

$\Lambda N \rightarrow NN$ potential with EFT & hypertriton weak decay

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FAIRNESS, Sitges, May 29th - June 2nd, 2017



Outline

- Motivation: Hypernuclei and hypertriton
- Effective field theory potentials
 - NN, YN
 - $\Lambda N \rightarrow NN$
- Hypertriton (non-mesonic) weak decay

Hyperons and hypernuclei

Hyperons: baryons with strangeness

Mesonic decay: ground state hyperons in free space decay weakly mainly to a baryon and a meson.

Mesonic decay of the Lambda: $\Lambda \rightarrow p \pi^-$ (64%) and $\Lambda \rightarrow n \pi^0$ (36%), $\tau \sim 260 \pm 2$ ps

Hypernuclei: Bound states of nucleons (p, n) and hyperons ($\Lambda, \Sigma, \Xi, \dots$)

Λ hypernuclei also decay **non-mesonically**: $\Lambda N \rightarrow NN$ (dominant for $A \geq 5$), $\Lambda NN \rightarrow NNN$ (DAΦNE, JParc)

Hypertriton

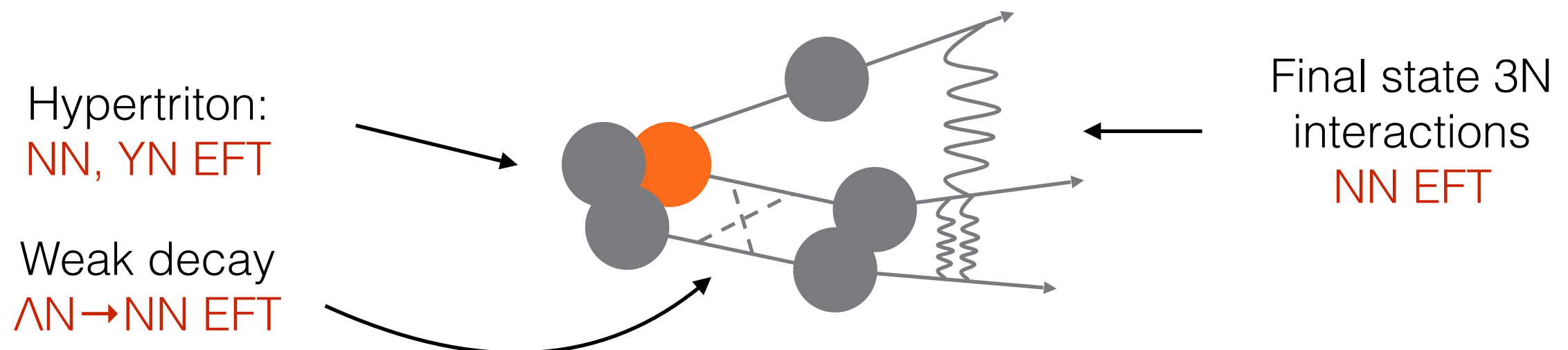
It's the *lightest hypernucleus*: Λ p n, Λ - Σ mix, $s=1/2$, $B=130\pm 50$ KeV

It decays faster than the Lambda! Rappold, Saito et al., PLB 728 (2013) 543

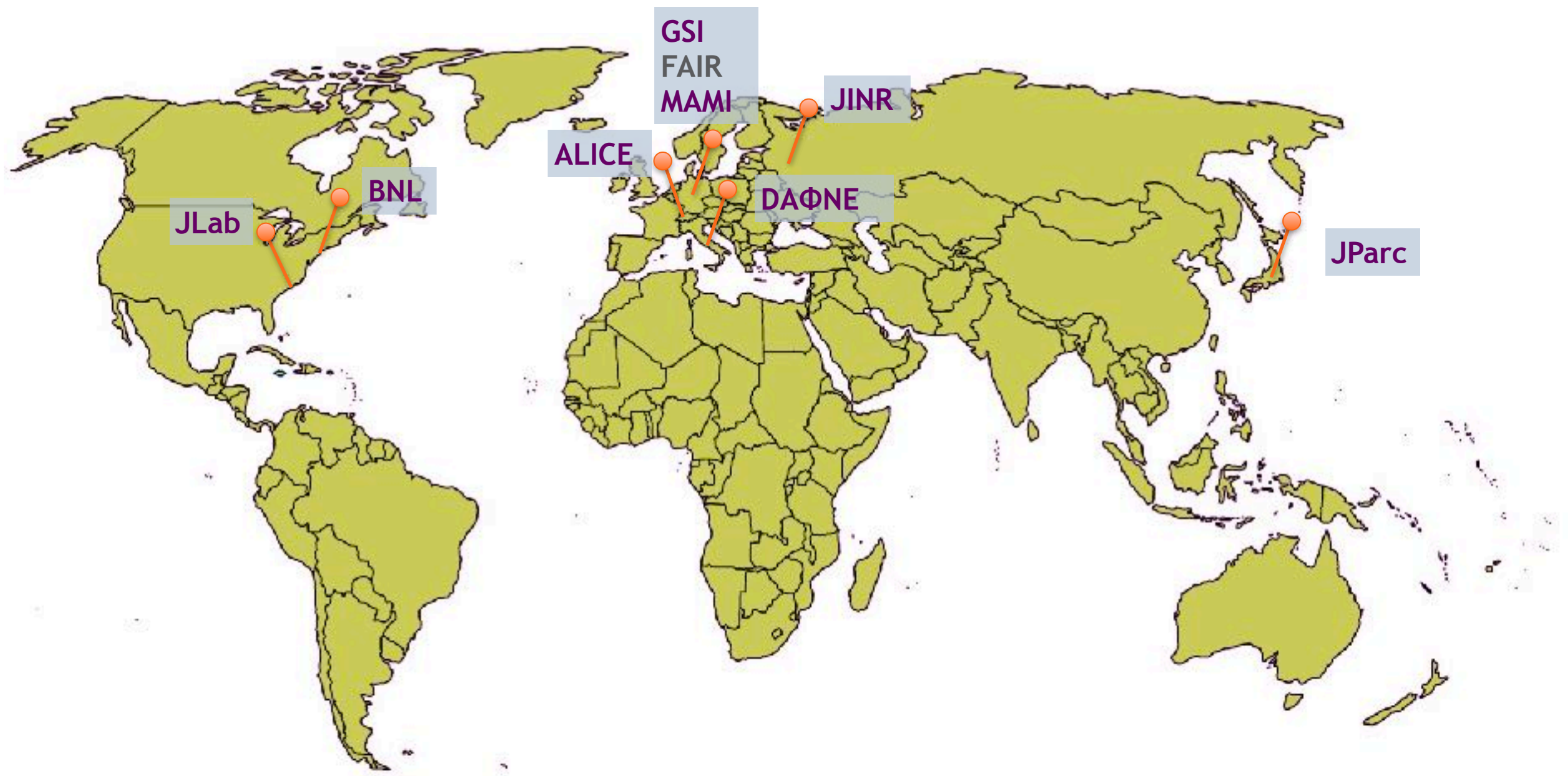
The study of its weak decays requires:

- The **strong forces** among hyperons and nucleons and the **weak decay mechanisms**.
- Few-body scheme to implement all the interactions

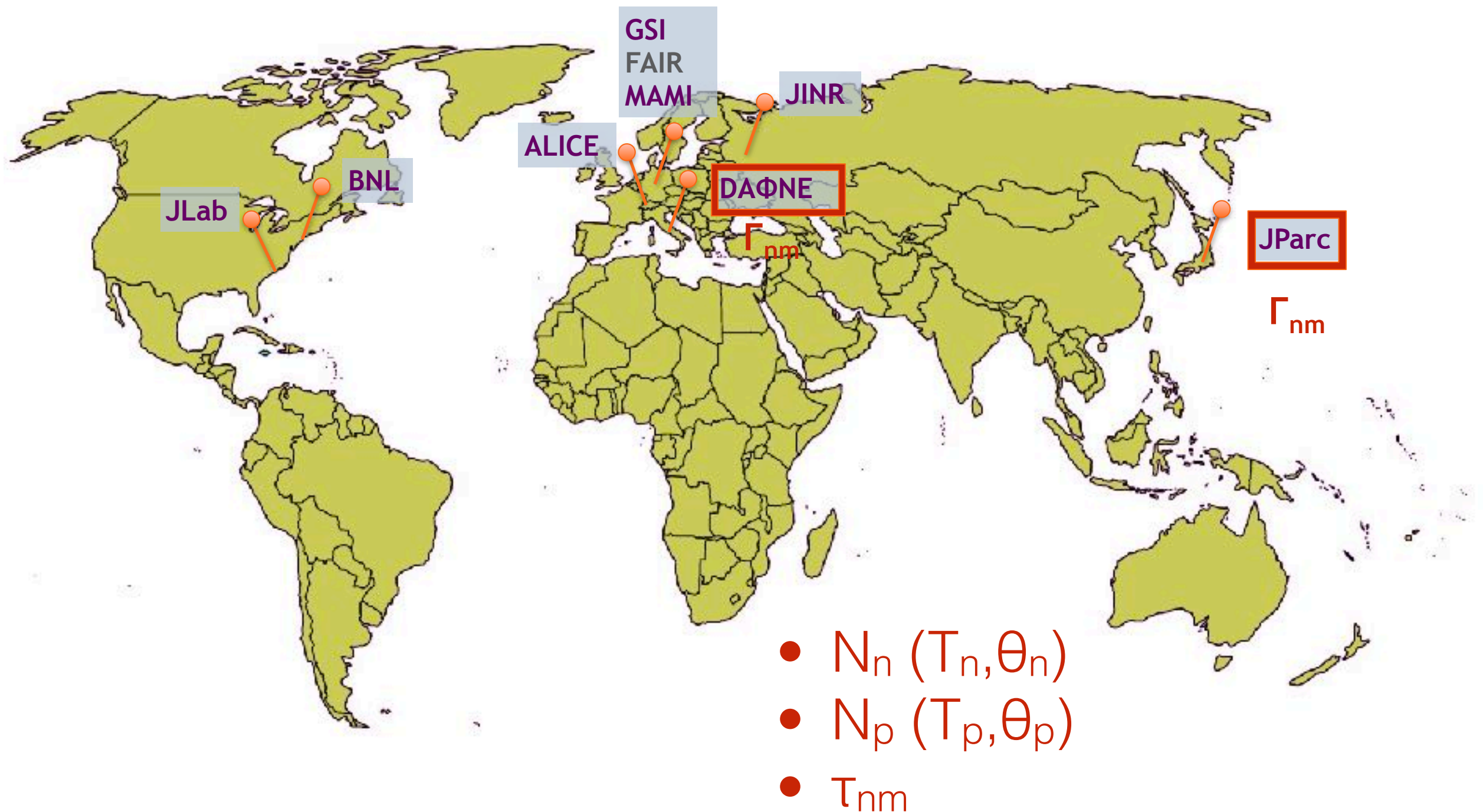
Non-mesonic weak decay with effective field theory (previous work with OME potentials: Golak et al., PRC56 (1997), 2892)



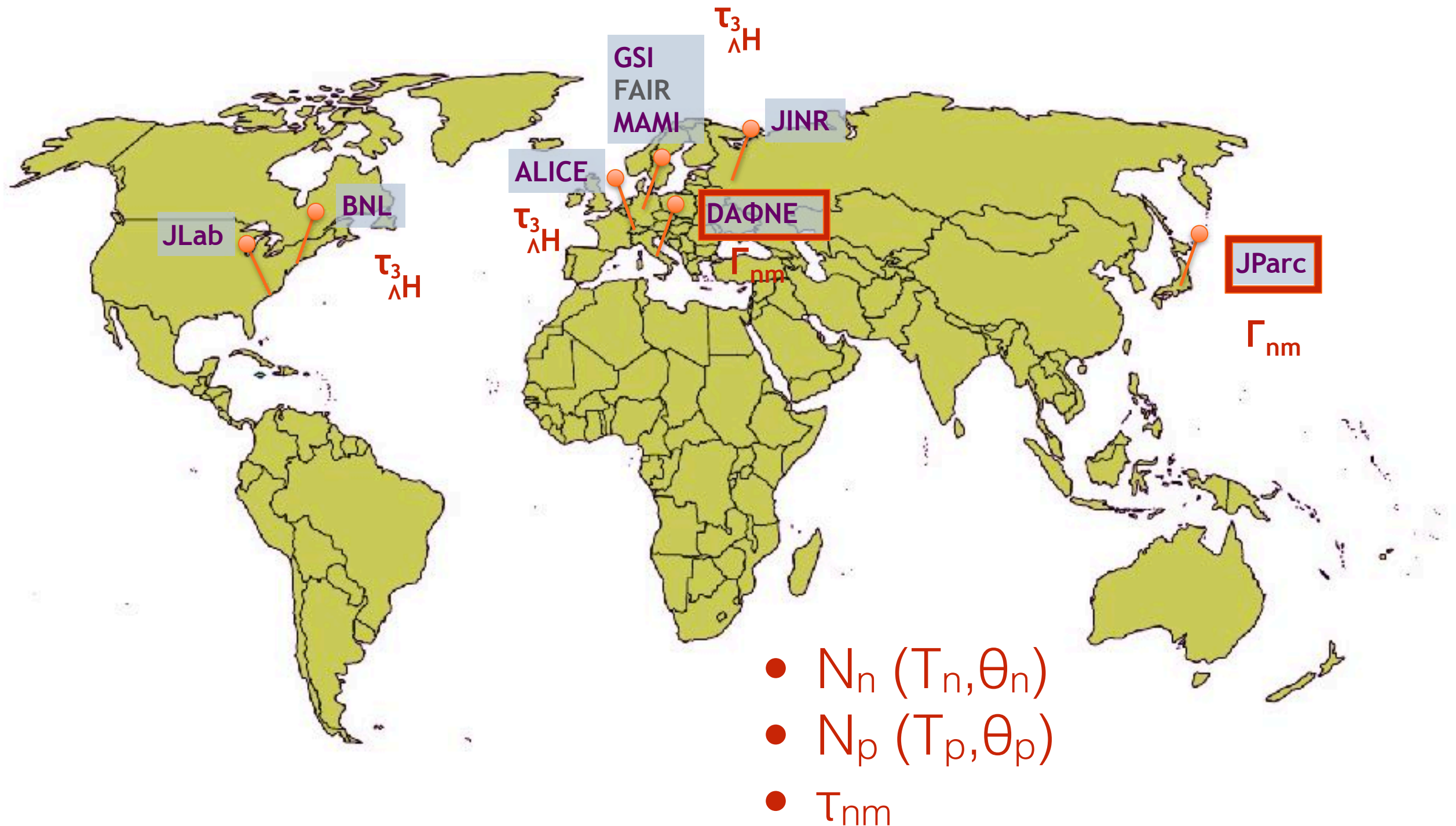
Hypernuclear experiments



Hypernuclear experiments



Hypernuclear experiments



Effective field theory

A systematic approximation to some underlying dynamics valid at a certain energy scale

Method:

1. Separate low and high energy scales and identify relevant degrees of freedom and symmetries
2. Construct organisational scheme (momentum expansion)
3. Build the most general Lagrangian with these d.o.f. and symmetry constraints
4. Calculate matrix elements up to a certain order in the expansion

Example: the nuclear force

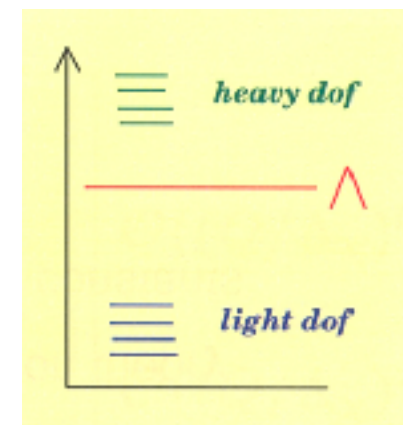
- Seminal work S. Weinberg, PLB 251 (1990) 288; NPB 363 (1991) 3. First application C. Ordoñez, L. Ray, and U. van Kolck, PRC 53 (1996) 2086. N3LO D.R. Entem and R. Machleidt, PRC 68 (2003) 041001 E. Epelbaum, W. Glöckle, Ulf-G. Meißner, NPA747 (2005) 362
- Separate low and high energy scales and identify relevant degrees of freedom and symmetries:

Low energy scale: $q < 300 \text{ MeV}$

Hard scale: $\Lambda_{\text{QCD}} \sim M_N \sim 1 \text{ GeV}$

Degrees of freedom: pions, $m_\pi \sim 138 \text{ MeV}$

Symmetries: chiral symmetry (π are goldstone bosons)



Example: the nuclear force

- Construct organisational scheme

Heavy baryon: expand in q/M_N and m_π/M_N
($q_0 \sim M_N \rightarrow$ treat nucleons non-relativistically)

Expand V chirally with Weinberg power counting
and use regularized Lippmann-Schwinger eq. (NN
bound states, large scattering lengths).

Example: the nuclear force

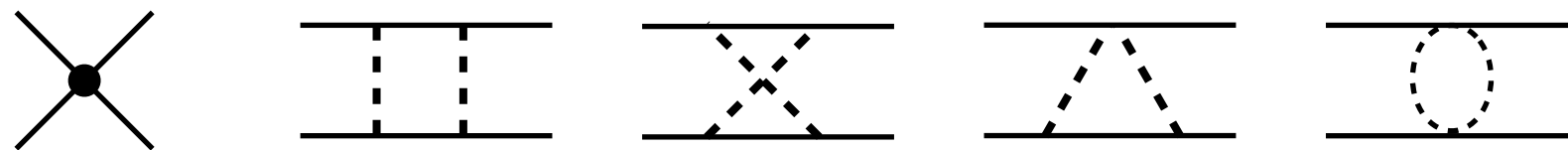
- Build the most general Lagrangian and calculate Feynman diagrams

$$\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\text{EFT}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \mathcal{L}_{3N} + \dots$$

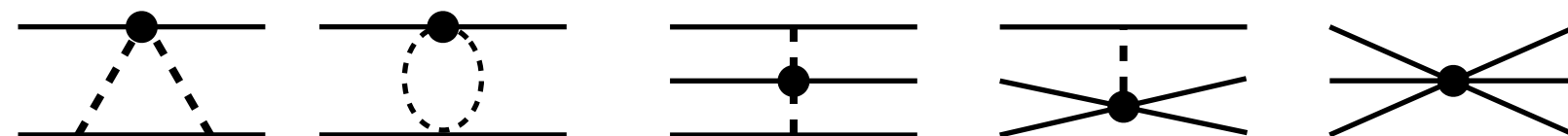
LO: contact interaction (unresolved short distance) + one-pion exchange (long range Yukawa potential)

$$C_1 + C_2 \sigma_1 \cdot \sigma_2 \quad \times \quad + \quad 1/(q^2 + m^2) \longrightarrow e^{-m \cdot r} \quad \text{---} \text{---} \text{---}$$

NLO: contact interactions plus two-pion exchanges (intermediate range attraction)



NNLO: sub-leading two-pion exchanges and **three-nucleon force**



EFT for YN

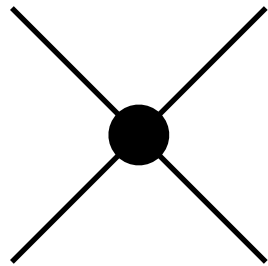
- **LO:** H. Polinder, J. Haidenbauer, U.-G. Meißner, NPA 779 (2006) 244; PLB 653 (2007) 29; J. Haidenbauer, U.-G. Meißner, A. Nogga, H. Polinder, Lect. Notes Phys. 724 (2007) 113; J. Haidenbauer, U.-G. Meißner, PLB 684 (2010) 275.
- **NLO:** J. Haidenbauer, S. Petschauer, N. Kaiser, U.-G. Meißner, A. Nogga, W. Weise, NPA 915 (2013) 24; S. Petschauer, N. Kaiser, NPA 916 (2013) 1
- **Other approaches:** C.L. Korpa, A.E.L. Dieperink, R.G.E. Timmermans, PRC 65 (2001) 015208
S.R. Beane, P.F. Bedaque, A. Parreño, M.J. Savage, NPA 747 (2005) 55
- New degrees of freedom (pseudoscalar mesons π , K , η)
- Larger symmetry: $SU(3)$
- Less data, constrain through hypernuclei

EFT for $\Lambda N \longrightarrow NN$

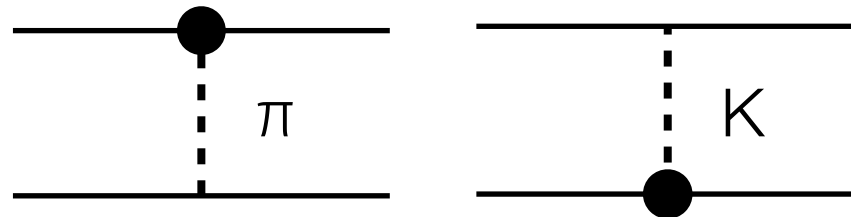
- LO: J.-H. Jun (2001); A. Parreño, C. Bennhold, B.R. Holstein (2004); Zhu, S.-lin, C.M. Maekawa, B.R. Holstein, U.V. Kolck (2005)
- NLO: Pérez-Obiol, Entem, Juliá-Díaz, Parreño, (2013, 2016)
- Low energy scale: $q \sim 400$ MeV (exothermic reaction)
- Degrees of freedom: π and K
- Symmetries: we also allow for PV
- Expansion in q/M_B , m_π/M_B and $\Delta M/M_B$

EFT for $\Lambda N \longrightarrow NN$

LO: contact interactions, one-pion and one-kaon exchanges



$$C_1 + C_2 \sigma_1 \cdot \sigma_2$$



$$V_\mu(\vec{q}) = -iG_F m_\pi^2 \frac{g_{BB\mu}}{2M} \left(\hat{A}_\mu - \frac{\hat{B}_\mu}{2M} \vec{\sigma}_1 \cdot \vec{q} \right) \frac{\vec{\sigma}_2 \cdot \vec{q}}{-q_0^2 + \vec{q}^2 + m_\pi^2}; \quad \mu = \pi, K.$$

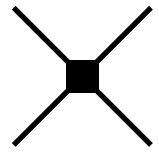
$$\hat{A}_\pi = A_\pi \vec{\tau}_1 \cdot \vec{\tau}_2; \quad \hat{A}_K = \frac{C_K^{PV}}{2} + D_K^{PV} + \frac{C_K^{PV}}{2} \vec{\tau}_1 \cdot \vec{\tau}_2;$$

$$\hat{B}_\pi = B_\pi \vec{\tau}_1 \cdot \vec{\tau}_2; \quad \hat{B}_K = \frac{C_K^{PC}}{2} + D_K^{PC} + \frac{C_K^{PC}}{2} \vec{\tau}_1 \cdot \vec{\tau}_2$$

EFT for $\Lambda N \longrightarrow NN$

NLO: higher order contact interactions:

Momentum space:



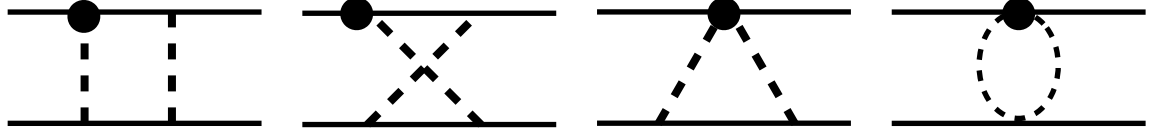
$\mathcal{O}(q)$	$\vec{\sigma}_1 \cdot \vec{q}, \vec{\sigma}_2 \cdot \vec{q}, \vec{\sigma}_1 \cdot \vec{p}, \vec{\sigma}_2 \cdot \vec{p}, (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{q}, (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{p},$
$\mathcal{O}(q^2)$	$\vec{q}^2, \vec{p}^2, \vec{q} \cdot \vec{p}, (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \vec{q}^2, (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \vec{p}^2, (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \vec{q} \cdot \vec{p}$ $\vec{\sigma}_1 \cdot (\vec{q} \times \vec{p}), \vec{\sigma}_2 \cdot (\vec{q} \times \vec{p}), (\vec{\sigma}_1 \cdot \vec{p})(\vec{\sigma}_2 \cdot \vec{p}),$ $(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{p}), (\vec{\sigma}_1 \cdot \vec{p})(\vec{\sigma}_2 \cdot \vec{q}), (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}).$

Coordinate
space:

$$\begin{aligned}
 V(r) = & V_1 + W_1(\vec{\tau}_1 \cdot \vec{\tau}_2) + V_{(\vec{\sigma}_1 \cdot \vec{\sigma}_2)}(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + W_{(\vec{\sigma}_1 \cdot \vec{\sigma}_2)}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2) \\
 & + V_{(\hat{r} \cdot \hat{p})}(\hat{r} \cdot \hat{p}) + W_{(\hat{r} \cdot \hat{p})}(\hat{r} \cdot \hat{p})(\vec{\tau}_1 \cdot \vec{\tau}_2) + V_{(\vec{\sigma}_1 \cdot \hat{r})}(\vec{\sigma}_1 \cdot \hat{r}) + W_{(\vec{\sigma}_1 \cdot \hat{r})}(\vec{\sigma}_1 \cdot \hat{r})(\vec{\tau}_1 \cdot \vec{\tau}_2) \\
 & + V_{(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\hat{r} \cdot \hat{p})}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\hat{r} \cdot \hat{p}) + W_{(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\hat{r} \cdot \hat{p})}(\vec{\sigma}_1 \cdot \vec{\sigma}_2)(\hat{r} \cdot \hat{p})(\vec{\tau}_1 \cdot \vec{\tau}_2) \\
 & + V_{(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_1 \cdot \hat{p})}(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_1 \cdot \hat{p}) + W_{(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_1 \cdot \hat{p})}(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_1 \cdot \hat{p})(\vec{\tau}_1 \cdot \vec{\tau}_2) \\
 & + V_{\vec{\sigma}_1 \cdot (\hat{r} \times \hat{p})} \vec{\sigma}_1 \cdot (\hat{r} \times \hat{p}) + W_{\vec{\sigma}_1 \cdot (\hat{r} \times \hat{p})} \vec{\sigma}_1 \cdot (\hat{r} \times \hat{p})(\vec{\tau}_1 \cdot \vec{\tau}_2) \\
 & + V_{\vec{\sigma}_2 \cdot (\hat{r} \times \hat{p})} \vec{\sigma}_2 \cdot (\hat{r} \times \hat{p}) + W_{\vec{\sigma}_2 \cdot (\hat{r} \times \hat{p})} \vec{\sigma}_2 \cdot (\hat{r} \times \hat{p})(\vec{\tau}_1 \cdot \vec{\tau}_2) \\
 & + V_{(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \hat{r}} (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \hat{r} + W_{(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \hat{r}} (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \hat{r} (\vec{\tau}_1 \cdot \vec{\tau}_2) \\
 & + V_{\hat{S}_{12}} \hat{S}_{12}(\hat{r}) + W_{\hat{S}_{12}} \hat{S}_{12}(\hat{r})(\vec{\tau}_1 \cdot \vec{\tau}_2).
 \end{aligned}$$

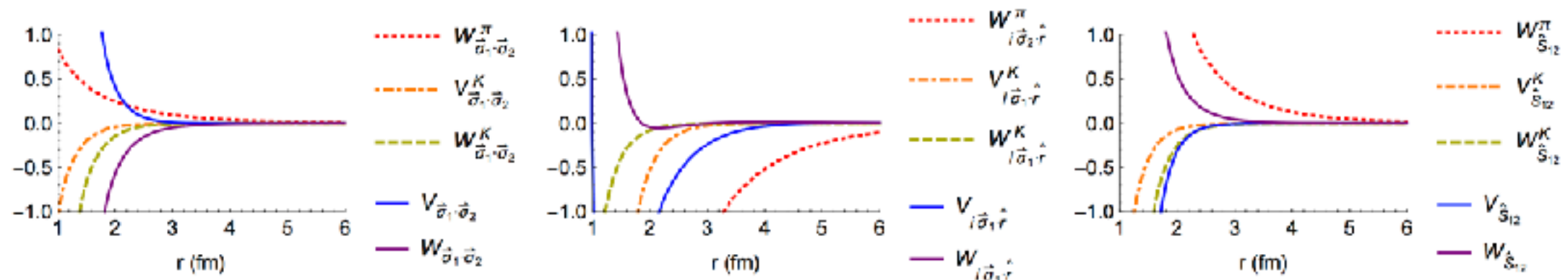
EFT for $\Lambda N \longrightarrow NN$

NLO: two-pion exchanges:

- 10 loop diagrams, e.g. 
- 4 kinds of loop integrals, e.g.

$$\int \frac{d^3q}{(2\pi)^3} e^{i\vec{q}\cdot\vec{r}} \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^2 - m^2 + i\epsilon} \frac{1}{(l+q)^2 - m^2 + i\epsilon} \frac{1}{-l_0 - q'_0 + i\epsilon} \frac{1}{-l_0 + i\epsilon} l_\mu \dots l_\nu$$

- Comparison between OPE, OKE and TPE



Non-mesonic weak decay of the hypertriton

- Use EFT YN, NN and $\Lambda N \rightarrow NN$ potentials
- Possible decay products: d+n and 3N
- Average over initial spins and integrate over all final states
- Example: decay to 3N

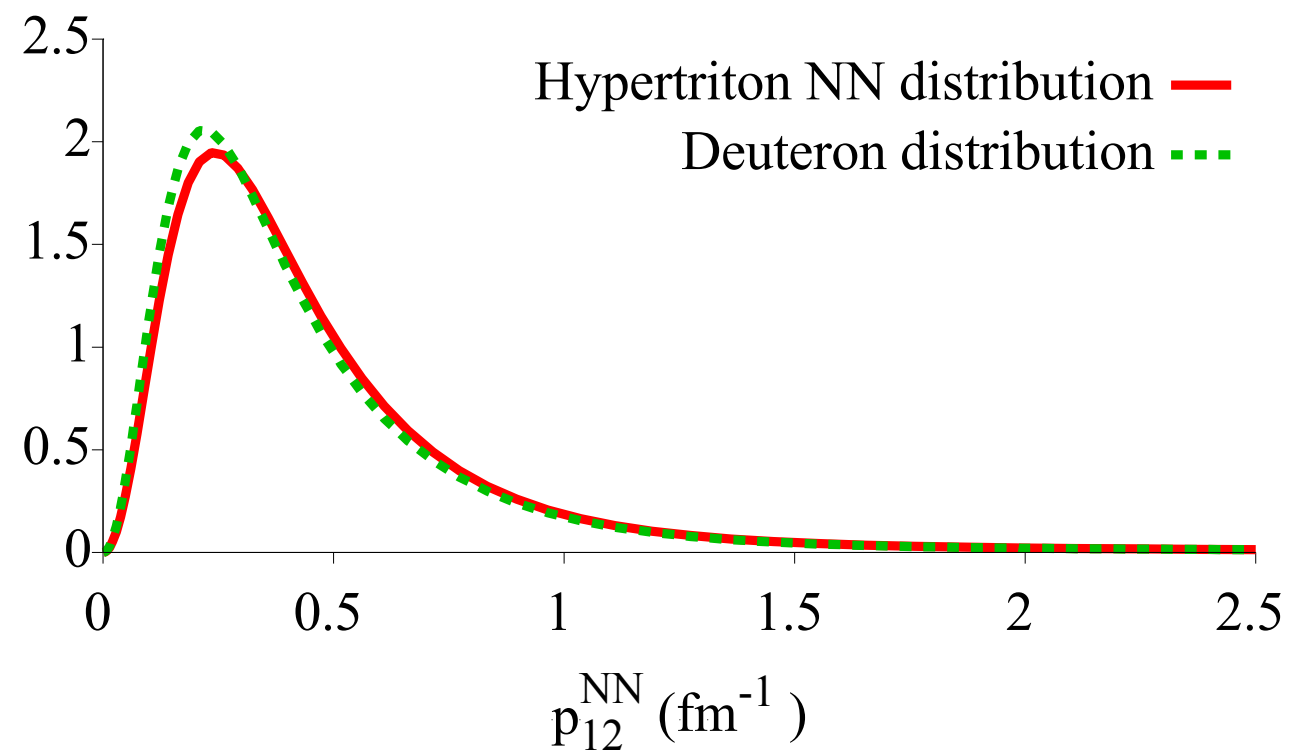
$$\Gamma^{3N} = \frac{128\pi^3 M_N}{9} \sum_{m_{J_3} m_i m_{t_i}} \int dp_{12} p_{12}^2 p_3 \int d\theta \sin(\theta) \left| \left\langle \psi_{m_{J_3} m_i m_{t_i}} \left| \left(1 + V_s \frac{1}{E + i\epsilon - H} \right) (1 + P) V_{12}^w \right| \psi_{\Lambda^3 H} \right\rangle \right|^2$$

Decay rate to 3N

$$\Gamma^{3N} = \frac{128\pi^3 M_N}{9} \sum_{m_{J_3} m_i m_{t_i}} \int dp_{12} p_{12}^2 p_3 \int d\theta \sin(\theta) \left| \left\langle \psi_{m_{J_3} m_i m_{t_i}} \left| \left(1 + V_s \frac{1}{E + i\epsilon - H} \right) (1 + P) V_{12} \right| \psi_{\Lambda^3 H} \right\rangle \right|^2$$

Hypertriton wave function:

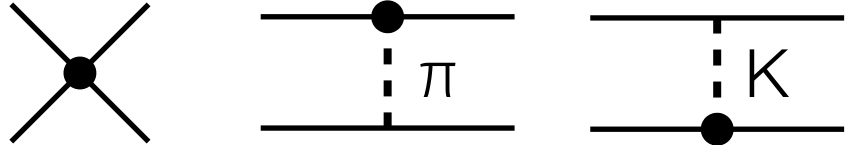
- YN and NN, NLO potentials
H. Polinder, J. Haidenbauer, U.G. Meissner, NPA, 779, 244-266, 2006
- Important of Σ component
- Similar to deuteron + Λ



Decay rate to 3N

$$\Gamma^{3N} = \frac{128\pi^3 M_N}{9} \sum_{m_{J_3} m_i m_{t_i}} \int dp_{12} p_{12}^2 p_3 \int d\theta \sin(\theta) \left| \left\langle \psi_{m_{J_3} m_i m_{t_i}} \left| \left(1 + V_s \frac{1}{E + i\epsilon - H} \right) (1 + \textcircled{P}) V_{12}^w \right| \psi_{\Lambda^3 H} \right\rangle \right|^2$$

Non-mesonic weak decay

- LO $\Lambda N \rightarrow NN$ EFT 
- Testing contribution of $\Sigma N \rightarrow NN$ EFT

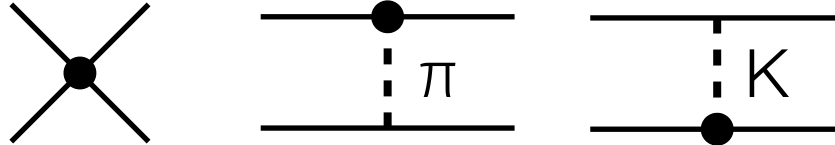
Final state interactions by directly solving the iterative equations

- Solve two body force $(1 + t_{12}G_0)(1 - V_{12}G_0) = 1$
- Solve rescattering state U
$$|U\rangle = t_{12}G_0(1 + P)V^w|\phi_{\Lambda^3 H}\rangle + (1 + t_{12}G_0)V_{123}^{(3)}G_0(1 + P)V^w|\phi_{\Lambda^3 H}\rangle + t_{12}G_0P|U\rangle + (1 + t_{12}G_0)V_{123}^{(3)}G_0(1 + P)|U\rangle$$

Decay rate to 3N

$$\Gamma^{3N} = \frac{128\pi^3 M_N}{9} \sum_{m_{J_3} m_i m_{t_i}} \int dp_{12} p_{12}^2 p_3 \int d\theta \sin(\theta) \left| \left\langle \psi_{m_{J_3} m_i m_{t_i}} \left| \underbrace{\left(1 + V_s \frac{1}{E + i\epsilon - H} \right) (1 + P) V_{12}^w}_{\text{Non-mesonic weak decay}} \right| \psi_{\Lambda^3 H} \right\rangle \right|^2$$

Non-mesonic weak decay

- LO $\Lambda N \rightarrow NN$ EFT 
- Testing contribution of $\Sigma N \rightarrow NN$ EFT

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$$|U\rangle = t_{12}G_0(1 + P)V^w|\phi_{\Lambda^3 H}\rangle + (1 + t_{12}G_0)V_{123}^{(3)}G_0(1 + P)V^w|\phi_{\Lambda^3 H}\rangle + t_{12}G_0P|U\rangle + (1 + t_{12}G_0)V_{123}^{(3)}G_0(1 + P)|U\rangle$$

Decay rate to 3N

Final state interactions through Stieltjes and Lorentz transformations

- Add spurious degree of freedom

$$\Gamma(w) = \frac{1}{2} \sum_m \int df \, 2\pi \delta(M_{\Lambda^3 H} - E_f + w) |\langle f | \hat{O} | \Psi_{\Lambda^3 H} m \rangle|^2$$

- Stieltjes transform and use completeness

$$\phi(\sigma) = \int_{w_0}^{\infty} dw \frac{\Gamma(w)}{w + \sigma} = \frac{1}{2} \sum_m \int df \, 2\pi \langle \Lambda^3 H | \hat{O} | f \rangle \frac{1}{E_f - M_{\Lambda^3 H} + \sigma} \langle f | \hat{O} | \Lambda^3 H \rangle$$

$$\phi(\sigma) = -\frac{1}{2} \sum_m 2\pi \langle \Lambda^3 H m | \hat{O} \left[\frac{1}{\Delta M + E_{\Lambda^3 H} - \sigma - H} + |^3 H \rangle \frac{1}{E_t - \Delta M - E_{\Lambda^3 H} + \sigma} \langle ^3 H | \right] \hat{O} | \Lambda^3 H m \rangle$$

- Solve iterative equation for large enough σ

$$|\phi\rangle = \frac{1}{\Delta M + E_{\Lambda^3 H} - \sigma - H_0} \left(1 - \langle ^3 H | \hat{O} | \Lambda^3 H m \rangle \right) \hat{O} | \Lambda^3 H \rangle + V |\phi\rangle$$

- Make inverse transformation with singular value decomposition

Decay rate to 3N

$$\Gamma^{3N} = \frac{128\pi^3 M_N}{9} \sum_{m_{J_3} m_i m_{t_i}} \int dp_{12} p_{12}^2 p_3 \int d\theta \sin(\theta) \left| \left\langle \psi_{m_{J_3} m_i m_{t_i}} \left| \left(1 + V_s \frac{1}{E + i\varepsilon - H} \right) (1 + P) V_{12}^w \right| \psi_{\Lambda^3 H} \right\rangle \right|^2$$

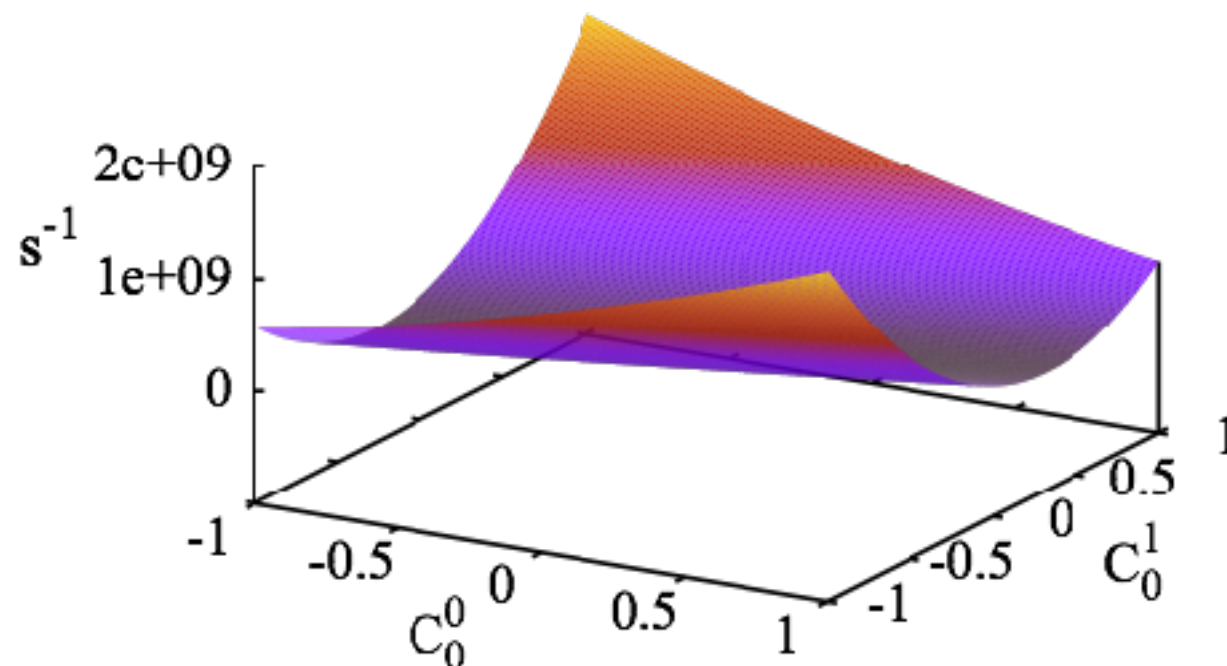
- $\Gamma_{3N} \sim 10\Gamma_{d+n}$
- K destructive interference

	$\Gamma_{d+n} \text{ (s}^{-1}\text{)}$	$\Gamma_{3N} \text{ (s}^{-1}\text{)}$
π	$0.54 \cdot 10^7$	$0.57 \cdot 10^8$
$\pi + K$	$0.15 \cdot 10^7$	$0.18 \cdot 10^8$

Decay rate to 3N

$$\Gamma^{3N} = \frac{128\pi^3 M_N}{9} \sum_{m_{J_3} m_i m_{t_i}} \int dp_{12} p_{12}^2 p_3 \int d\theta \sin(\theta) \left| \left\langle \psi_{m_{J_3} m_i m_{t_i}} \left| \left(1 + V_s \frac{1}{E + i\varepsilon - H} \right) (1 + P) V_{12}^w \right| \psi_{\Lambda^3 H} \right\rangle \right|^2$$

- Explore the parameter space (LECs C_0 and C_1)
- LECs with opposite signs interfere constructively



Hypertriton decay rates

- 5 possible weak decays: 2 non-mesonic + 3 mesonic

$$\Gamma_{nm}^{d+n} = \frac{32\pi^2 M_N}{9} p_3^{d+n} \sum_{m_{J_3} m_{j_{12}} m_3} \left| \left\langle \psi_{m_{j_{12}} m_3} \left| \left(1 + (V_s - V_{12}) \frac{1}{E + i\epsilon - H} \right) (1 + P) V_{12}^w \right| \psi_{\Lambda^3 H} \right\rangle \right|^2$$

$$\Gamma_{nm}^{3N} = \frac{128\pi^3 M_N}{9} \sum_{m_{J_3} m_i m_{t_i}} \int dp_{12} p_{12}^2 p_3 \int d\theta \sin(\theta) \left| \left\langle \psi_{m_i m_{t_i}} \left| \left(1 + V_s \frac{1}{E + i\epsilon - H} \right) (1 + P) V_{12}^w \right| \psi_{\Lambda^3 H} \right\rangle \right|^2$$

$$\Gamma_m^{He} = \frac{M_{He} q^{(He)}}{12\pi(M_{He} + E_\pi)} \sum_{m_{J_3} m_{He}} \left| \left\langle \psi_{m_{He}} \left| (1 + P) V_3^w \right| \psi_{\Lambda^3 H} \right\rangle \right|^2$$

$$\Gamma_m^{d+n} = \frac{M_N}{6M_N + 2E_\pi} q^{(d+n)} \sum_{m_{J_3} m_i m_{t_i}} \int dp_3 p_3^2 \int d\theta \sin(\theta) \left| \left\langle \psi_{m_i m_{t_i}} \left| \left(1 + (V_s - V_{12}) \frac{1}{E + i\epsilon - H} \right) (1 + P) V_3^w \right| \psi_{\Lambda^3 H} \right\rangle \right|^2$$

$$\Gamma_m^{3N} = \frac{M_N}{3M_N + E_\pi} \sum_{m_{J_3} m_i m_{t_i}} \int dp_{12} p_{12}^2 \int dp_3 p_3^2 \int d\theta_{12} \sin(\theta_{12}) \int d\theta_3 \sin(\theta_3) q^{(3N)} \left| \left\langle \psi_{m_i m_{t_i}} \left| \left(1 + V_s \frac{1}{E + i\epsilon - H} \right) (1 + P) V_3^w \right| \psi_{\Lambda^3 H} \right\rangle \right|^2$$

Summary and conclusions

- The **hypertriton can be studied with precise few-body techniques**. Its weak non-mesonic decay has been studied complementing exact wave functions obtained solving Faddeev equations with the leading order piece of the weak $\Lambda N \rightarrow NN$ transition
- We have developed an **EFT for the two-body $\Lambda N \rightarrow NN$ transition** driving the decay of hypernuclei.
- The **2π exchange mechanism** has been incorporated systematically in the EFT. It has a sizeable effect at medium and short ranges and should play an important role once more experimental data are available.
- Due to the lack of experimental data we have explored the decay rate as **a function of the two low-energy constants** appearing at leading order.

Current/future work

- Explore the effect of final state interactions and the Σ contribution.
 - Through Stieltjes and Lorentz transformations
- Compute the mesonic decays, beginning with $3\text{LH} \longrightarrow \text{He} + \pi^-$
 - FSI through pionic distorted waves

$$\Gamma_m^{\text{He}} = \frac{M_{\text{He}} q^{(\text{He})}}{12\pi(M_{\text{He}} + E_\pi)} \sum_{m_{J_3} m_{\text{He}}} \left| \langle \psi_{m_{\text{He}}} | (1+P) V_3^w | \psi_{\Lambda^3\text{H}} \rangle \right|^2$$

- A=4 hypernuclei.