Electromagnetic Decays of the π^0

Esther Weil

Gernot Eichmann, Christian Fischer and Richard Williams

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Fairness Conference 2017, Sitges

Justus-Liebig-Universität Gießen

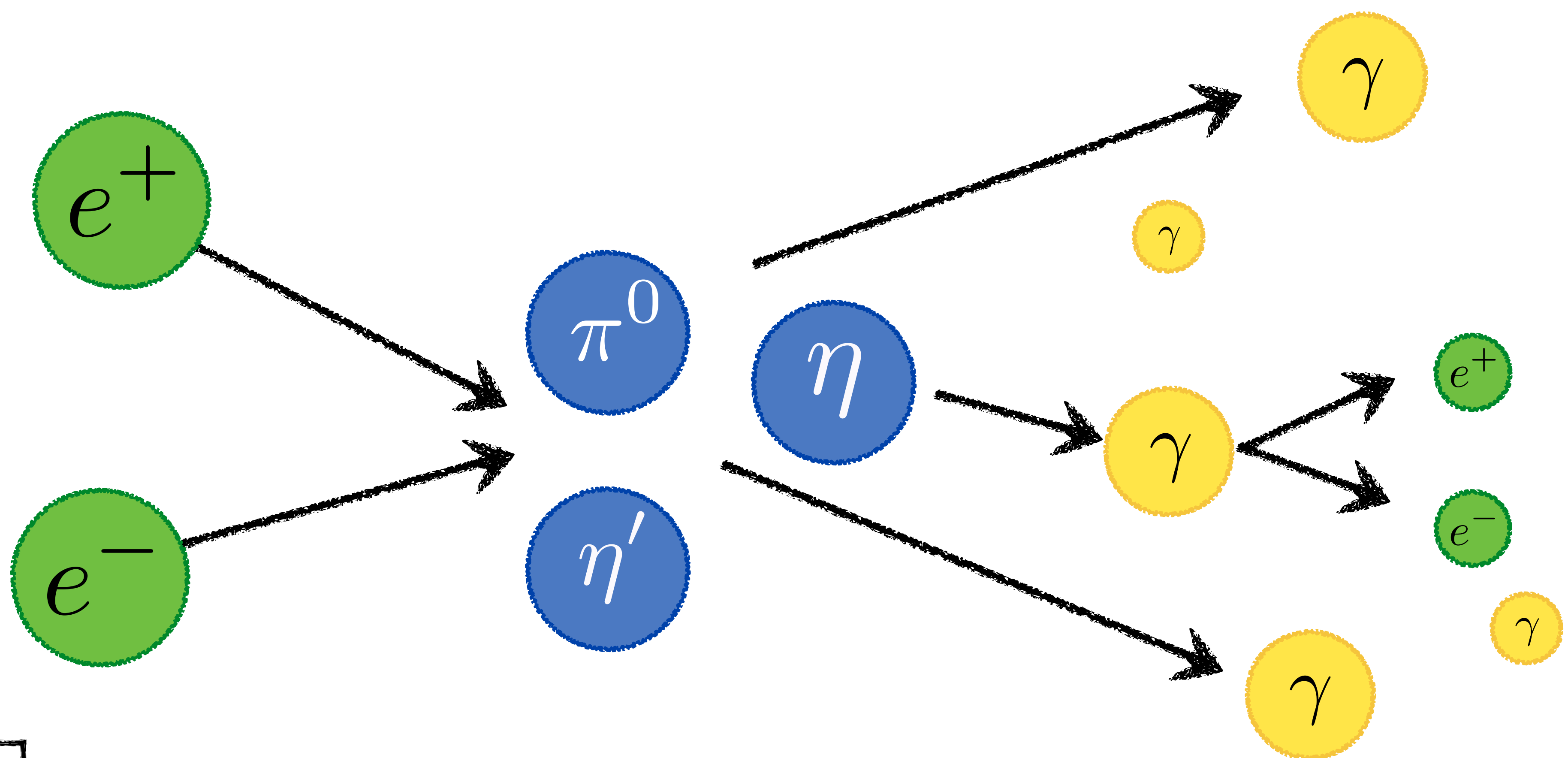
Institut für Theoretische Physik

based on recent publications

G. Eichmann et al. (2017), **arXiv:1704.05774** [hep-ph].

E. Weil et al. (2017), **arXiv:1704.06046** [hep-ph].

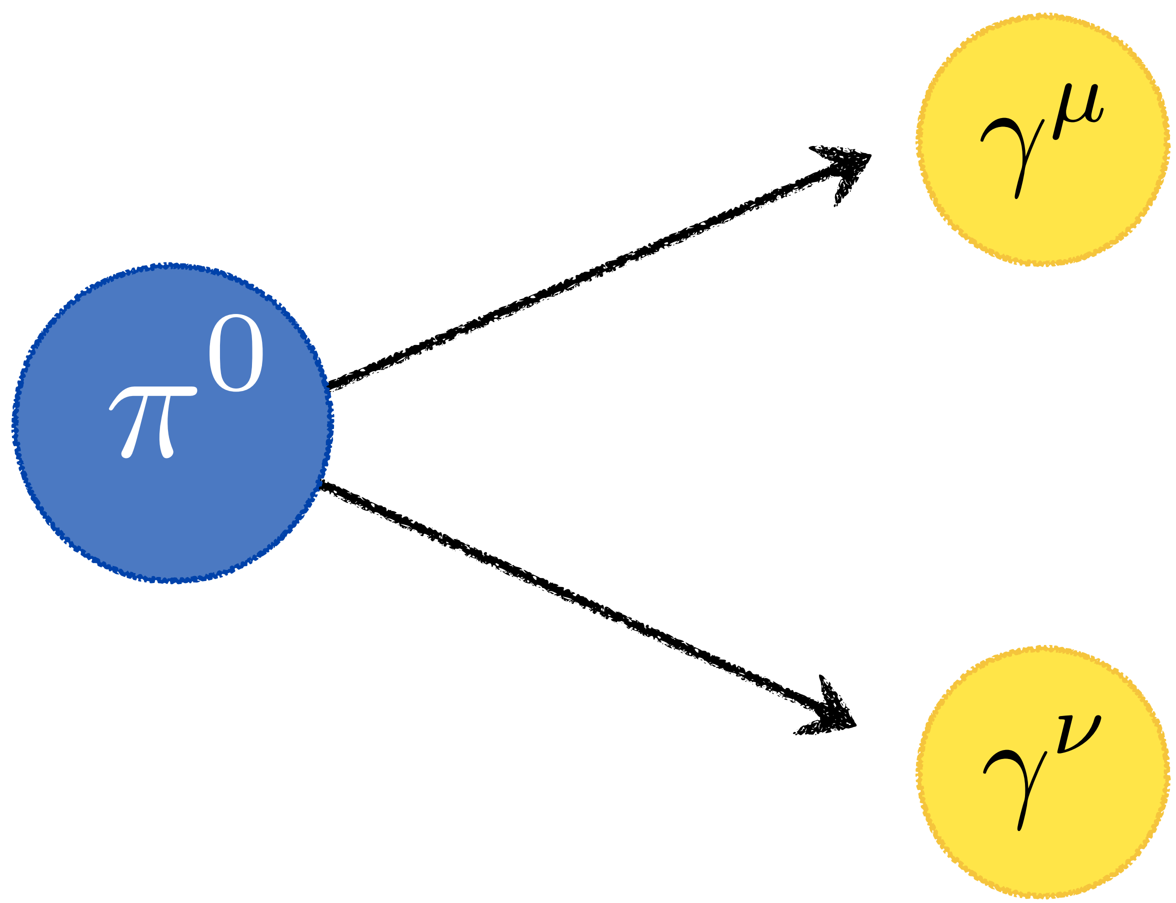
Transition form factor



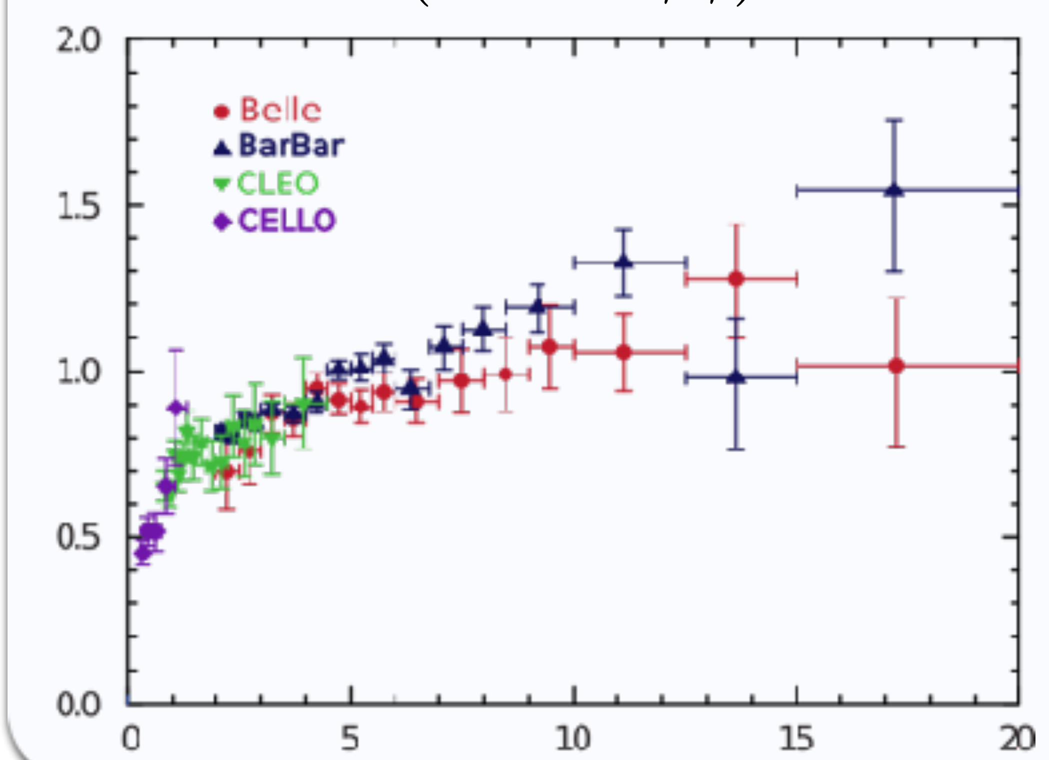
eg. Belle

e^+e^--collider

Transition form factor



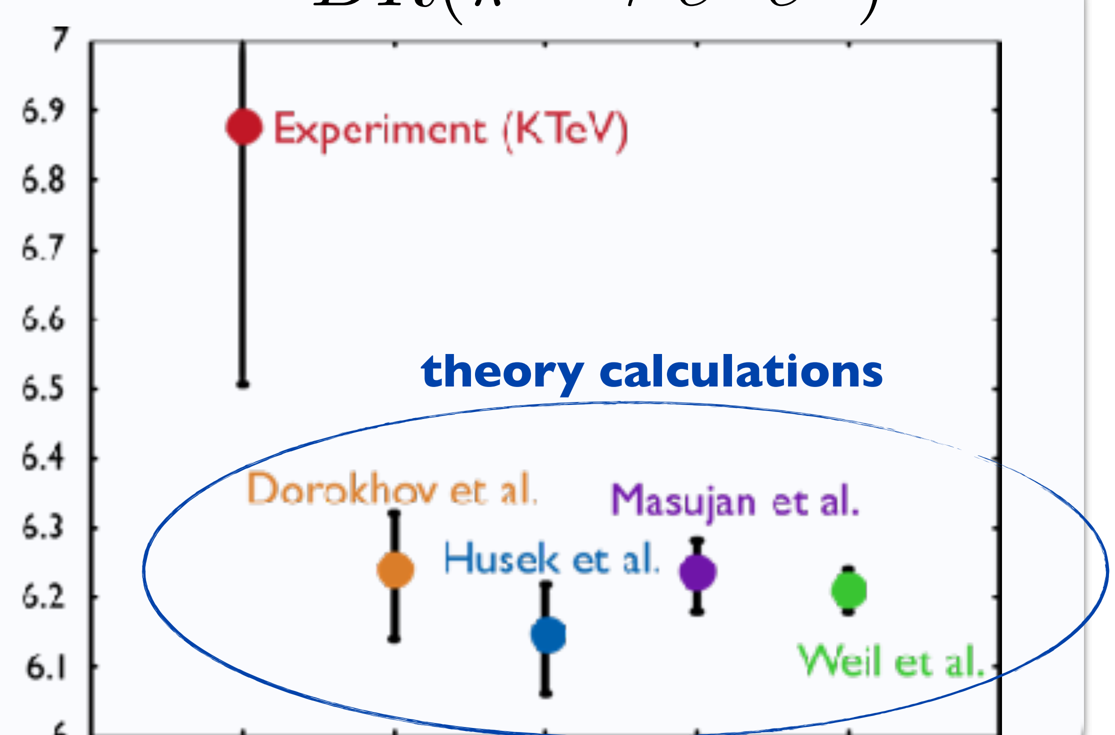
$\Gamma(\pi^0 \to \gamma\gamma)$



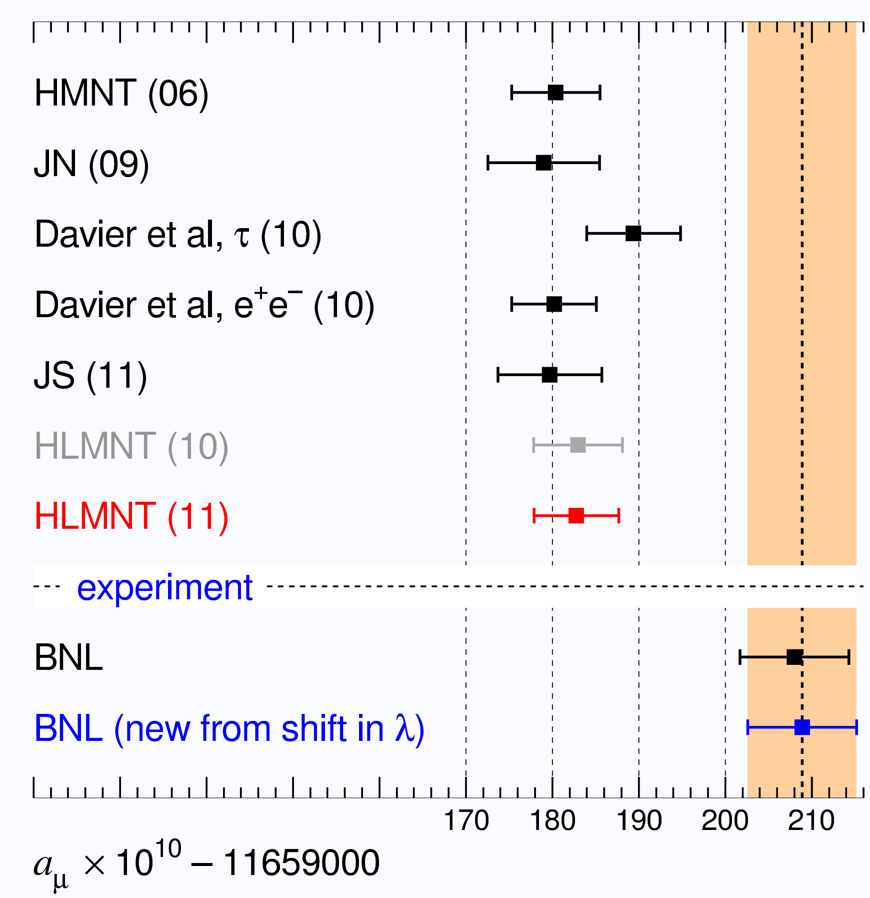
Motivation

discrepancies of experiment and theory ~ 2σ

$BR(\pi^0 \to e^+e^-)$



μ (g-2)



A. Anastasi (2017), **DOI** 10.1051/epjconf/201714201002.

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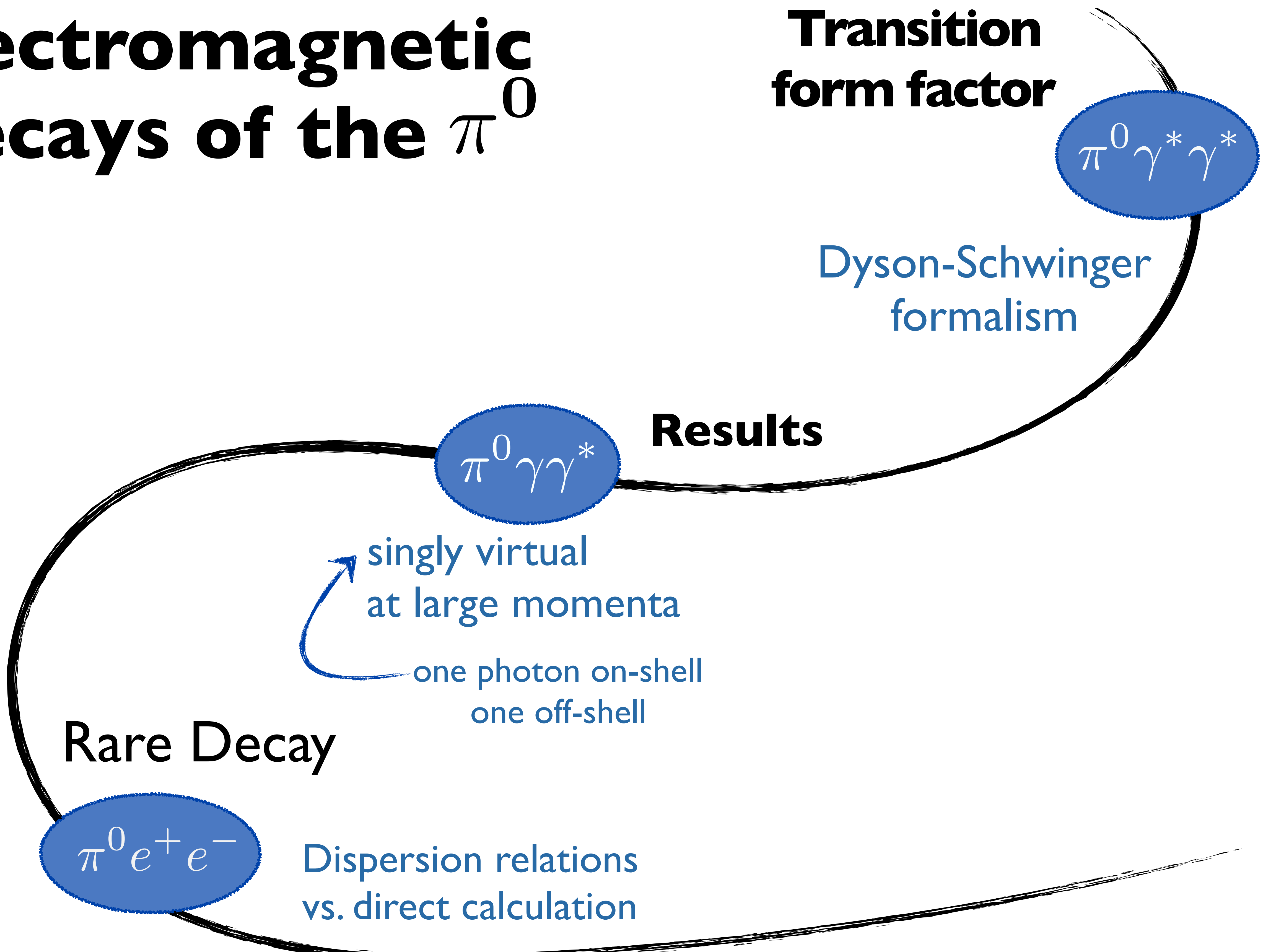
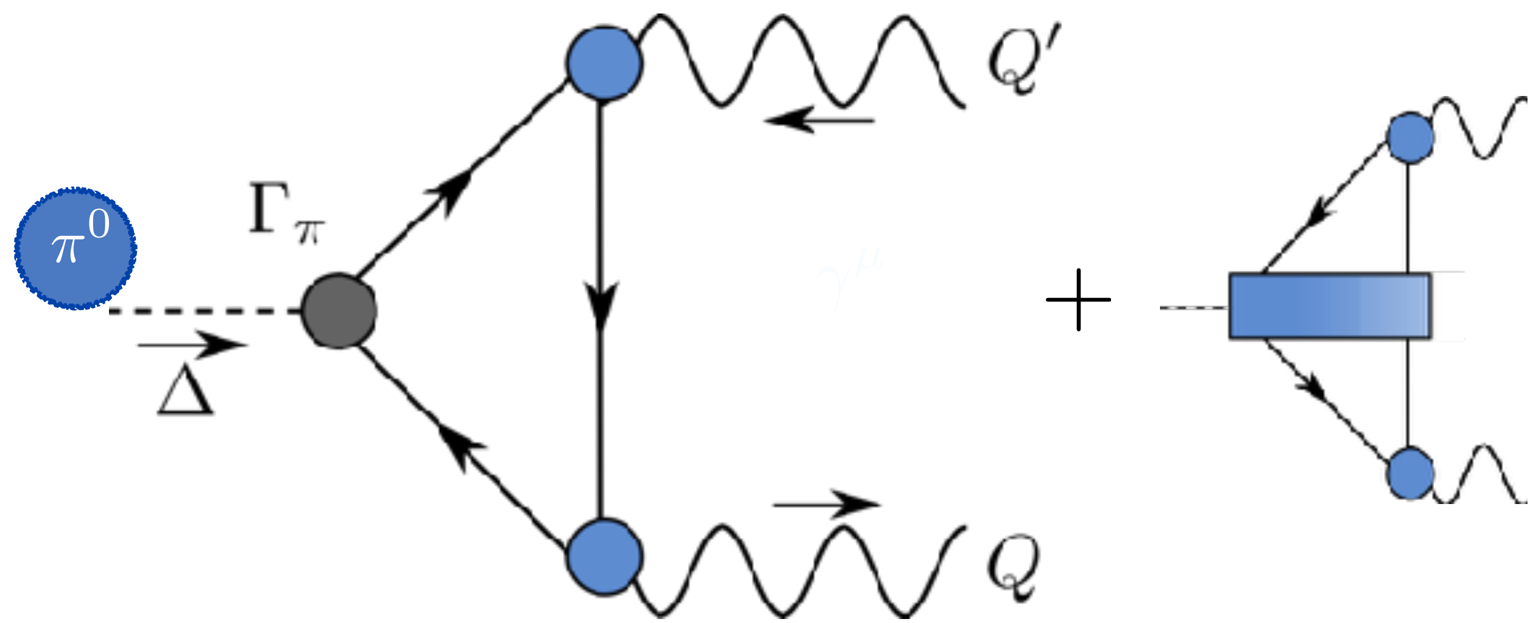


Transition form factor $\Lambda^{\mu\nu} = \langle \gamma^\mu \gamma^\nu | \pi^0 \rangle$ (TFF)



The Tensor decomposition is given by

$$\Lambda^{\mu\nu}(Q, Q') = e^2 \frac{F(Q^2, Q'^2)}{4\pi^2 f_\pi} \varepsilon^{\mu\nu\alpha\beta} Q'^\alpha Q^\beta$$

SU(3) flavor anomaly

ADLER, BELL, JACKIW, 1969

From the anomaly we know

$$F(0,0) = 1$$

for $m_\pi = 0$ and on-shell photons

$$\boxed{\Gamma(\pi^0 \to \gamma\gamma) = 7.77\mathrm{eV}}$$

Experimental value

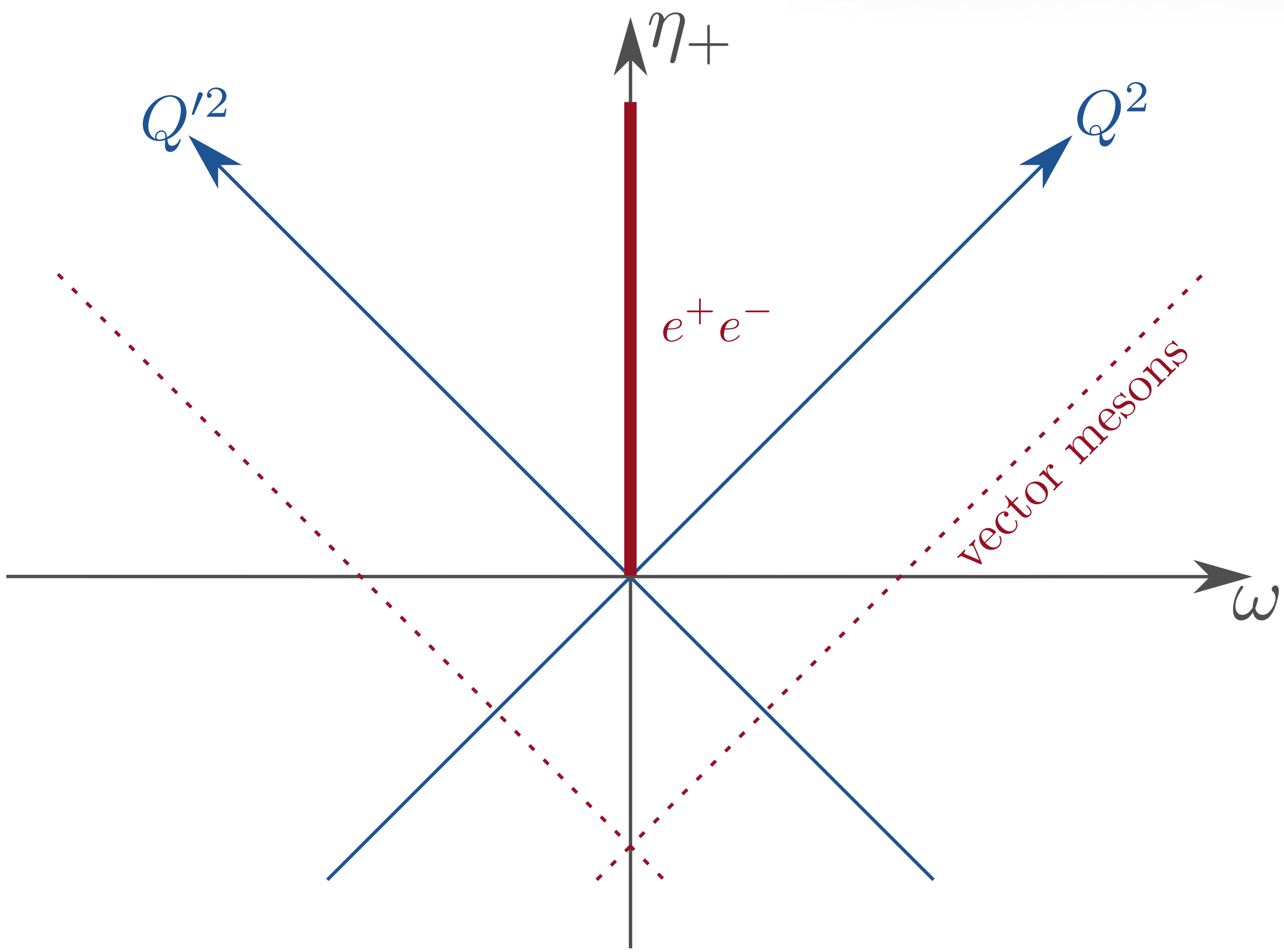
$$\Gamma_{\exp}(\pi^0 \to \gamma\gamma) = 7.8\mathrm{eV}$$

Want to know TFF for all kinematic regions in Q^2 and Q'^2 (photon momenta)

Transition form factor $\Lambda^{\mu\nu} = \langle \gamma^\mu \gamma^\nu | \pi^0 \rangle$

kinematic domains:

$$\eta_+ = \frac{Q^2 + Q'^2}{2} \quad \omega = \frac{Q^2 - Q'^2}{2}$$



space-like region: $Q^2 > 0\,,\ Q'^2 > 0$

- singly-virtual limit for large Q^2, which is measured in experiment.
- symmetric limit $Q^2 = Q'^2$ for the rare decay $(\pi^0 \to e^+e^-)$

time-like region: $Q^2 < 0\,,\ Q'^2 < 0$

- contains physical singularities, feels enhancement of the vector-meson poles.

What can we access using the **Dyson-Schwinger formalism**?

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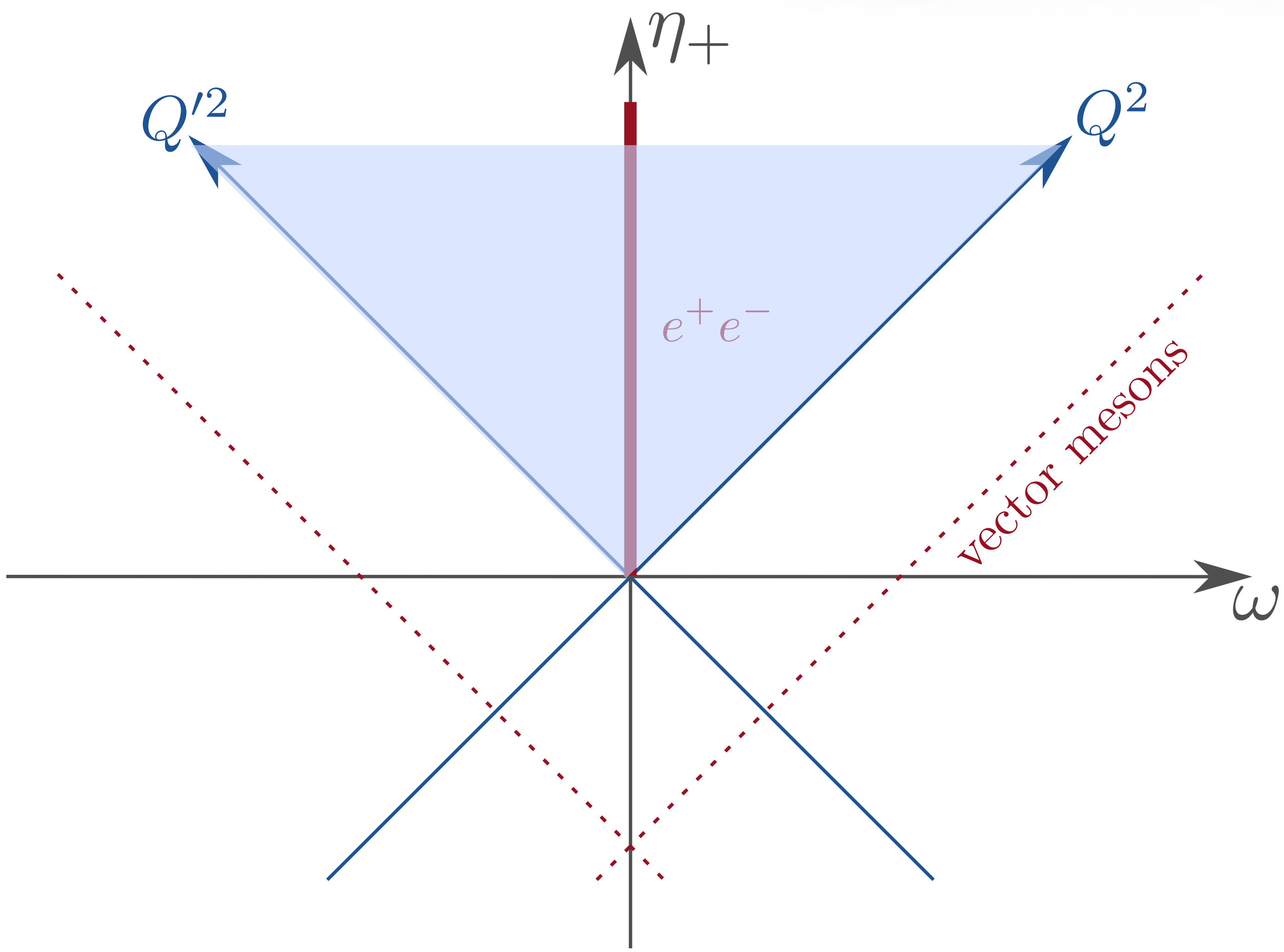

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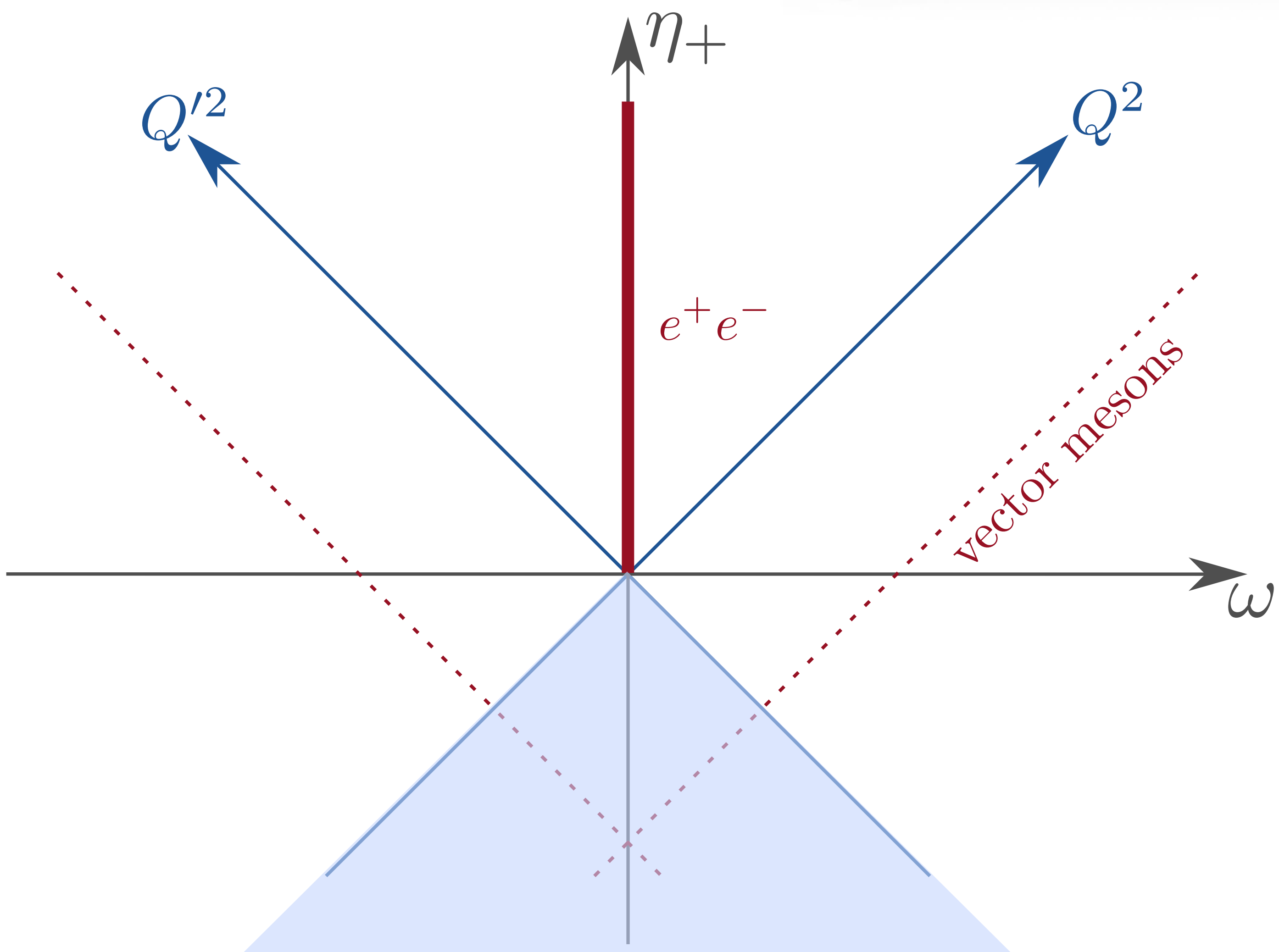

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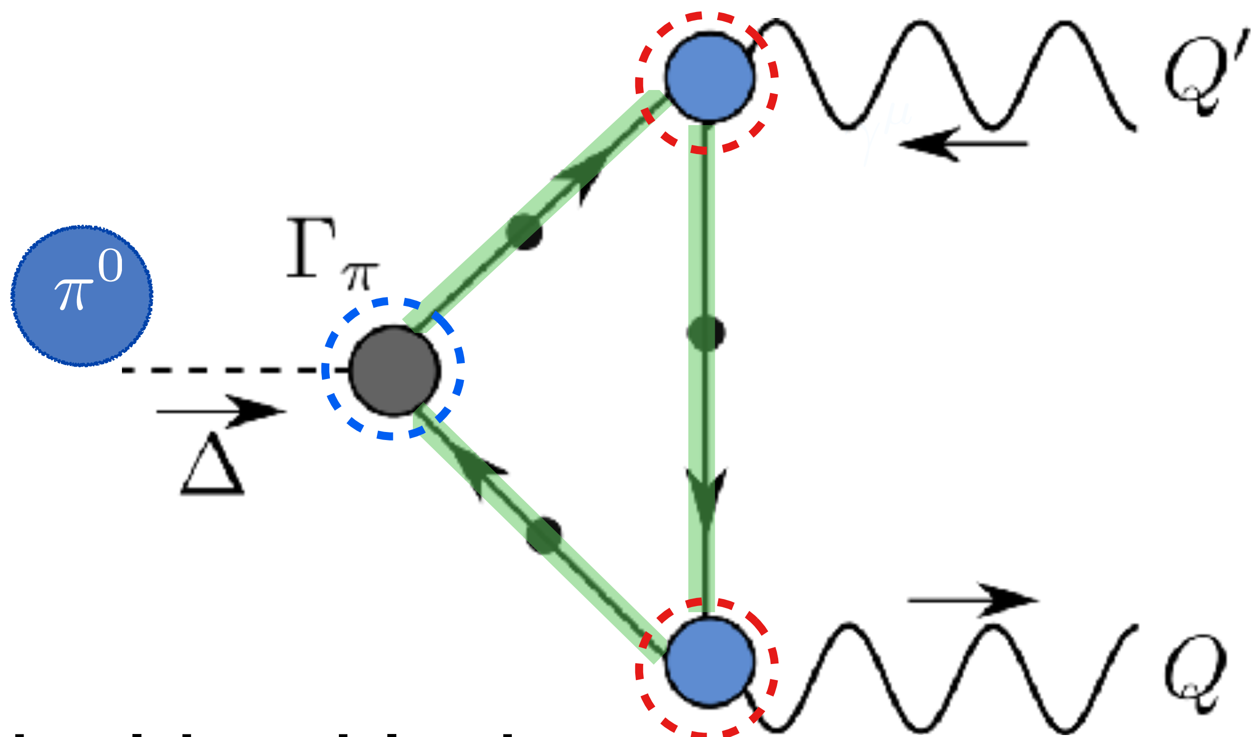
The Dyson-Schwinger approach

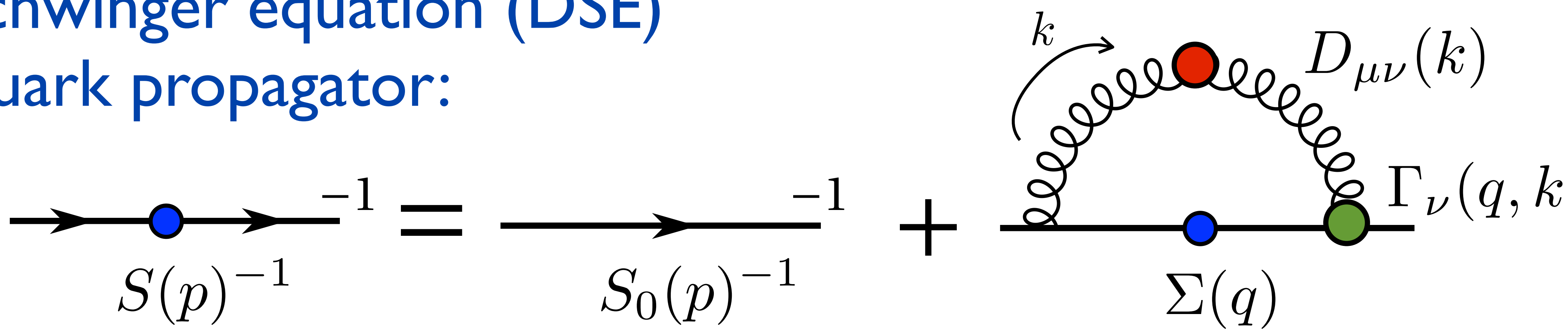
$$\Lambda^{\mu\nu}(Q,Q') = e^2 \frac{F(Q^2,Q'^2)}{4\pi^2 f_\pi} \varepsilon^{\mu\nu\alpha\beta} Q'^{\alpha} Q^{\beta}$$



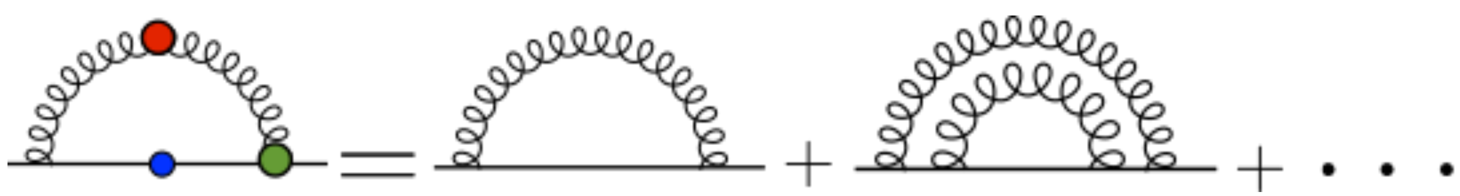
We need these building blocks

- Quark propagator
- Pion amplitude
- Quark-photon vertex

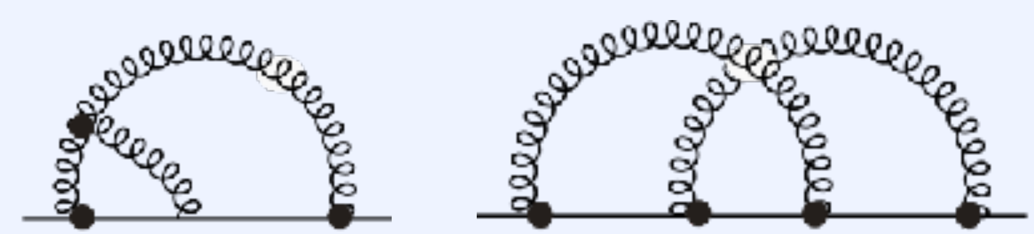
Dyson-Schwinger equation (DSE) for the quark propagator:



(a) Perturbative meaning



Are these included?



(b) non-perturbative origin

effective action

$$\Gamma[\tilde{\varphi}] = \ln(Z[J]) + \int_x \tilde{\varphi}(x) J(x)$$

The invariance of the action under

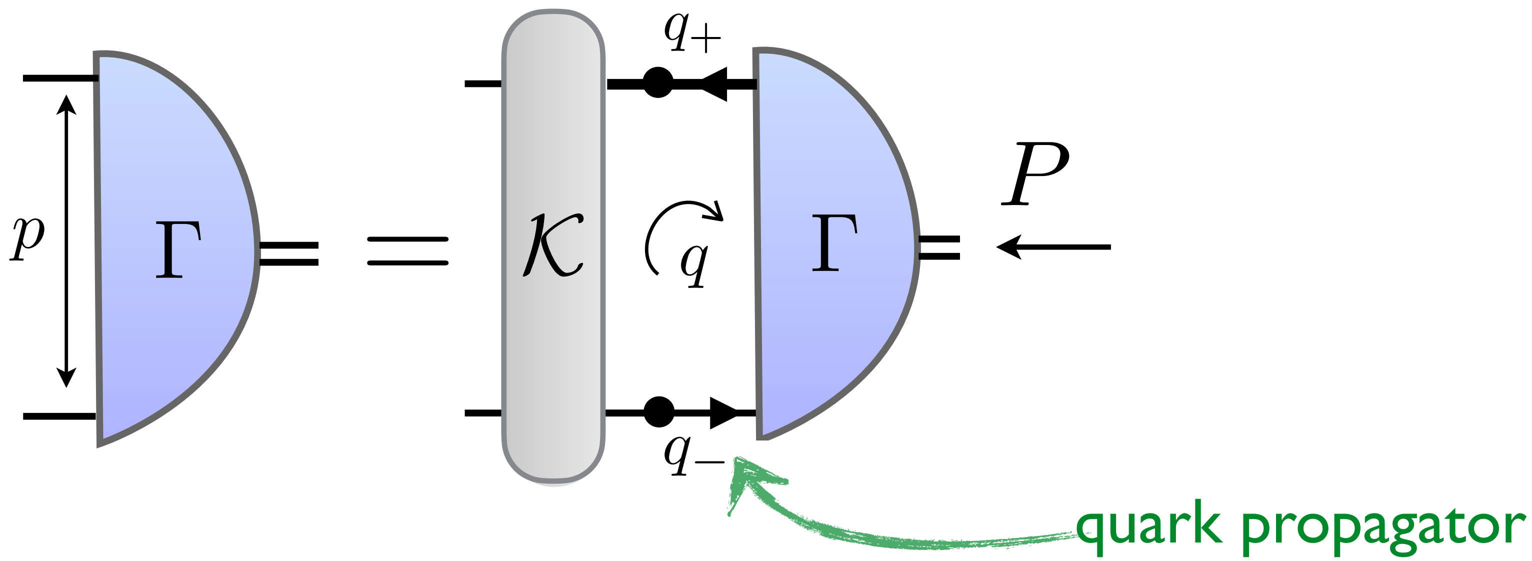
$$\varphi(x) \to \varphi(x) + \varepsilon(x)$$

DSE

$$\frac{\delta \Gamma[\tilde{\varphi}]}{\delta \tilde{\varphi}(x)} = \frac{\delta}{\delta \varphi} S \left[\tilde{\varphi}(x) + \int_y \Gamma''[\tilde{\varphi}]^{-1}_{xy} \frac{i\delta}{\delta \tilde{\varphi}(y)} \right]$$

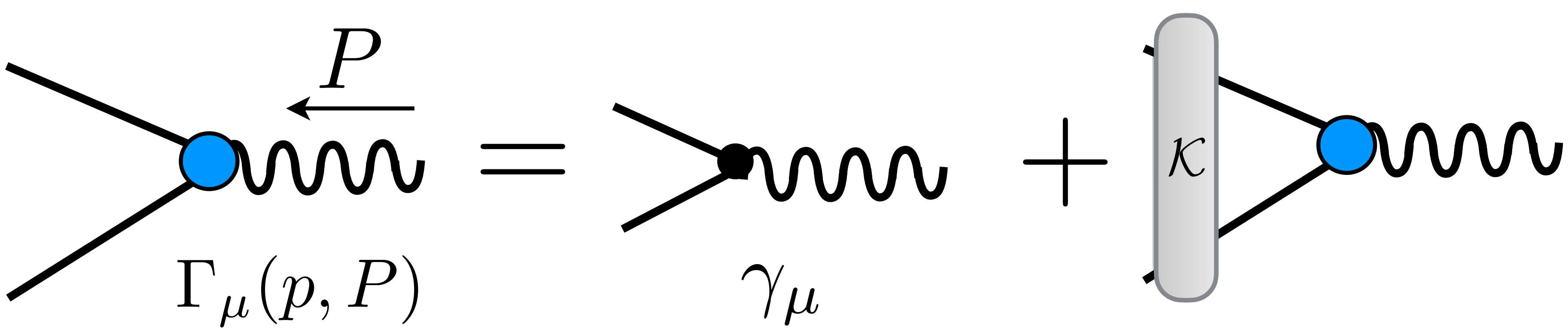
Bethe-Salpeter(BSE) equation for a bound-states (meson):

- **4** tensor structures

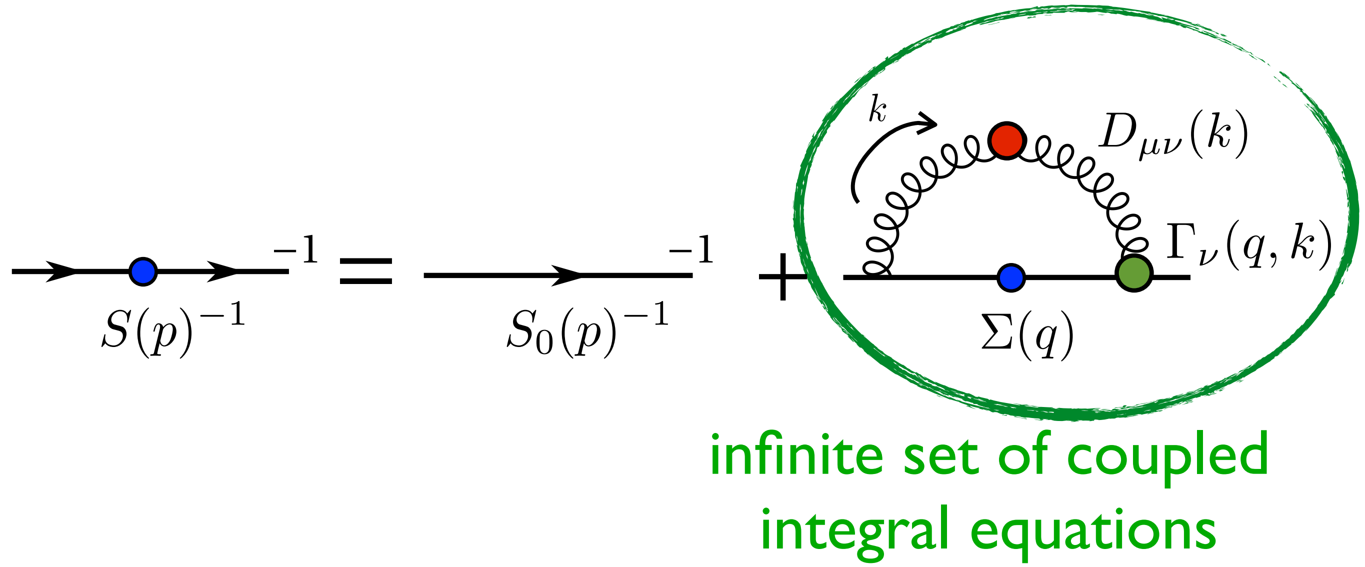


inhomogeneous BSE for quark-photon vertex (QPV):

- **12** tensor structures



- Vector meson poles are included directly in the QPV through solving the BSE

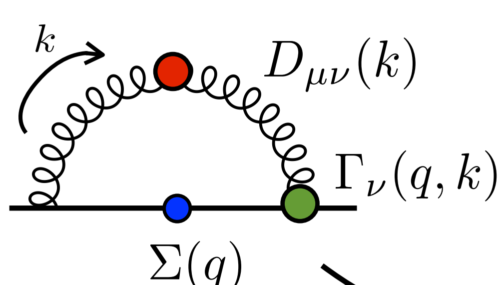


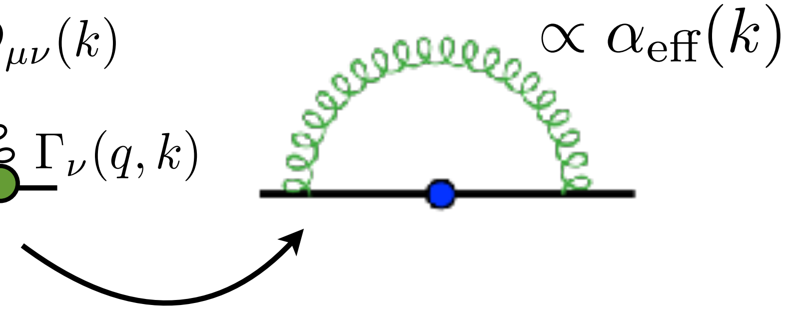
Truncate, but how?

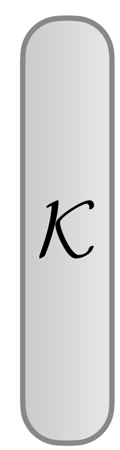
Quark self-energy $\Sigma(q)$ and the quark-anti-quark kernel $\mathcal{K}$ are related through an identity (AVWTI)

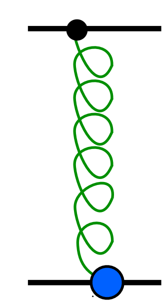
Rainbow-ladder truncation

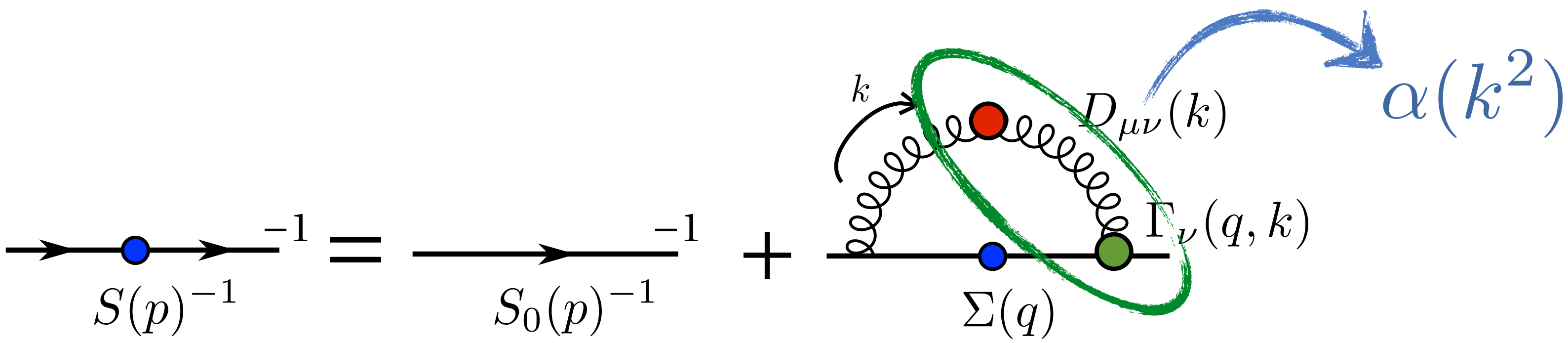
to ensure chiral symmetry, we need to choose accordingly

$\Sigma = \frac{\partial \Gamma}{\partial S} =$ 





$= \frac{\partial \Sigma}{\partial S} =$ 



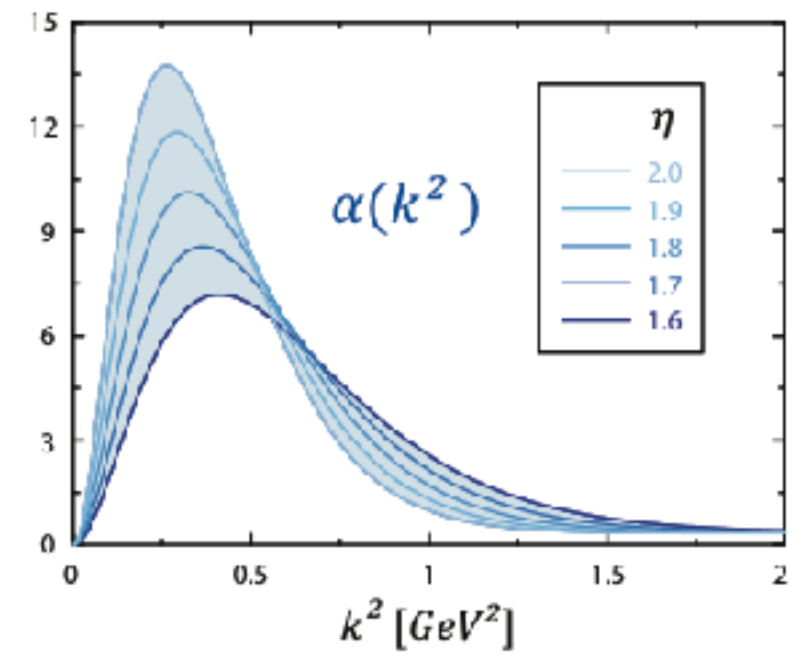
Combine gluon with quark-gluon vertex:

effective coupling

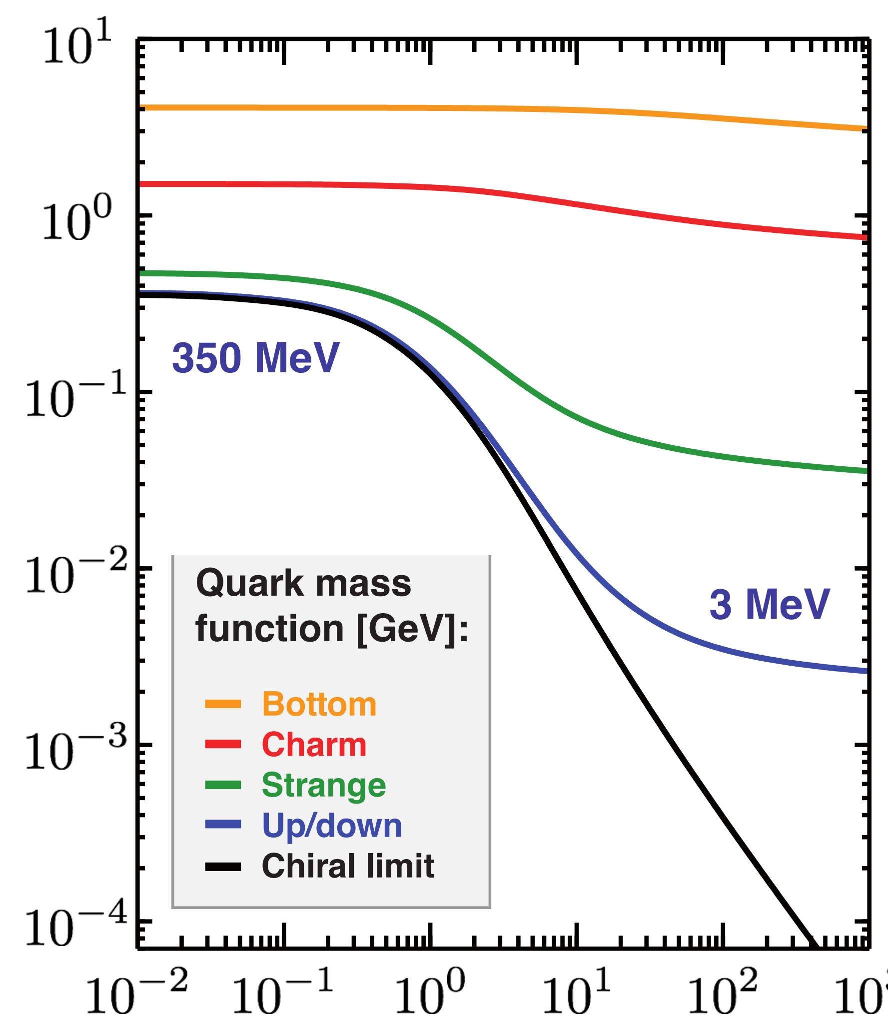
$$\alpha(k^2) = \alpha_{\rm IR}(k^2/\Lambda^2, \eta) + \alpha_{\rm UV}(k^2)$$

$$\Lambda = 0.74 \quad \eta = 1.85 \pm 0.2$$

- scale Λ is adjusted to observables, like f_π .
- masses m_u=m_d, m_s from m_π, m_K
- $\alpha_{\rm UV}$ from perturbation theory
- parameter η : band of results



Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)



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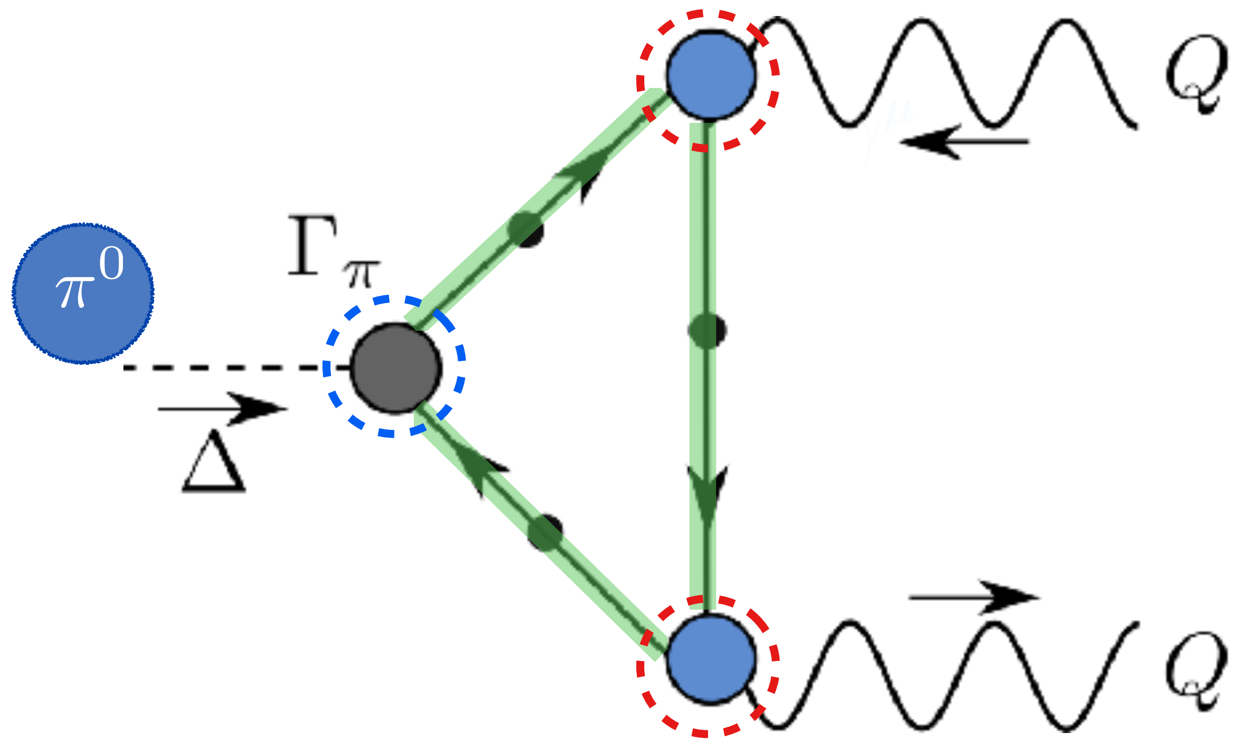

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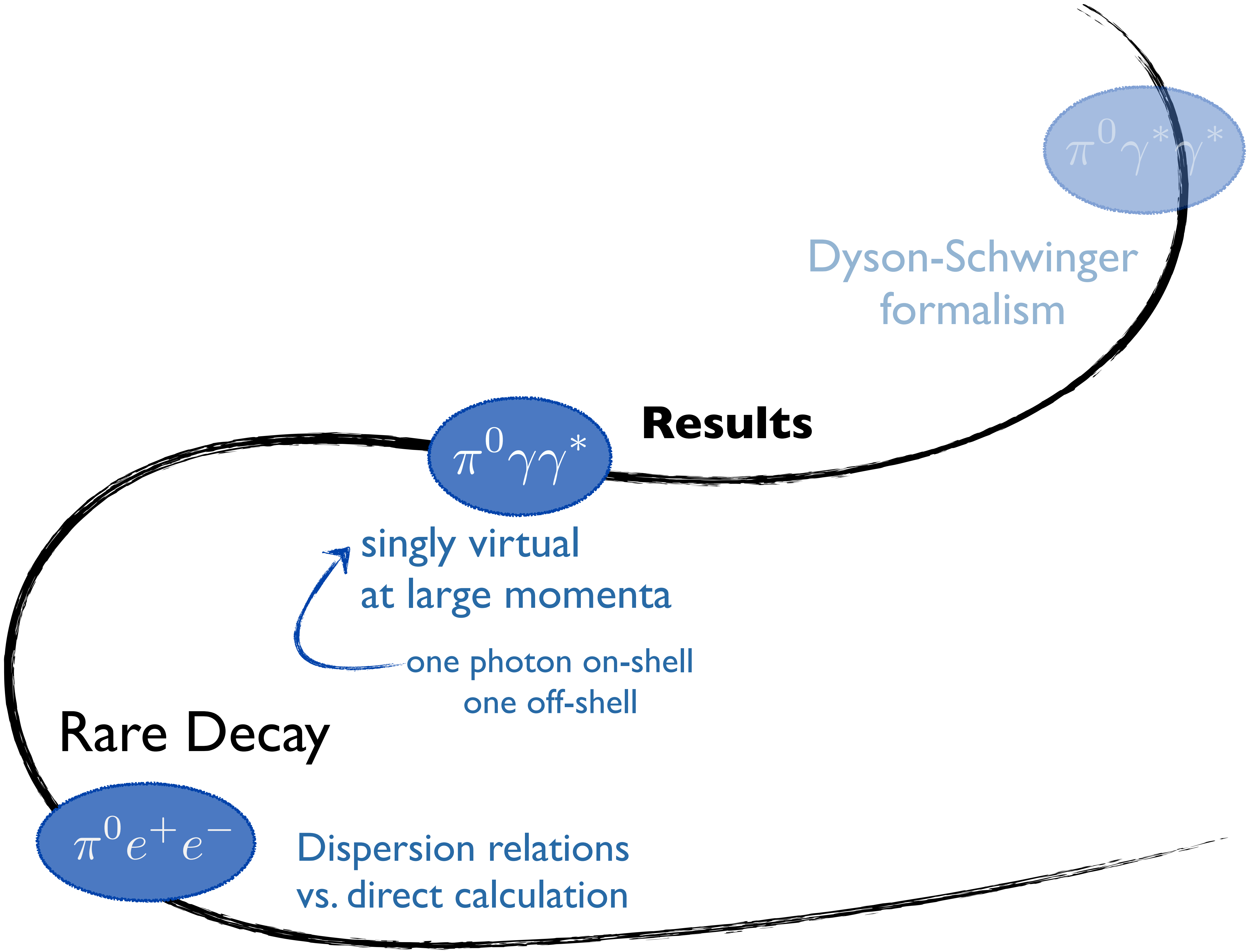
Ready to calculate form factor


$\pi^0\gamma^*\gamma^*$
Dyson-Schwinger
formalism
Results
$\pi^0\gamma\gamma^*$
singly virtual
at large momenta
one photon on-shell
one off-shell
Rare Decay
$\pi^0 e^+ e^-$
Dispersion relations
vs. direct calculation

What can be done using the
Dyson-Schwinger formalism?

and what are our limitations?

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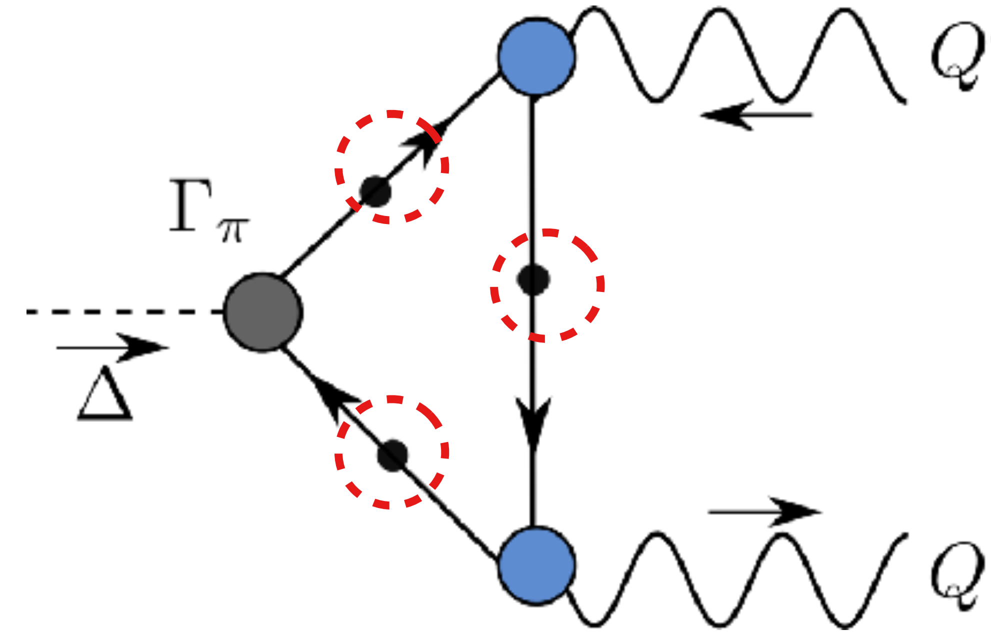
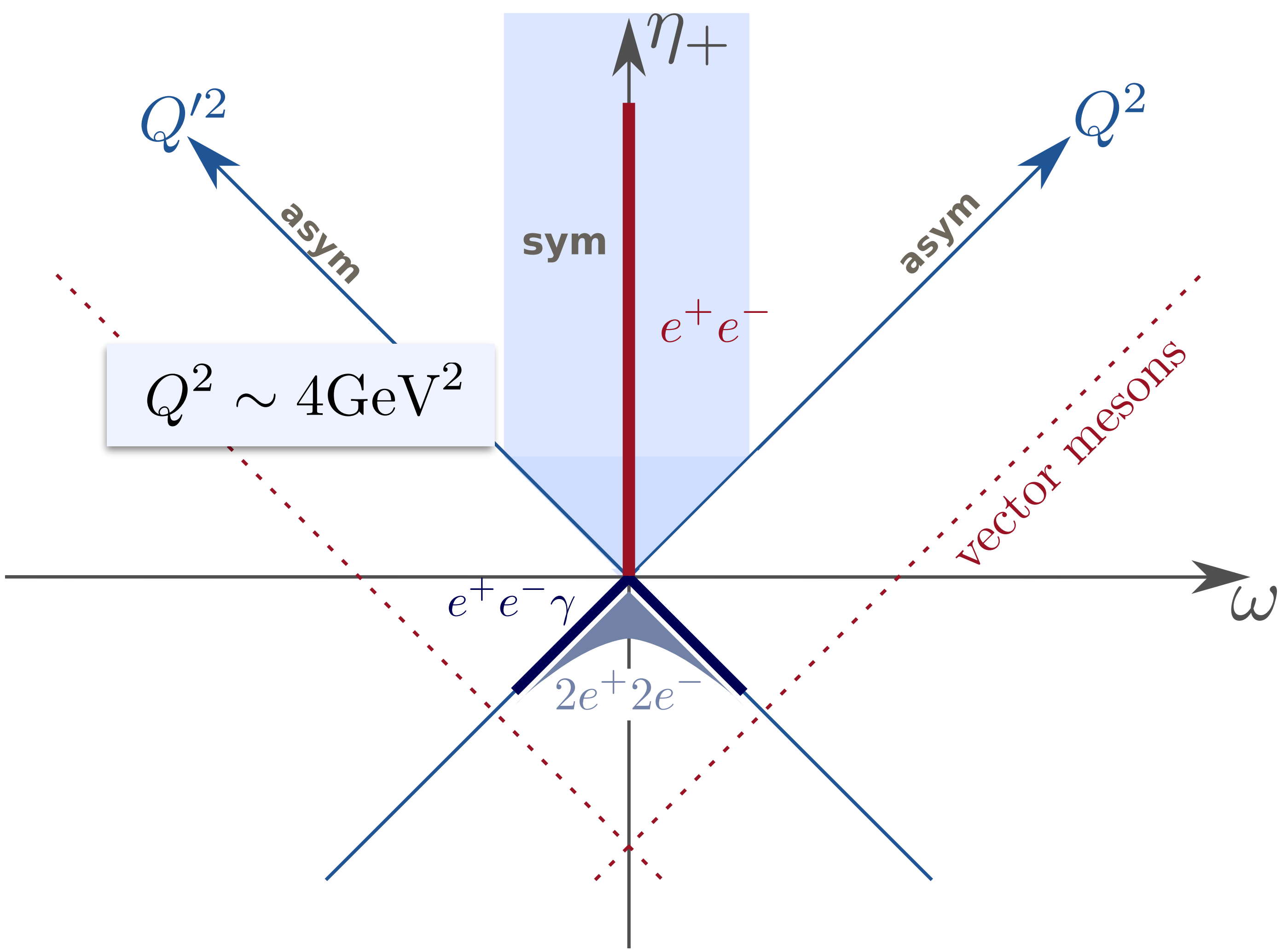
Dyson-Schwinger formalism?
and what are our limitations?

Caveats: kinematic regions are limited by the poles of the different blocks





- symmetric limit for all Q^2
- asymmetric till $Q^2 \sim 4\text{GeV}^2$

- **Dispersion relation**

M. Hoferichter et al.(2014), **arXiv:1410.4691**[hep-ph]

- **Lattice**

estimate of the continuum limit

A. Geradin et al. (2016), **arXiv:1508.07178** [hep-lat]

singly-virtual up to $\sim 2\text{GeV}^2$

Scaling Brodsky-Lepage (BL) scaling limit tells us

$$\eta_+ = \frac{Q^2 + Q'^2}{2}$$

$$\widetilde{F}(Q^2, Q'^2) = \frac{\eta_+ F(Q^2, Q'^2)}{4\pi^2 f_\pi^2} \xrightarrow{\eta_+ \to \infty} j(\omega)$$

$$\omega = \frac{Q^2 - Q'^2}{2}$$

is reproduced

symmetric case $j(0) = \frac{2}{3}$

asymmetric case $j(\infty) = 1$

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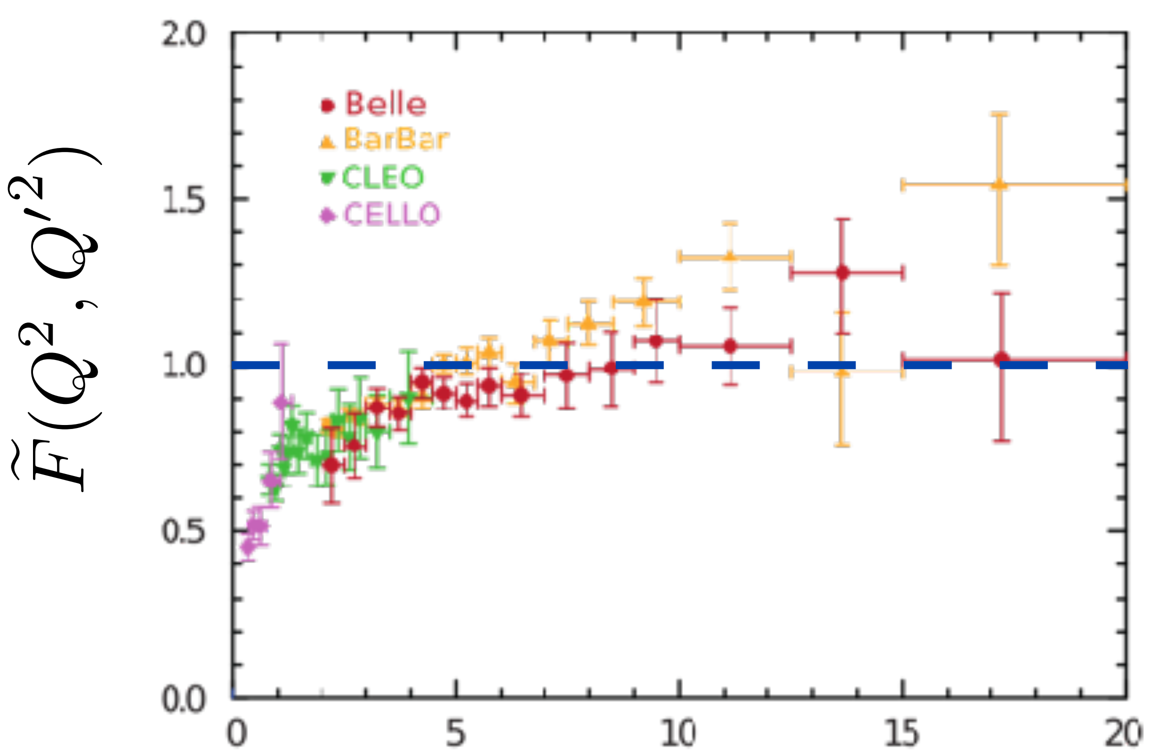
singly virtual form factor: $F(0, Q^2)$

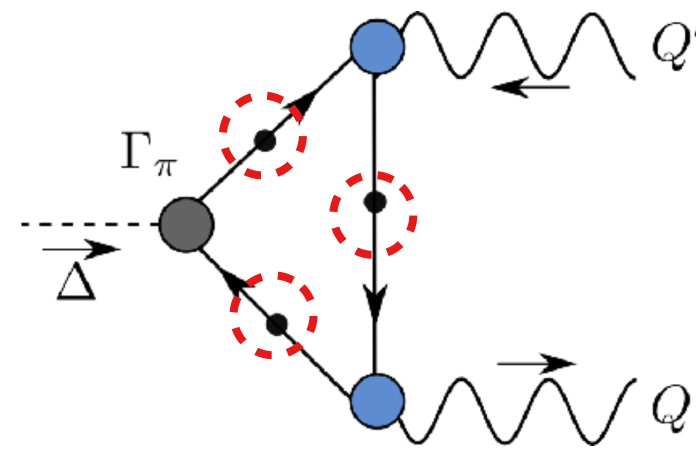


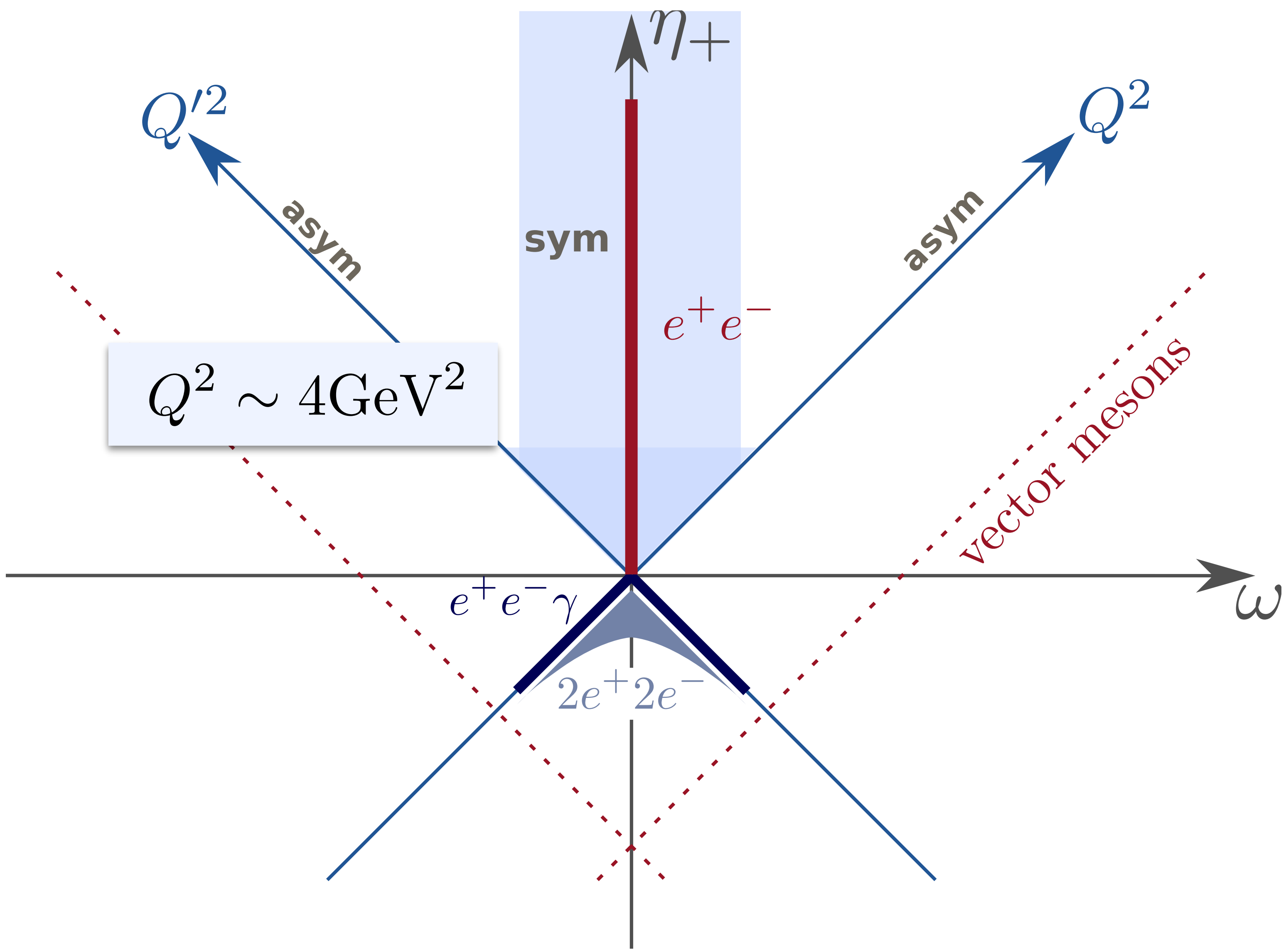
Does the limit hold?

- Experimental data indicated scale violation
- non-perturbative interaction missing?

Work around our limitations

For $Q^2 \gtrsim 4\mathrm{GeV}^2$ singularities from the quark propagator appear in the integrand making a direct calculation difficult.





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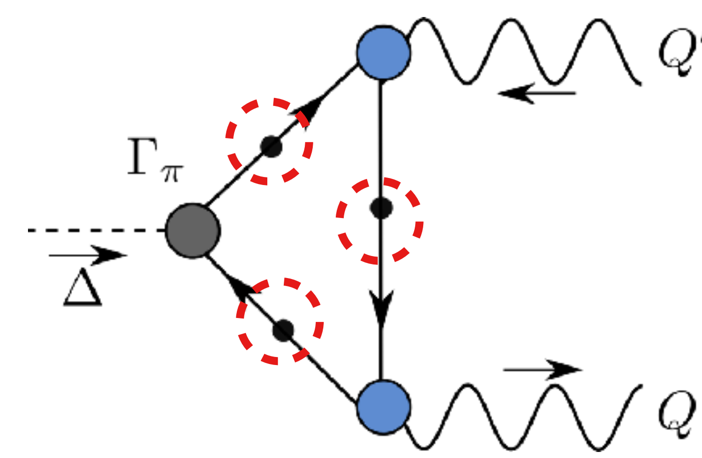
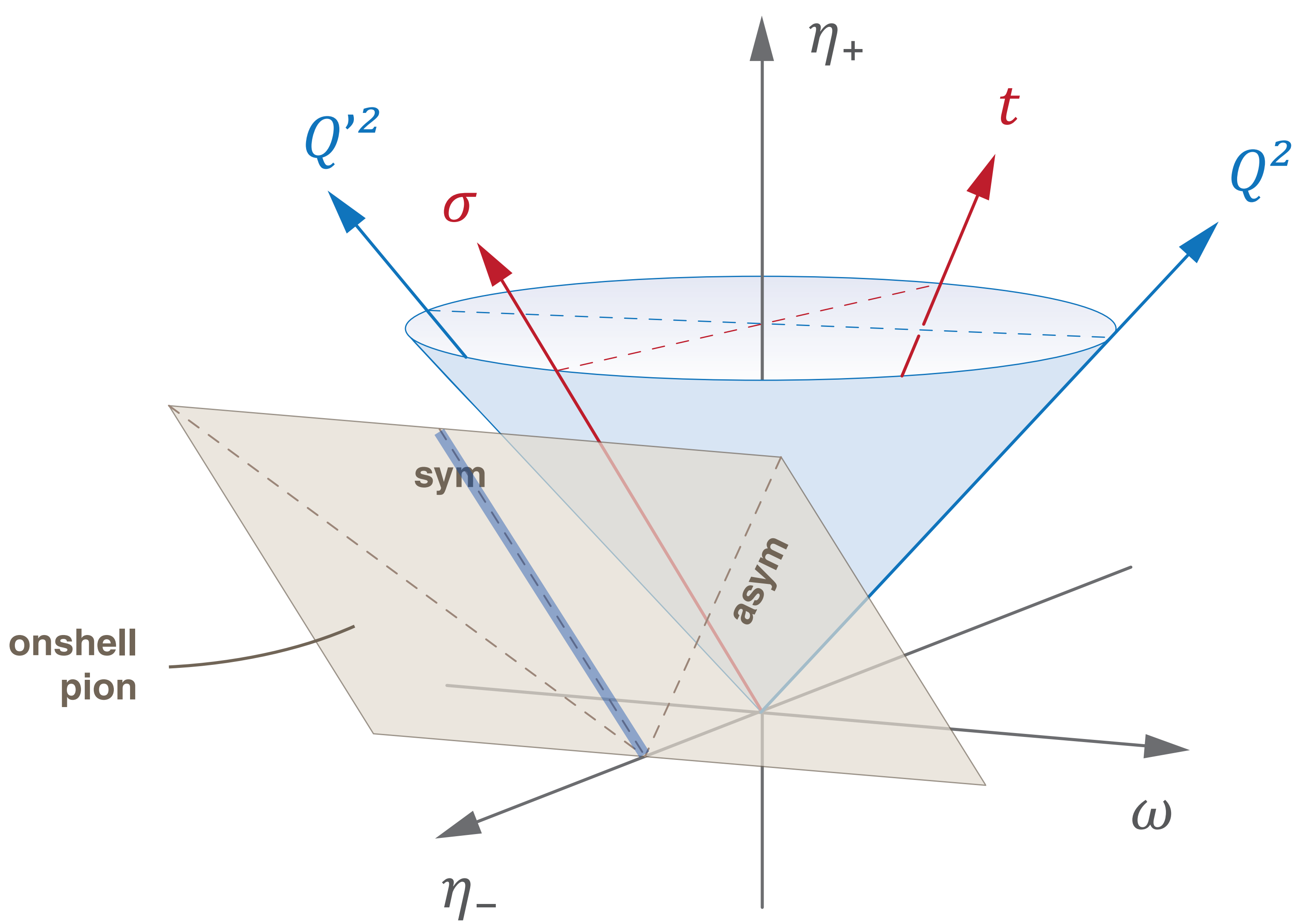




$$\eta_+ = \frac{Q^2 + Q'^2}{2} \qquad \omega = \frac{Q^2 - Q'^2}{2}$$

$$\eta_- = Q \cdot Q' \qquad t = \Delta^2/4$$

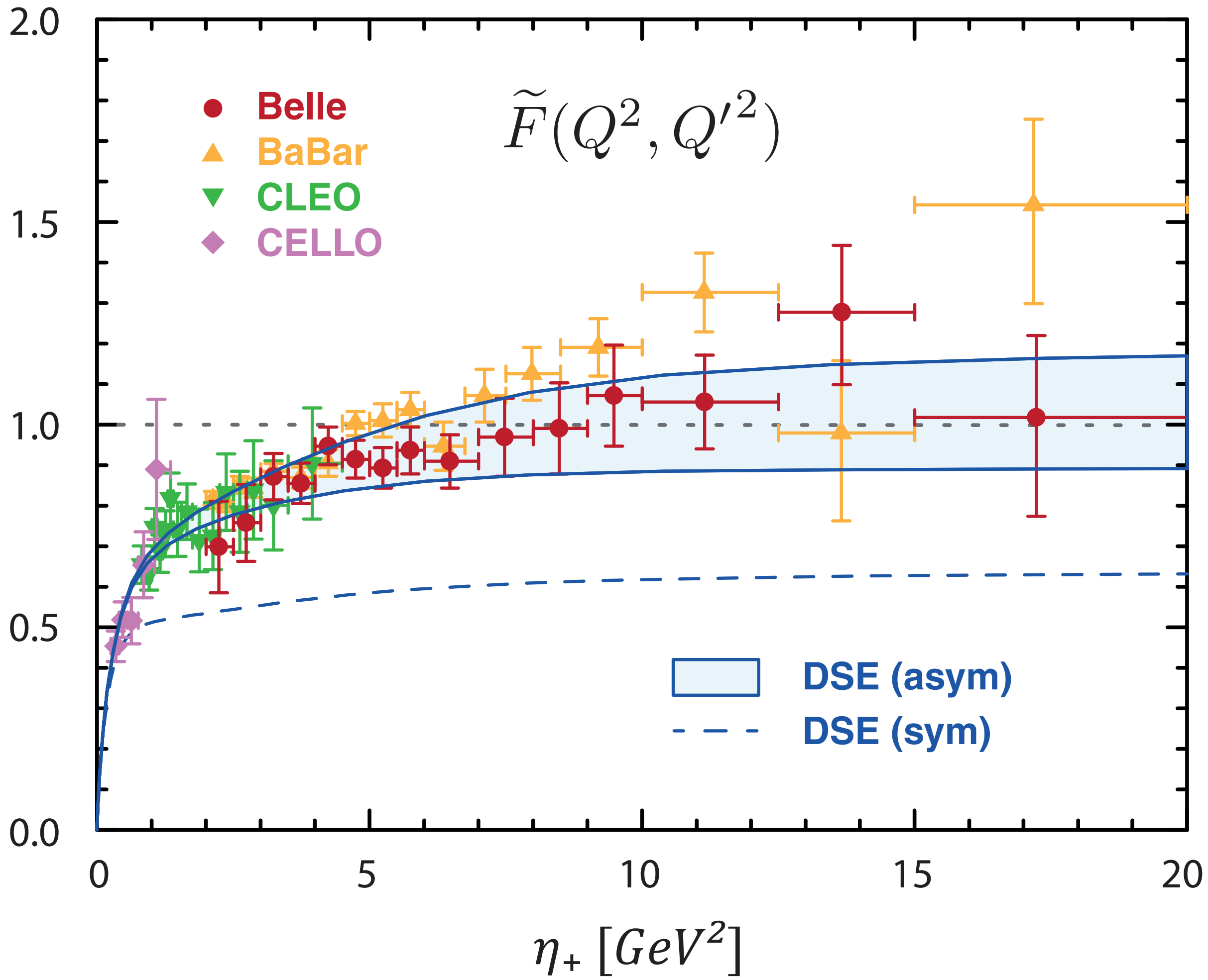
- Calculation of the off-shell TFF and **extrapolation** to on-shell point.

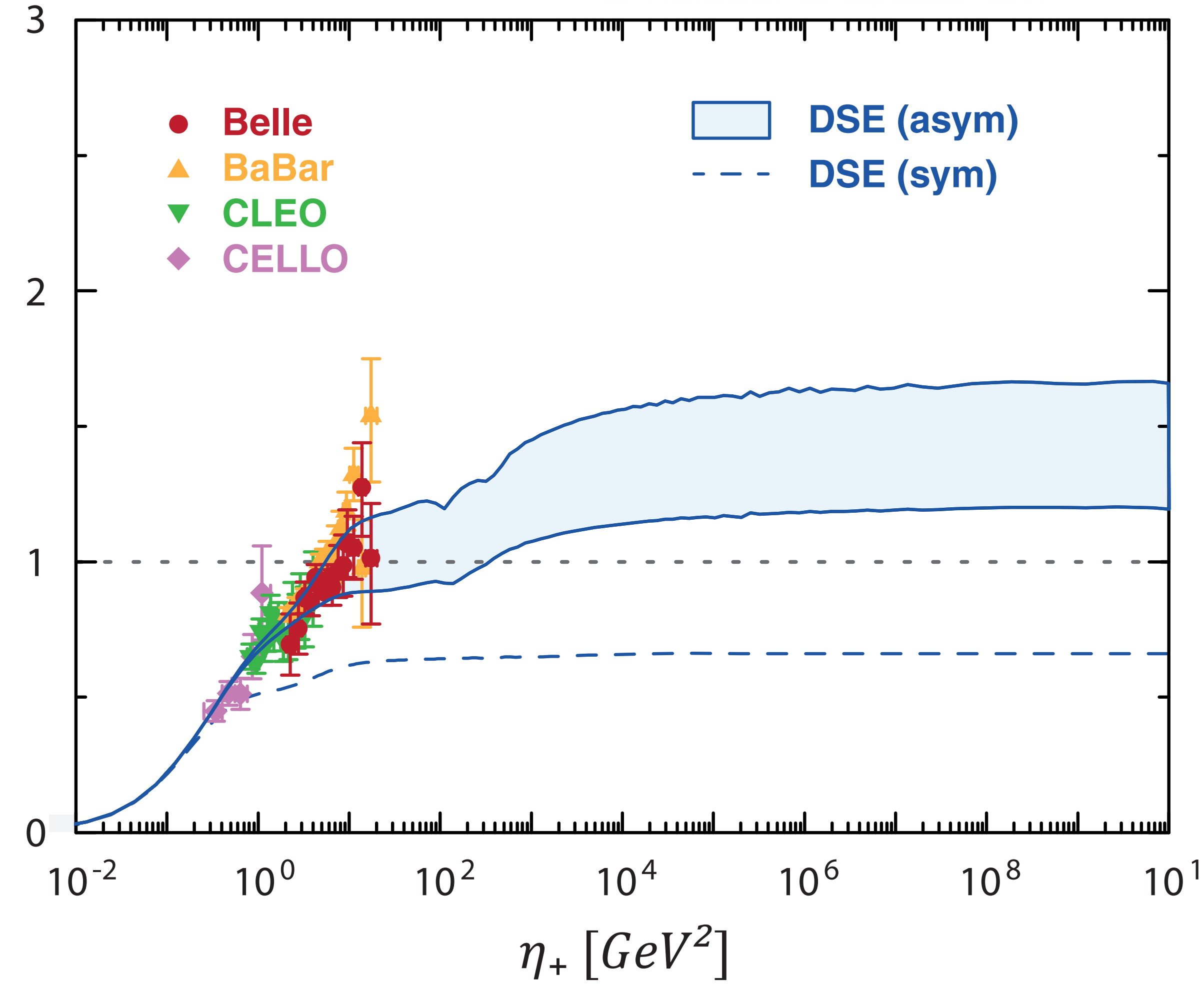
Extrapolation Fit:

$F(Q^2, Q'^2)$ = polynominal + pole

Results

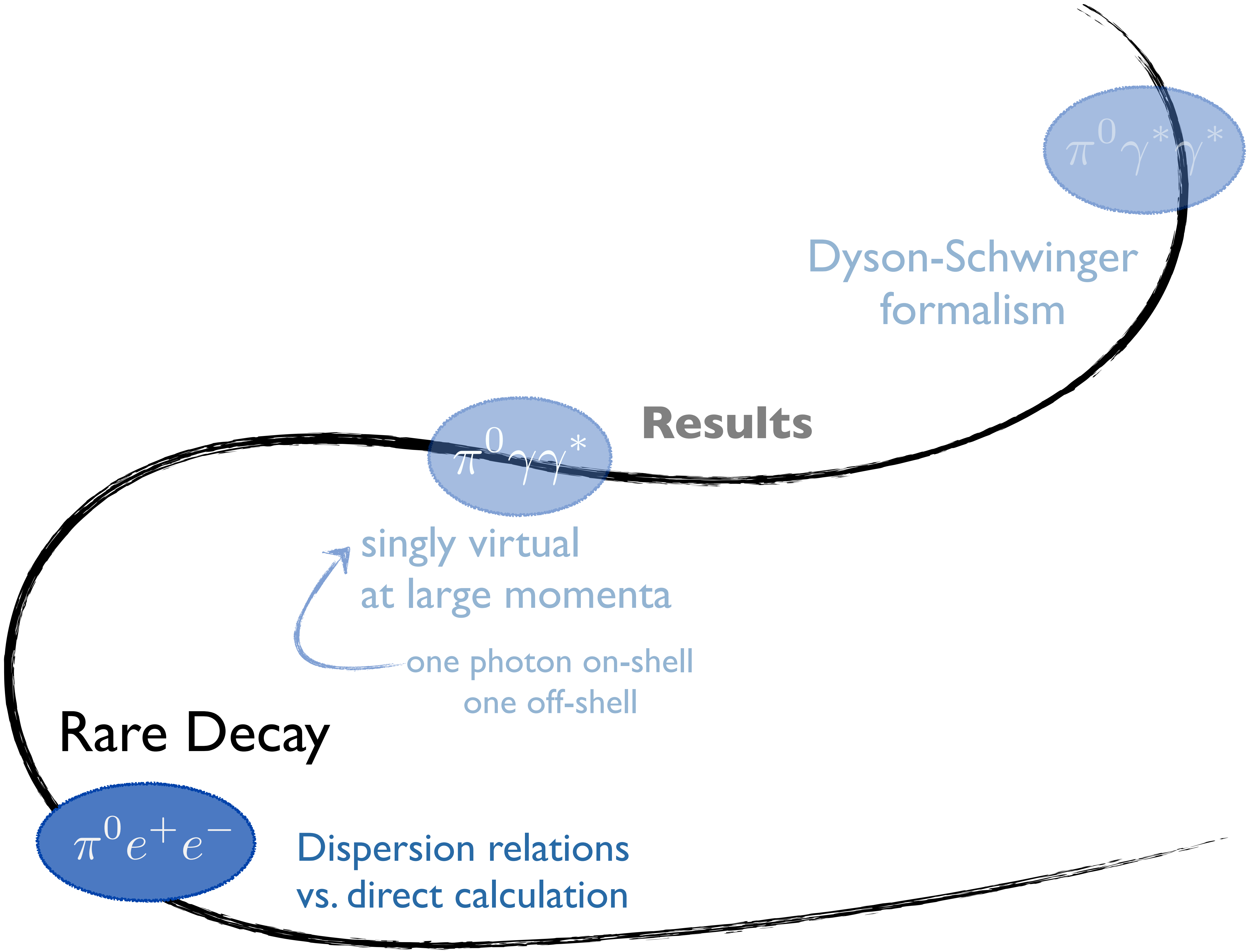
$$\eta_+ = \frac{Q^2 + Q'^2}{2}$$





- TFF is sensitive to non-perturbative effects at every momentum range and generates natural correction to the BL limit.

BL scaling limit is violated!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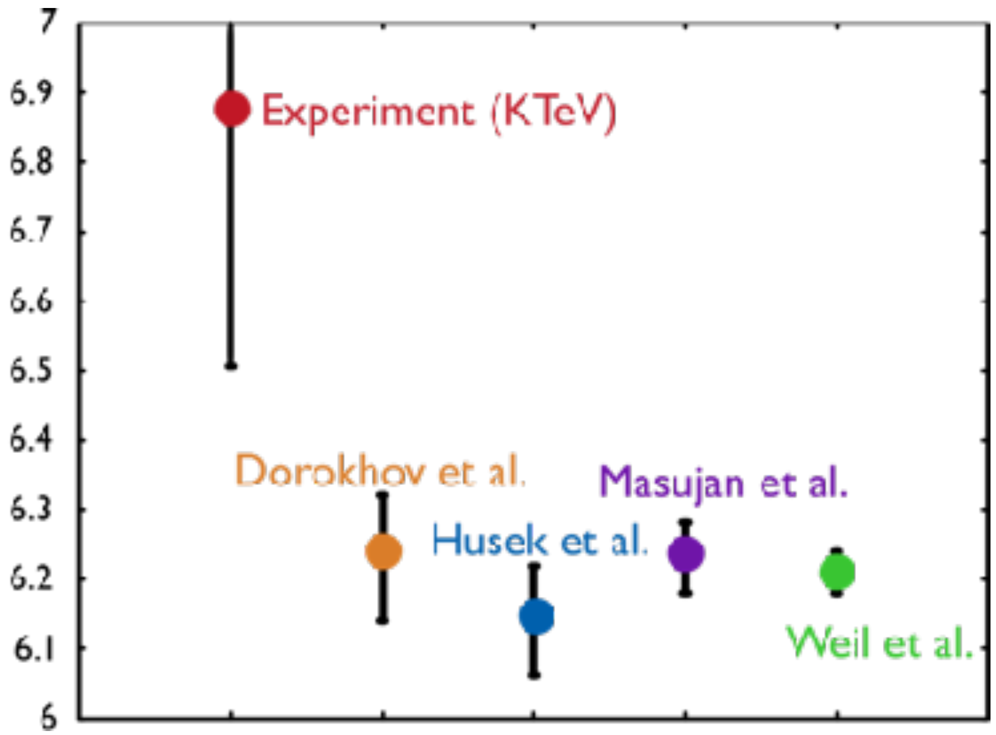
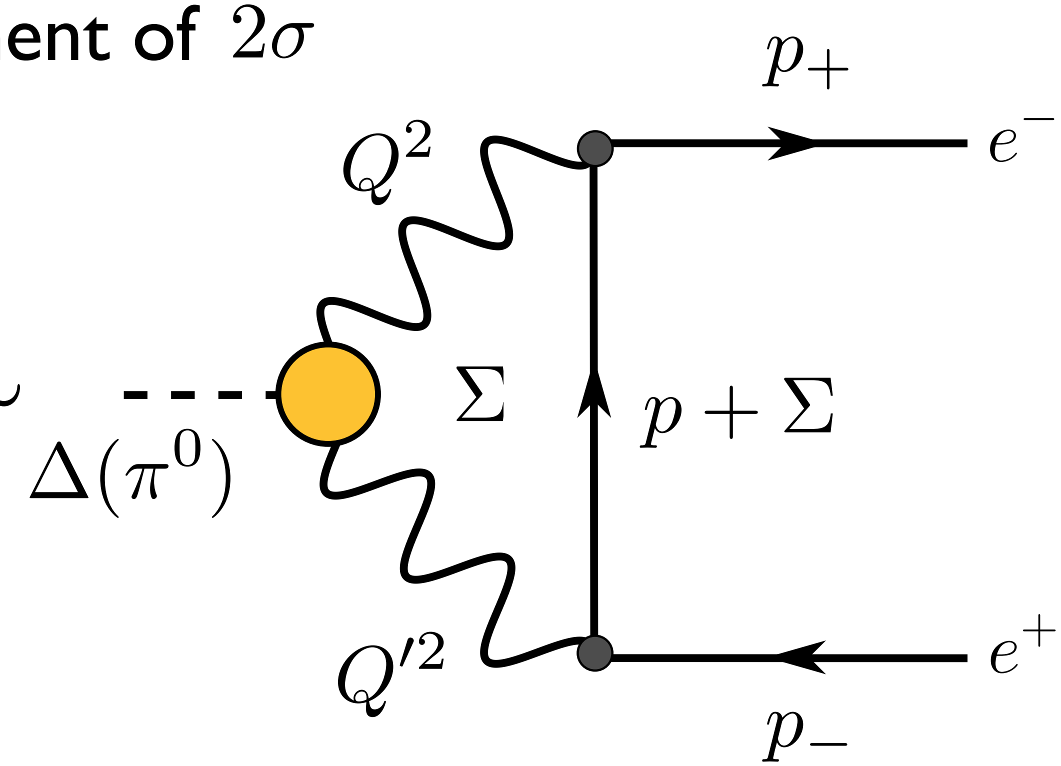
$\boxed{BR_{\text{exp}}(\pi^0_{e^+e^-}) \approx 6.87(36) \times 10^{-8}}$

Discrepancy between theory and experiment of 2σ



$\mathcal{A}(t) \sim$



The branching ratio is given by

$$BR = 2\left(\frac{m_e \alpha_{em}}{\pi m_\pi}\right)^2 \sqrt{1+\frac{m_e^2}{t_0}}\,|\mathcal{A}(t_0)|^2 \text{ with } \quad t_0 = -m_\pi^2/4$$

and the scalar amplitude

$$\mathcal{A}(t) = \frac{1}{2\pi^2 t}\int d^4\Sigma\, \frac{(\Sigma\cdot\Delta)^2 - \Sigma^2\Delta^2}{(p+\Sigma)^2 + m_e^2}\, \frac{F(Q^2, Q'^2)}{Q^2\, Q'^2}\,.$$

Rare Decay $\pi^0 \to e^+e^-$

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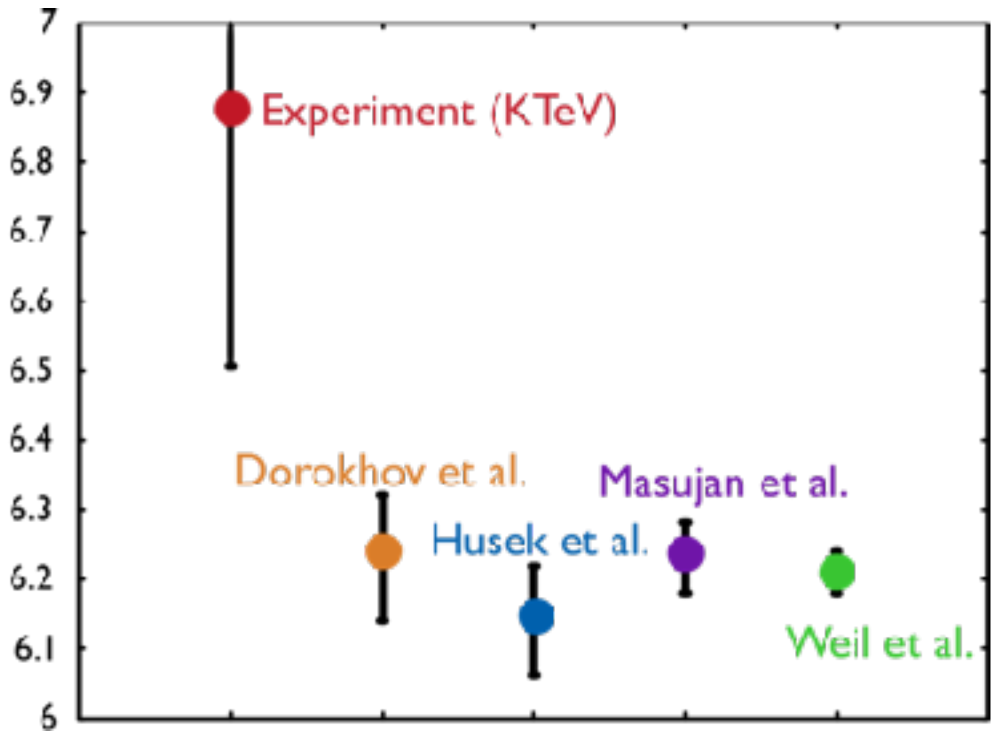
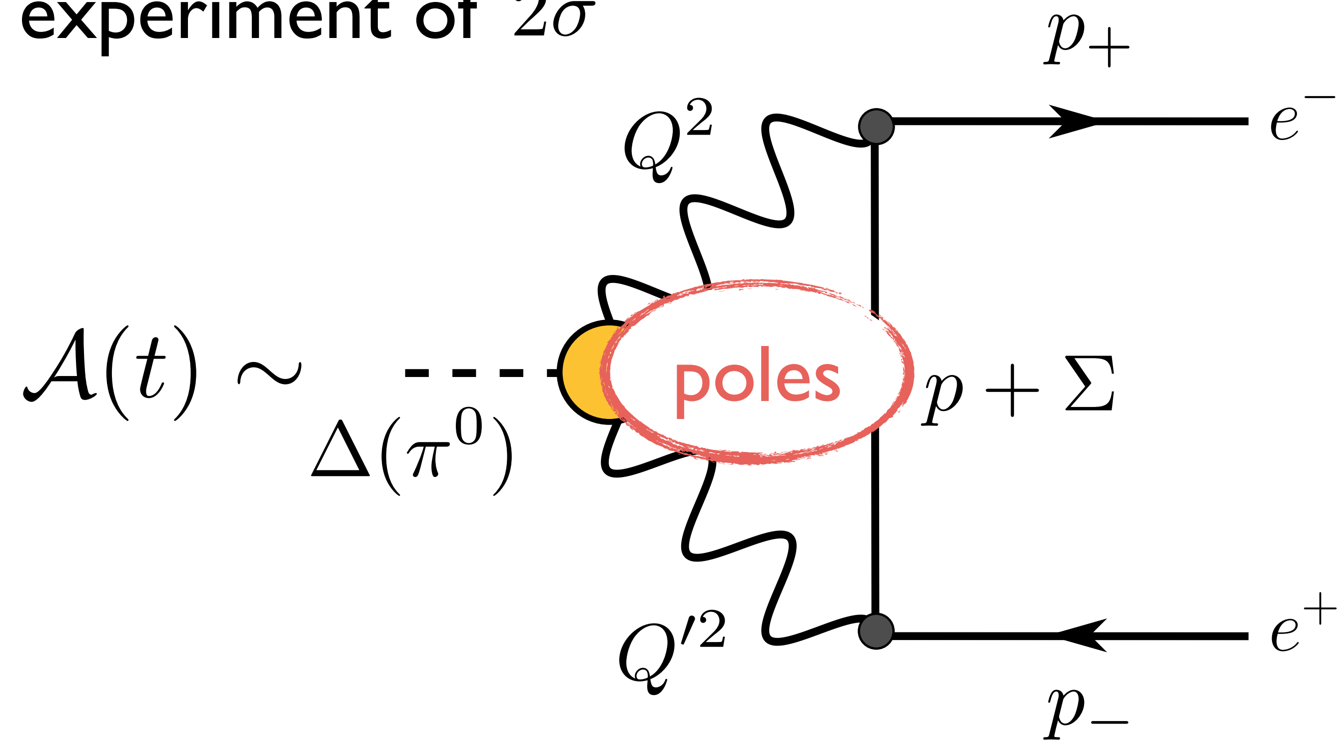
Discrepancy between theory and experiment of 2σ





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Rare Decay

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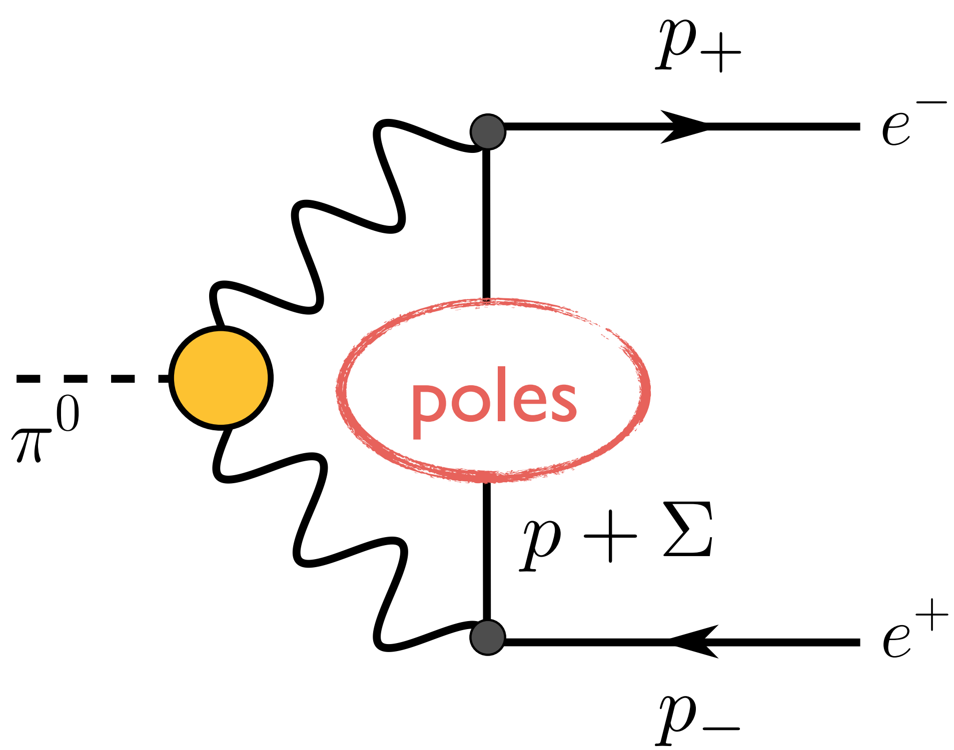


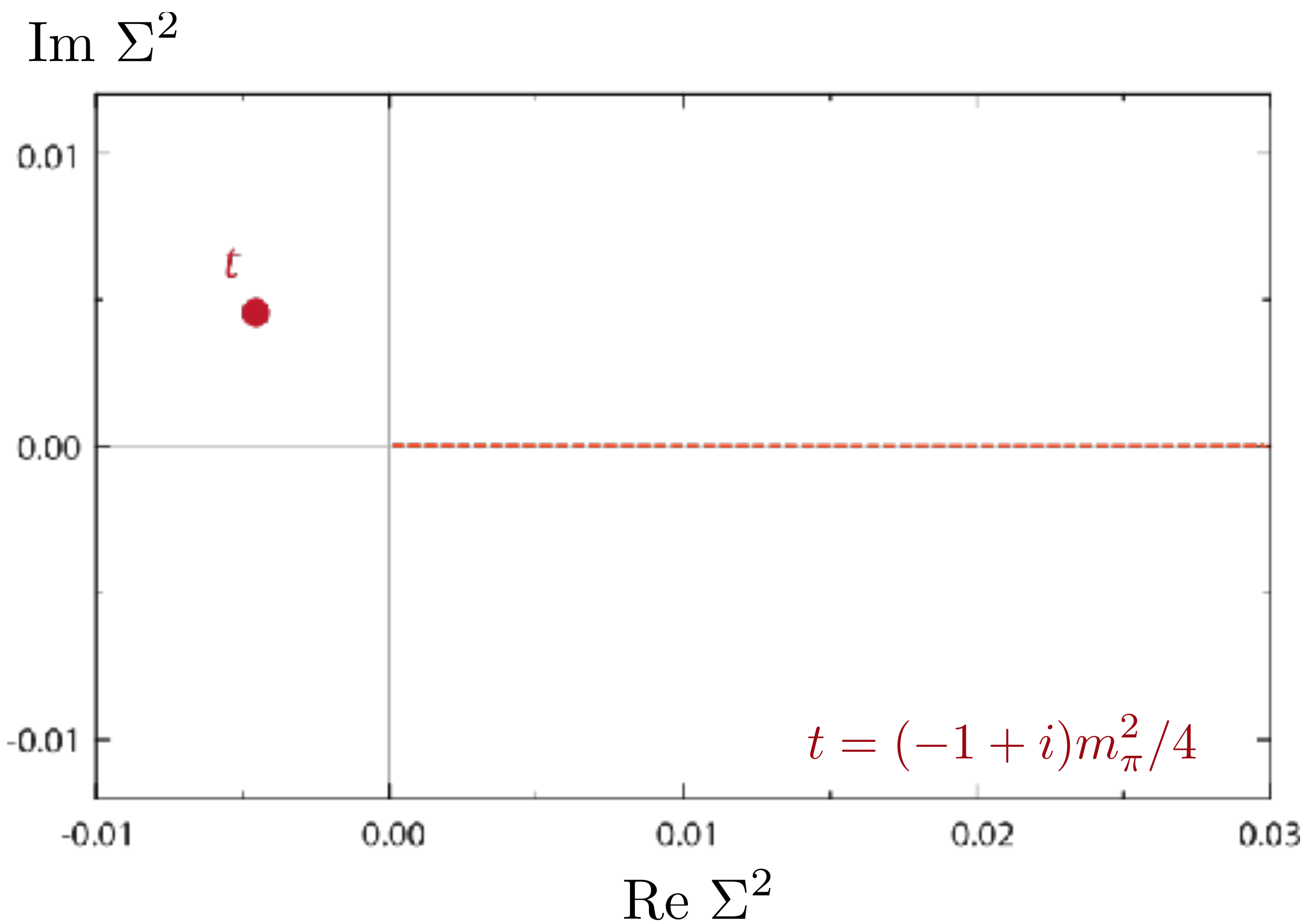
(a) Dispersion relation (DR)

(b) Direct calculation

L.Bergström al.(2014), Phys Let. B126, 8117(19839)

Rare Decay

$$\mathcal{A}(t) = \frac{1}{2\pi^2 t}\int_0^\infty d^2\Sigma^2 \int d\theta \int d\phi \, \frac{(\Sigma\cdot\Delta)^2 - \Sigma^2\Delta^2}{(p+\Sigma)^2 + m_e^2}\, \frac{F(Q^2,Q'^2)}{Q^2\,Q'^2}\,.$$



integration path in the complex plane

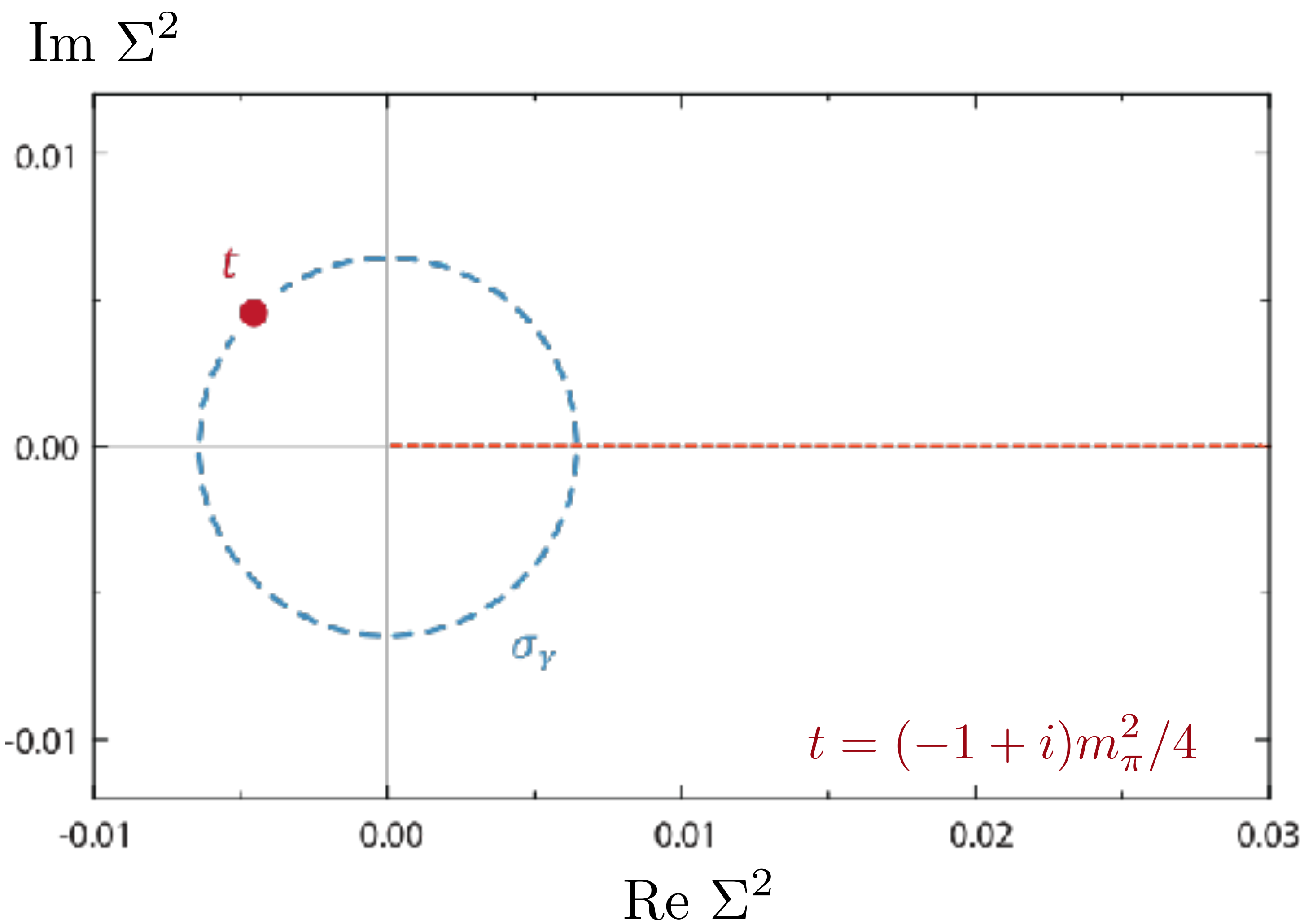
(b) Direct calculation

1. Using path deformation to calculate $\mathcal{A}(t)$ directly.

- Euclidean integration form 0 to ∞
- photon and lepton poles produce cuts in the complex Σ^2-plane:

Rare Decay

$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int_0^\infty d^2\Sigma^2 \int d\theta \int d\phi \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p+\Sigma)^2 + m_e^2} \, \frac{F(Q^2, Q'^2)}{Q^2 \, Q'^2} \, .$$



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(b) Direct calculation

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- Euclidean integration form 0 to ∞
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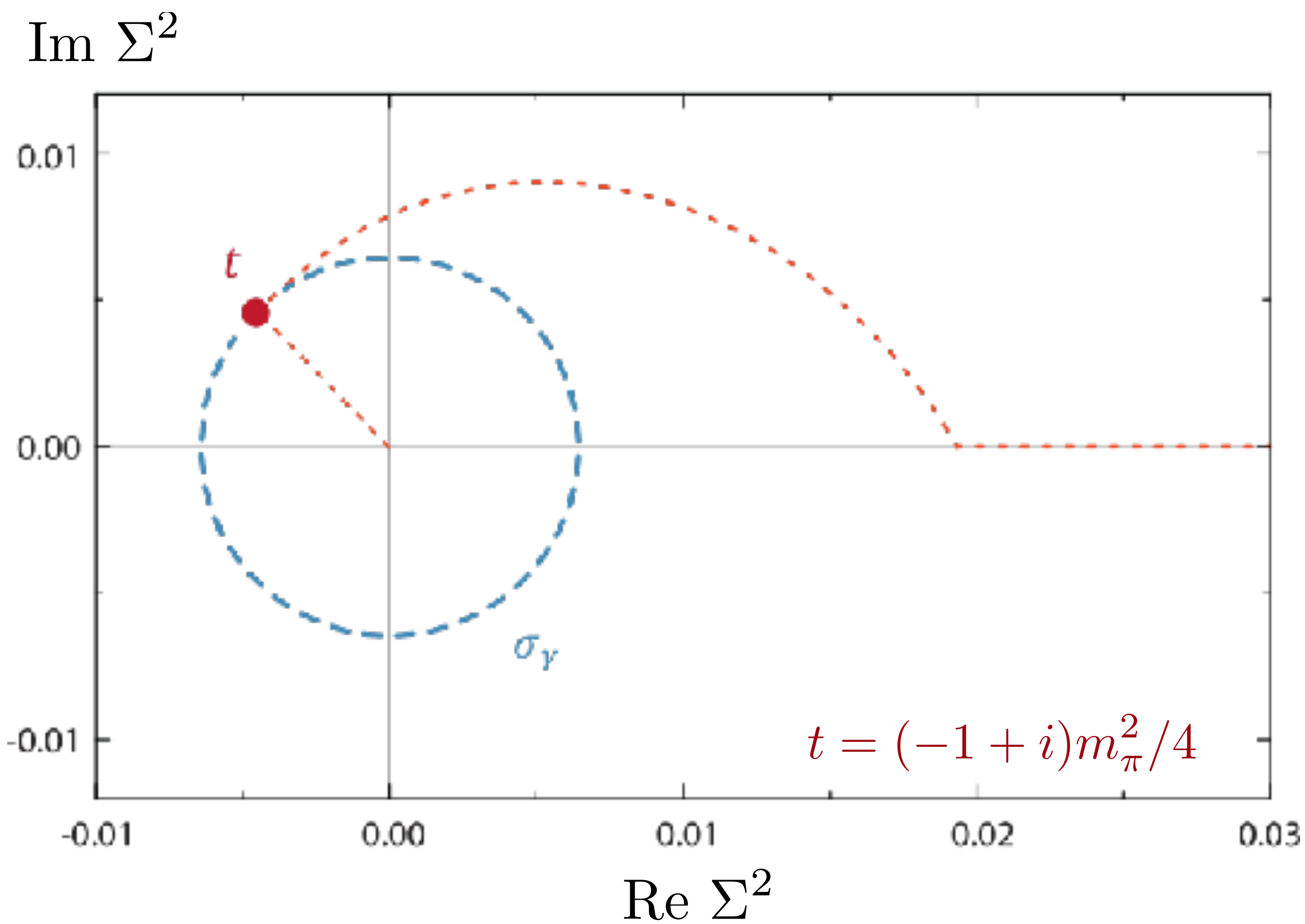
Photon poles
(opens at $\Sigma^2 = t$)

Rare Decay

$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int_0^\infty d^2\Sigma^2 \int d\theta \int d\phi \, \frac{(\Sigma\cdot\Delta)^2 - \Sigma^2\Delta^2}{(p+\Sigma)^2 + m_e^2} \, \frac{F(Q^2, Q'^2)}{Q^2\, Q'^2} \, .$$



······ integration path in the complex plane

(b) Direct calculation

1. Using path deformation to calculate $\mathcal{A}(t)$ directly.

- Euclidean integration form 0 to ∞
- photon and lepton poles produce cuts in the complex Σ^2-plane:

Photon poles
(opens at $\Sigma^2 = t$)

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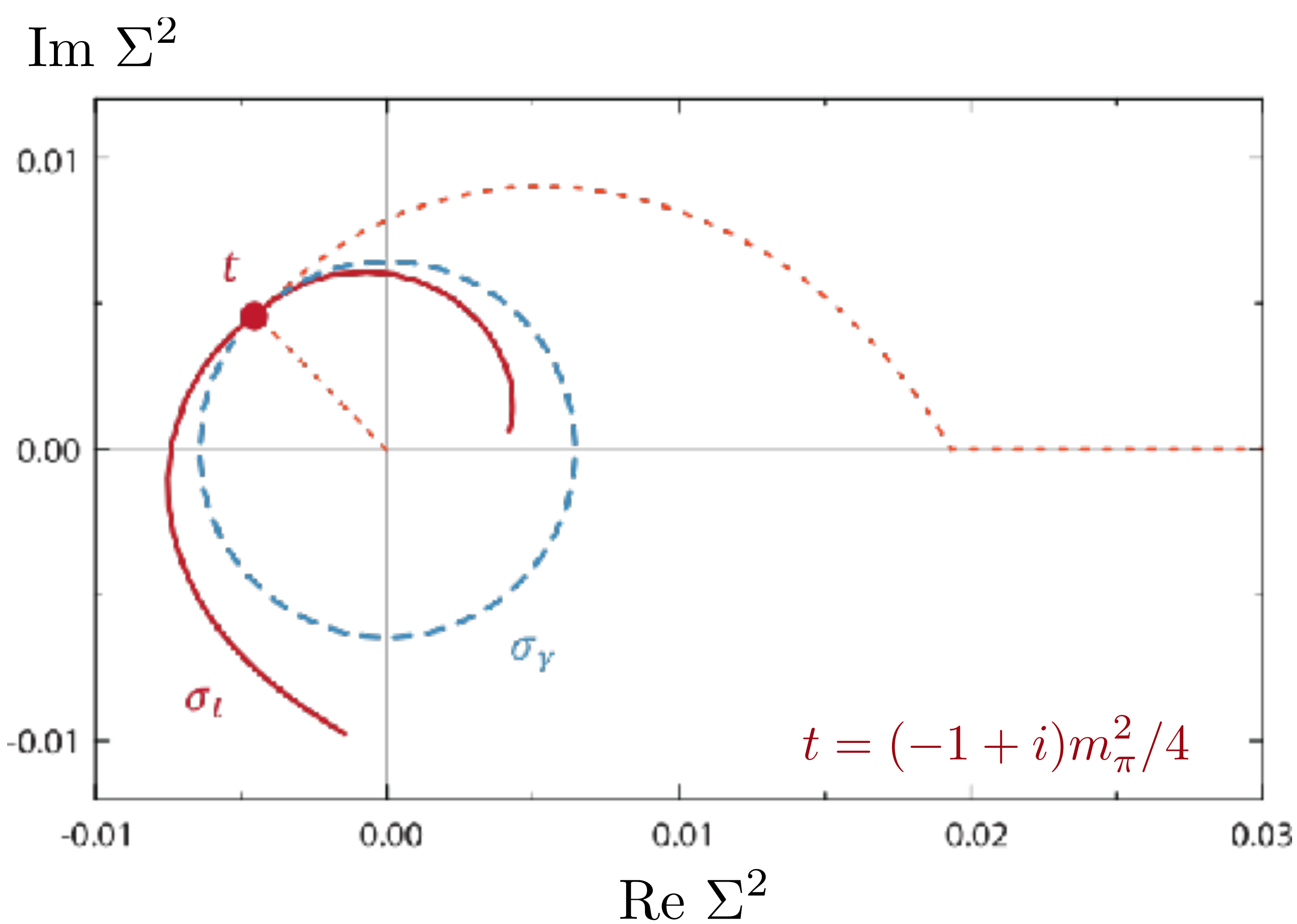

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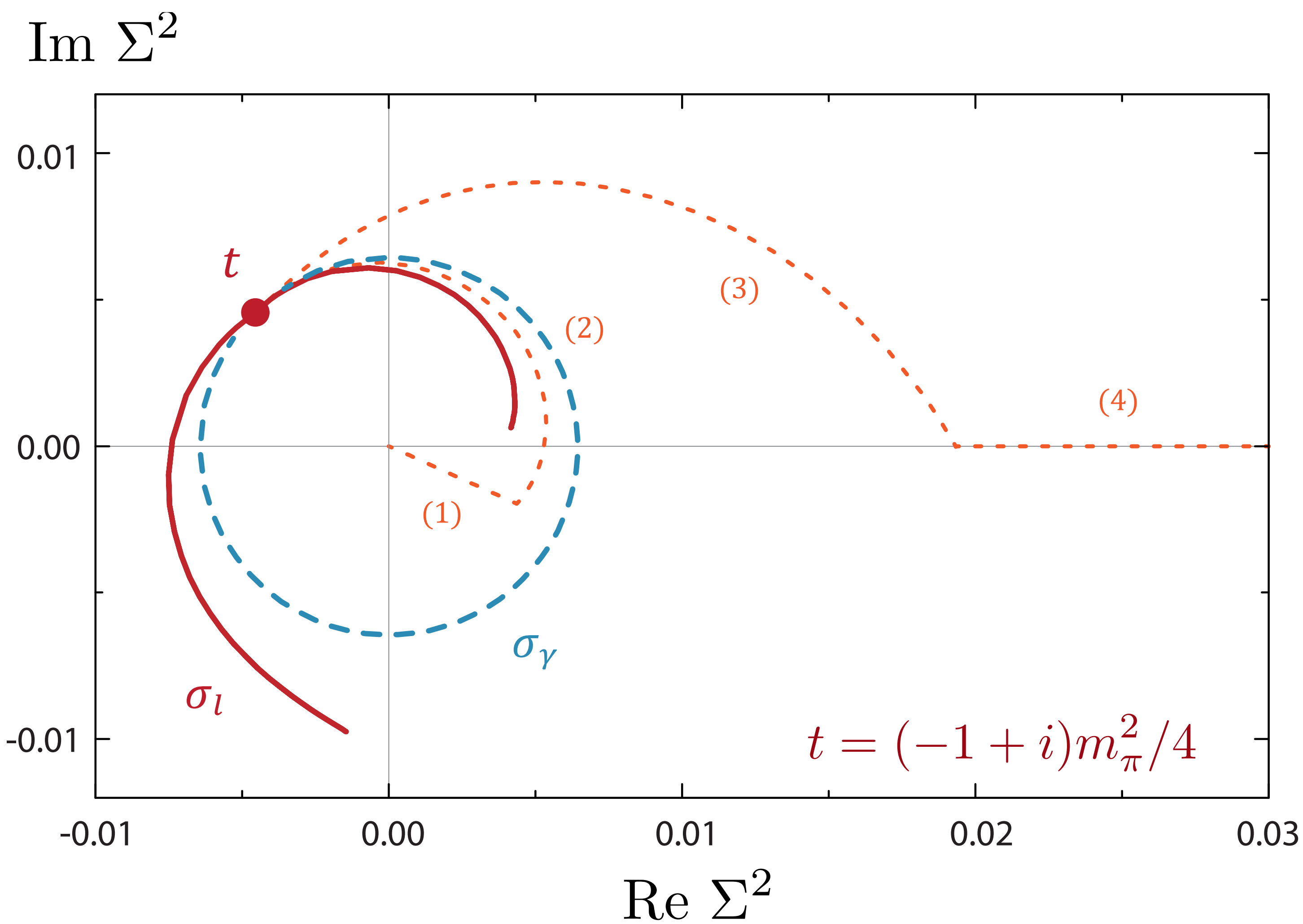

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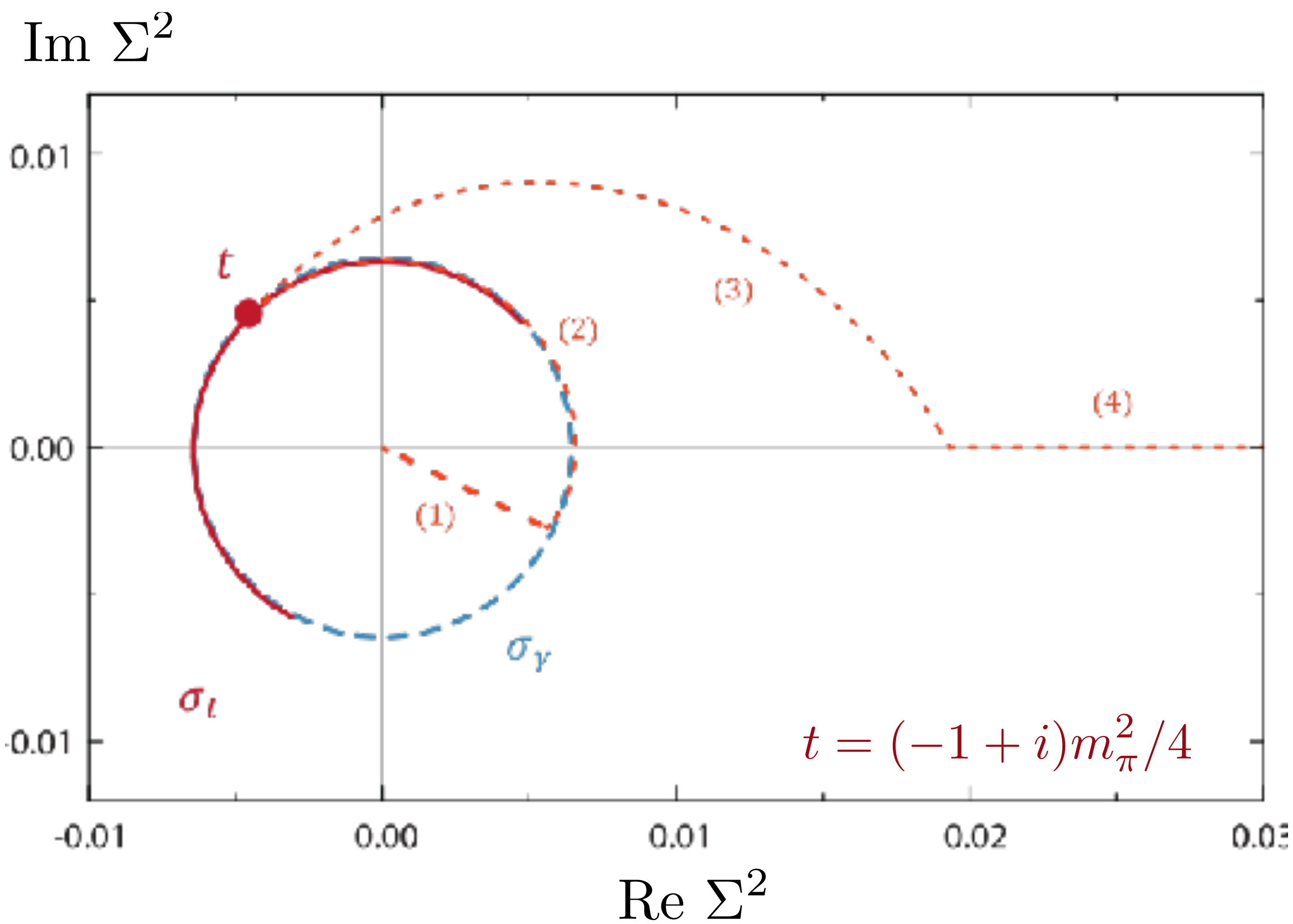

...... integration path in the complex plane

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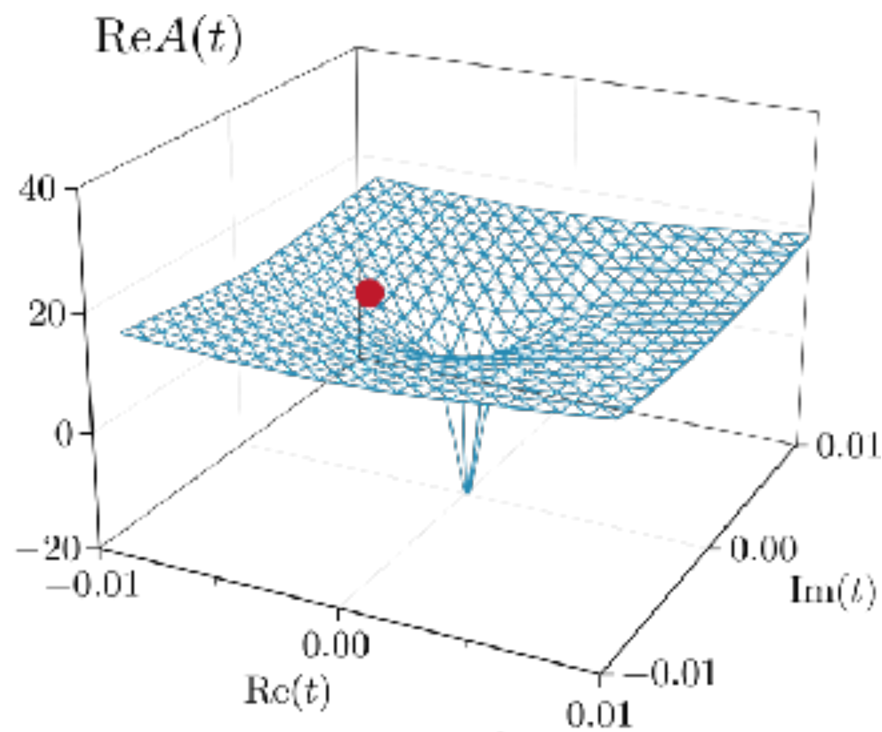
- photon and lepton poles produce cuts in the complex Σ^2-plane:

Photon poles
(opens at $\Sigma^2 = t$)

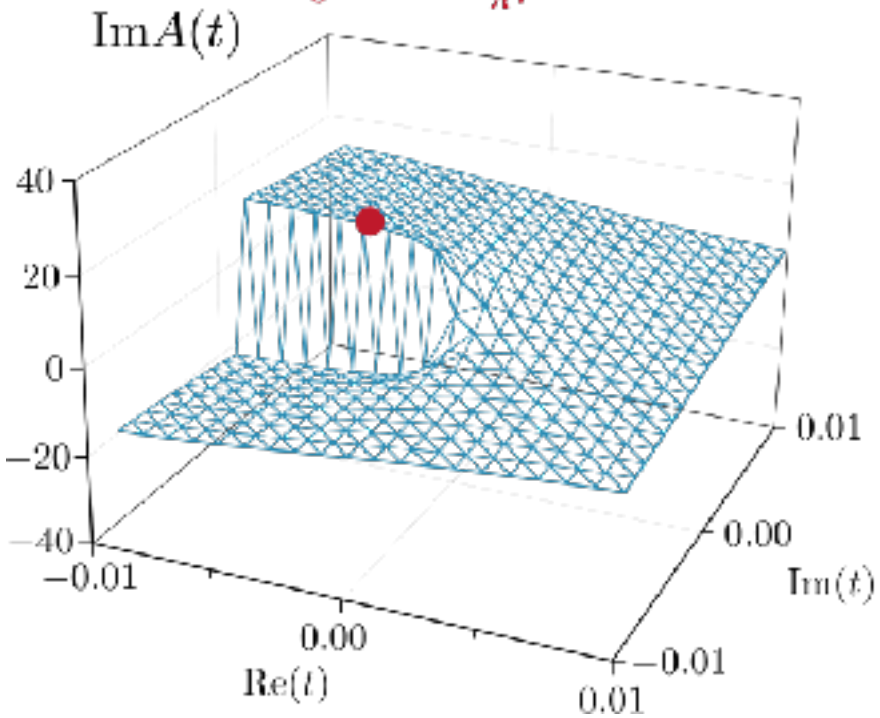
Lepton pole
(open at $\Sigma^2 = -t$)

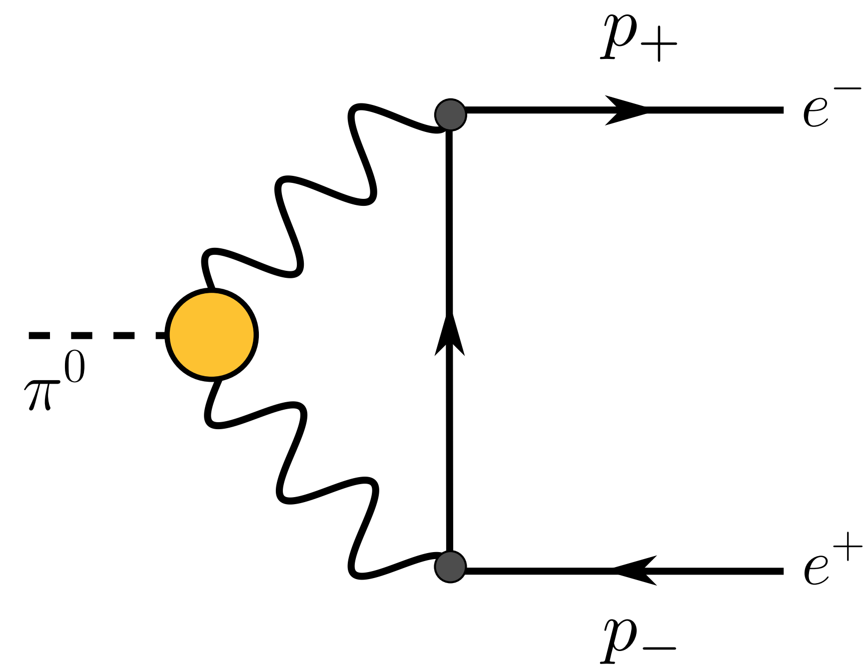
<u>technical challenge</u>: cuts are almost congruent, due to the small electron mass

Results $\mathcal{A}(t)$ in the whole complex plane



$t_0 = -m_\pi^2/4$





Compare to former calculations:

Collaboration	$B(\pi^0 \to e^+e^-)\,[10^{-8}]$
Experiment	6.87(36)
Dorokhov et al. (DR)	6.23(9)
Husek et al. (DR)	6.14(8)
Masjuan et al. (DR)	6.23(5)
Our result (DR)	6.21(3)
Our result (direct)	6.22(3)

- direct and dispersive calculation agree ✓
- discrepancy: theory and experiment ✓

Summary

- Overview of the progress in calculation of the pions transition form factor using the DSE formalism.

- New and interesting scaling results for singly-virtual form factor.

- Contour deformation to solve 1-loop diagram in Euclidean space.

- Thank you for your attention -

our papers

G. Eichmann et al. (2017), **arXiv:1704.05774** [hep-ph].

E. Weil et al. (2017), **arXiv:1704.06046** [hep-ph].

and Reference therein

Back-up

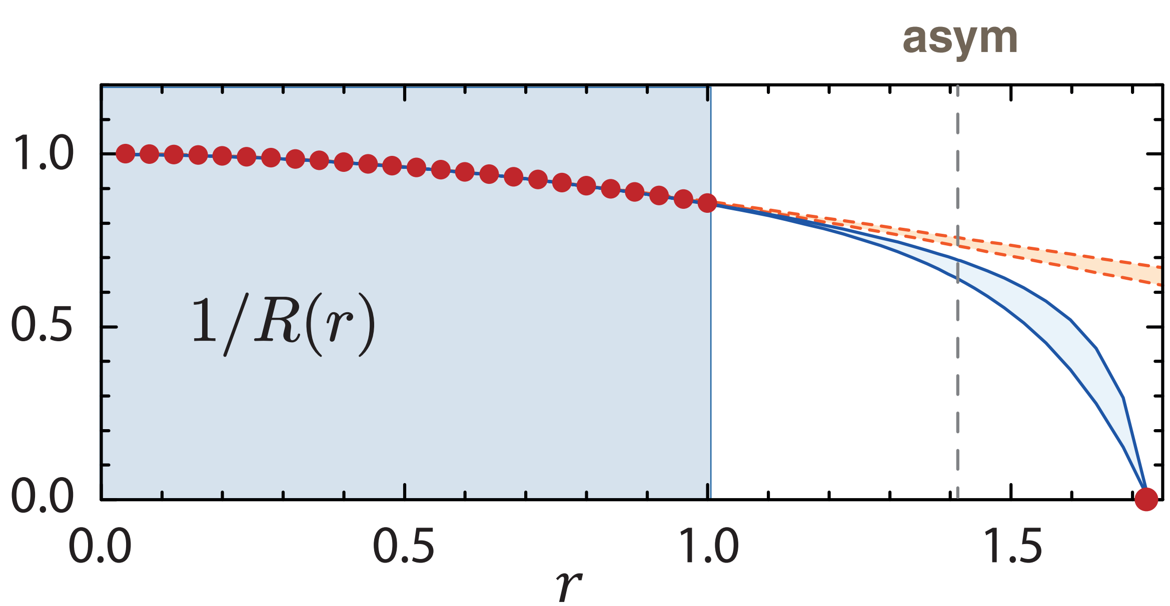
More about the Extrapolation of the TFF

$$\eta_+ = \frac{Q^2 + Q'^2}{2} \qquad \omega = \frac{Q^2 - Q'^2}{2}$$
$$\eta_- = Q \cdot Q' \qquad t = \Delta^2/4$$

we define for fixed η_+

$$R(r) = \frac{F(\eta_+, \eta_-, \omega)}{F(\eta_+, \eta_-, 0)} \qquad r = \sqrt{2}\omega/\eta_+$$

symmetric result

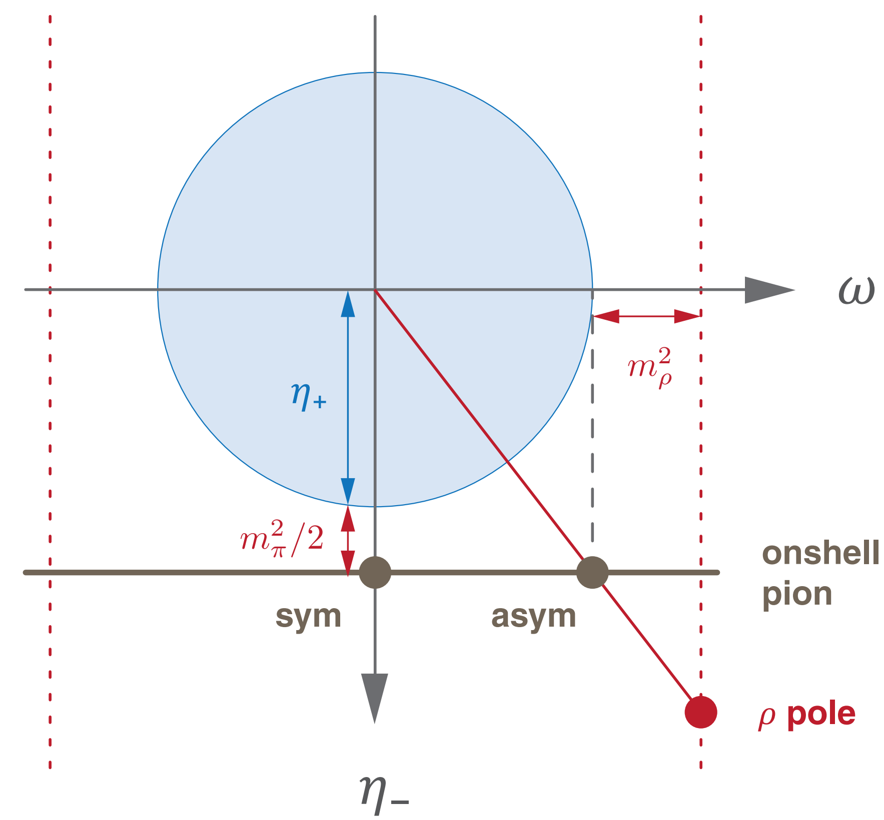


Fit: $R(r)$ = polynominal + pole

only polynomial

with vector-meson pole constraints

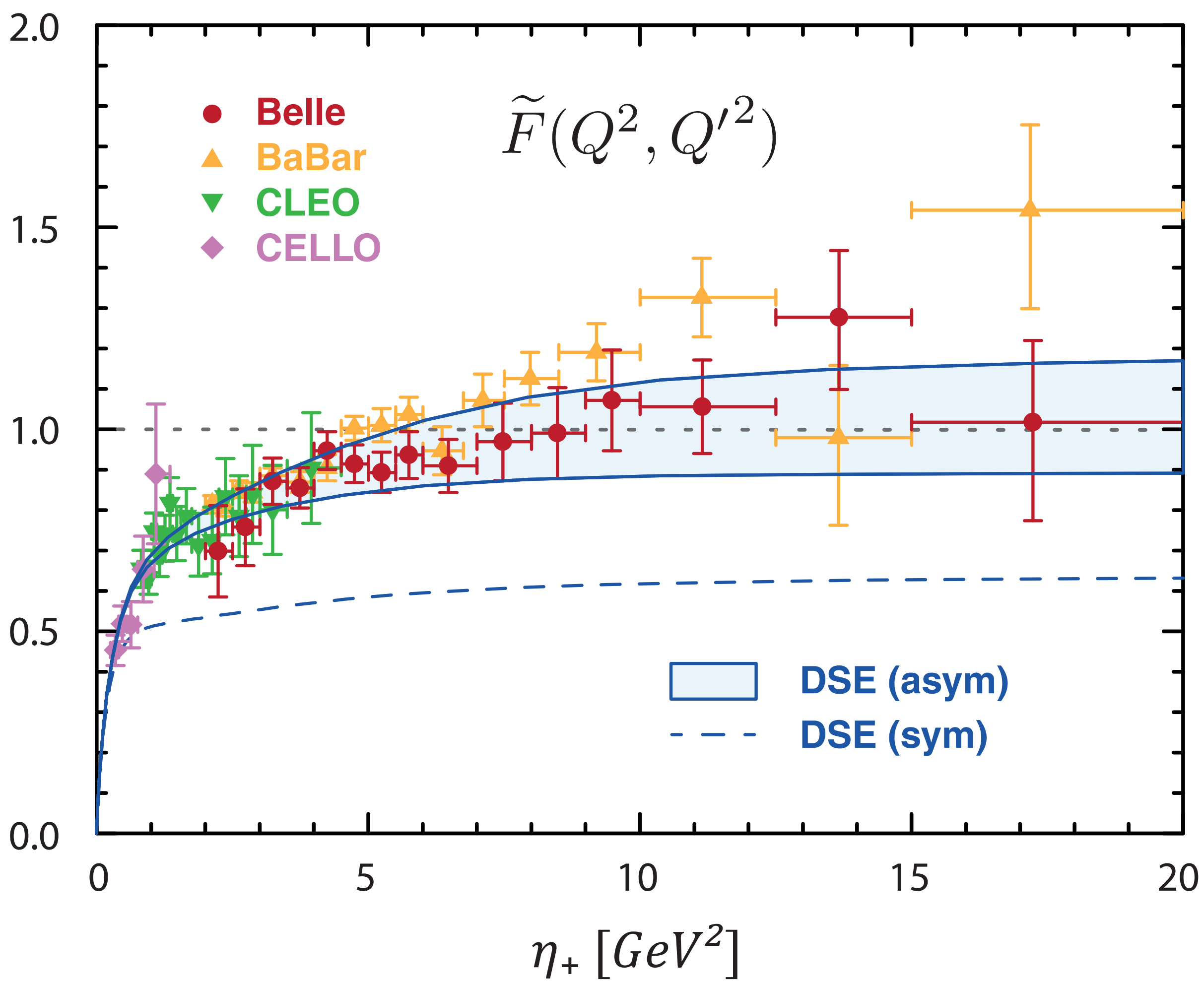
$$\eta_-^2 + \omega^2 = \eta_+^2 r^2 =$$

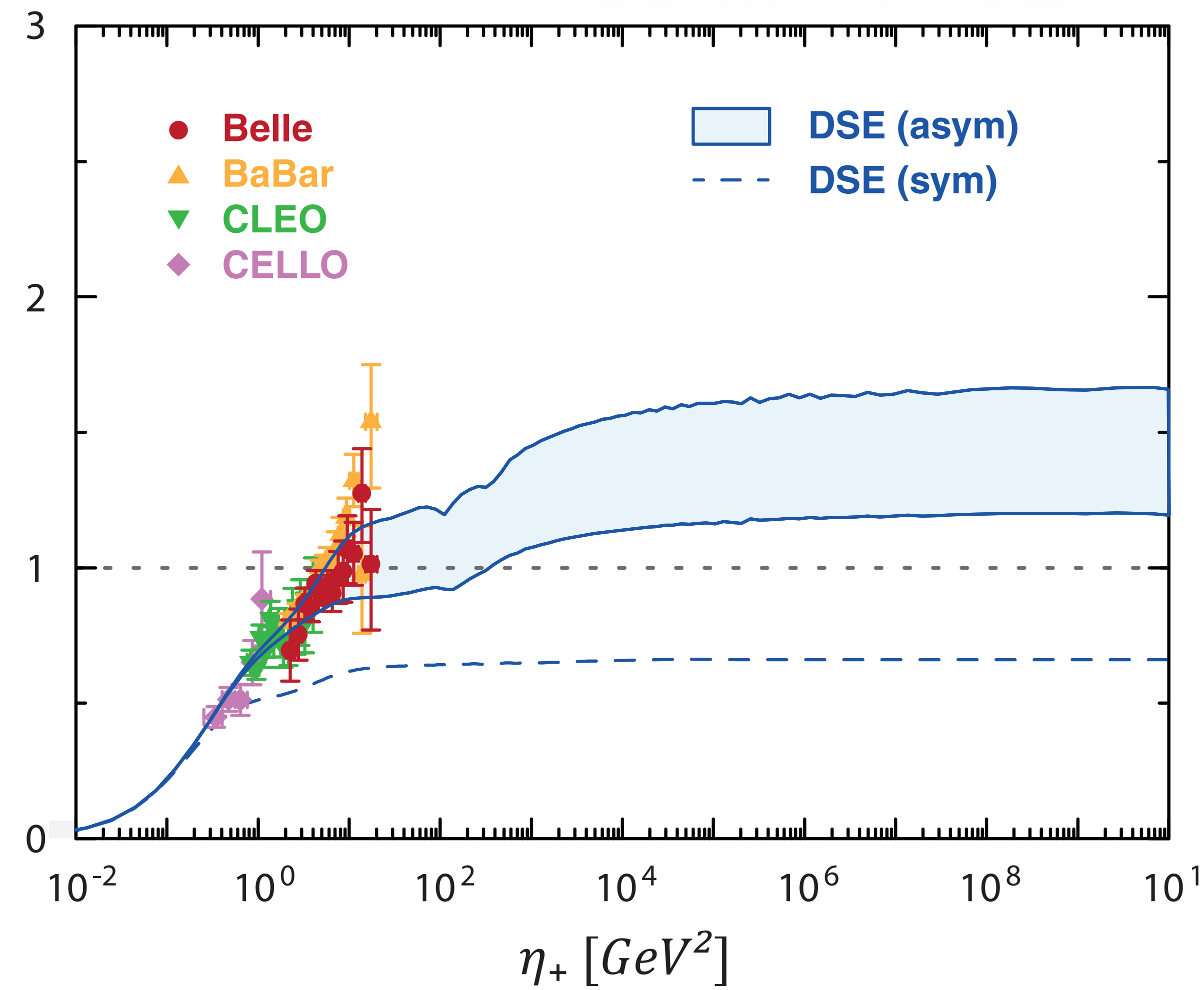


Extrapolation was checked against the direct calculation for $Q^2 \lesssim 4\mathrm{GeV}^2$

Results

$$\eta_+ = \frac{Q^2 + Q'^2}{2}$$

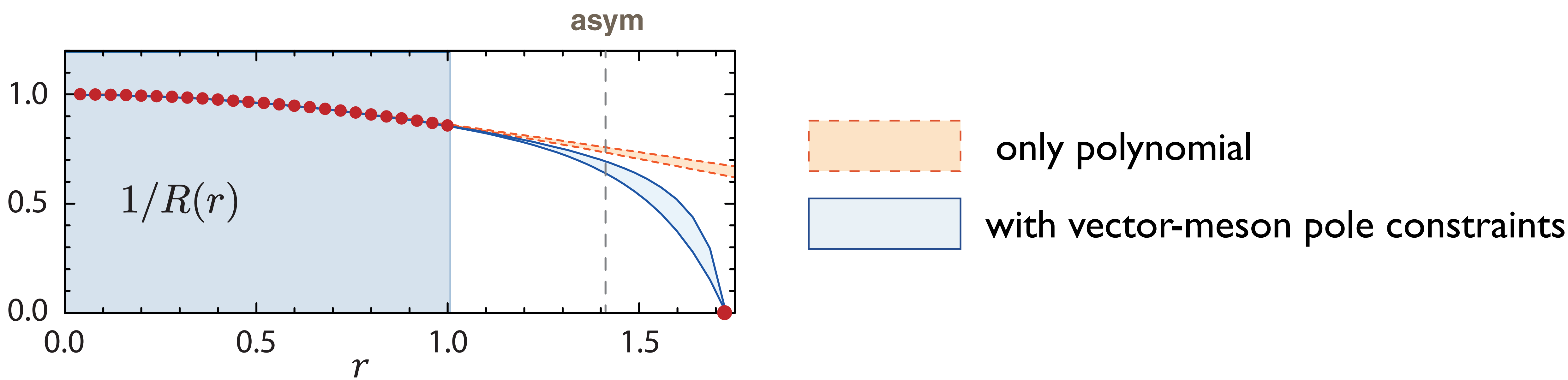




- Fit = polynominal, recovers the BL limit.
- Fit = polynominal + poles exceeds the BL limit.
- TFF is sensitive to the vector-meson pole at every momentum range and generates natural correction to the BL limit.

BL scaling limit is violated! VM pole is important

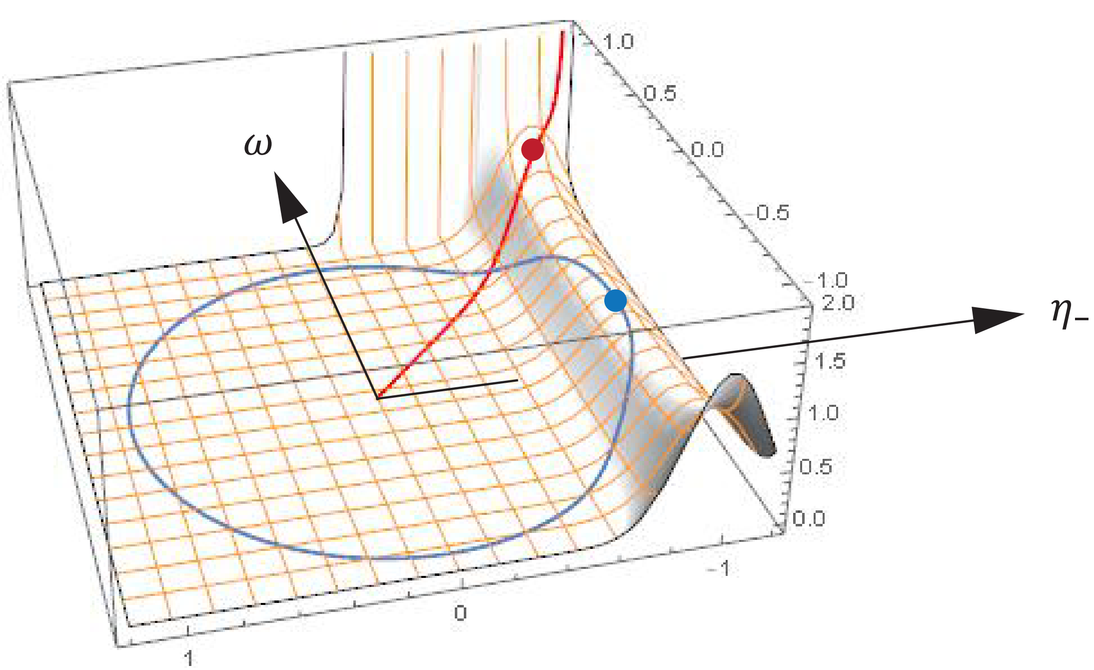
More about the Extrapolation of the TFF



Fit: $R(r)$ = polynominal + pole

$$R(r) = c_0 + c_1\, r^2 + c_2\, r^4 + c_3 \frac{m_\rho^2}{\eta_+(r_v^2 - r^2)}$$

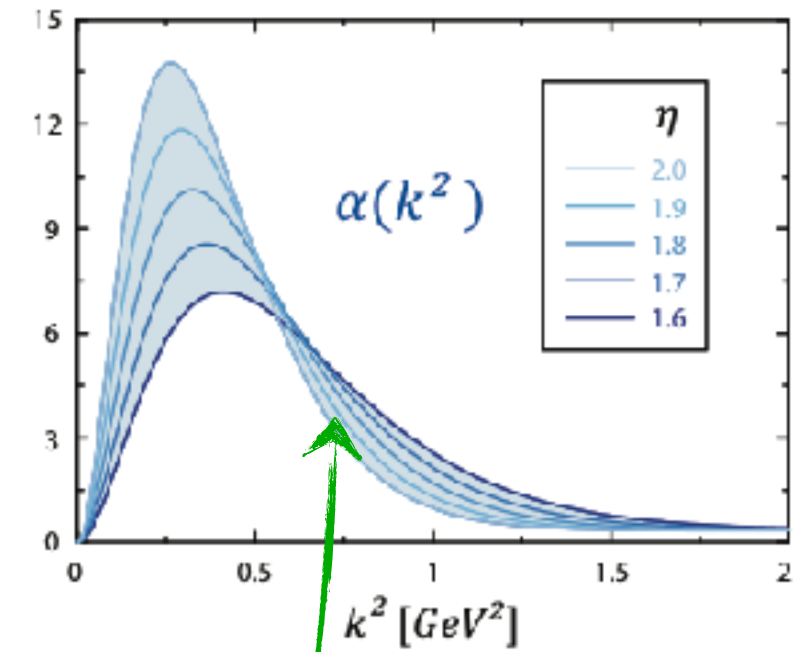
= error band



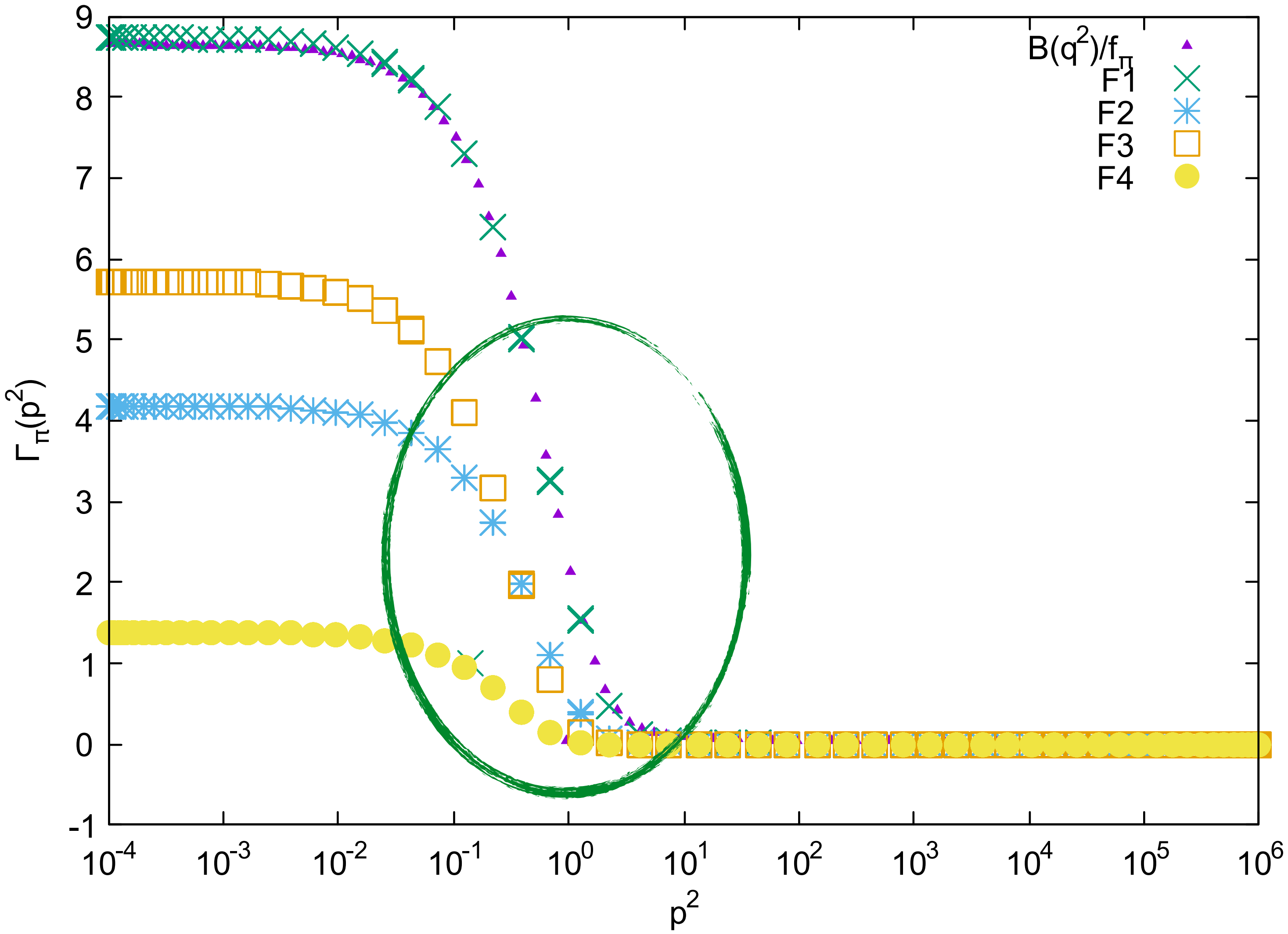
$$R(r) = \frac{F(\eta_+, \eta_-, \omega)}{F(\eta_+, \eta_-, 0)}$$

symmetric result

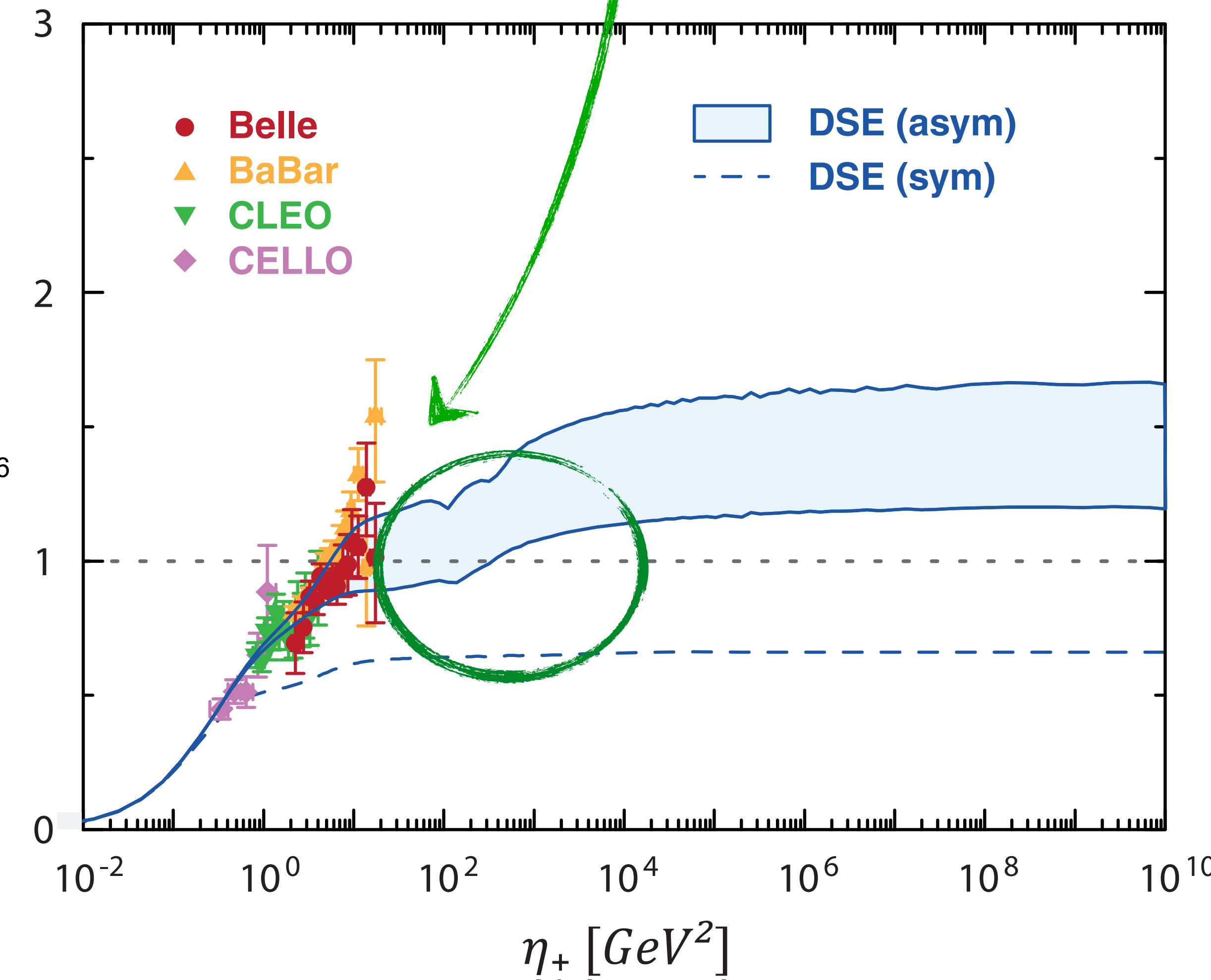
effective coupling



Pion BSE



form factor



Wiggels in the singly-virtual

A Fit for the TFF $\quad F(Q^2, Q'^2):\quad \pi^0 \to \gamma^*\gamma^*$

$$F(Q^2, {Q'}^2) = \frac{\mathcal{A}(v) + v(1-z^2)\,\mathcal{B}_1(v)\,(1+\mathcal{B}_2(v)z^2)}{(1+v)^2 - v^2 z^2}$$

$$\eta_+ = \frac{Q^2 + Q'^2}{2} \qquad \omega = \frac{Q^2 - Q'^2}{2}$$
$$z = \omega/\eta_+ \qquad v = \eta_+/m_\rho^2$$

$\mathcal{A}(v)$, $\mathcal{B}_i(v)$

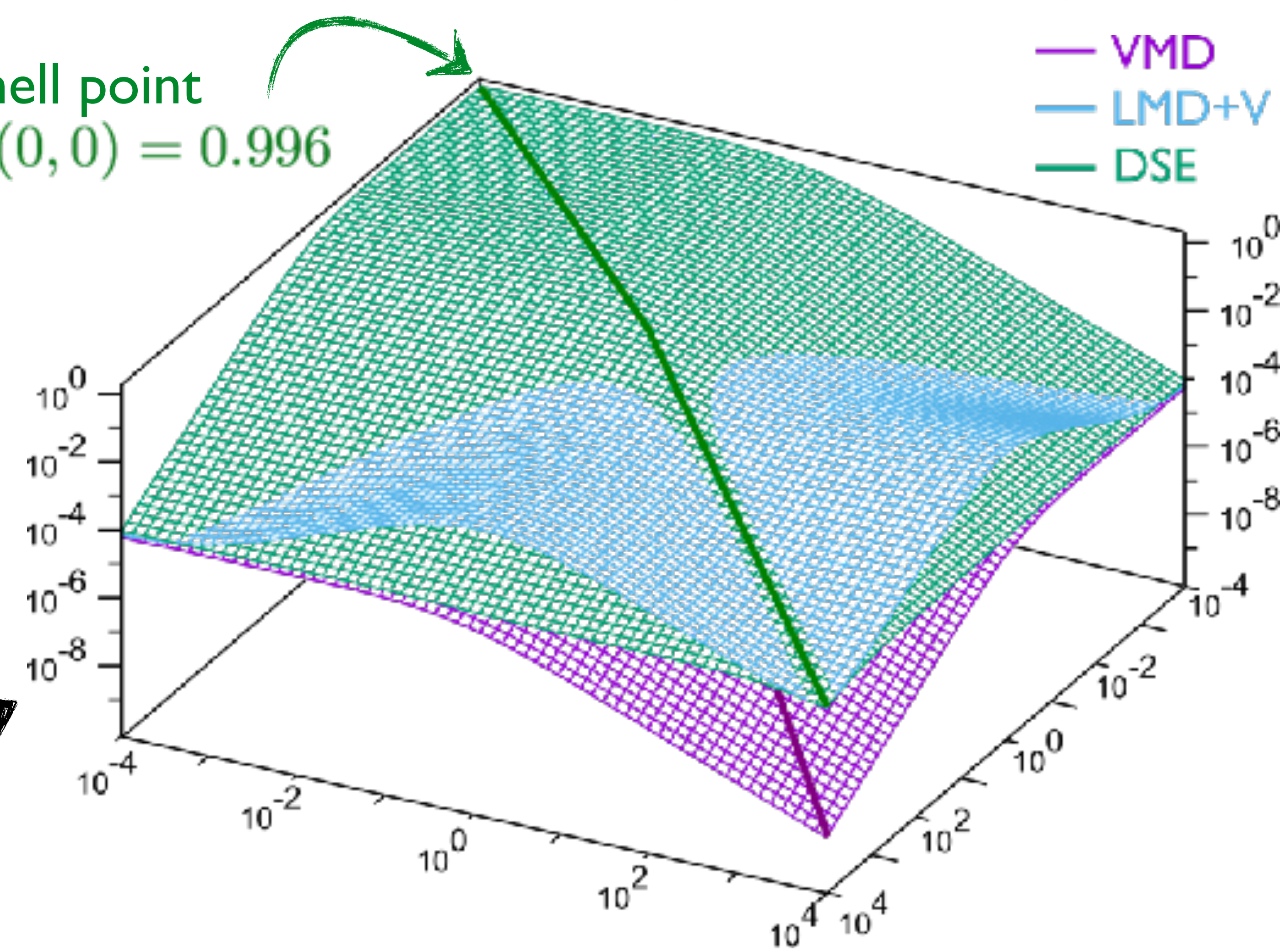
= ratio of polynominals with 12 fit parameters

uncertainty through variation of MT interaction + fit uncertainty.

on-shell point :

- bare vertices $F_{\text{bare}}(0,0) = 0.29$
- Ball-Chiu vertex $F_{\text{BC}}(0,0) = 0.86$

on-shell point
$F_{\text{DSE}}(0,0) = 0.996$

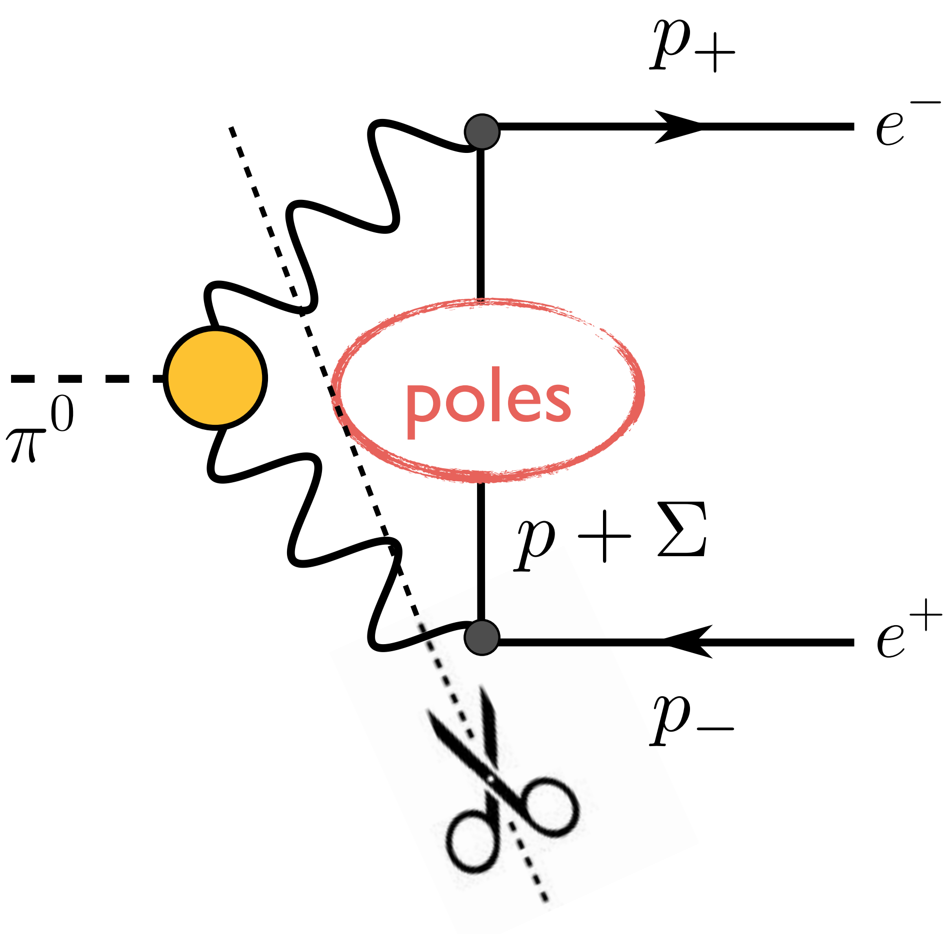


Compared to VMD & LMD+V

- agrees for symmetric and low asymmetric Q^2.
- varies for large η_+ .

Rare Decay

$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p+\Sigma)^2 + m_e^2} \, \frac{F(Q^2, {Q'}^2)}{Q^2 \, {Q'}^2} \, .$$



(a) Dispersion relation (DR)

1. cutting the photon lines 

$$\text{Im } \mathcal{A}^{\text{LO}}(t) = \frac{\pi \, \ln \gamma(t)}{2\beta(t)} F(0,0)$$

2. once-subtracted dispersion relation

$$\text{Re } \mathcal{A}(t) = \mathcal{A}(0) + f(t)$$

3. Mellin-Barnes representation to calculate $\mathcal{A}(0)$ using $F(Q^2, Q^2)$:

$$\mathcal{A}(0) \approx -\frac{5}{4} + \frac{3}{2} \int_0^\infty dx \ln(4x) \, \frac{d}{dx} F(Q^2, Q^2)$$

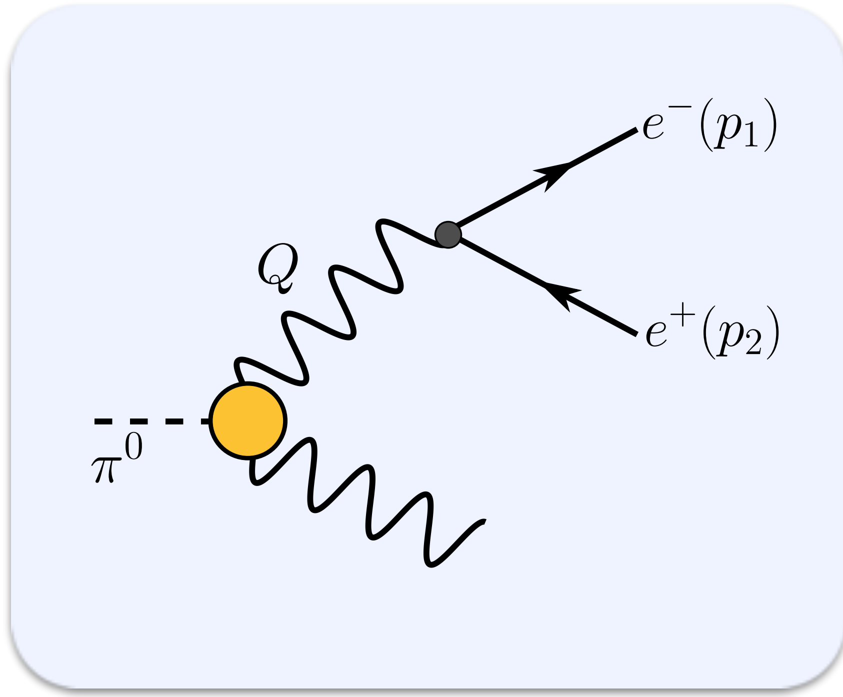
with $Q^2 = 4m^2 x$

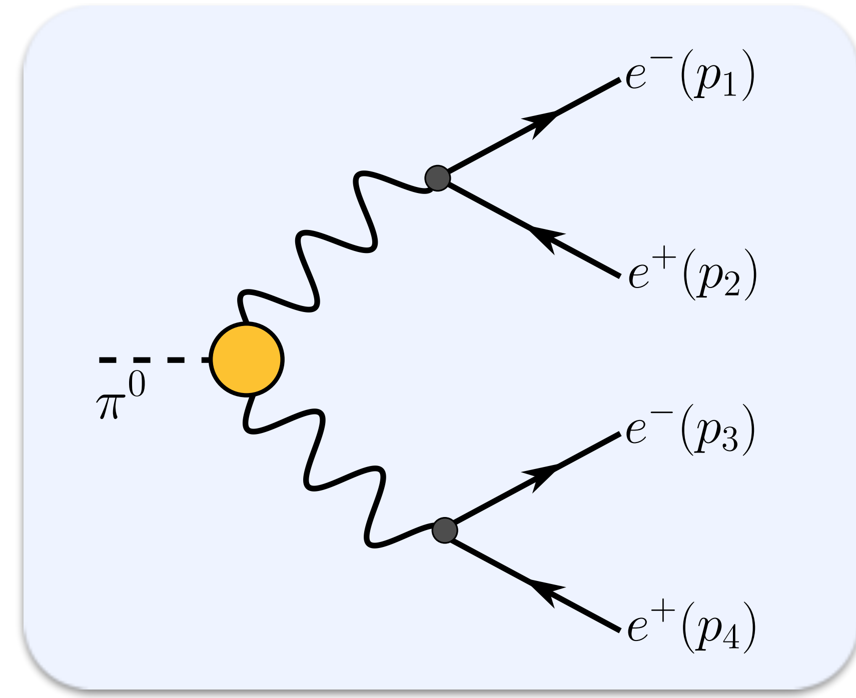
L.Bergström al.(2014), Phys Let. B126, 8117(19839)

Dalitz Decays

$\pi^0 \to \gamma e^+ e^-$

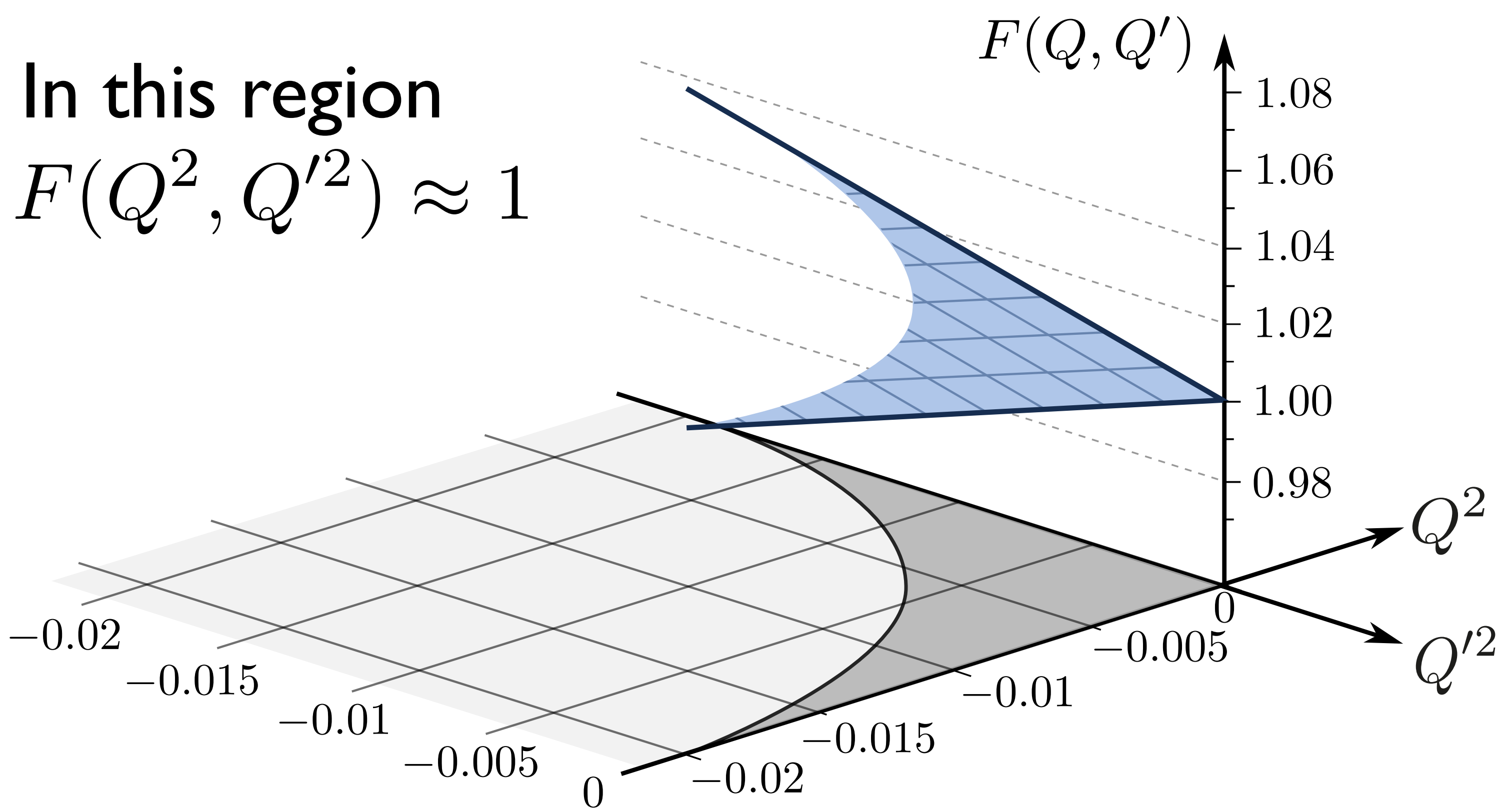
$\pi^0 \to 2e^+ 2e^-$





- $F(Q^2, Q'^2)$ evaluated at time-like photon momenta $-m_\pi^2 < Q^2 < -4m_e^2$

In this region $F(Q^2, Q'^2) \approx 1$



- In our full calculation $F(0,0) = 0.996$
- Only bare QP vertices $F(0,0) = 0.29$
- With Ball-Chiu $F(0,0) = 0.86$

Dalitz Decays

$\pi^0 \gamma e^+ e^-$

Collaboration	$\Gamma_{\pi^0 \to e^+ e^- \gamma}$ $[10^{-11}$ GeV$]$
PDG	9.06(18)
Terschlüsen et al.	9.26
Hoferichter et al.	9.065
Our result	9.11(4)

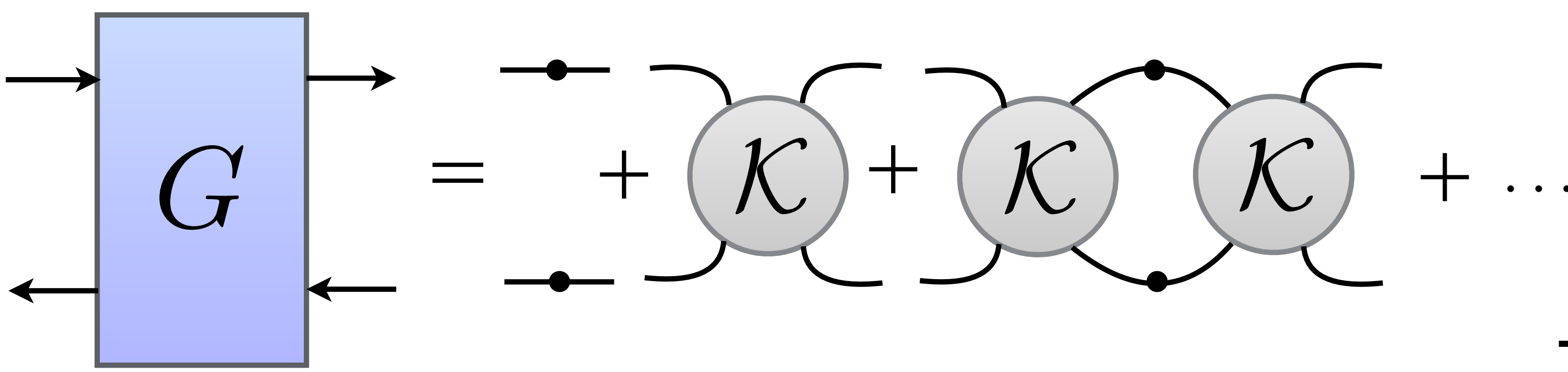
Result in agreement with experiment

as expected since integrated region is not sensitive to the form factor structure

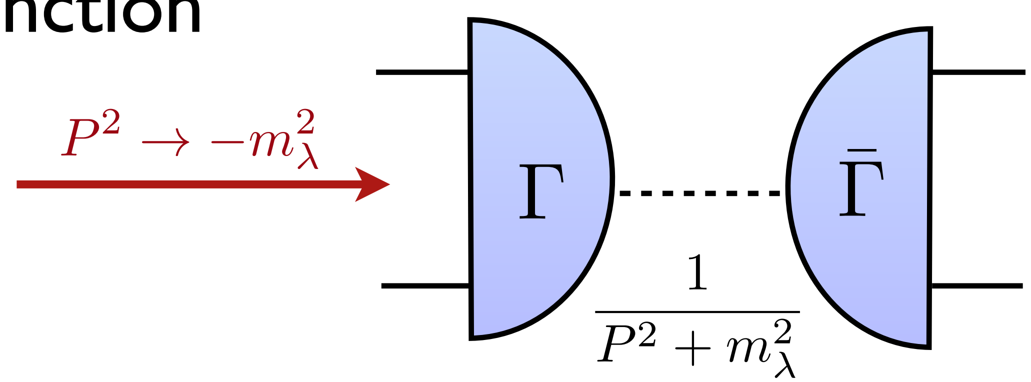
$\pi^0 2e^+ 2e^-$

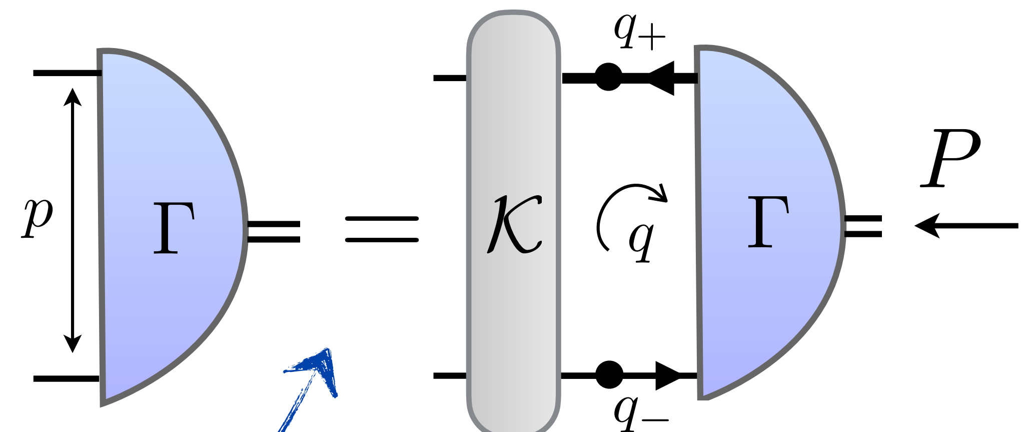
Collaboration	$\Gamma_{\pi^0 \to 2e^+ 2e^-}$ $[10^{-13}$GeV$]$
PDG	2.58(12)
Terschlüsen et al.	2.68
Escribano et al.	2.62
Our result	2.63(1)

Bethe-Salpeter(BSE) equation for a bound-states (meson):



four point function

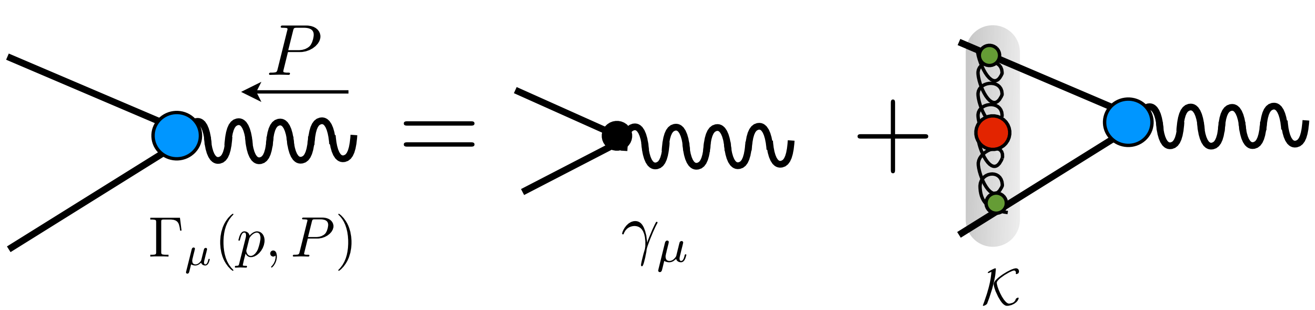




BSE

- **4** basis components

inhomogeneous BSE for quark-photon vertex :



- decomposed in into **12** tensor structures