Recent theoretical advances regarding α-spectroscopy

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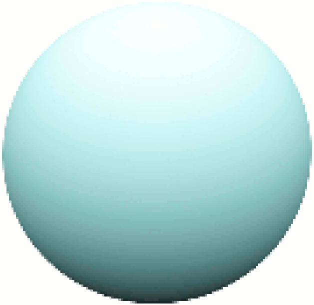
Key Points

- Nuclear structure
- The α-emission spectrum

1. Nuclear structure

- coherent states
- the energy spectrum of an even-even nucleus

The deformation of a nuclear ground state



The Coherent State Model[1] (CSM) I

- The intrinsic state of an axially-deformed even-even nucleus is given by a coherent superposition of quadrupole phonons $b_{2\mu}^{\dagger}$, with $\mu = 0$, acting on the vacuum

$$
\begin{aligned}
|\psi_g\rangle &= e^{d\left(b_{20}^{\dagger} - b_{20}\right)}|0\rangle \\
|\psi_g\rangle &= \left(1 + d\left(b_{20}^{\dagger} - b_{20}\right) + \frac{d^2}{2!}\left(b_{20}^{\dagger} - b_{20}\right)^2 + \dots\right)|0\rangle
\end{aligned}
$$

- The deformation parameter is proportional with the expectation value of the quadrupole operator on the intrinsic state

$$\langle\psi_g|\left(b_{20}^{\dagger} + b_{20}\right)|\psi_g\rangle = 2d$$

[1] A.A. Răduță *et al.*, Nucl. Phys. **A** 381 (1982).
A.A. Răduță, *Nuclear Structure with Coherent States*, Springer International Publishing, Cham, Switzerland, 2015.

The Coherent State Model II

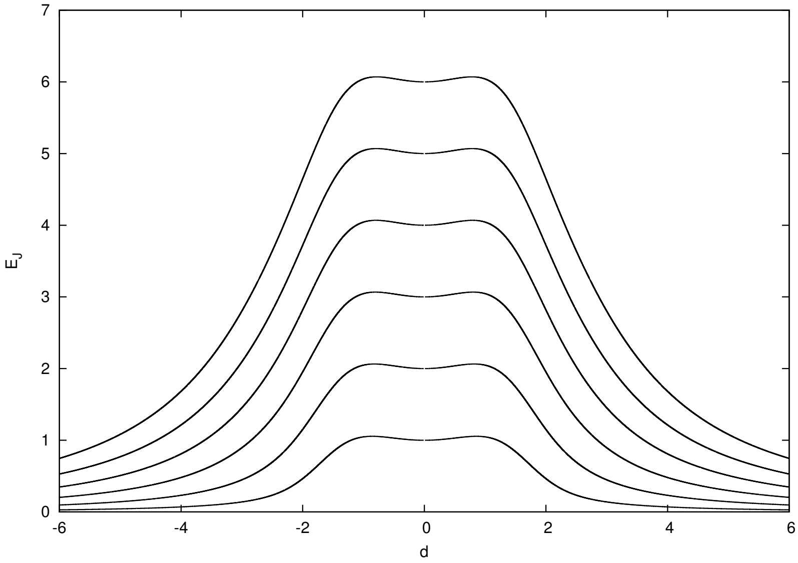
- The physical states that form the ground band are obtained through angular momentum projection

$$|\varphi_J^{(g)}\rangle = \mathcal{N}_J^{(g)} \hat{P}_{M0}^J |\psi_g\rangle$$

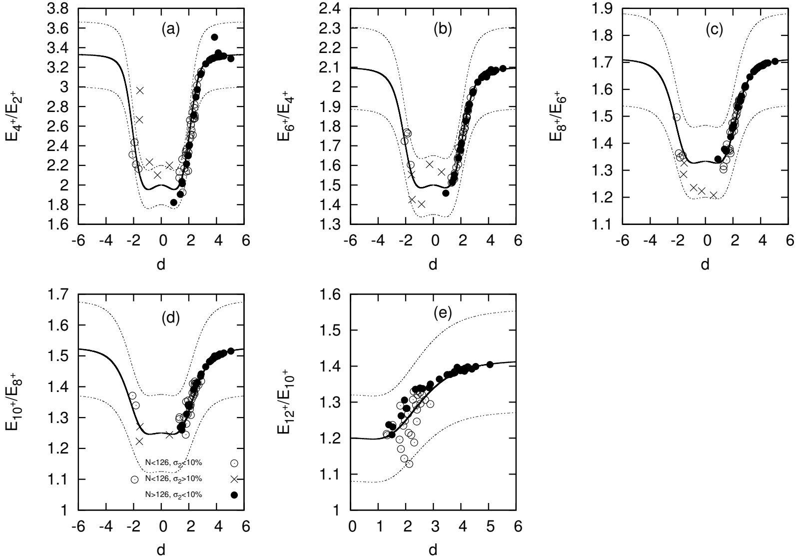
- The corresponding energy levels are the expectation values of a harmonic Hamiltonian

$$H = A_1 b_2^\dagger \cdot b_2$$

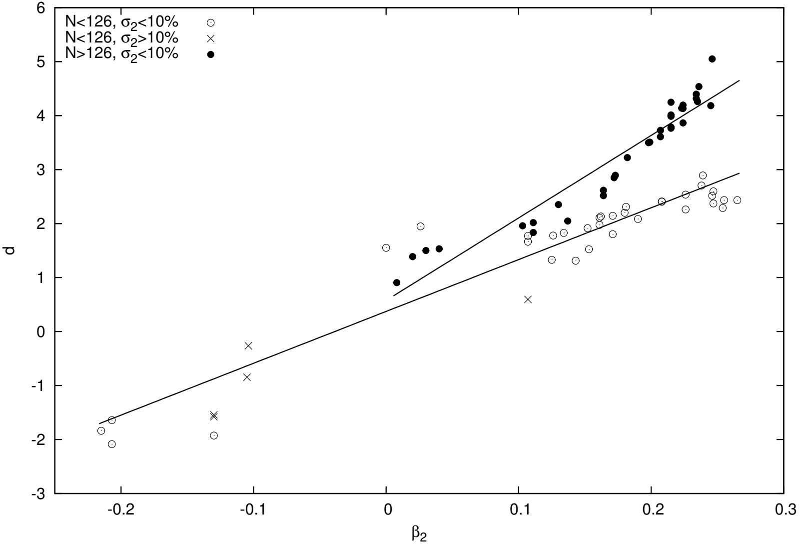
The CSM spectrum versus deformation



Consecutive ratios for energy levels versus the deformation parameter



The CSM deformation versus the standard quadrupole deformation[2]



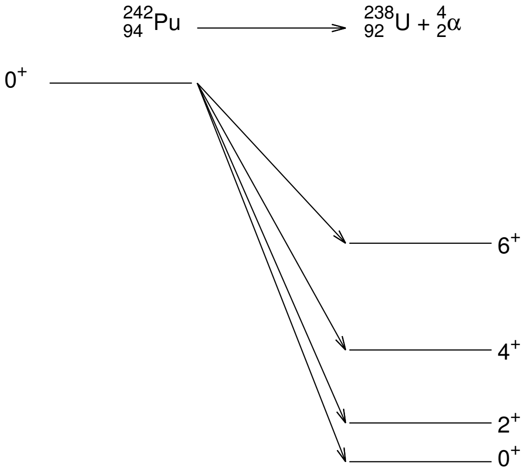
[2] P. Möller și J.R. Nix, Nucl. Phys. **A** 272, 502 (1995).

2. The α-emission spectrum[3]

- The Schrödinger equation for the emission process
- The α–nucleus interaction potential
- Application for α–emission in even-even nuclei

[3] D.S. Delion, Theory of Particle and Cluster Emission, Springer-Verlag, Berlin, 2010.
A.M. Lane, R.G. Thomas, Rev. Modern Phys. 30, 257 (1958).

The fine structure of the α-emission spectrum



The Coupled Channels Method I

- The α-decay process

$$P \to D(J) + \alpha(J)$$

can be described by a separable wave function depending on the degrees of the freedom of the daughter nucleus $b_2^\dagger$ and the relative distance $\mathbf{R}$ between the fragments

$$\Psi(b_2^\dagger, \mathbf{R}) = \sum_J \frac{f_J(R)}{R} \mathcal{Y}_J(b_2^\dagger, \hat{R})$$

- The core-angular harmonic describes the relative rotation between the fragments

$$\mathcal{Y}_J(b_2^\dagger, \hat{R}) = \left[\varphi_J(b_2^\dagger) \otimes Y_J(\hat{R})\right]_0$$

The Coupled Channels Method II

- The α–daughter dynamics is governed by a stationary Schrödinger equation

$$H\Psi(b_2^\dagger, \mathbf{R}) = Q_\alpha \Psi(b_2^\dagger, \mathbf{R})$$

with a Hamiltonian containing three components: the kinetic term, the structure term of the daughter nucleus and the α–daughter interaction term

$$H = -\frac{\hbar^2}{2\mu}\nabla_{\mathbf{R}}^2 + H_D(b_2^\dagger) + V(b_2^\dagger, \mathbf{R})$$

- The interaction contains of a spherically-symmetric term and a deformed component [4]

$$V(b_2^\dagger, \mathbf{R}) = V_0(R) + V_d(b_2^\dagger, \mathbf{R})$$

[4] D.S. Delion *et al.*, Phys. Rev. **C** 73, 014315 (2006).

The Coupled Channels Method III

- The standard procedure of separating variables leads to a system of coupled differential equations for the radial functions

$$\frac{d^2 f_J(R)}{d\rho_J^2} = \sum_{J'} A_{JJ'}(R) f_{J'}(R) \ ,$$

$$\rho_J = k_J R, \ \ k_J = \frac{\sqrt{2\mu\left(Q_\alpha - E_J\right)}}{\hbar}$$

with the coupling matrix given by

$$\begin{aligned} A_{JJ'}(R) &= \left[\frac{J(J+1)}{\rho_J^2} + \frac{V_0(\xi_D, R)}{Q_\alpha - E_J} - 1\right]\delta_{JJ'} \\ &+ \frac{\langle \mathcal{Y}^{(J)} | V_d(\xi_D, \mathbf{R}) | \mathcal{Y}^{(J')} \rangle}{Q_\alpha - E_J} \end{aligned}$$

The Coupled Channels Method IV

- At large distances, the system is decoupled

$$\left[-\frac{d^2}{d\rho_J^2}+\frac{J(J+1)}{\rho_J^2}+\frac{\chi_J}{\rho_J}-1\right]f_J(\chi_J,\rho_J)=0$$

- The asymptotic solution is proportional in each channel with the outgoing Coulomb-Hankel wave

$$f_J(\chi_J,\rho_J)\rightarrow N_J H_J^{(+)}(\chi_J,\rho_J)$$

- χ_J is the channel-dependent Coulomb parameter

$$\chi_J=\frac{2Z_D Z_\alpha e^2}{\hbar v_J}\sim\frac{2Z_D Z_\alpha e^2}{\sqrt{Q_\alpha-E_J}},\ \ v_J=\frac{\hbar k_J}{\mu}$$

The Coupled Channels Method V

- The total decay width is a sum of partial widths supplied by the radial functions

$$\begin{aligned}\Gamma &= \sum_J \Gamma_J = \sum_J \hbar v_J \lim_{R\to\infty} |f_J(R)|^2 \\ &= \sum_J \hbar v_J |N_J|^2\end{aligned}$$

- The decay intensity:

$$\Upsilon_J = \log_{10} \frac{\Gamma_0}{\Gamma_J}$$

- The logarithm of the hindrance factor:

$$\log_{10} HF_J = \Upsilon_J^{exp} - \Upsilon_J^{teor}$$

The interaction potential[5] I

- The usual method of estimating the interaction consists in integrating a nucleon-nucleon potential over the fragment densities $\rho_{D,\alpha}$:

$$V(\mathbf{R},\omega_D) = \int d\mathbf{r}_D \int d\mathbf{r}_\alpha \rho_D(\mathbf{r}_D)\,\rho_\alpha(\mathbf{r}_\alpha)\,\nu(\mathbf{R}+\mathbf{r}_D-\mathbf{r}_\alpha)$$

- The resulting interaction is decomposed in spherically-symmetric and deformed components:

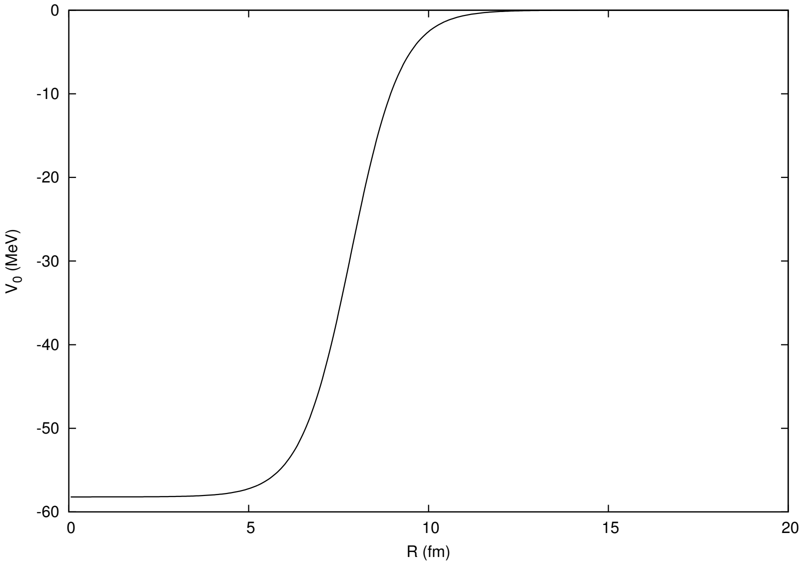
$$V(\mathbf{R},\omega_D) = V_0(R) + V_d(\mathbf{R},\omega_D)$$

- The deformed term follows from the multipole expansion of the nuclear densities:

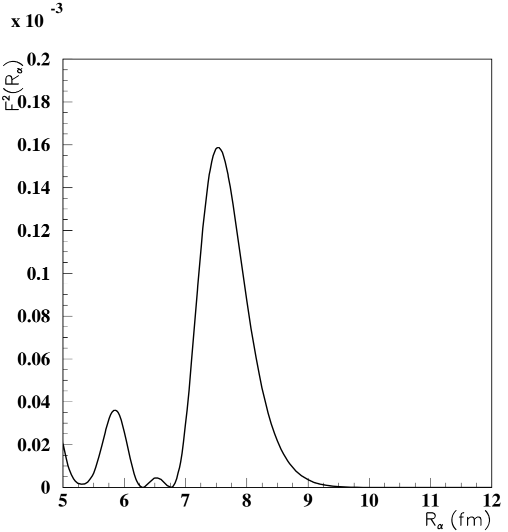
$$V_d(\mathbf{R},\omega_D) = \sum_{\lambda>0} V_\lambda(R)\,\mathcal{Y}_\lambda(\omega_D,\omega_\alpha)$$

[5] F. Cârstoiu, R. J. Lombard, Ann. Phys. **217**, 279 (1992).
G. Bertsch *et al.*, Nucl. Phys. A **284**, 399 (1977).

The Woods-Saxon Potential



The α-particle formation probability[6]



[6] D.S. Delion, Theory of Particle and Cluster Emission, Springer-Verlag, Berlin, 2010.

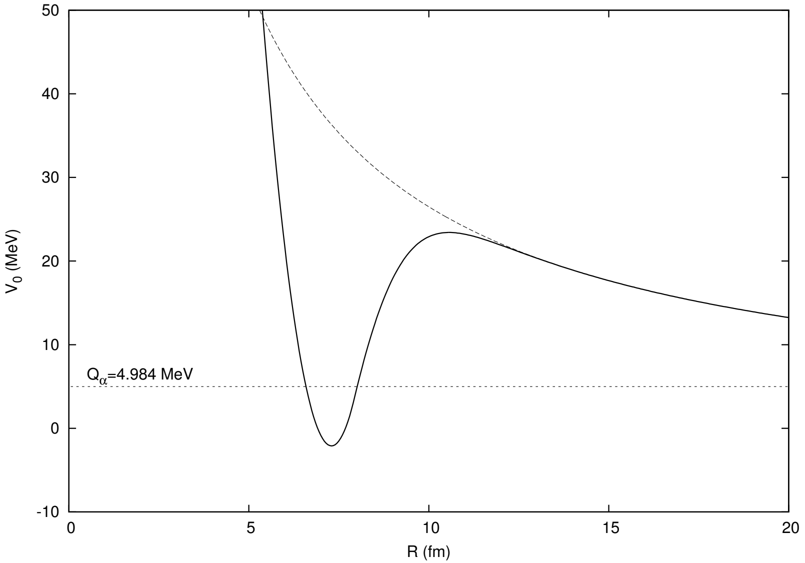
The interaction potential II

- The monopole component:

$$\begin{aligned} V_0(R) &= v_a \overline{V}_0(R), \; R > R_m \\ &= a(R - R_{min})^2 - v_0, \; R \leq R_m \end{aligned}$$

- $\overline{V}_0$ is obtained through the double folding integration of the nucleon-nucleon M3Y plus Coulomb force.
- v_a is a quenching factor that be used to reproduce the total half-life.
- a is the harmonic oscillator strength, fixed at $a = 50\ \mathrm{MeV}$.
- v_0 is the minimum of the oscillator potential; it is used to fix the experimental reaction energy Q_α.

The interaction potential III



The interaction potential IV

- The deformed term[7] is given by the quadrupole-quadrupole component (QQ)

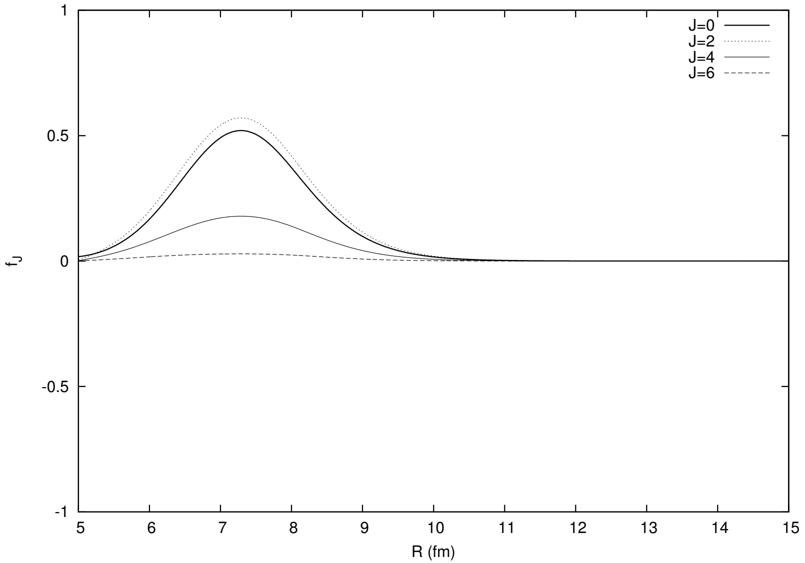
$$V_d(b_2^\dagger, \mathbf{R}) = -C_0(R - R_{min})\frac{dV_0(R)}{dR}\sqrt{5}\left[Q_2(b_2^\dagger) \otimes Y_2(\hat{R})\right]_0$$

- C_0 is the interaction strength
- Q_2 is the quadrupole operator of the daughter nucleus
- Y_2 is the spherical harmonic representing the α-particle
- The effective strength of the QQ coupling is a linear function of deformation

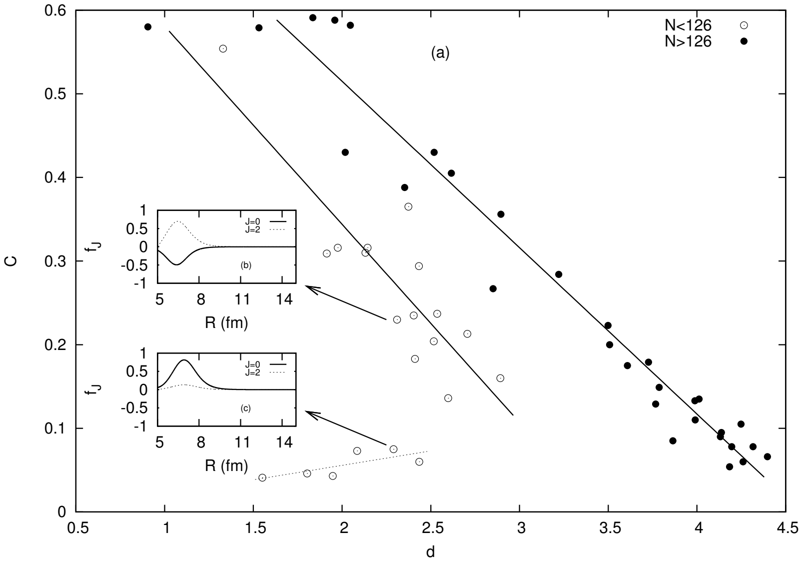
$$C = C_0\left(1 - \sqrt{\frac{2}{7}}a_\alpha d\right)$$

[7] J. M. Eisenberg, W. Greiner, *Nuclear Theory, Vol. I*, North-Holland Publishing Company, 1975.

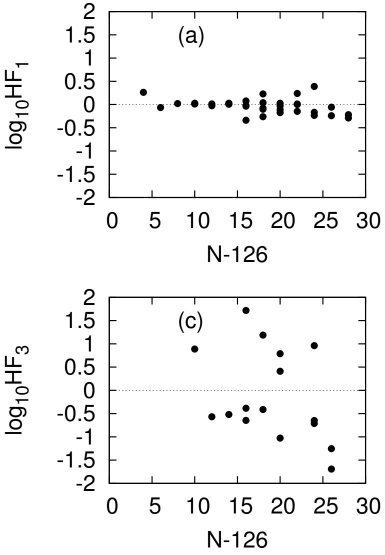
The radial functions



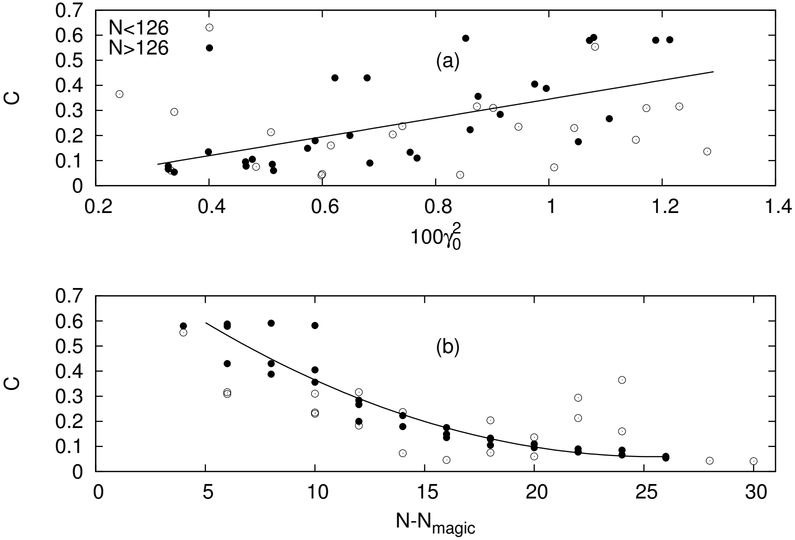
The QQ interaction strength versus deformation



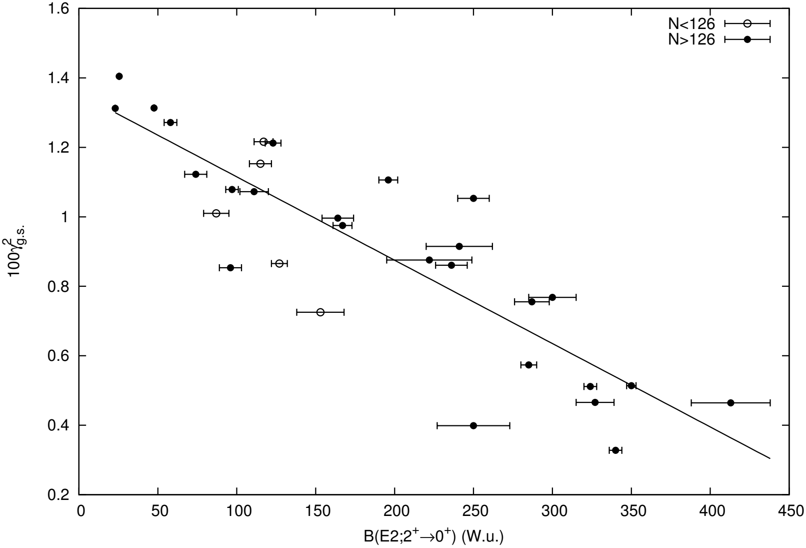
Hindrance factors versus neutron number, in the actinides series



The physical interpretation of the QQ strength



The reduced decay width versus the reduced transition probability



Thank you!