

Two-pole structure of the $D_0^*(2400)$

Pedro Fernandez-Soler

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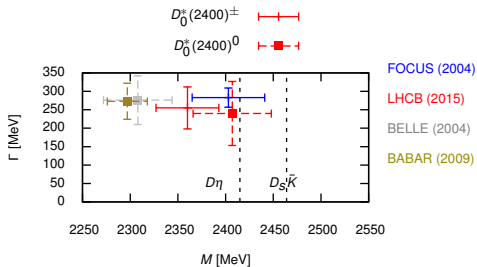
M. Albaladejo – P. Fernandez-Soler – F-K Guo – J. Nieves

2 of June 2017

- I Introduction
- II Formalism: $D\pi$ $D\eta$ $D_s\bar{K}$ interaction
- III Predictions: comparison with Lattice QCD
- IV $SU(3)$ considerations
- V Summary

I. Introduction (1)

- Lowest lying $(S, l) = (0, 1/2)$,
 $J^P = 0^+$

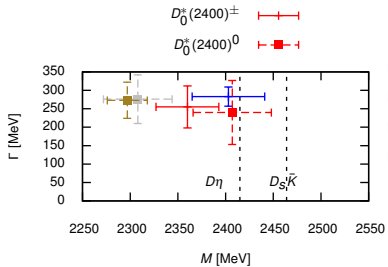
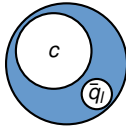


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- ▶ Naive picture:

$$|D_0^*(2400)\rangle \propto$$



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LHCb (2015)

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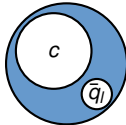
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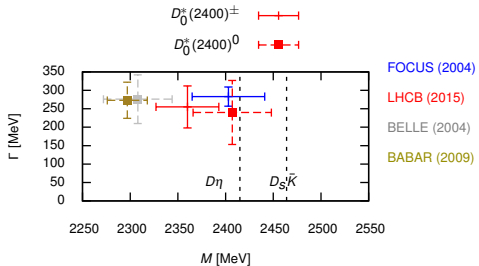
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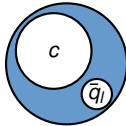


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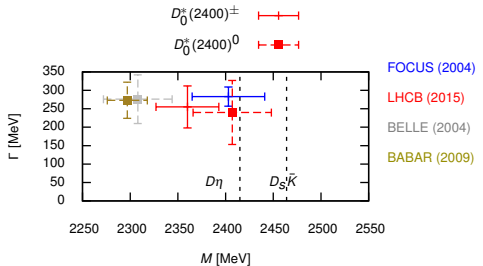
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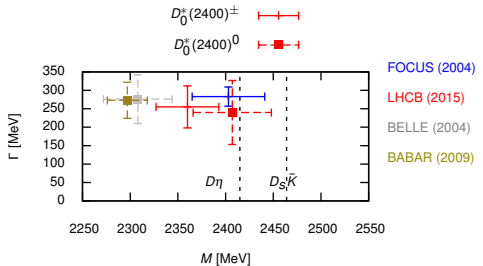
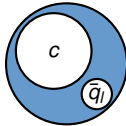


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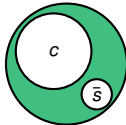
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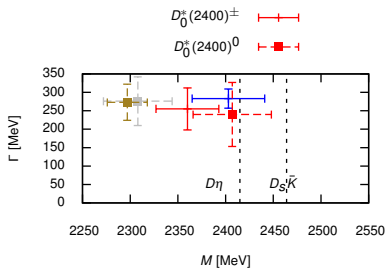
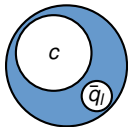


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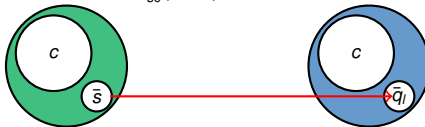
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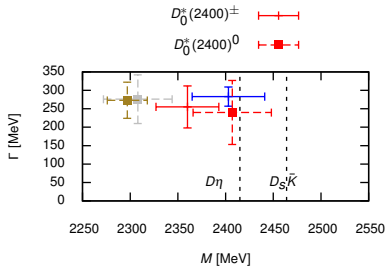
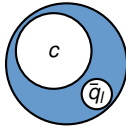


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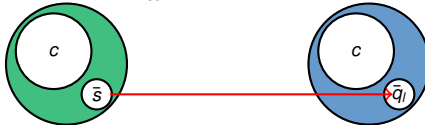
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- ▶ Its presence influences the shape of the f_0 and f_+ semileptonic form factors

Measurements

- ▶ C. Patrignani, et al., Particle Data Group, Chin. Phys. C 40 (2016) 100001
- ▶ B. Aubert, et al., BaBar Collaboration, Phys. Rev. Lett. 90 (2003) 242001
- K. Abe, et al., Belle Collaboration, Phys. Rev. D 69 (2004) 112002. (discovery)

Lattice QCD

- ▶ G.S. Bali, Phys. Rev. D 68 (2003) 071501.
- ▶ A. Dougall, et al., UKQCD Collaboration, Phys. Lett. B 569 (2003) 41.
- ▶ D. Mohler, S. Prelovsek, R.M. Woloshyn, Phys. Rev. D 87 (2013) 034501.
- D. Mohler, C.B. Lang, L. Leskovec, S. Prelovsek, R.M. Woloshyn, Phys. Rev. Lett. 111 (2013) 222001 (include meson correlators)
- ▶ G. Moir, M. Peardon, S.M. Ryan, C.E. Thomas, D.J. Wilson, J. High Energy Phys. 1610 (2016) 011 (used here!)

Here

- ▶ E.E. Kolomeitsev, M.F.M. Lutz, Phys. Lett. B 582 (2004) 39
- F.-K. Guo, P.-N. Shen, H.-C. Chiang, R.-G. Ping, B.-S. Zou, Phys. Lett. B 641 (2006) 278
- F.-K. Guo, C. Hanhart, U.-G. Meißner, Eur. Phys. J. A 40 (2009) 171
- ▶ L. Liu, K. Orginos, F.-K. Guo, C. Hanhart, U.-G. Meißner, Phys. Rev. D 87 (2013) 014508

I. Introduction (3)

LQCD simulation of $(S, I) = (0, 1/2)$ sector J. High Energy Phys. 1610 (2016) 011.

- ▶ First principle calculations
- ▶ Quarks and gluons in a grid
- ▶ Periodic boundary conditions
 $\psi(\vec{x}_0 + \vec{n}L) = \psi(\vec{x}_0)$
 $\vec{k} = \frac{2\pi}{L}(n_1, n_2, n_3) \rightarrow \vec{k}^2 = n^2 \frac{4\pi^2}{L^2}$
- ▶ Produce hadron states $|D\pi(n)\rangle$
 - ▶ Volume dependence of energy levels

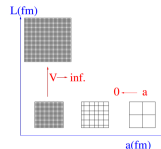
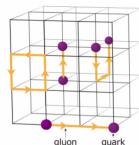


Figure: Taken from [S.Ryan MESON2016](#)

$$\omega_{D\pi}(n, L) = \sqrt{\vec{k}^2 + m_\pi^2} + \sqrt{\vec{k}^2 + m_D^2} \text{ vs } E_{D\pi}(L) \approx \omega_{D\pi}(n, L) + \Delta E(L)$$

▶ m_i

	LQCD	Phys
m_π	391	138
m_K	550	496
m_η	588	548
m_D	1886	1867
m_{D_s}	1952	1968

Comparing with LQCD $L \rightarrow \infty$

- ▶ Lüscher formalism $\delta(L \rightarrow \infty) \leftrightarrow \Delta E(L)$ Commun. Math. Phys. 105, 153 (1986); Nucl. Phys. B354, 531 (1991)
- ▶ Finite volume calculations Eur. Phys. J. A (2011) 47: 139; Phys. Rev. D 85, 014027 (2012)

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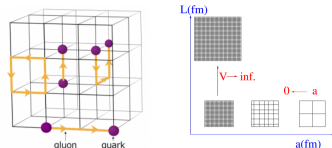
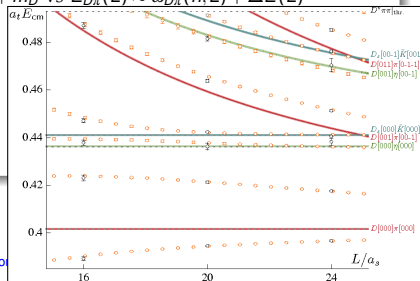


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II. Formalism (1): Unitarized scattering amplitudes

$$H(p_1)\phi(p_2) \rightarrow H'(p_3)\phi'(p_4)$$

$$H = \{D^0, D^+, D_s\}, \phi = \begin{pmatrix} -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta}{\sqrt{6}} \end{pmatrix}$$

$\mathcal{L}$ at NLO: $\mathcal{O}(p^2)$

S	I	channels
-1	0	$D\bar{K}$
-1	1	$D\bar{K}$
2	1/2	$D_s K$
0	3/2	$D\pi$
1	1	$D_s \pi, DK$
1	0	$DK, D_s \eta$
0	1/2	$D\pi, D\eta, D_s \bar{K}$

$$\mathcal{M}(s, t, u) = \frac{C_0}{4F^2}(s - u) + \mathcal{F}(s, t, u)$$

$$\mathcal{F}(s, t, u) = h_0 \mathcal{F}_0(s, t, u) + h_1 \mathcal{F}_1(s, t, u) + \dots + h_5 \mathcal{F}_5(s, t, u)$$

- 6 LEC describe all (S, I) sectors
- S-wave amplitudes unitarized

$$\begin{aligned} \mathcal{T}(s) &= \mathcal{M}(s) + \mathcal{M}(s)G(s)\mathcal{M}(s) + \mathcal{M}(s)G(s)\mathcal{M}(s)G(s)\mathcal{M}(s) + \dots \\ &= (1 - \mathcal{M}(s)G(s))^{-1} \mathcal{M}(s) \Rightarrow \boxed{\mathcal{T}^{-1}(s) = \mathcal{M}(s)^{-1} - G(s)} \end{aligned}$$

- h_{1-5} LECS constrained by LQCD [PHYSICAL REVIEW D 87, 014508 (2013)]

II. Formalism (2): Unitarized scattering amplitudes

$$(S, I) = (0, 1/2)$$

- S-matrix S-wave parametrization

$$S_{i,j}(s) = \delta_{i,j} - i \sqrt{\frac{q_i}{4\pi\sqrt{s}}} \mathcal{T}_{i,j} \sqrt{\frac{q_j}{4\pi\sqrt{s}}} \quad S_{j,j}(s) = \eta_j e^{i\delta_j(s)}$$

- Unitarity! $(SS^\dagger) = \mathbb{I} \Rightarrow \text{Im} \mathcal{T}_{jj}^{-1}(s) = q_j / 8\pi\sqrt{s} \Theta(\sqrt{s} - (m_j + M_j))$

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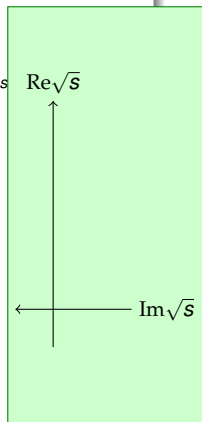
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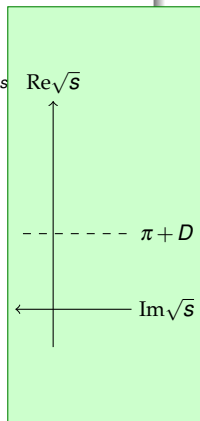
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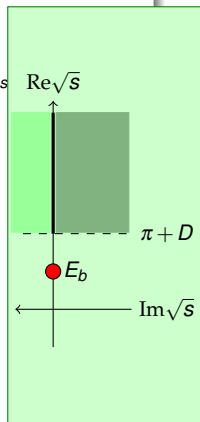
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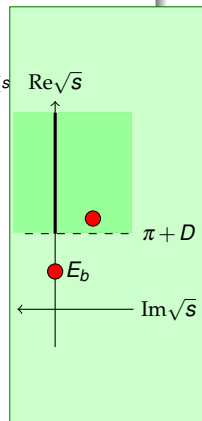
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Analytical continuation

- $G_{D\pi}(s) \rightarrow G_{ij}^{\text{II}}(s) = G_{D\pi}(s) + i\frac{k}{4\pi\sqrt{s}}$

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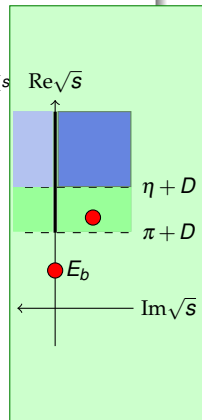
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- $\xi = 0, 1 \quad G_{j,j} \rightarrow G_{j,j}(s) + i\frac{k}{4\pi\sqrt{s}} \xi_j, (\xi_1, \xi_2, \xi_3) \quad (0,0,0) - (1,0,0) - (1,1,0) - (1,1,1)$

$$\xrightarrow{D\pi \quad D\eta \quad D_s \bar{K}} \text{Re}\sqrt{s}$$

II. Formalism (4): finite volume

- In finite volume

$$G_j(s) = \int_{\mathbb{R}^3} \frac{d^3q}{(2\pi)^3} \frac{f(\vec{q})}{s - \omega_j(\vec{q})^2} \rightarrow \tilde{G}_j(s, L) = \sum_{\vec{q}(L)} \frac{1}{L^3} \frac{f(\vec{q})}{s - \omega_j(\vec{q})^2}$$

- Energy levels $\leftrightarrow$ eigen-energies

$$S_{i,j}(s) \rightarrow \tilde{S}_{i,j}(s, L)$$

$$\mathcal{T}_{i,j}(s) \rightarrow \tilde{\mathcal{T}}_{i,j}(s, L)$$

Let us denote them as $E(L)$,

$$\det \tilde{\mathcal{T}}^{-1} \left(s = E(L)^2, L \right) = 0, \text{ i.e., } \det(\mathcal{M}^{-1}(s) - \tilde{G}(s, L)) = 0$$

II. Formalism (4): finite volume

- In finite volume

$$G_j(s) = \int_{\mathbb{R}^3} \frac{d^3 q}{(2\pi)^3} \frac{f(\vec{q})}{s - \omega_j(\vec{q})^2} \rightarrow \tilde{G}_j(s, L) = \sum_{\vec{q}(L)} \frac{1}{L^3} \frac{f(\vec{q})}{s - \omega_j(\vec{q})^2}$$

- Energy levels $\leftrightarrow$ eigen-energies

$$S_{i,j}(s) \rightarrow \tilde{S}_{i,j}(s, L)$$

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No interaction

Take $\mathcal{M}_{i,j} \rightarrow 0$. Recall $\omega_i(n, L) = \sqrt{n(\frac{2\pi}{L})^2 + M_i^2} + \sqrt{n(\frac{2\pi}{L})^2 + m_i^2}$

- $\tilde{G}_{i,i}(s, L) \sim \sum_{\vec{q}(L)} \frac{1}{s - [\omega_i(\vec{q}(L))]^2} = \frac{1}{s - \omega_i(0)^2} + \frac{1}{s - \omega_i(1)^2} + \frac{1}{s - \omega_i(2)^2} + \dots$
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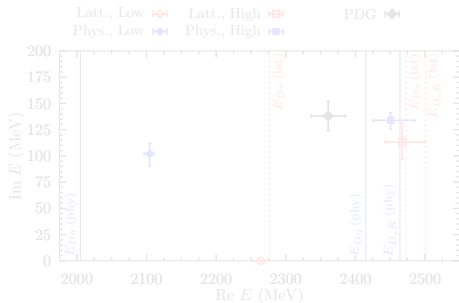
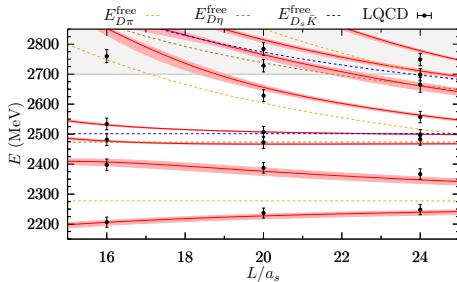
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- $\det(\mathcal{M}^{-1} - \tilde{G}(s, L)) = 0 \Rightarrow s = \omega_i(n, L)^2$
- $\mathcal{M} \neq 0$: deviation of $E(L)$ from $\omega_i(n, L)$

III. Results (1)

Energy levels and poles

No fit is performed here!

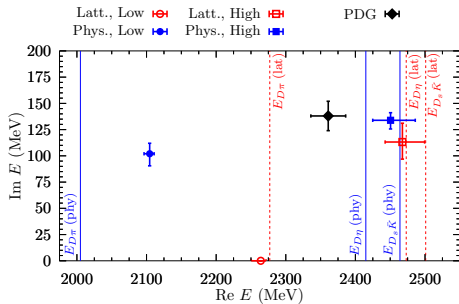
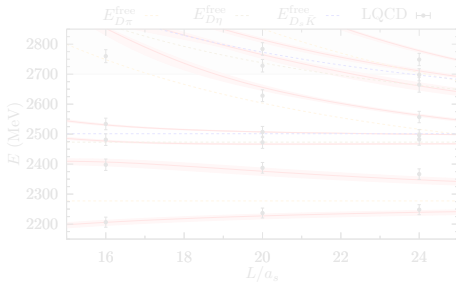


Masses	M (MeV)	$\Gamma/2$ (MeV)	RS	$ g_{D\pi} $	$ g_{D\eta} $	$ g_{D_s\bar{K}} $
lattice	2264^{+8}_{-14}	0	(000)	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	2468^{+32}_{-25}	113^{+18}_{-16}	(110)	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$
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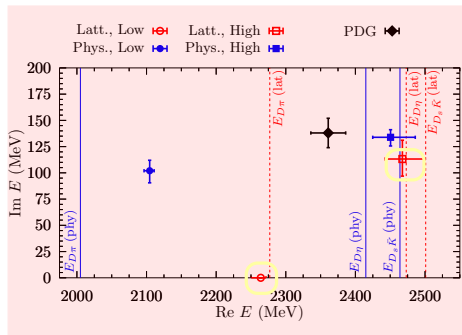
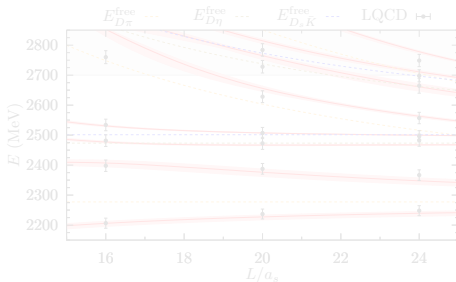


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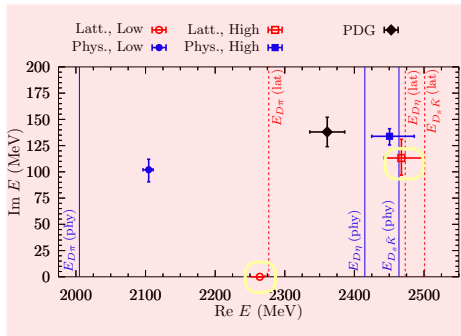
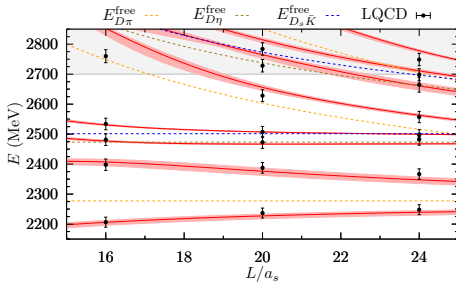
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	LQCD	Phys
m_π	391	138
m_K	550	496
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m_D	1886	1867
m_{D_s}	1952	1968

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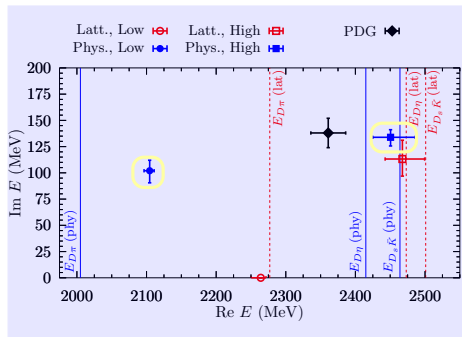
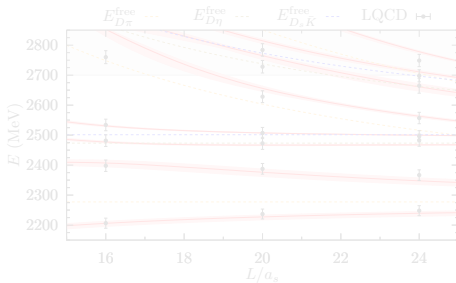
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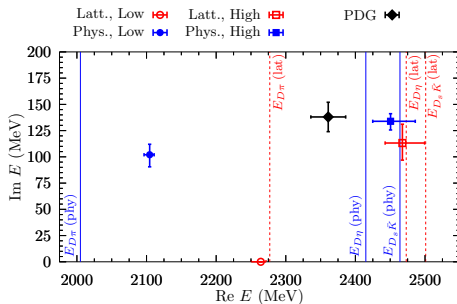
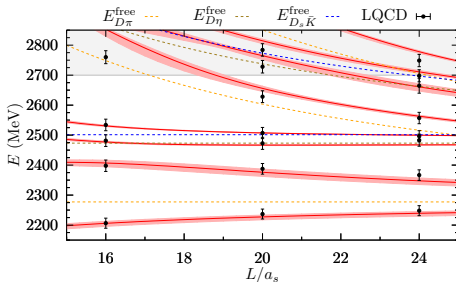
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III. Results (2)

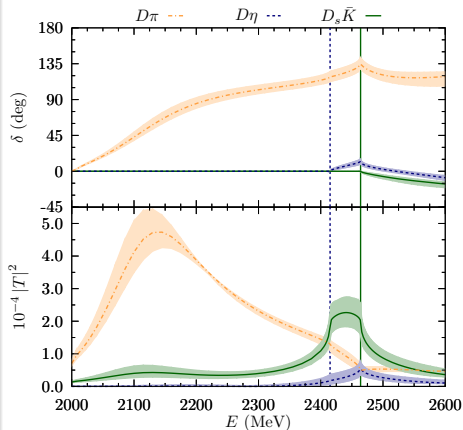
Amplitudes and phase-shifts

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- ▶ Lower pole: $D\pi \rightarrow D\pi$ amplitude clearly peaks at $E \in (2.1, 2.2)$ GeV



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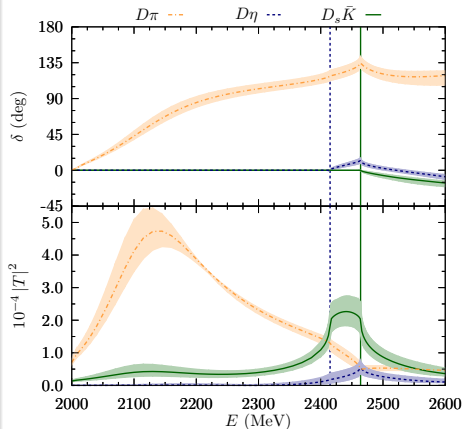
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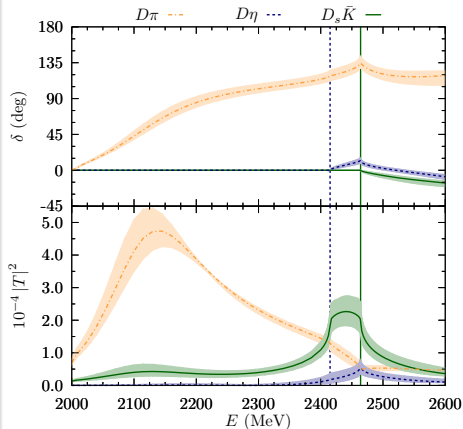
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Babar: [Phys.Rev.Lett. 100 \(2008\) 171803](#)
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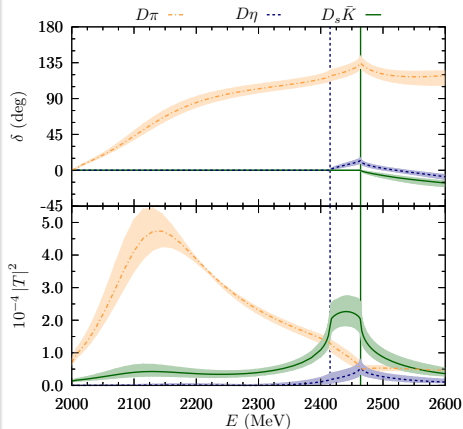
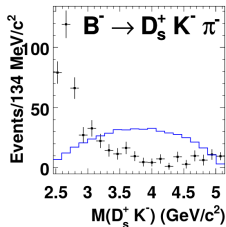
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The two pole $SU(3)$ origin

- We take the $SU(3)$ limit

$$m_i \rightarrow m$$

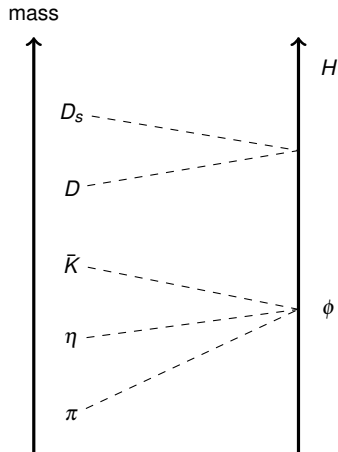
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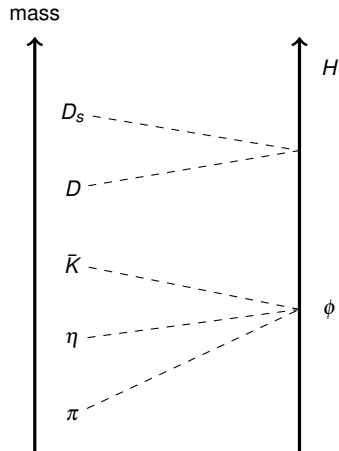
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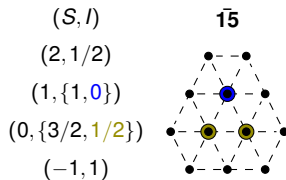
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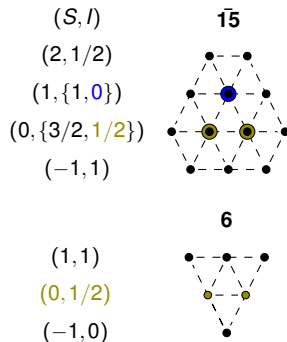
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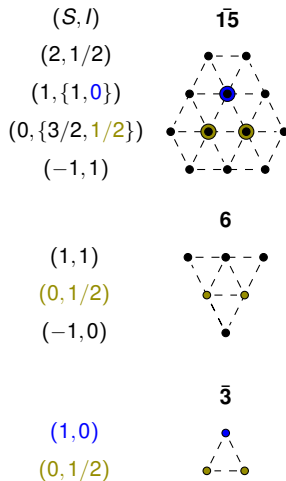
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III. Results (3)

$SU(3)$ limit

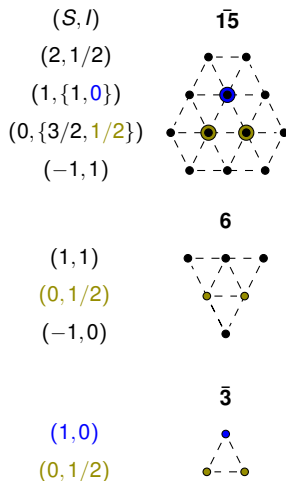
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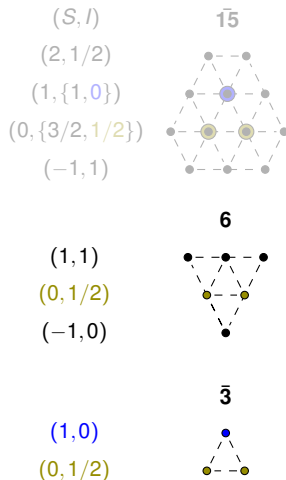
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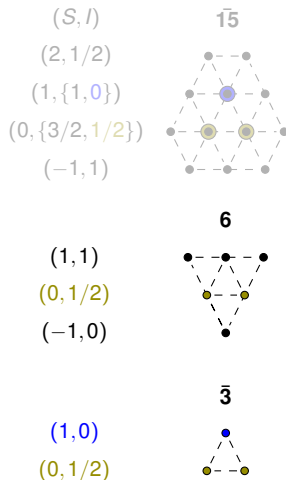
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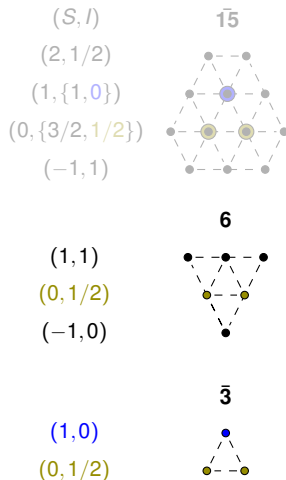
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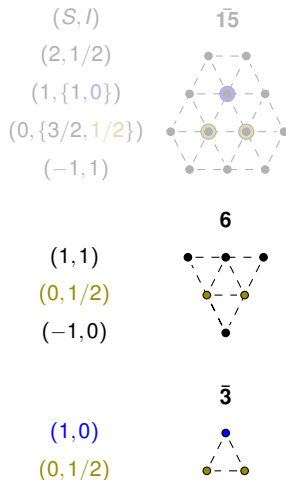
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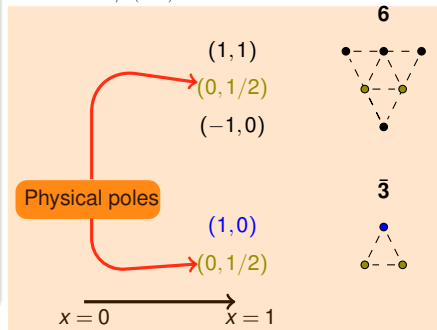
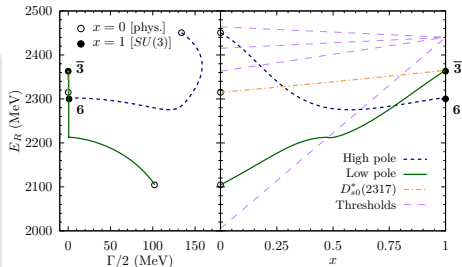
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Summary

- ▶ We have used unitarized HM chiral amplitudes previously constrained in different channels and sectors (S, I) to compute the scattering in $(0, 1/2) J^P = 0^+$
- ▶ The predictions in finite volume are in agreement with the results of a recent lattice simulation
- ▶ Our scattering amplitudes show a two-pole structure in the range of energies of the $D_0^*(2400)$ ($2.1 \sim 2.4$) GeV
- ▶ We have studied the $SU(3)$ origin of the two-poles, finding that the lighter (2.1 GeV) and the $D_{s0}^*(2317)$ build the $\bar{3}$ triplet (mass puzzle)
- ▶ This two-pole structure also predicted in $J^P = 1^+$ and in the bottom 0^+ and 1^+ sectors
- ▶ Thank you for your attention!!