Calculating hadron properties from dynamical hadronization within the FRG

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- Analytical continuation
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Motivation

Observable properties of hadrons are difficult to extract from QCD's degrees of freedom:

- Need theoretical assumptions:
- Bound states and QCD (at hadronic energies) are not perturbative.

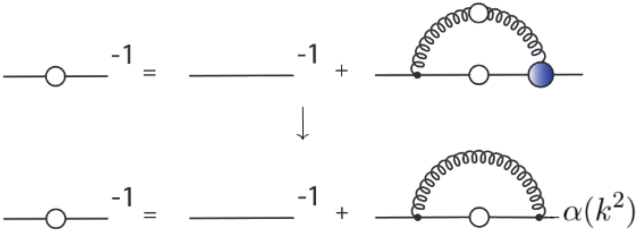
Many approaches and models are built to solve these problems.

- Lattice QCD.
- Functional methods.
  - **Dyson-Schwinger equations (DSE)**.
  - **Bethe-Salpeter equations (BSE)**.

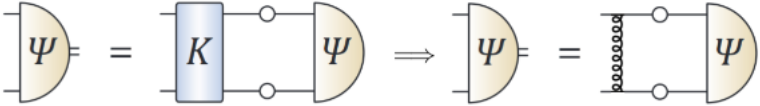
Both equations need to be truncated to be numerically solved.

- Rainbow-Ladder truncation:

Quark propagator:



Bethe-Salpeter equation:



Results for the pion and other ground state mesons are well understood. However:

- The solution relies strongly on the truncation.
- For more complex systems:
  - Other terms appear in the DSE.
  - Rainbow-Ladder truncation is not good enough.
  - New technical issues appear.

Work in a different approach:

- Use of the **Functional Renormalization Group (FRG)** to find properties of mesons.
- The FRG approach is consistent with BSE.

Introduction to the FRG

The properties of the scale-dependent effective action lead to the **1-loop** integral-differential equation:

$$\partial_t \Gamma_k = \frac{1}{2}\ \mathrm{Tr}\ \left(\partial_t R_k \left[\Gamma_k^{(2)} + R_k \right]^{-1} \right)$$

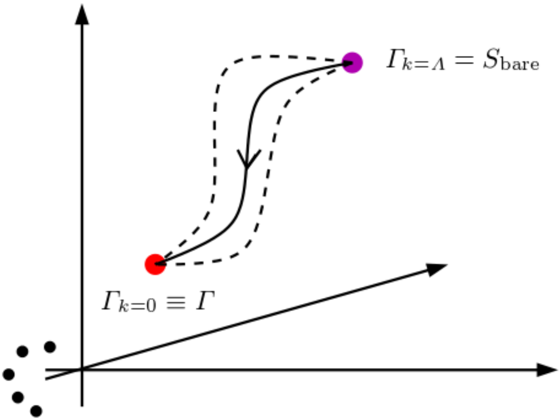
Wetterich's Flow Equation

with

$$t = \ln\ \left(\frac{k}{\Lambda_{UV}} \right) \qquad \partial_t = k \frac{d}{dk}$$

The properties of this exact flow equation are very convenient for physical calculations since it is an Euclidean **1-loop** integral-differential equation.

- Initial and final conditions are fixed in theory space:

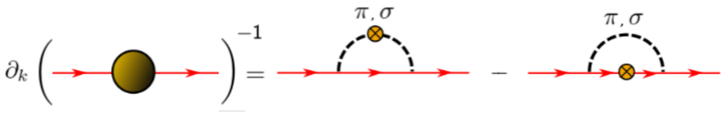


- The choice of the regulator is not unique.

Wetterich's equation can be solved applying vertex expansion:

$$\Gamma_k[\phi] = \sum_{n=0}^{\infty} \frac{1}{n!} \int_{p_1 \ldots p_n} \Gamma_k^{(n)}(p_1, \ldots p_n)\phi(p_1)\ldots\phi(p_n)$$

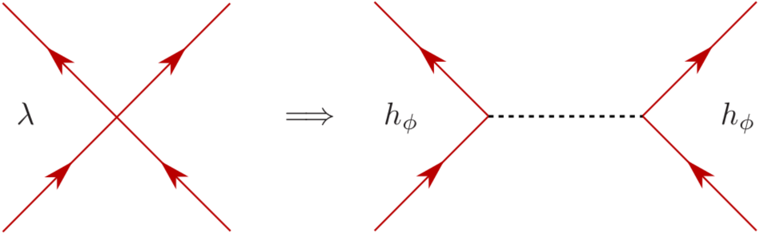
Applying $n-$derivatives and averaging the fields one obtains the flow of the momentum dependent vertex functions.



Remark: a truncation/approximation is needed.

Dynamical Hadronization

Macroscopic QCD degrees of freedom are mesons and baryons. Introduced in the effective action through 4-Fermi Hubbard-Stratonovich transf.:

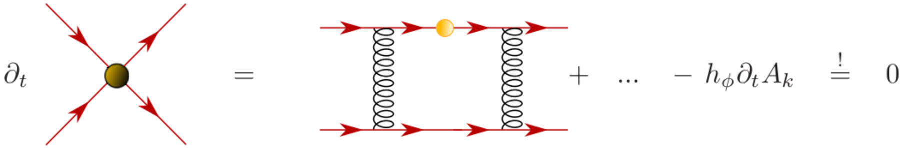


- **Problem**: 4-Fermi interaction flow non-zero, H-S transfomation must be applied in every RG-step $\Longrightarrow$ Solved by Dynamical Hadrnization.

Introduction of scale dependent bosonic field:

$$\partial_t \phi_k(p) = \partial_t A_k(p)(\bar{\psi}\tau\psi)(p) + \partial_t B_k(p)\phi_k(p)$$

with $\partial_t A_k$ and $\partial_t B_k$ defined such that 4-Fermi flow is cancelled:



- This generalizes Hubbard-Stratonovich transf. for every RG-step.
- Green's functions computed with meson exchange diagrams.

Analytical Continuation

Search for bound state properties through real-time Green's functions: analytical continuation must be performed.

Since the inverse propagator of 2-point function is proportional to $(p^2 + M^2)$ in Euclidean Space, goal is to continue to purely imaginary p_0.

- **Extrapolation** by fitting Euclidean momenta p^2 data to a parametrized function and evaluating it at Minkowski momenta $-p^2$.
- **Direct calculation** using the properties of the regulators.

Analytical continuation by extrapolation:

- Padé approximant:

$$R^{(m\,n)}(x) = \frac{\sum_{i=0}^{m} c_i x^i}{1 + \sum_{j=1}^{n} d_j x^j}$$

- Schlessinger point method:

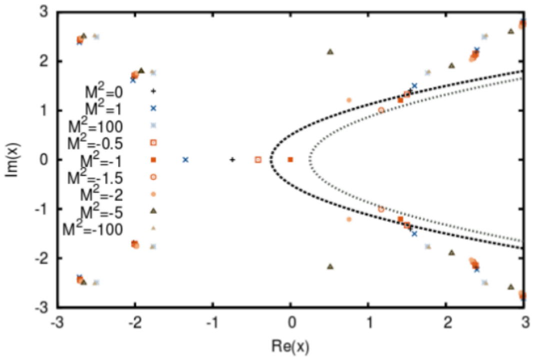
$$C(x) = \cfrac{F(x_1)}{1 + \cfrac{z_1(x-x_1)}{1 + \cfrac{z_2(x-x_2)}{\vdots\,z_M(x-x_M)}}}$$

Analytical continuation by direct calculation:

- Complex momenta accessed by direct calculation, not extrapolation. Already achieved using 3D-regulators [R-A Tripolt et al, arXiv:1311.0630v2].
- Successfully performed with a 4D modified regulator for zero temperature $O(N)$ model [J. M. Pawlowski and N. Strodthoff, arXiv:1508.01160v3].

$$R_{k;\Delta m_r^2}(p^2) = \left(\Delta\Gamma_k^{(2)}(p^2)|_{\phi=\phi_0} + \Delta m_r^2\right) r\left(\frac{p^2+\Delta m_r^2}{k^2}\right)$$

- Modified regulator moves poles out the integrating region $\Longrightarrow$ physical implications induced.

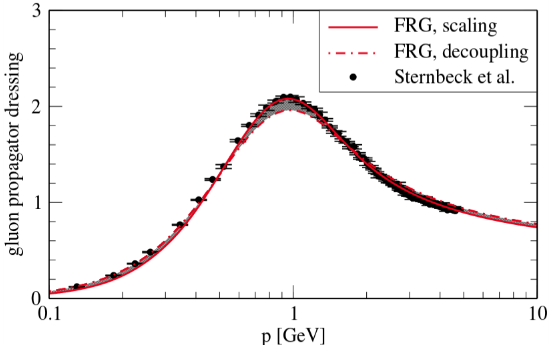


Physical and regulator poles in the complex $x = (p_0^2 + \vec{p}^2)/k^2$ for values of $M^2 = (m^2 - \Delta m_r^2)/k^2$.

[J. M. Pawlowski and N. Strodthoff, arXiv:1508.01160v3]

Preliminary results in the Quark-Meson model

Why using QM as starting point?



A.Cyrol et al, arXiv:1605.01856

Low-energy effective theory with gluons decoupled.

Free massless quark effective action + 4-Fermi interaction:

$$\Gamma_k[\bar{\psi},\psi] = \int \frac{d^4p}{(2\pi)^4}\, Z_{k,\psi}\; \bar{\psi}_a^A\, i\not{p}\, \psi_a^A \qquad + \qquad \Gamma_k^{4-int}[\bar{\psi},\psi]$$

Applying Hubbard-Stratonovich transformation introducing σ and $\vec{\pi}$ fields:

$$\begin{aligned}\Gamma_k\left[\bar{\psi},\psi,\sigma,\vec{\pi}\right] \;=\; \int_p \Big\{ & Z_{\psi,k}\,\bar{\psi}_a^A\, i\not{p}\,\psi_a^A + \\ & + \frac{1}{2}\left(Z_{k,\phi}p^2 + \overline{m}^2_{k,\phi}\right)\left(\sigma^2 + \pi^z\pi_z\right) - c\sigma + \\ & + \int_q h_k\bar{\psi}_a^A\left(\frac{\sigma}{2}\delta_{ab} + i\gamma_5(\tau_z)_{ab}\pi^z\right)\psi_b^A \Big\}\end{aligned}$$

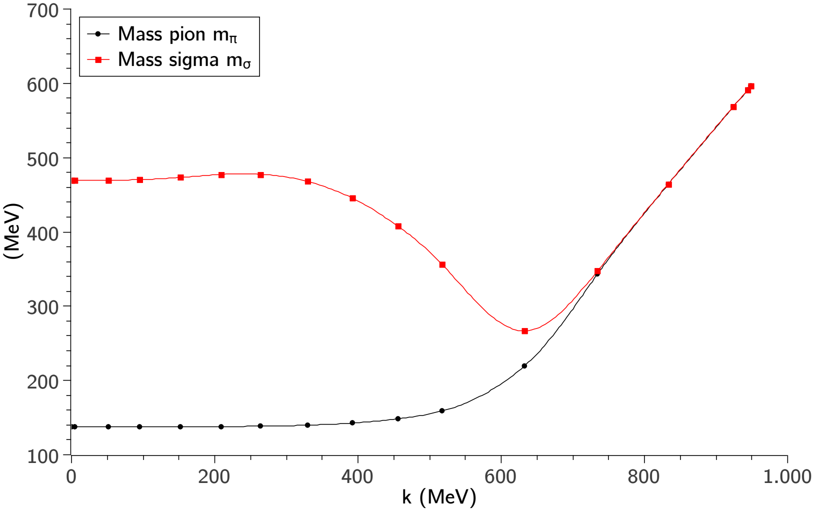
Local potential approximation (LPA)

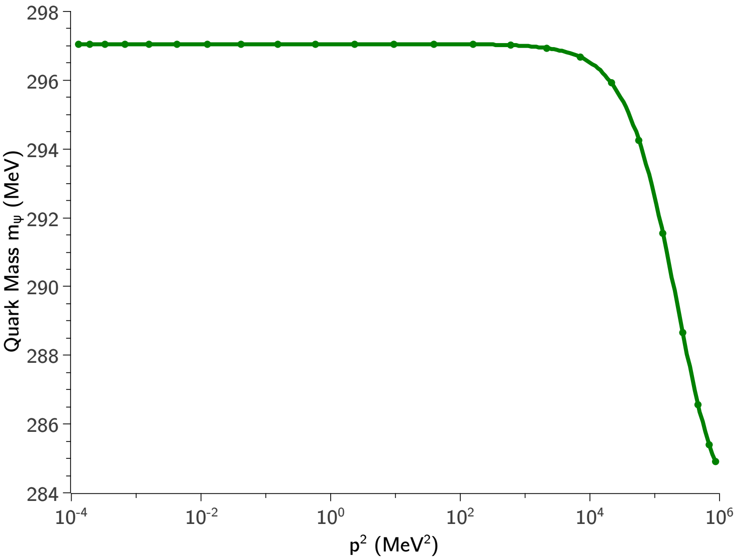
Wave-function renormalizations $Z_{k,i} \to 1$ and non-kinetic bosonic terms rewritten in a O(N) potential:

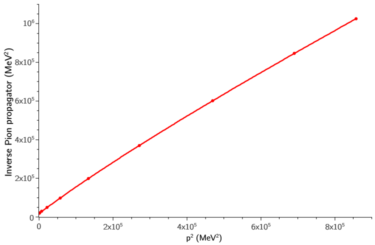
$$V_k(\rho) = \sum_{n=0}^{\infty} \frac{V_k^{(n)}}{n!} (\rho - \rho_0)^n$$

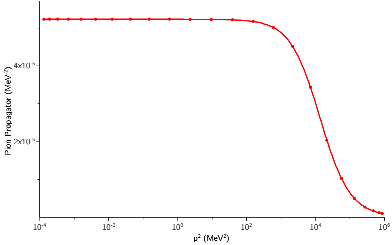
with $\rho = \frac{1}{2}(\sigma^2 + \pi_z \pi^z)$ and ρ_0 constant expansion point related to $\langle \sigma \rangle \neq 0$.

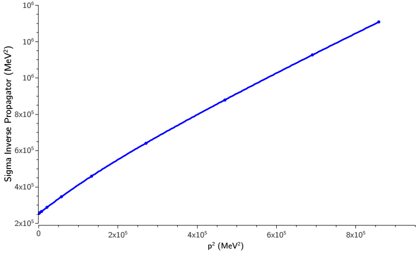
Infinite set of flow equations for $V_k^{(n)}$, but result unchanged for $n > 6$.

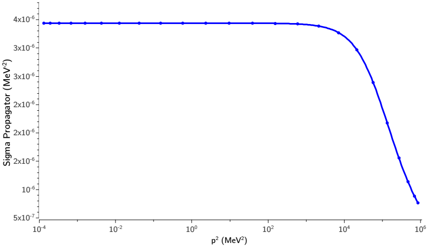

Mass pion m_π
Mass sigma m_σ
(MeV)
700
600
500
400
300
200
100
0
200
400
600
800
1.000
k (MeV)


298
296
294
292
290
288
286
284
Quark Mass m_ψ (MeV)
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
p^2 (MeV^2)


Inverse Pion propagator (MeV²)
p² (MeV²)


Pion Propagator (MeV⁻²)
p² (MeV²)


Sigma Inverse Propagator (MeV²)
p² (MeV²)


Sigma Propagator (MeV⁻²)
p² (MeV²)

Applying analytical continuation using the Padé method with numerical noise, common poles appear for different orders at:

- Pole $m_\pi^2 \approx 120$ MeV
- Pole $m_\sigma^2 \approx 400$ MeV

Results agree with Schlessinger point method.

LPA values for mesonic pole masses differ from curvature masses and quark mass does not behave as expected. **Different approximations need to be used.**

Current status:

Quark-Meson model in a different approximation (LPA'), where:

- $Z_{k,i} \neq 1$, therefore anomalous dimensions appear.
- A potential $V_k(\rho)$ is still used.
- The expansion point at $\langle\sigma\rangle$ is no longer constant.

Once results are obtained in the LPA', the gauge bosons will be introduced and start working with QCD effective action.

Summary and Outlook

- The FRG provides an alternative procedure to the BSE/Faddeev equation to obtain resonance masses and decay widths.
- Analytical continuation of the 4-fermi interaction to timelike momenta are to be performed in the Quark-Meson model.
- Big numerical effort and tools are needed to obtain accurate results.
- Provided that results in the FRG are compatible with DSE/BSE, the viability of this method should be analysed.

THANK YOU FOR YOUR ATTENTION