





Exploring The QCD Phase Diagram

Lattice QCD

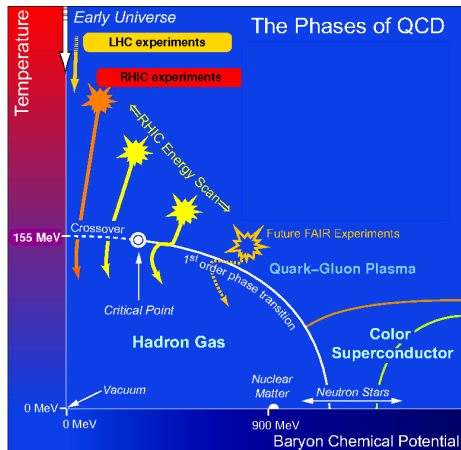
Perform calculations at $\mu_B = 0$, and extrapolate via Taylor expansion to finite μ_B

Black Hole Engineering

Based on Lattice data at $\mu_B = 0$, allows us to calculate observables at finite density.

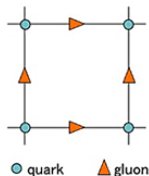
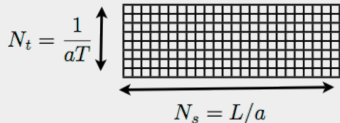
Susceptibilities of Conserved Charges

Are sensitive to the critical point and can be measured





QCD on a discretized lattice



- Study QCD from first principles in the **non-perturbative** region (Performs path integral using Monte-Carlo technique).
- Calculate equilibrium properties at $\mu_B = 0$ or at imaginary- μ_B (**sign problem!**). Calculations can be extrapolated to a small regime of μ_B .
- Has technical difficulties to compute **transport properties**



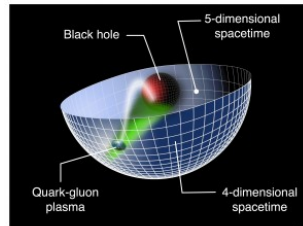
Holography (gauge/string duality at Strong Coupling)

Quantum Field Theory in
4- dimensions



Classical Gravity in at
least
5-dimensions

- Coupling $\gg 1$ in QFT \rightarrow vanishing string coupling
- (T, μ_B) in QFT \rightarrow black hole solution
- Holography \rightarrow Near Perfect fluidity



J M Maldacena 1999 Int. J. Theor. Phys. (38) 1113

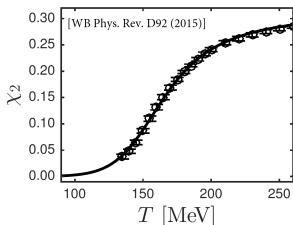
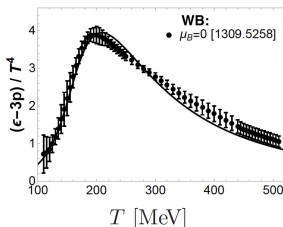


Non-conformal holographic gravity
dual in 5 dimensions



Black Hole
Solution

$$\mathcal{S} = \frac{1}{16\pi G_5} \int dx^5 \sqrt{-g} \left[\mathcal{R} - \frac{1}{2} (\partial_M \phi)^2 - \underbrace{V(\phi)}_{\text{nonconformal}} - \frac{1}{4} \underbrace{f(\phi) F_{MN}^2}_{\mu_B \neq 0} \right]$$



- Input parameters are fixed by lattice QCD results at $\mu_B = 0$
- Finite T and $\mu_B \rightarrow$ Predictions

O DeWolfe, S S Gubser, and C Rosen, Phys. Rev. D **83**, (2011)

R Rougemont, A Ficnar, S Finazzo and J Noronha, JHEP (2016) **102**.

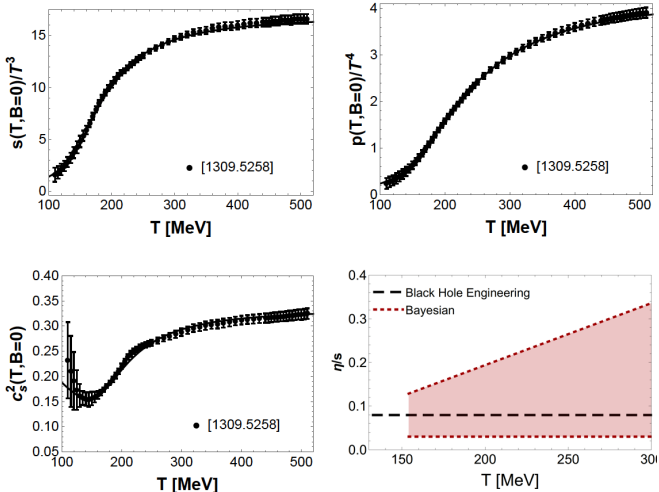


Black Hole Model

- Input parameters are fixed by Lattice data at $(T, \mu_B = 0)$
- Non-conformal Equation of State
 - at finite T and finite μ_B
 - with a critical end point
 - agrees with lattice data at small μ_B
- Near perfect fluidity
 - Ability to compute transport coefficients near the crossover and at large μ_B



Model Predictions at $\mu_B = 0$



R Rougemont, A Ficnar, S Finazzo and J Noronha, JHEP (2016) 102.



- A system in thermal equilibrium is characterized by

$$Z = \text{Tr} \left[-\frac{H - \sum_i \mu_i Q_i}{T} \right]$$

- The Pressure

$$P = \frac{T}{V} \ln Z$$

- The Baryonic Susceptibilities χ_n^B are defined as

$$\chi_n^B(T, \mu_B) = \frac{\partial^n}{\partial (\mu_B/T)^n} \left(\frac{P}{T^4} \right)$$



Susceptibilities of Conserved Charges

- The susceptibilities $\chi_n = \chi_n^B(T, \mu_B)$ are related directly to the moments of the distribution.
- The volume-independent ratios are useful quantities to compare to experimental data.

$$\text{mean : } M = \chi_1$$

$$\text{variance : } \sigma^2 = \chi_2$$

$$\text{skewness : } S = \chi_3 / \chi_2^{3/2}$$

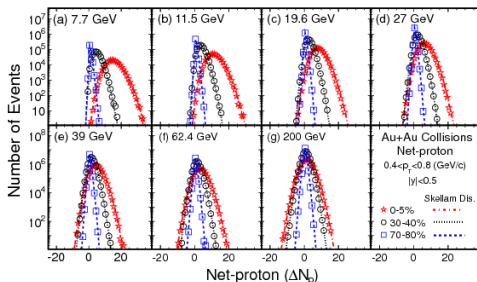
$$\text{kurtosis : } \kappa = \chi_4 / \chi_2^2$$

$$M / \sigma^2 = \chi_1 / \chi_2$$

$$S \sigma = \chi_3 / \chi_2$$

$$\kappa \sigma^2 = \chi_4 / \chi_2$$

$$S \sigma^3 / M = \chi_3 / \chi_1$$



[STAR] Phys. Rev. Lett. **112** (2014) 032302

Karsch Central Eur. J. Phys. **10** (2012)

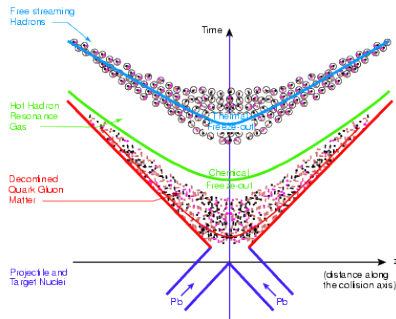




Evolution of heavy ion collisions

- **Chemical freeze-out:** all inelastic interactions cease. The chemical composition of the system is fixed

- **Kinetic freeze-out:** all elastic interactions cease: the spectra of the particles are fixed

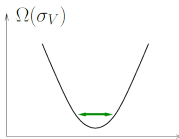


- We want to study the chemical freeze-out
- **Observables:** susceptibilities of conserved charges
 - They are fixed at the freeze-out
 - They can be measured and calculated
 - They are sensitive to the critical point

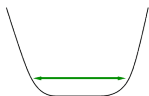


Fluctuations at the Critical End Point

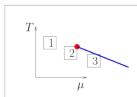
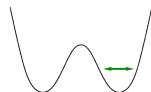
1

 $\mu < \mu_{\text{CP}}$ 

2

 $\mu = \mu_{\text{CP}}$ 

3

 $\mu > \mu_{\text{CP}}$ 

The probability distribution for the order parameter

$$P[\sigma] \sim \exp \{ -\Omega[\sigma]/T \}$$

$$\Omega = \int d^3x \left[\frac{1}{2} (\nabla \sigma)^2 + \frac{m_\sigma^2}{2} \sigma^2 + \frac{\lambda_3}{3} \sigma^3 + \dots \right]$$

The **correlation length** ($\xi = m_\sigma^{-1}$)

$$\xi \sim |T - T_c|^{-\nu} \text{ where } \nu > 0$$

$$\chi_2 = VT\xi^2$$

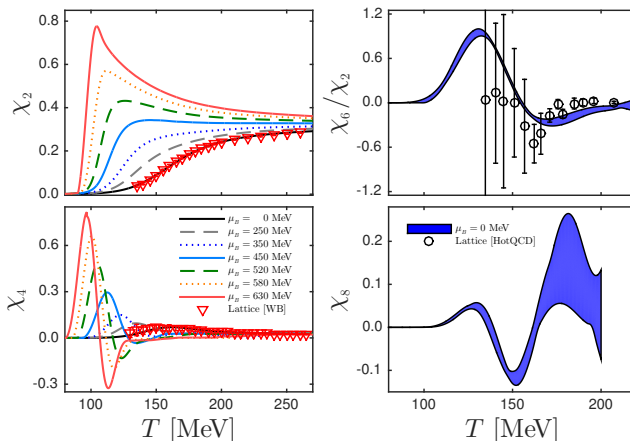
$$\chi_3 = 2VT^{3/2}\hat{\lambda}_3\xi^{9/2}$$

$$\chi_4 = 6VT^2[2\hat{\lambda}_3^2 - \hat{\lambda}_4]\xi^7$$

M. A. Stephanov, Phys. Rev. Lett. **102** (2009) 032301



Black Hole Susceptibilities



BH curves: R. Critelli, I. P. et al., to appear.

Lattice results: [WB] Phys. Rev. D, **92**, 114505 (2015).

[HotQCD] Phys. Rev. D, **95**, 054504 (2017).



Observables at finite μ_B

Taylor expansion of observables in terms of susceptibilities

$$\chi_n = \chi_n^B(T, \mu_B = 0)$$

■ Pressure

$$\frac{p(T, \mu_B) - p(T, \mu_B = 0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n}$$

■ Baryonic density

$$\frac{\rho_B(T, \mu_B)}{T^3} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n-1)!} \left(\frac{\mu_B}{T}\right)^{2n-1}$$

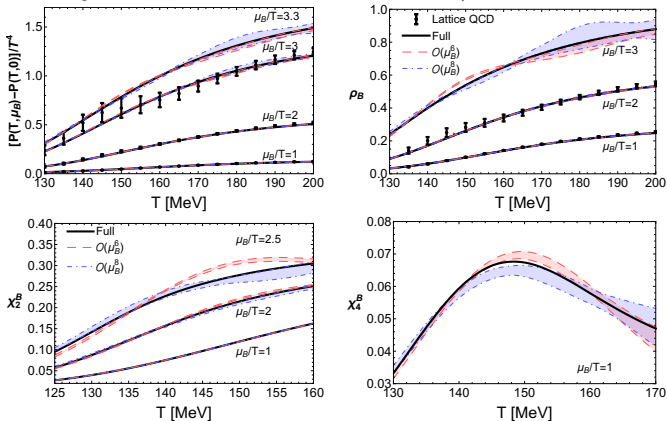
■ Susceptibilities χ_2 and χ_4

$$\chi_2(T, \mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+2}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n} \quad \chi_4(T, \mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+4}}{(2n!)} \left(\frac{\mu_B}{T}\right)^{2n}$$



Taylor Reconstruction up to $\mathcal{O}(\mu_B^8)$

Reconstruction of thermodynamic quantities at different values of μ_B/T via Taylor series from calculations at $\mu_B = 0$.



BH curves: R. Critelli, I. P. et al., to appear.

Lattice results: [HotQCD] Phys. Rev. D, **95**, 054504 (2017).

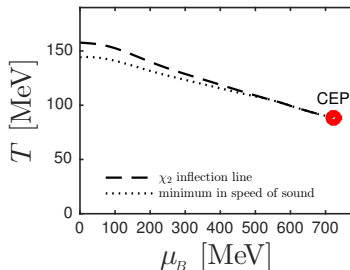
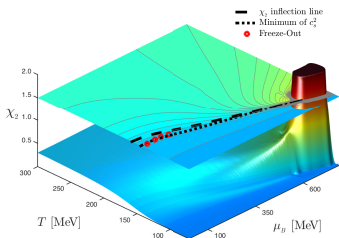


Black Hole Critical End Point

The black hole model contains a critical end point at

■ $\mu_B = 723 \pm 36 \text{ MeV}$

■ $T = 89 \pm 11 \text{ MeV}$

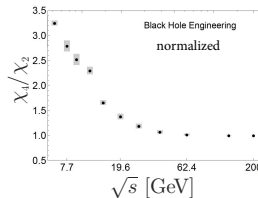
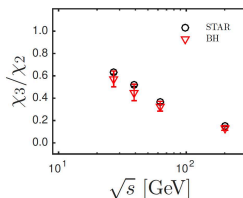
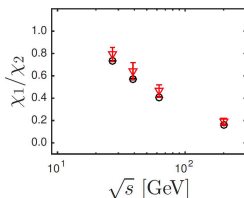


R. Critelli, I. P. et al., to appear.



Connection to Experiment

- We compare the baryonic BH susceptibilities ratios with the net-proton moments measured at STAR
- Freeze-out parameters are extracted by fitting the experimental values for χ_1/χ_2 and χ_3/χ_2
- χ_4/χ_2 predicted at the minimum in speed of sound

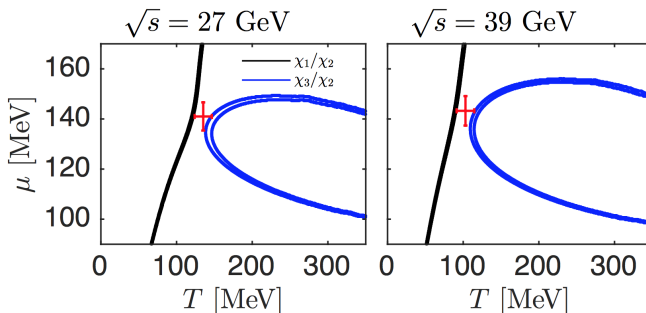


R. Critelli, I. P. et al., to appear. [STAR] Phys. Rev. Lett. **112** (2014)



Freeze out parameters from the Black Hole model

Trajectories in the $[T - \mu]$ plane that satisfy the experimental values

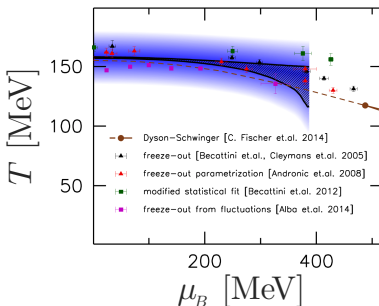


- Freeze out points $[T - \mu_B]$ are extracted from the line made by the closest points between χ_1/χ_2 and χ_3/χ_2



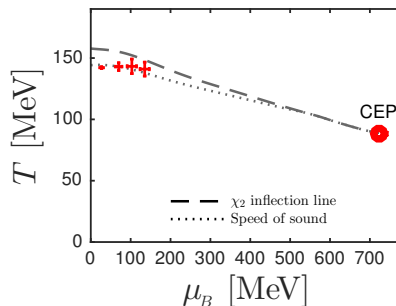
Freeze-out Line

Lattice QCD



[WB:] R. Bellwied *et. al.*,
Phys. Lett. B **751** (2015) 053

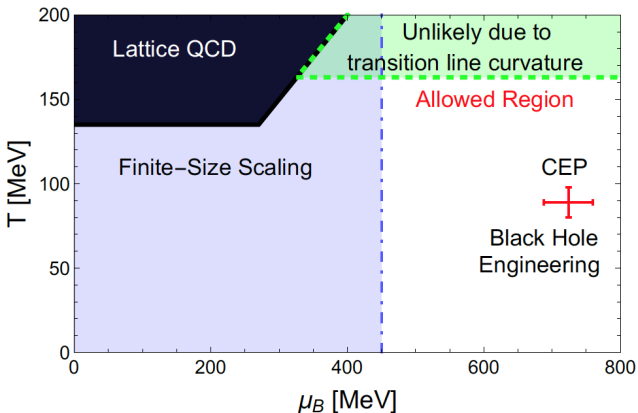
BH Model



R. Critelli, I. P. et al., to appear.



Black Hole Model CEP



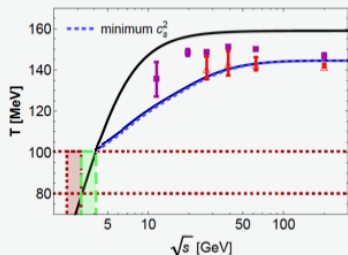
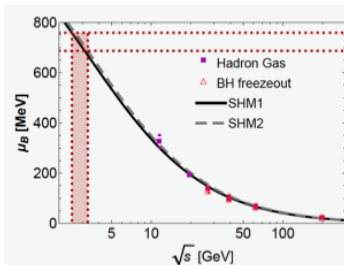
R. Critelli, I. P. et al., to appear.



Collision Energy Estimates

We estimate a collision energy needed to hit the CEP

■ $\sqrt{s} = 2.5 - 4.1 \text{ GeV}$



- The collision energy is reachable by the next generation of colliders

[BH] R. Critelli, I. P. et al., to appear.

[HRG] Paolo Alba et al. Phys. Lett. B738 (2014),

[SHM1] A. Andronic et al. Phys. Lett. B673 (2009).

[SHM2] J. Cleymans et al. Phys. Rev. C73 (2006).



The holographic Black Hole Model

- Reproduces lattice data at $\mu_B = 0$
- Contains a critical end point at $\mu_B = 723 \pm 36$ MeV and $T = 89 \pm 11$ MeV
- Allows us to compute baryonic susceptibilities, and extract freeze-out parameters
- Estimates that the collision energy needed to hit the CEP should be $\sqrt{s} = 2.5 - 4.1$ GeV