viscosity is a measure of the friction between layers of a fluid



Wikipedia-Viscosity

...and why do we need it?

Viscosity is a measure of the friction between layers of a fluid



Wikipedia-Viscosity

- Hydrodynamics is conservation laws:

$$\boxed{\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N^\mu = 0}$$

- With 1st order dissipative corrections (Navier-Stokes):

$$\boxed{T^{\mu\nu} = eu^\mu u^\nu - (p + \zeta\theta)\Delta^{\mu\nu} + 2\eta\sigma^{\mu\nu}, \quad N^\mu = nu^\mu + \varkappa\partial^\mu \frac{\mu}{T}}$$

...and why do we need it?

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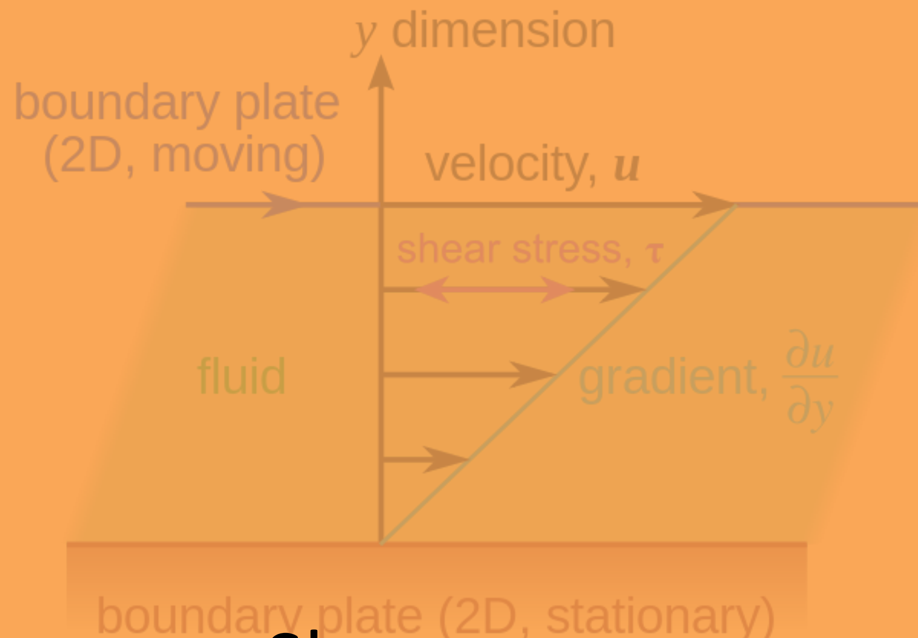

Bulk

Shear

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$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N^\mu = 0$$

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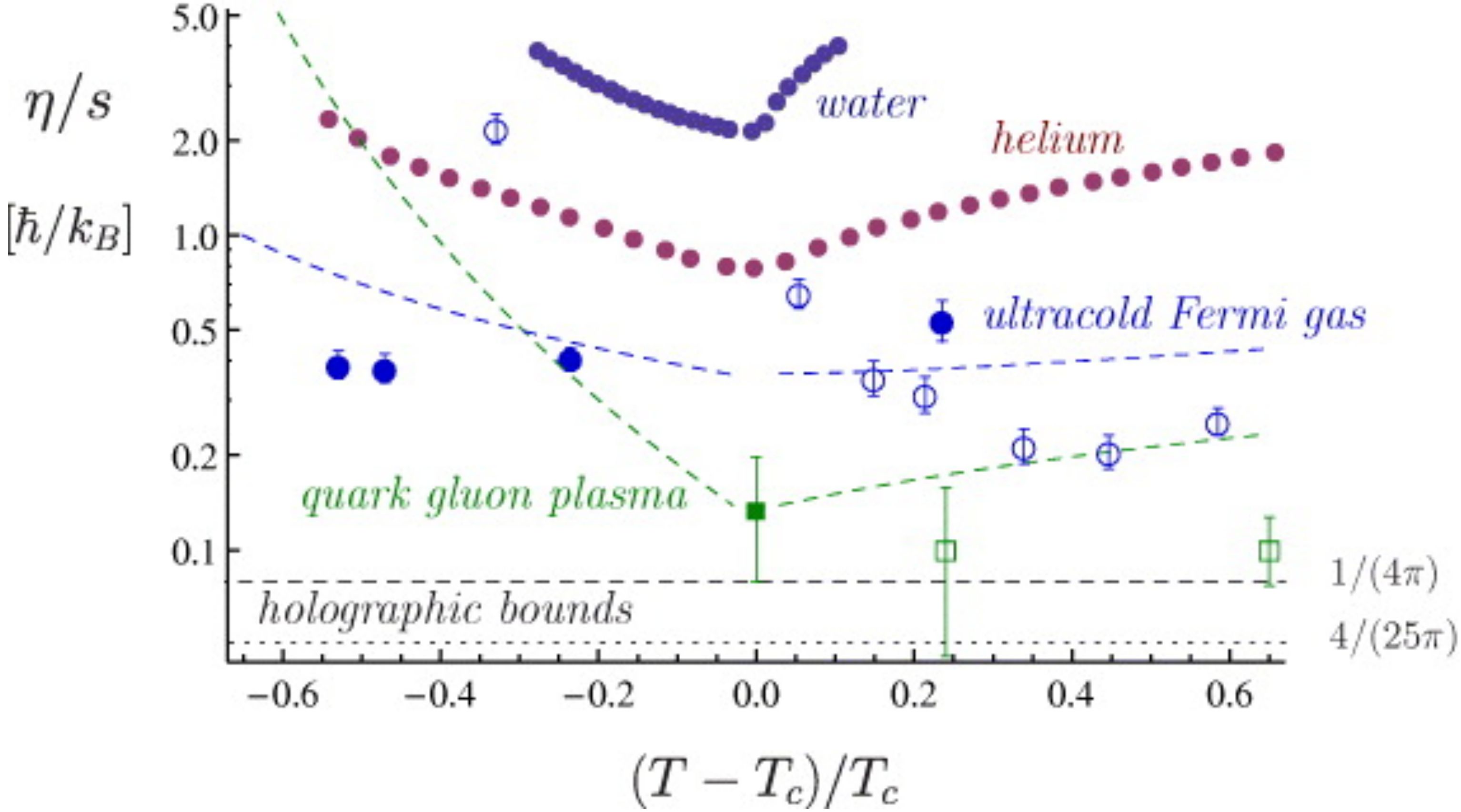
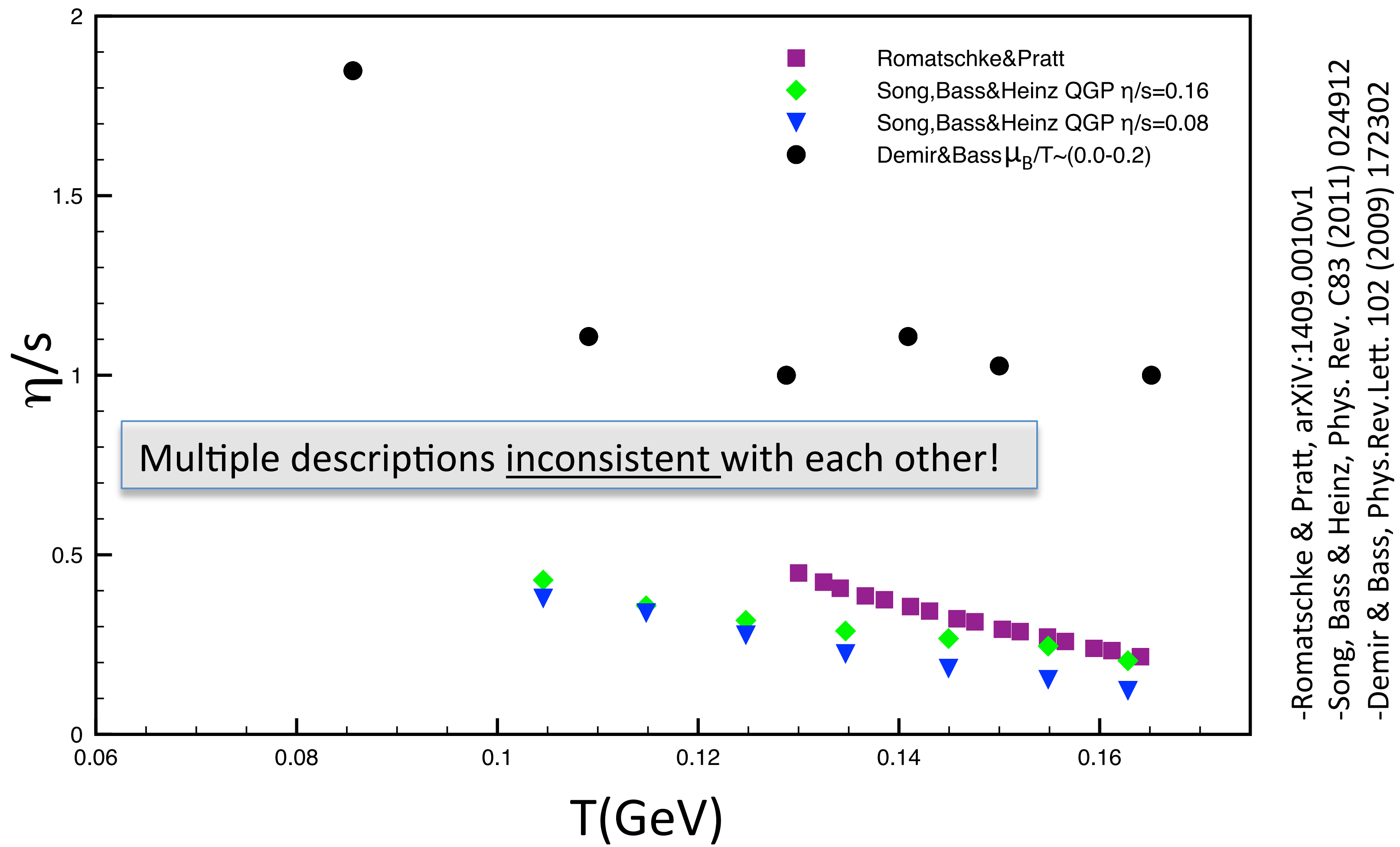
How low is low?



A. Adams, L. D. Carr, T. Schäfer, P. Steinberg, J E Thomas, New J. Phys. 15 (2013) 045022

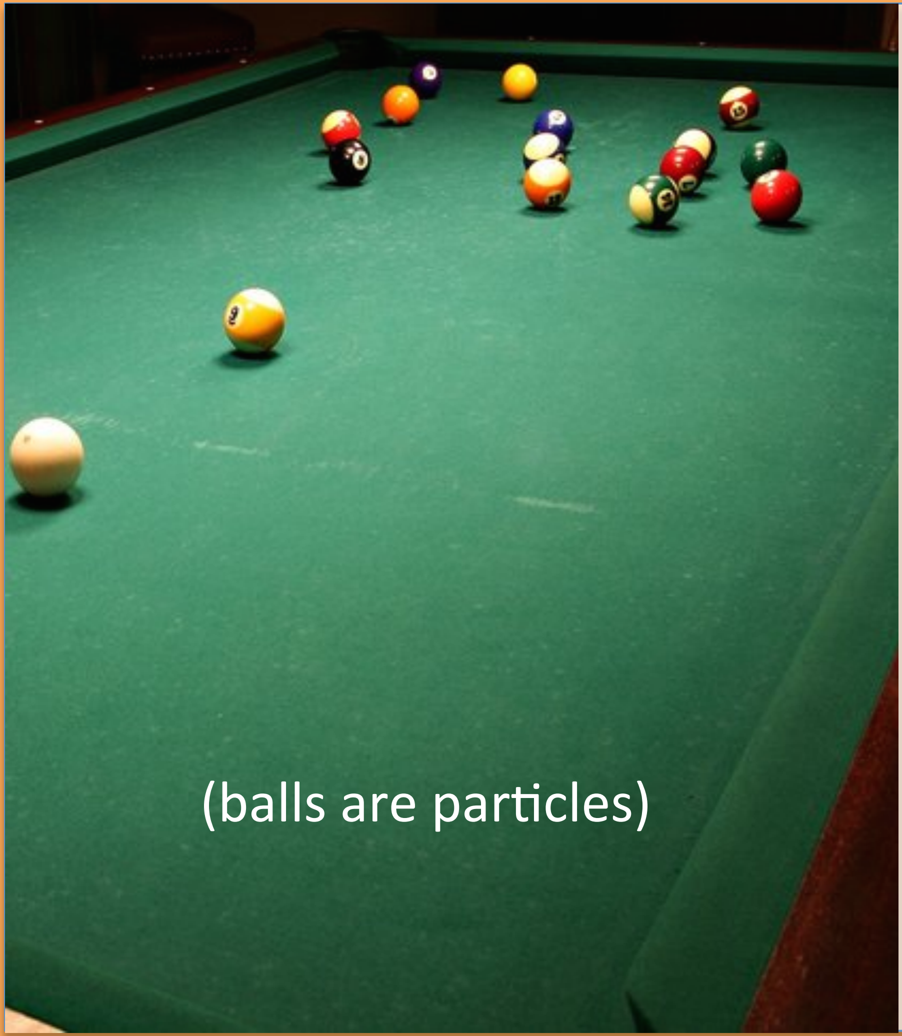
Previous HG viscosity calculations



Transport approaches

- **Transport models are 3D billiard tables**
- **But...**
  - Balls do not see each other as being the same size
  - Balls can annihilate
  - Balls can decay
  - Balls can become other balls on collision

Transport approaches



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Transport approaches

- **More fundamentally, transport *effectively* solves the Boltzmann equation:**

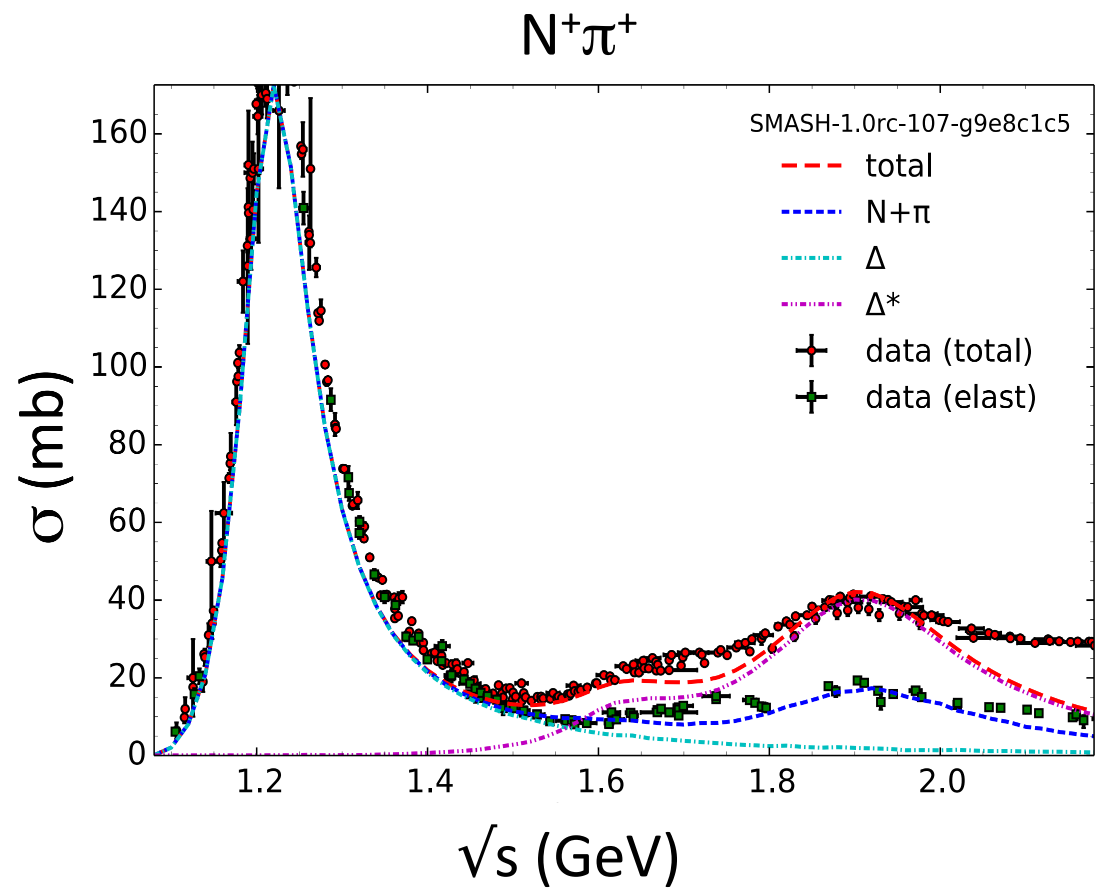
$$p^{\mu}\partial_{\mu}\, f_i\,(x,\,p) + m_i\, F^{\alpha}\,\partial^{p}_{\alpha}\, f_i\,(x,\,p) = \mathrm{C}^{i}_{coll}$$

where $f_i(x,p)$ is the one-particle distribution function, F^{α} the force experienced by particles and C^{i}_{coll} is the collision term.

- **Each particles species is represented with point-like test particles**
  - The default is 1 test particle, which then corresponds to the physical particle
  - As the number of test particles per physical particle increases, we recover the smooth distributions of the Boltzmann equation

The transport code : SMASH

- **SMASH is a new transport code**
  - Geometric collision criterion
  - Cross-sections built from a resonance model
  - All PDG resonances up to a mass of 2.3 GeV
  - 2-to-1 and 2-to-2 collisions

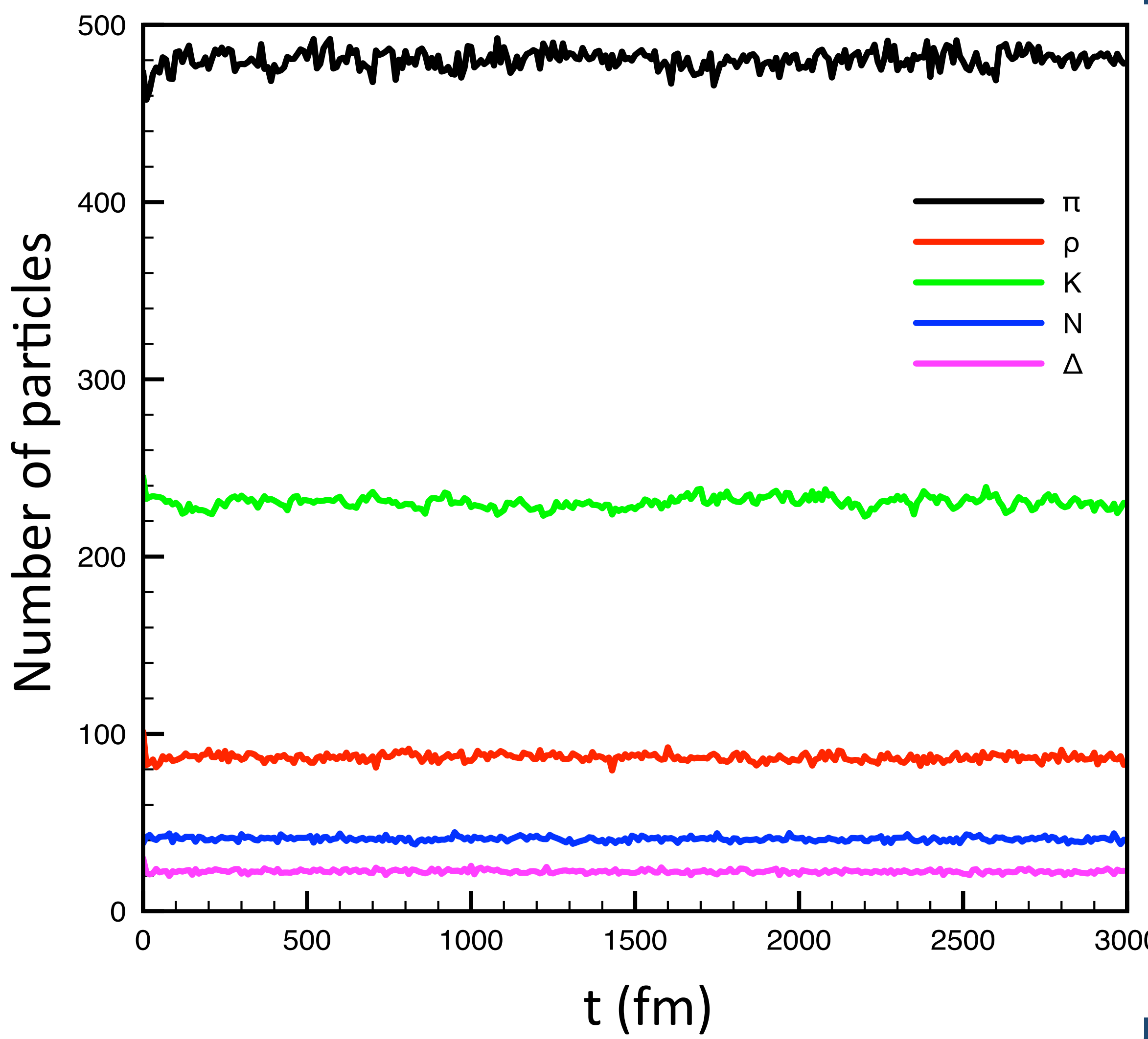


Weil et al., Phys. Rev. C94 (2016) no.5, 054905

Viscosity in SMASH

- **Box calculations simulating infinite matter to apply the Green-Kubo procedure**

- **MUST have thermal & chemical equilibrium**
  - Baryon/antibaryon annihilation implemented to conserve detailed balance via an average decay to 5π



Green-Kubo Formalism

The shear viscosity is calculated from

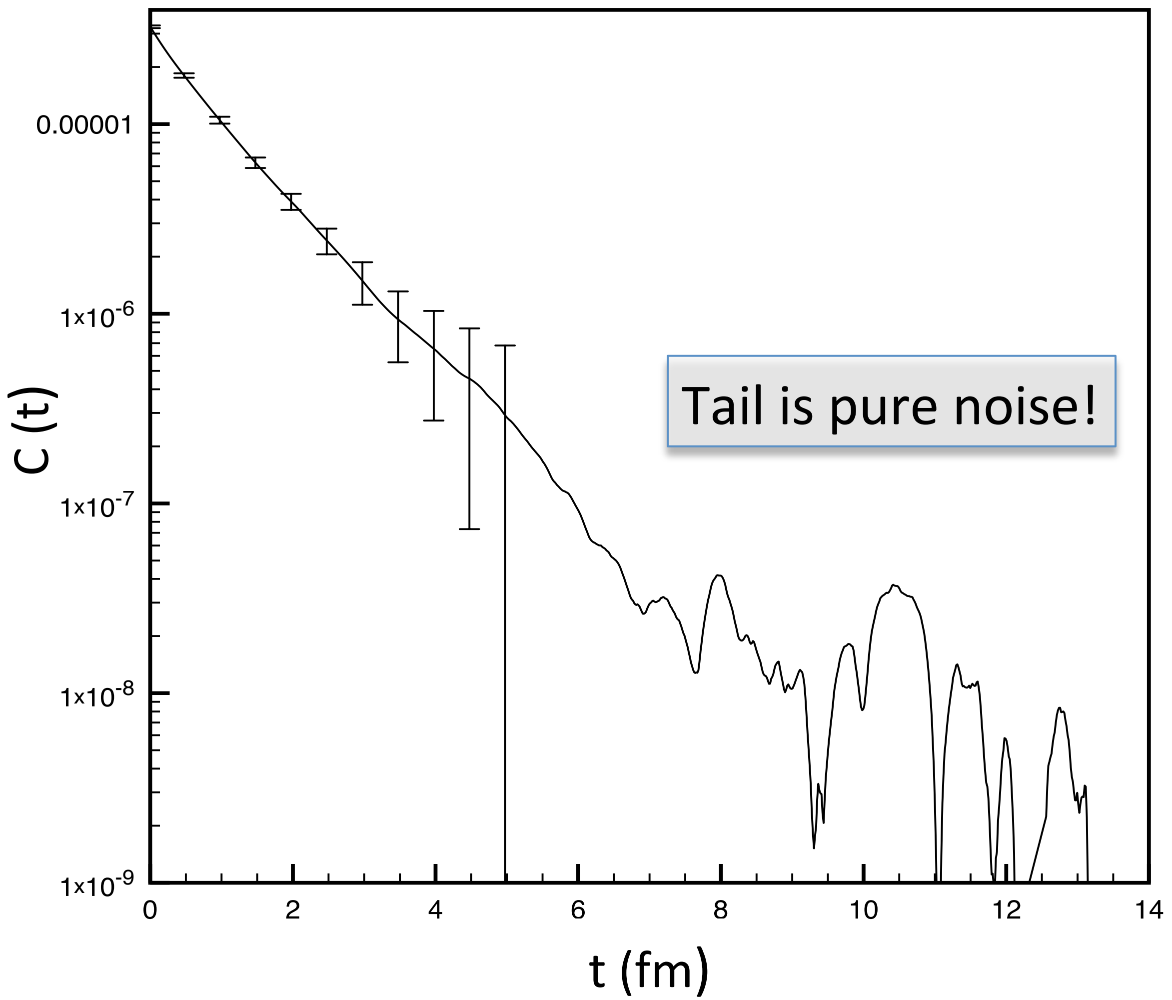
$$\eta = \frac{V}{T}\int_0^\infty C^{xy}(t)\,dt$$

where

$$C^{xy}(t) = \frac{1}{N}\sum_s^N T^{xy}(s)\,T^{xy}(s+t)$$

and

$$T^{\mu\nu} = \frac{1}{V}\sum_i^{N_{part}} \frac{p_i^\mu p_i^\nu}{p_i^0}$$



N is the number of time steps, and N_{part} the number of particles

Green-Kubo Formalism

It has been shown that the correlation

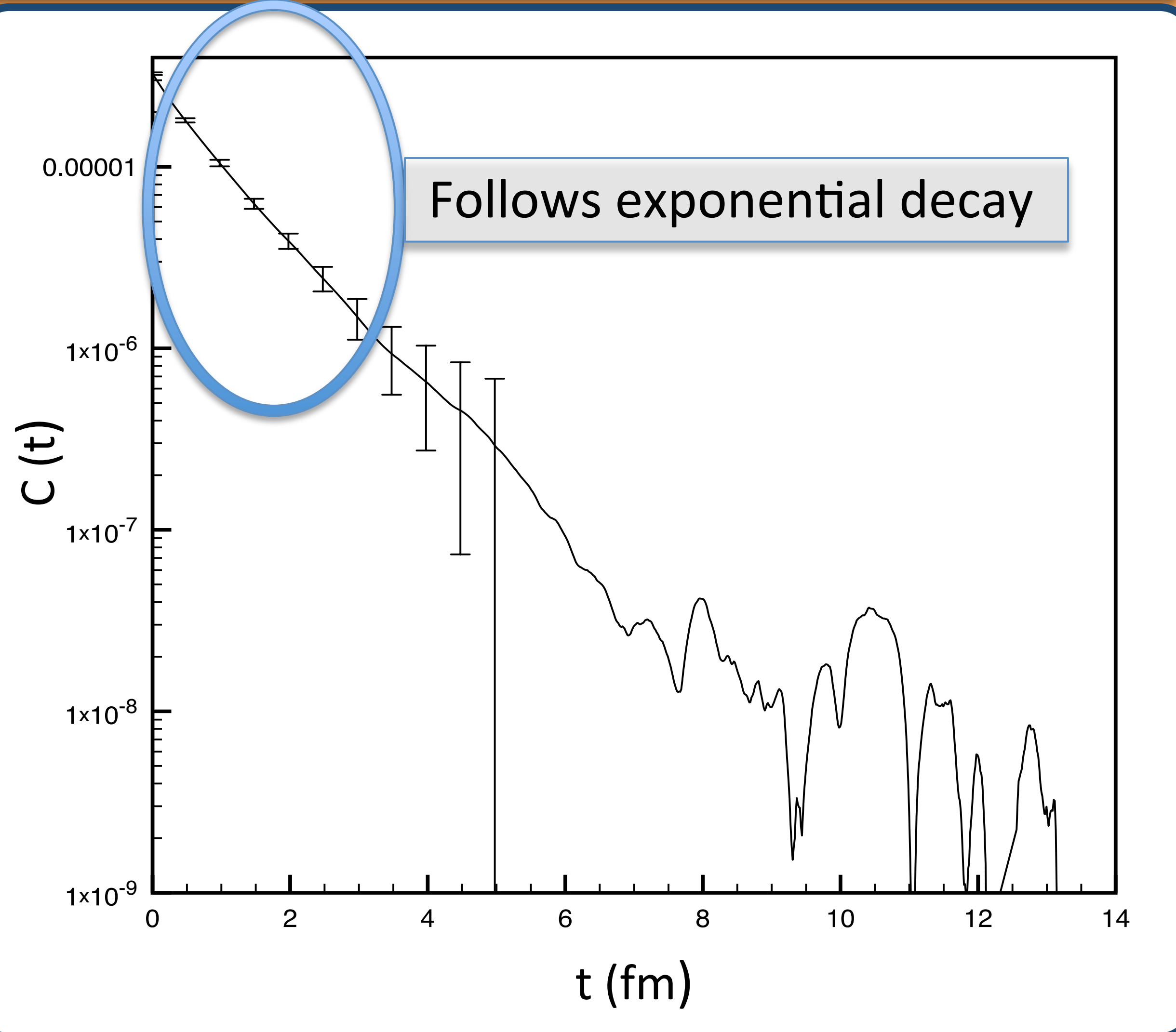
$$\eta = \frac{V}{T}\int_0^\infty C^{xy}(t)\,dt$$

Follows

$$C^{xy}(t) = C^{xy}(0)\exp\left(-\frac{t}{\tau}\right)$$

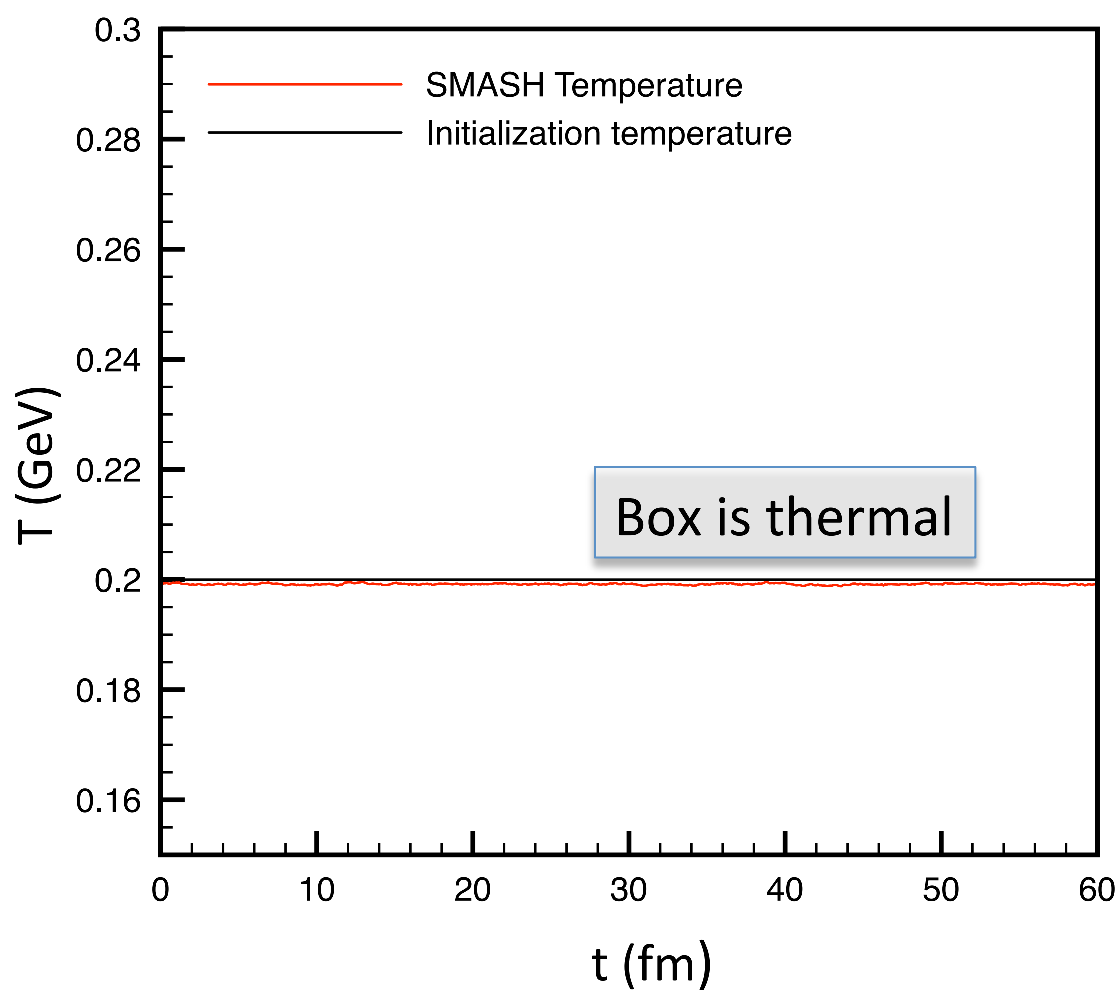
So that

$$\eta = \frac{V C^{xy}(0)\tau}{T}$$

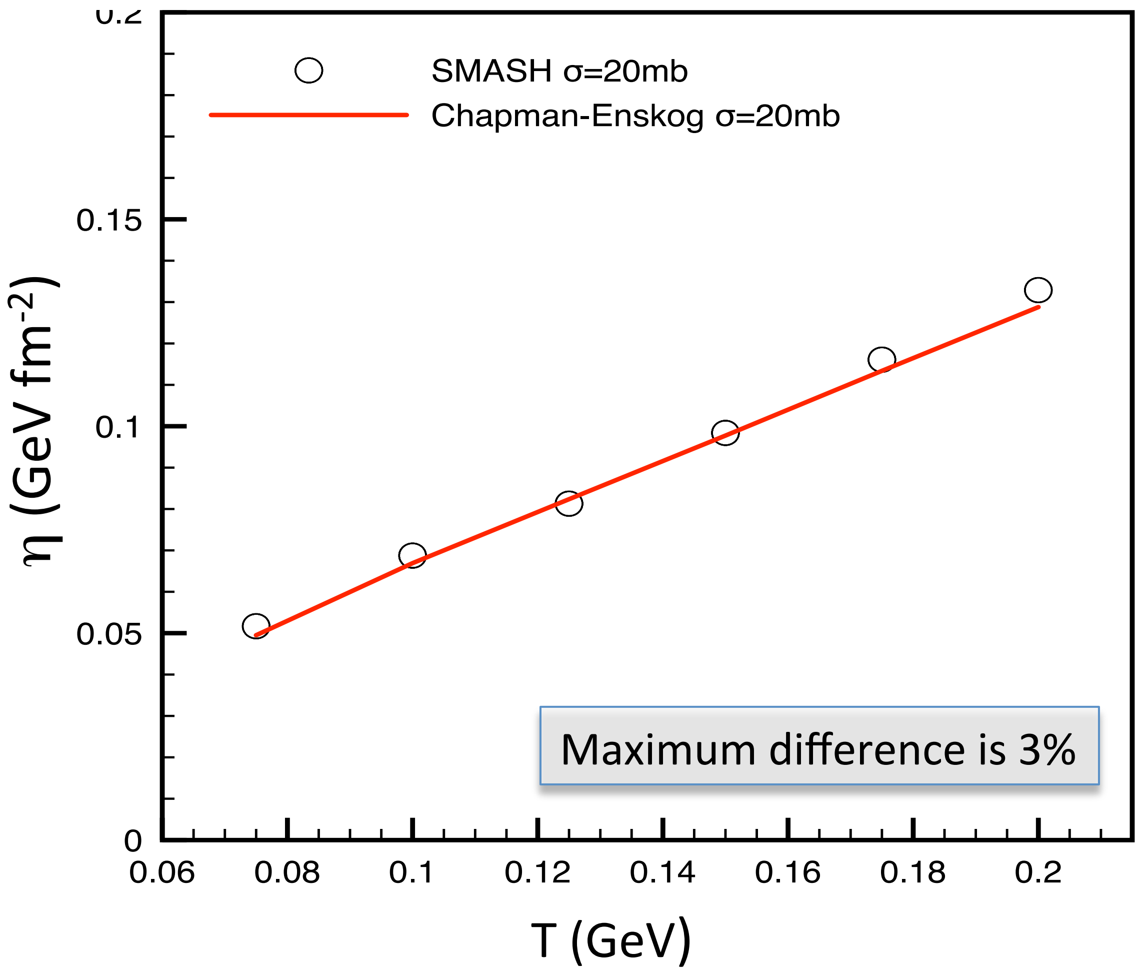


Test case : Pion box

- Pions in a $(20\text{ fm})^3$ box simulating infinite matter
- Constant, isotropic σ
- Runs for t_{max}=200 fm
- Initialized with initial densities consistent with Boltzmann ideal gas

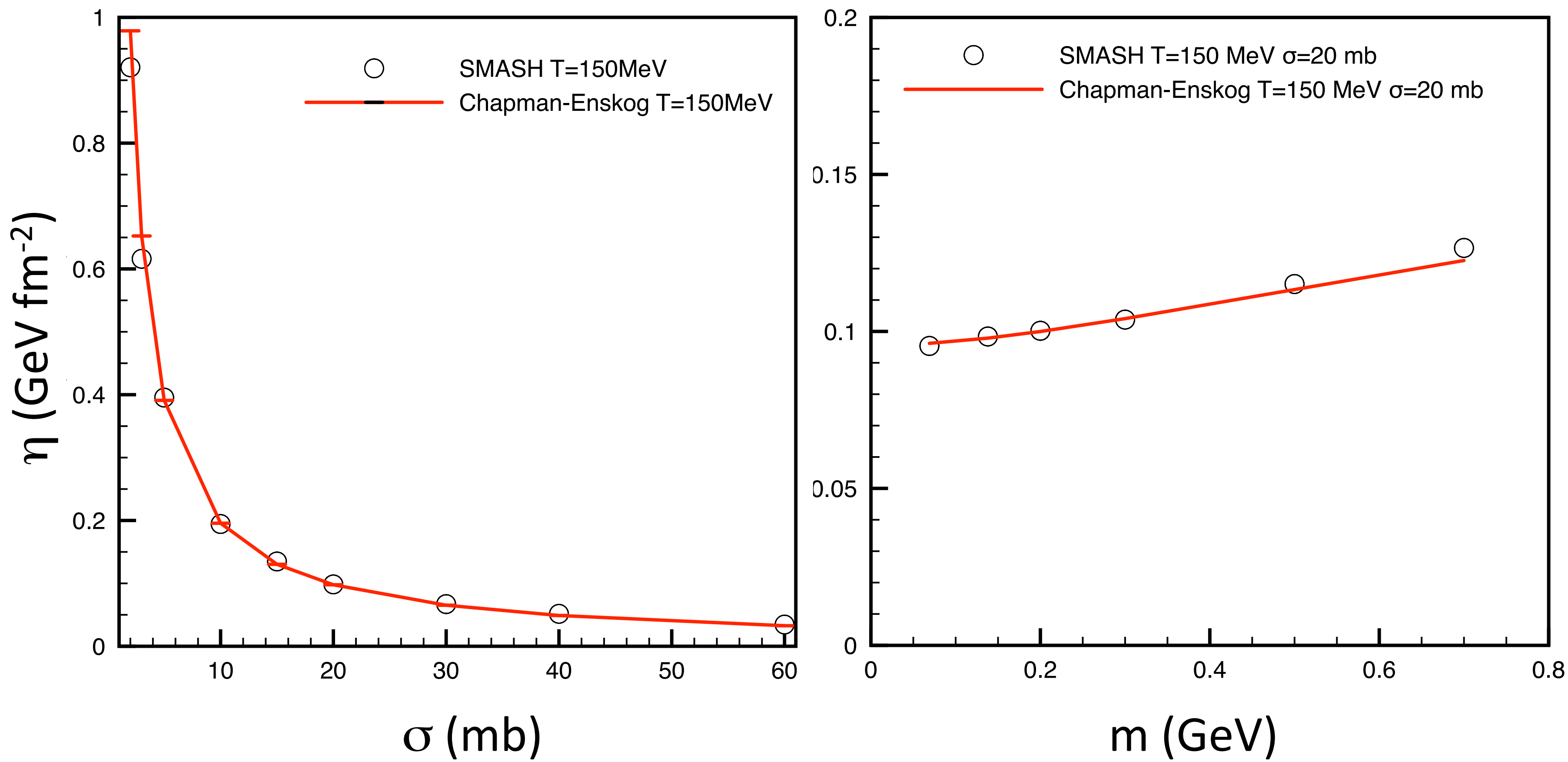


Pion box : Temperature dependence



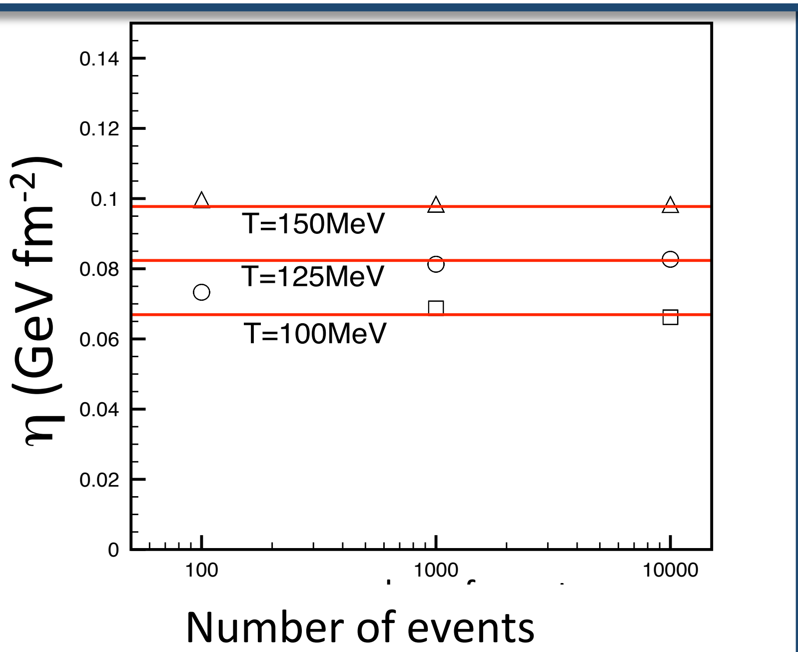
J. Torres-Rincon, PhD dissertation (2012), *Hadronic Transport Coefficients from Effective Field Theories*

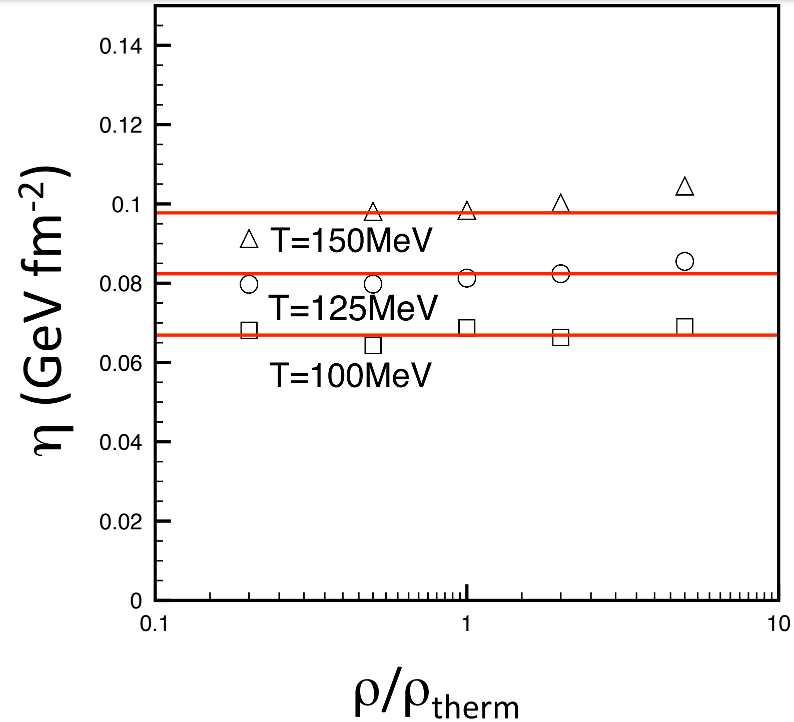
Pion box : Cross-section/mass dependence

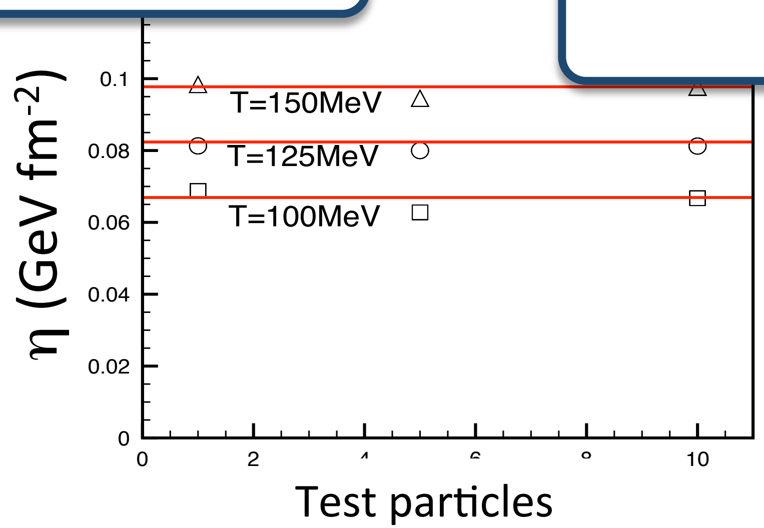


Very good agreement with analytical calculations!

Pion box : Systematics







Main take-away:

The method is relatively inelastic to variations of most parameters; maximum error is less than 10%

What about entropy?

The entropy density can be calculated from the Gibbs formula:

$$s = \frac{e + p - \mu n}{T}$$

where the energy density and pressure can be taken from the average shear-stress tensor according to:

$$T^{\mu\nu} = diag(e, p, p, p)$$

Assuming a nearly ideal gas, one can fit the temperature and chemical potential with momentum distributions:

$$\frac{dN}{dp} = \frac{g}{2\pi^2} V p^2 \exp\left(- \frac{\sqrt{p^2 + m^2} - \mu}{T} \right)$$

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When is this correct?

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Energy density and pressure

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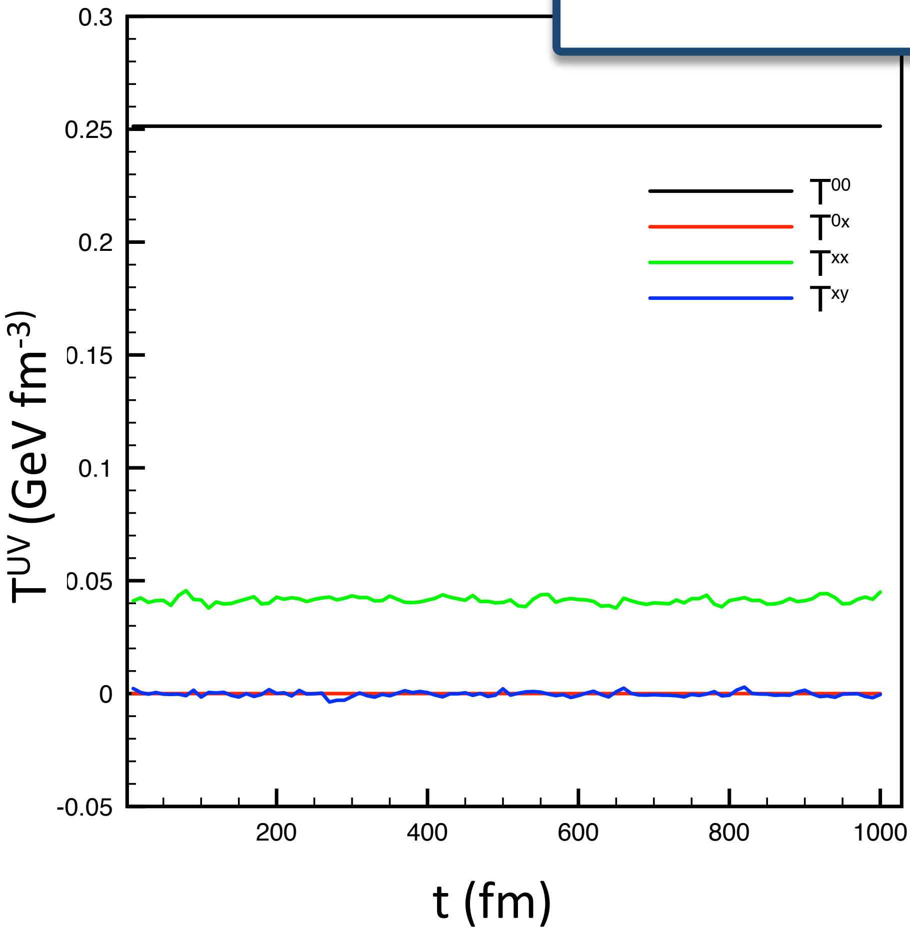
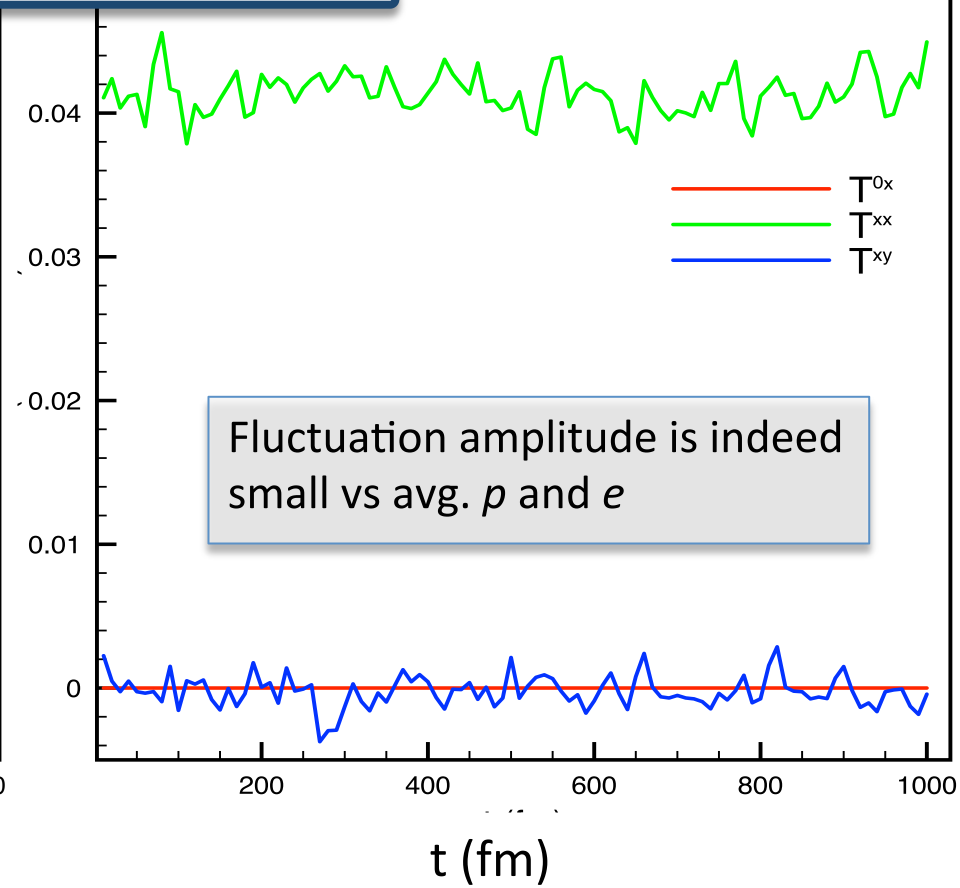
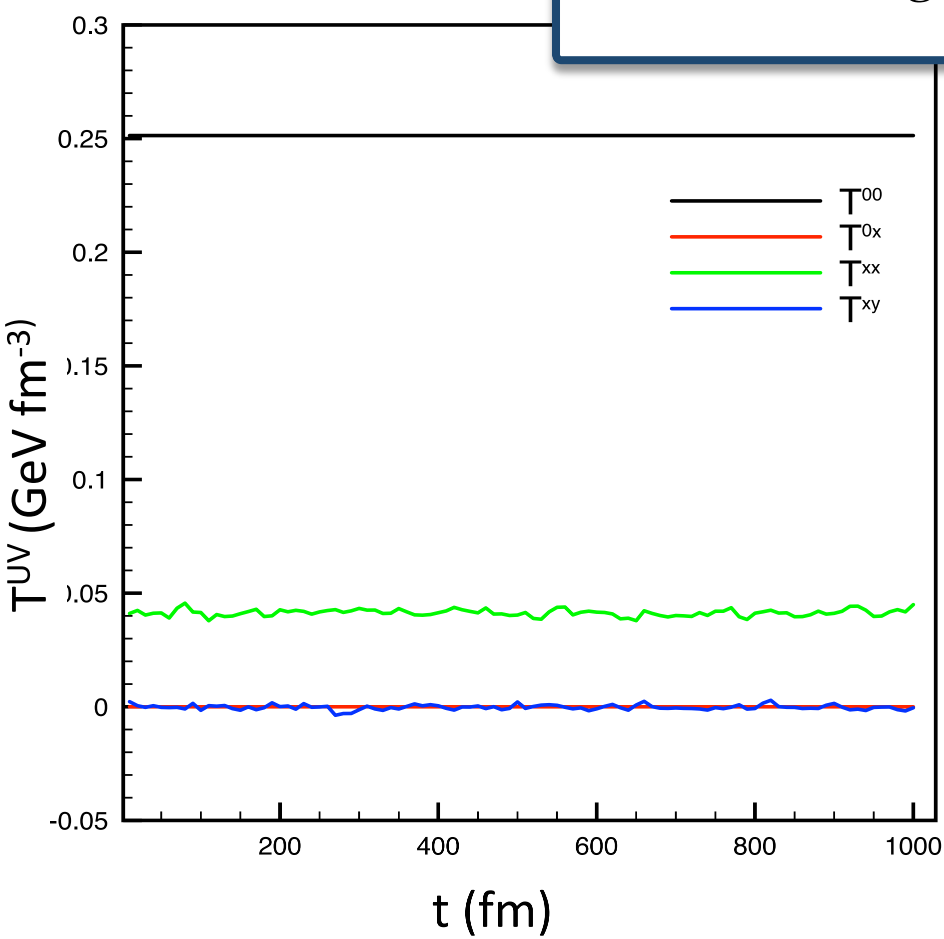




Fluctuation amplitude is indeed small vs avg. *p* and *e*

Energy density and pressure

$$T^{\mu\nu} = diag(e, p, p, p)$$

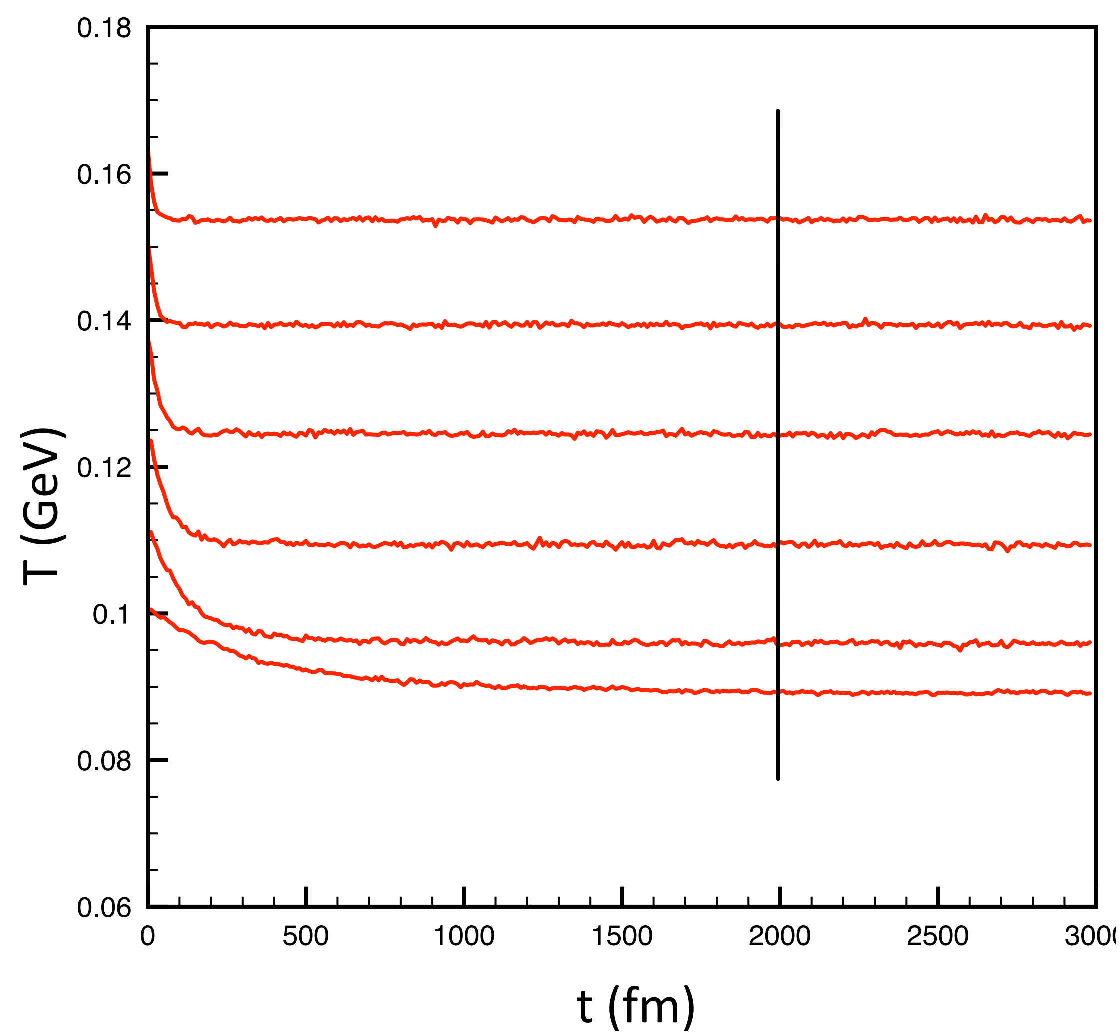


$$\frac{p}{e} \sim \frac{1}{6}$$

In the range of typical hadron gas equation of state

Hadron Gas (HG)

- All particles and resonances initialized to thermal multiplicities
- Must wait for equilibration and compute *T, μ* once in equilibrium from most abundant particles
  - T fitted from weighted momentum spectra of *π, K & N*
  - μ_B obtained from *N* / anti-*N* ratio

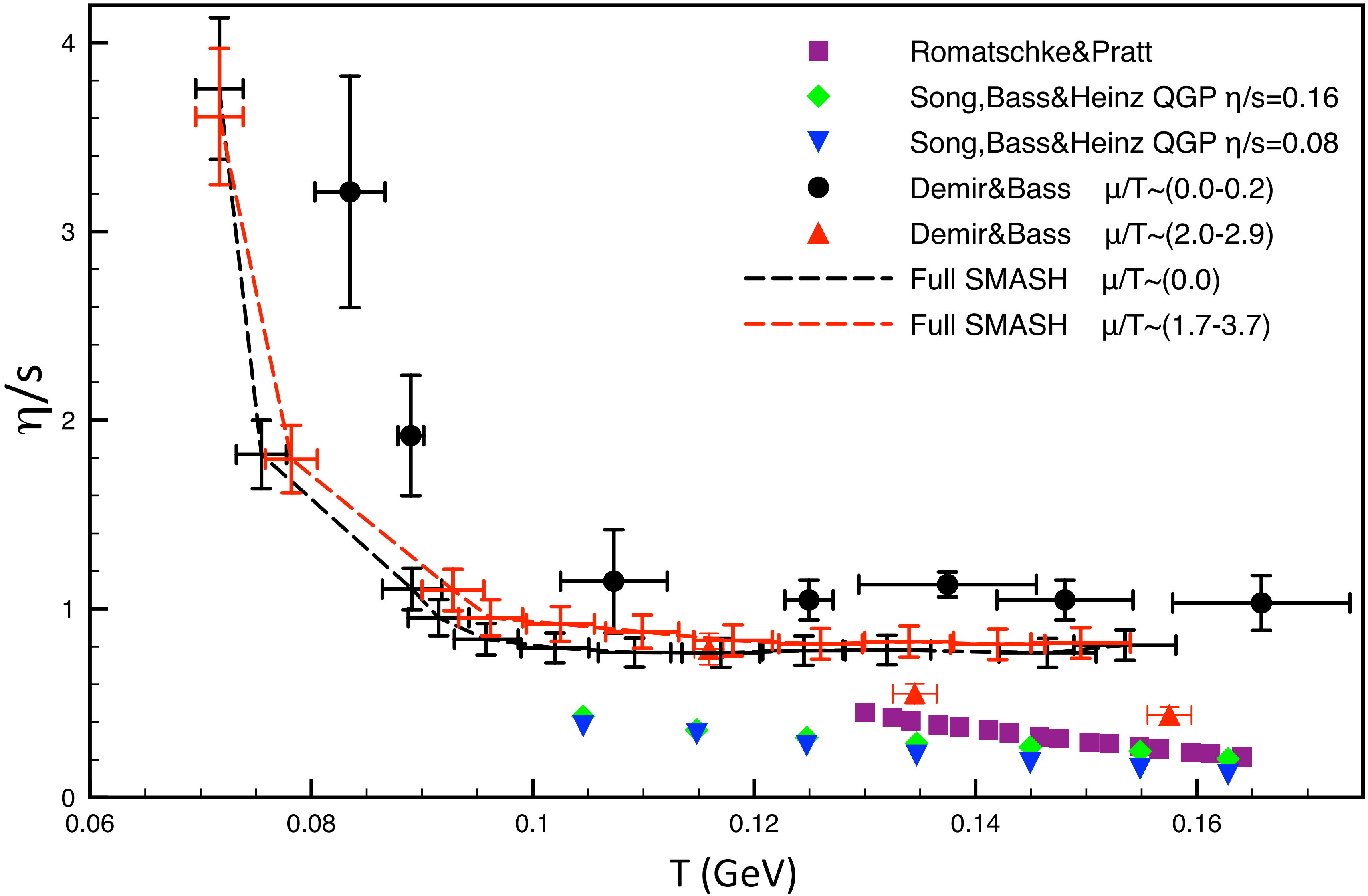


HG: Particle list

N	Δ	Λ	Σ	Ξ	Ω
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^{-}_{1672}
N_{1462}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1532}	Ω^{-}_{2252}
N_{1515}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}	
N_{1535}	Δ_{1905}	Λ_{1600}	Σ_{1670}	Ξ_{1820}	
N_{1655}	Δ_{1910}	Λ_{1670}	Σ_{1750}	Ξ_{1950}	
N_{1675}	Δ_{1920}	Λ_{1690}	Σ_{1775}	Ξ_{2030}	
N_{1685}	Δ_{1930}	Λ_{1800}	Σ_{1915}		
N_{1700}	Δ_{1950}	Λ_{1810}	Σ_{1940}		
N_{1710}		Λ_{1820}	Σ_{2030}		
N_{1720}		Λ_{1830}	Σ_{2250}		
N_{1875}		Λ_{1890}			
N_{1900}		Λ_{2100}			
N_{1990}		Λ_{2110}			
N_{2000}		Λ_{2350}			
N_{2190}					
N_{2220}					
N_{2250}					

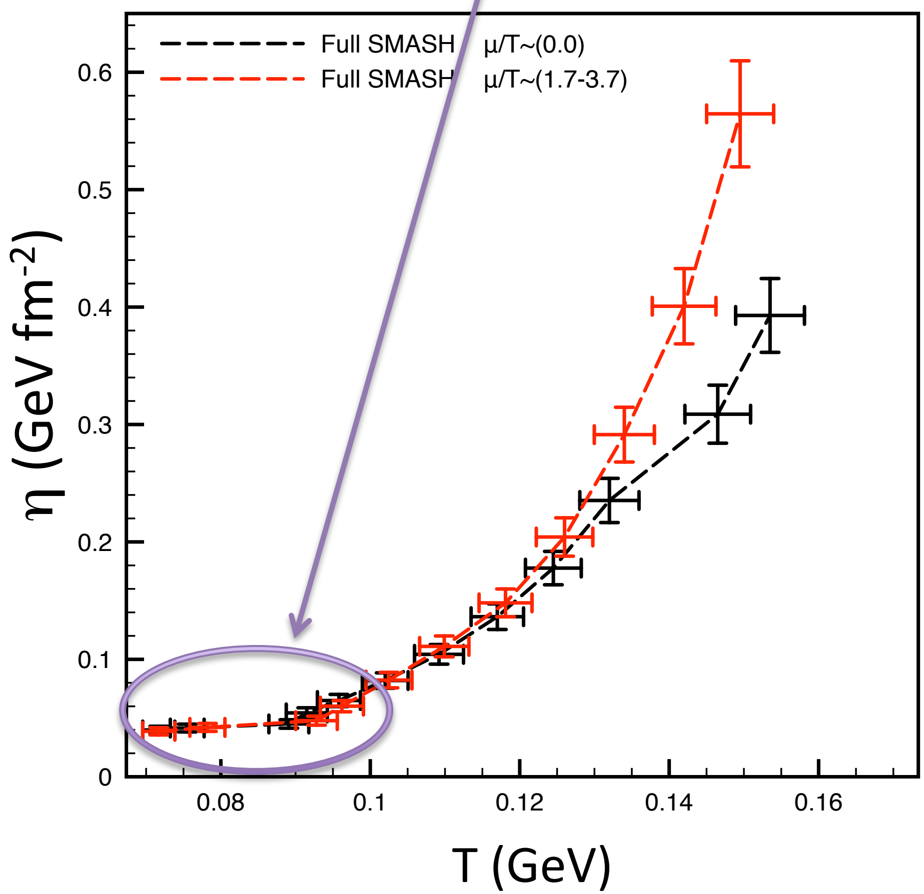
Unflavored				Strange
π_{138}	$f_{0\,980}$	$f_{2\,1275}$	$\pi_{2\,1670}$	K_{494}
π_{1300}	$f_{0\,1370}$	$f'_{2\,1525}$		K^*_{892}
π_{1800}	$f_{0\,1500}$	$f_{2\,1950}$	$\rho_{3\,1690}$	$K_{1\,1270}$
	$f_{0\,1710}$	$f_{2\,2010}$		$K_{1\,1400}$
η_{548}		$f_{2\,2300}$	$\phi_{3\,1850}$	K^*_{1410}
η'_{958}	$a_{0\,980}$	$f_{2\,2340}$		$K_0^*{}_{1430}$
η_{1295}	$a_{0\,1450}$		$a_{4\,2040}$	$K_2^*{}_{1430}$
η_{1405}		$f_{1\,1285}$		K^*_{1680}
η_{1475}	ϕ_{1019}	$f_{1\,1420}$	$f_{4\,2050}$	$K_{2\,1770}$
	ϕ_{1680}			$K_3^*{}_{1780}$
σ_{800}		$a_{2\,1320}$		$K_{2\,1820}$
	$h_{1\,1170}$			$K_4^*{}_{2045}$
ρ_{776}		$\pi_{1\,1400}$		
ρ_{1450}	$b_{1\,1235}$	$\pi_{1\,1600}$		
ρ_{1700}				
	$a_{1\,1260}$	$\eta_{2\,1645}$		
ω_{783}				
ω_{1420}		$\omega_{3\,1670}$		
ω_{1650}				

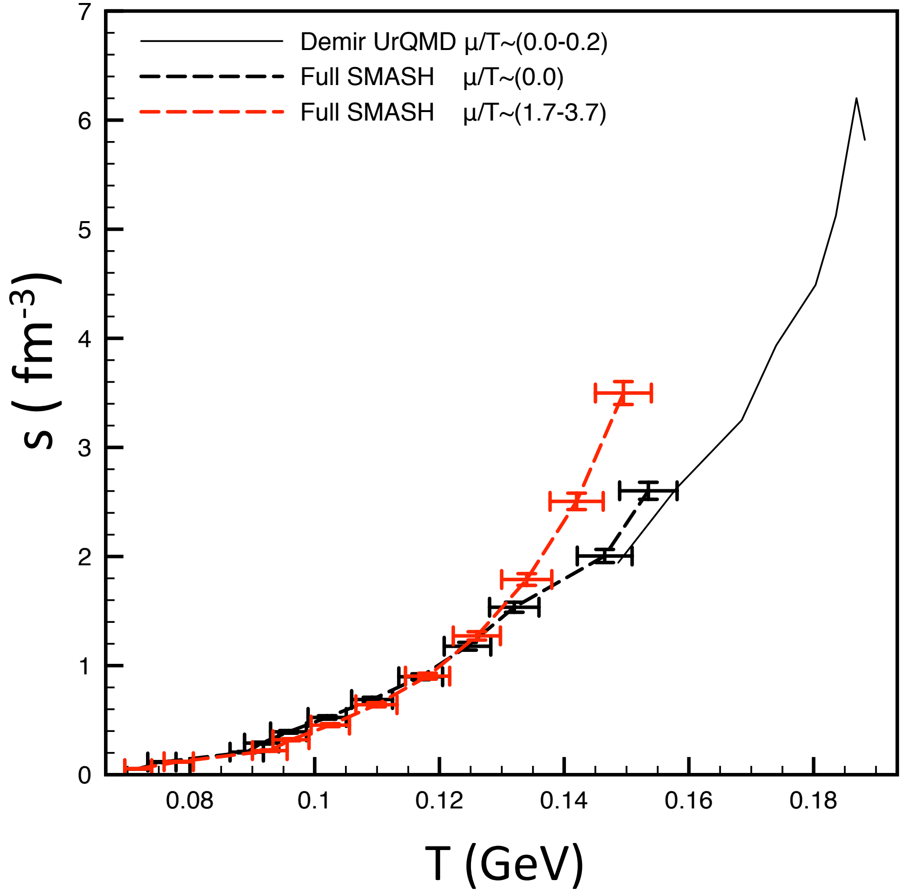
HG: Viscosity Comparison



Comparing η and s

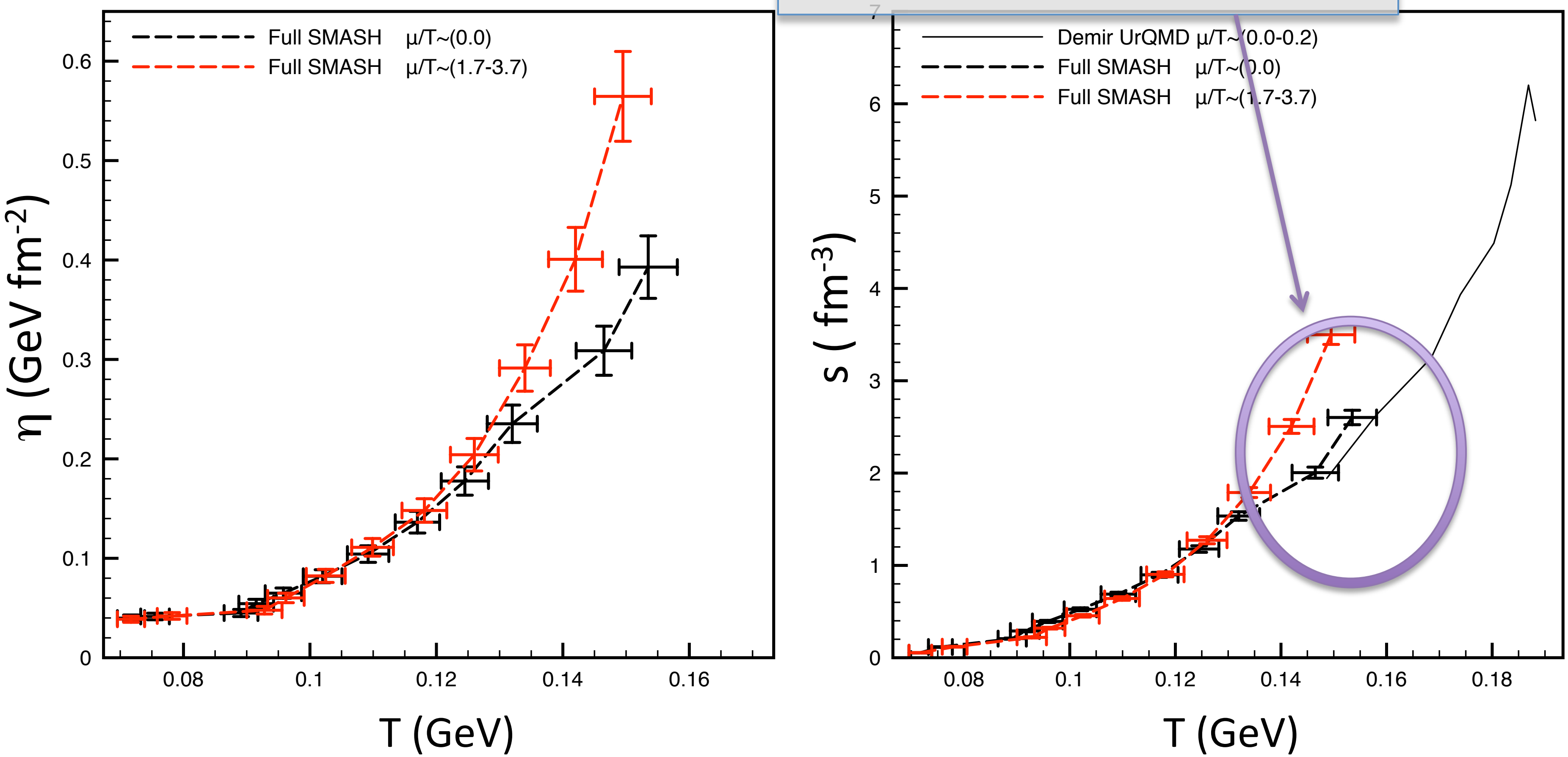
Viscosity decreases slower at small temperatures; explains rise of *η/s*





Comparing η and s

Although comparison is difficult, SMASH entropy density slightly higher than previous calculation



Summary & Outlook

- **Investigated temperature, cross-section and mass dependence of the shear viscosity in an elastic pion box**
  - Very good agreement with Chapman-Enskog approximation (within 3%)
  - Systematics show that the method is robust to variation of most parameters

- **Full hadron gas η/s calculated**
  - Has the expected decreasing profile
  - Final results are in qualitative agreement with previous calculations, but not in full agreement with any

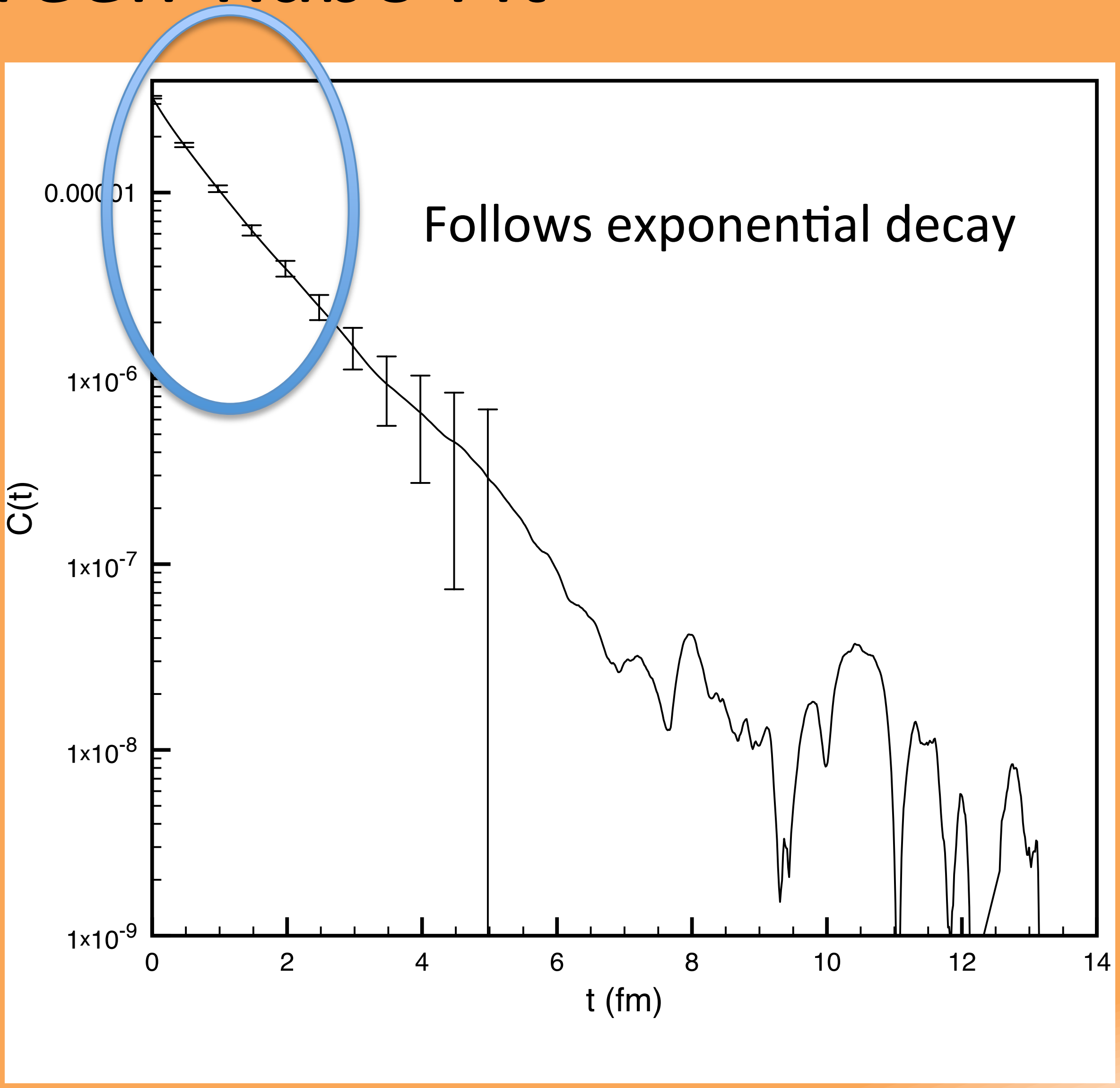
- **Outlook:**
  - Investigation of a π-ρ-σ box and comparison with Chapman-Enskog analytical calculations
  - Comparison of SMASH/UrQMD (viscosity, entropy, cross-sections)
  - More thorough investigation of the μ_B, μ_S parameter space
  - Other transport coefficients (electrical conductivity, bulk viscosity, etc.)

Backup

Green-Kubo Fit

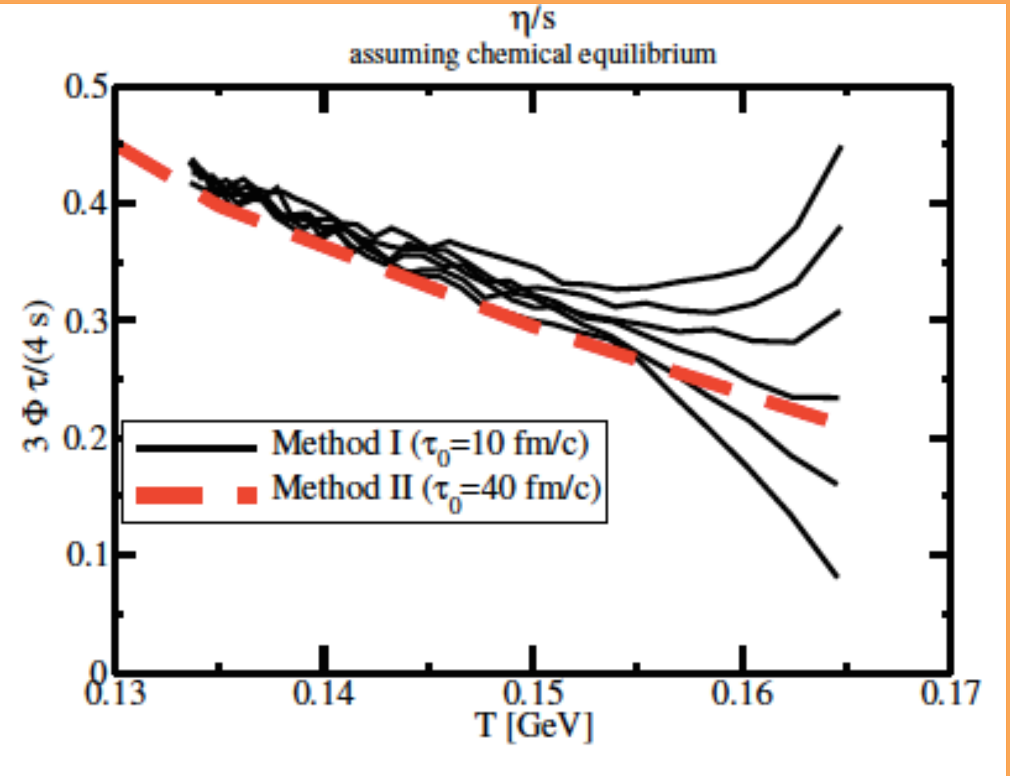
The exponential fit:

1. Calculate errors on an averaged autocorrelator for every time difference
2. Determine a maximal relative error tolerance (4% in this case)
3. Calculate C(0), τ, and finally η



Viscosity in the Hadron gas

What about low temperatures?



- Cascade code B3D, initialize over large 2D area at mid rapidity, with $T^{\mu\nu}$ modified such that

$$T_{ij} = \sum_{species\ l} \int \frac{d^3p}{(2\pi)^3} \frac{p_i p_j}{E(p)} f_l(p)$$

$$f(p) = f_{eq}(p)\left[1 + C(p) p_i p_j \pi_{ij}^{(s)}\right]$$

- Writing evolution equation using $\Phi = -\pi^{zz}$

$$\Phi = \frac{1}{3}(T_{xx} + T_{yy} + T_{zz}) - T_{zz} = \frac{4\eta}{3\tau} + \ldots$$

initialized where $d\Phi/d\tau = 0$.

Romatschke & Pratt, arXiV:1409.0010v1

Viscosity in the Hadron gas

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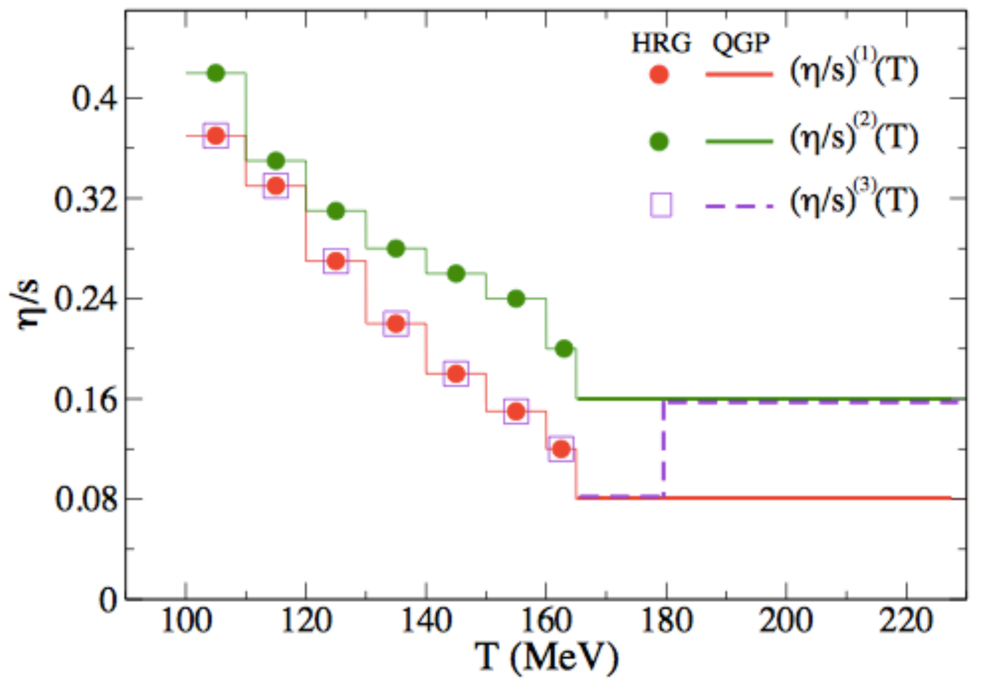

Song, Bass & Heinz, Phys. Rev. C83 (2011) 024912

- UrQMD coupled with VISH2+1
- Progressively lowering the coupling temperature
- Each step, the η/s of VISH2+1 is adjusted so that there is no pion v_2 buildup
- Take this η/s to be the effective UrQMD η/s at this temperature
- Non-universal: changing the QGP η/s changes the results

Viscosity in the Hadron gas

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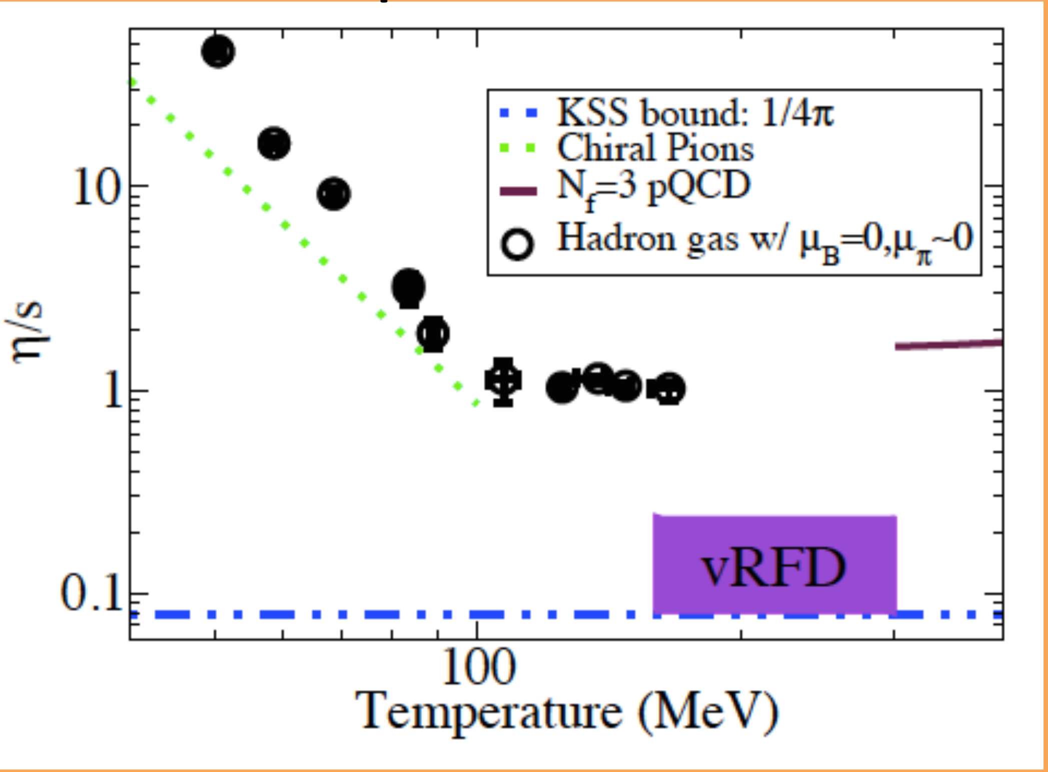

- UrQMD
- Box calculation
- Green-Kubo formalism
- Essentially the same procedure that we used

Demir & Bass, Phys.Rev.Lett. 102 (2009) 172302