Extraction of Polarization Parameters from the $\bar{p}p \to \bar{\Omega}^+\Omega^-$ Reaction

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FAIRNESS 2017 at Sitges, Spain

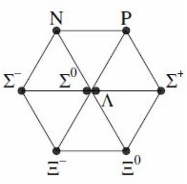
28 May – 3 June

Outline

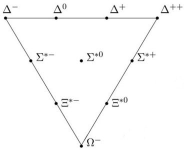
Before starting: Hyperons

- Nucleons siblings:

  $\rightarrow$ replace up/down by strange quark $\rightarrow$ octet hyperons ($J^P = \frac{1}{2}^+$)



  $\rightarrow$ replace up/down by strange quark + flip spin $\rightarrow$ decuplet hyperons ($J^P = \frac{3}{2}^+$)



(Gell-Mann's multiplet assignments)

Motivation

The PANDA experiment will allow to study for the first time the reaction:

$$\bar{p}p \rightarrow \bar{\Omega}^{+}\Omega^{-}$$

Ω's features:

- spin 3/2 particle according to quark model (not yet confirmed by experiment!)
- triple strangeness hyperon
- can be used to test baryon CP violation in weak decays

GOAL:

The **polarization parameters** r^L_M of the Ω can be extracted making use of the angular distributions of:

Ω decay + subsequent decay of the daughter Λ

i.e. $\Omega \rightarrow \Lambda K$ and $\Lambda \rightarrow p\pi$

The Density Matrix

For a pure state the expectation value of an observable E is given by

$$\langle E \rangle = \langle \Psi \mid E \mid \Psi \rangle$$

In an orthonormal basis $\{|a_k\rangle\}$ can be rewritten as

$$\langle E \rangle = \langle \Psi| \left(\sum_k |a_k\rangle \langle a_k| \right) E|\Psi\rangle = \sum_k \langle \Psi|a_k\rangle \langle a_k|E|\Psi\rangle$$
$$= \sum_k \langle a_k|E|\Psi\rangle \langle \Psi|a_k\rangle = \mathrm{Tr}\left(E\,|\Psi\rangle\langle\Psi|\right)$$

Define $\rho = |\Psi\rangle \langle \Psi|$, then

$$\langle E \rangle = \mathrm{Tr}\left(E\rho\right)$$

Generalization for a mixed state:

$$\langle E \rangle = \mathrm{Tr}\left(E \sum_i a_i \,|\Psi_i\rangle \langle \Psi_i| \right)$$

where ρ is now defined as $\rho = \sum_i a_i \,|\Psi_i\rangle \langle \Psi_i|$

The Density Matrix

For a particle with spin j the **density matrix** is given by

$$\rho = \frac{1}{2j+1}\mathcal{I} + \sum_{L=1}^{2j} \rho^L$$

$$\text{with} \qquad \rho^L = \frac{2j}{2j+1} \sum_{M=-L}^{L} Q^L_M r^L_M$$

where $\mathcal{I}$ is the identity matrix, Q^L_M is a set of hermitian matrices and r^L_M are the polarization parameters.

$$\text{Particle's degree of polarization:} \qquad d(\rho) = \sqrt{\sum_{L=1}^{2j} \sum_{M=-L}^{L} (r^L_M)^2}$$

- Spin 1/2: **3** polarization parameters (L=1 -L<M<L)
- Spin 3/2: **15** polarization parameters (L=1,2,3 -L<M<L)

Doncel *et al.*, Phys.Rev. D7, 815 (1973)

The Density Matrix (spin 1/2)

- $Q^1_M \rightarrow$ Pauli matrices $\bar{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$
- $r^1_M \rightarrow$ vector polarization $\bar{P} = (P_x, P_y, P_z)$

$$\rho(1/2) = \begin{bmatrix} \rho_{11} & \rho_{1-1} \\ \rho_{-11} & \rho_{-1-1} \end{bmatrix} = \frac{1}{2}(\mathcal{I} + \bar{P}\cdot\bar{\sigma}) = \frac{1}{2}\begin{bmatrix} 1+P_z & P_x - iP_y \\ P_x + iP_y & 1-P_z \end{bmatrix}$$

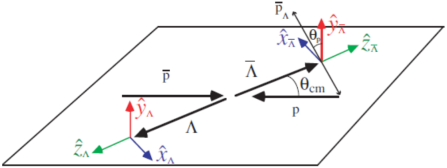
For hyperons produced at e.g. PANDA:

- $\bar{\mathrm{p}}\mathrm{p} \rightarrow \bar{\mathrm{Y}}\mathrm{Y}$
- strong interaction
- unpolarized beam and target

$\longrightarrow$ the density matrix must fulfil symmetries due to parity conservation

$$\rho(1/2) = \begin{bmatrix} \rho_{11} & \rho_{1-1} \\ -\rho_{1-1} & \rho_{11} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & -iP_y \\ iP_y & 1 \end{bmatrix}$$

i.e. $P_x = P_z = 0$



Polarization normal to the production plane!

The Density Matrix (spin 3/2)

It gets more complicated now...

- 15 r^L_M parameters (L=1,2,3)

However thanks to the symmetries imposed on the ρ by parity conservation in the creation mechanism:

$\rightarrow$ 8 of the 15 r^l_s can be set to zero

We are left with:

$$\rho(3/2) = \frac{1}{4}\begin{bmatrix} 1+\sqrt{3}r^2_0 & -i\frac{3}{\sqrt{5}}r^1_{-1}+\sqrt{3}r^2_1-i\sqrt{\frac{6}{5}}r^3_{-1} & \sqrt{3}r^2_2-i\sqrt{3}r^3_{-2} & -i\sqrt{6}r^3_{-3} \\ i\sqrt{\frac{6}{5}}r^3_{-1}+i\frac{3}{\sqrt{5}}r^1_{-1}+\sqrt{3}r^2_1 & 1-\sqrt{3}r^2_0 & -i2\sqrt{\frac{3}{5}}r^1_{-1}+i3\sqrt{\frac{2}{5}}r^3_{-1} & \sqrt{3}r^2_2+i\sqrt{3}r^3_{-2} \\ \sqrt{3}r^2_2+i\sqrt{3}r^3_{-2} & i2\sqrt{\frac{3}{5}}r^1_{-1}-i3\sqrt{\frac{2}{5}}r^3_{-1} & 1-\sqrt{3}r^2_0 & -i\frac{3}{\sqrt{5}}r^1_{-1}+\sqrt{3}r^2_1-i\sqrt{\frac{6}{5}}r^3_{-1} \\ i\sqrt{6}r^3_{-3} & \sqrt{3}r^2_2-i\sqrt{3}r^3_{-2} & i\frac{3}{\sqrt{5}}r^1_{-1}+\sqrt{3}r^2_1+i\sqrt{\frac{6}{5}}r^3_{-1} & 1+\sqrt{3}r^2_0 \end{bmatrix}$$

$\rightarrow$ To get the polarization, the **7** remaining r coefficients need to be measured

Erik Thomé, Ph.D. Thesis, Uppsala University (2012)

Angular Distributions for Hyperons Decays

- Introduce decay matrix T such that $T|\Psi_i\rangle = |\Psi_f\rangle$
- Recall that $\rho = |\Psi\rangle\langle\Psi|$

The transformation of the density matrix is performed by the T matrix :

$$\rho_{\text{final}} = T\rho_{\text{initial}}T^{\dagger}$$

The angular distribution of the daughter particle is given by:

$$I = \text{Tr}(T\rho_{\text{initial}}T^{\dagger})$$

spin $1/2 \to$ *spin* $1/2$ *spin* 0

For weak decay both the parity conserving P state and the parity violating S state are allowed:

$$T(1/2 \to 1/2\ 0) = \frac{1}{\sqrt{4\pi}} \begin{bmatrix} T_s + T_p \cos\theta & T_p \sin\theta e^{-i\phi} \\ T_p \sin\theta e^{i\phi} & T_s - T_p \cos\theta \end{bmatrix}$$

Using cyclicity of the trace: $\mathrm{Tr}(T\rho T^\dagger) = \mathrm{Tr}(\rho T^\dagger T)$, where:

$$T^\dagger T = A(1/2 \to 1/2\ 0) = \frac{1}{4\pi} \begin{bmatrix} 1 + \alpha\cos\theta & \alpha \sin\theta e^{-i\phi} \\ \alpha \sin\theta e^{i\phi} & 1 - \alpha\cos\theta \end{bmatrix}$$

Asymmetry parameters:

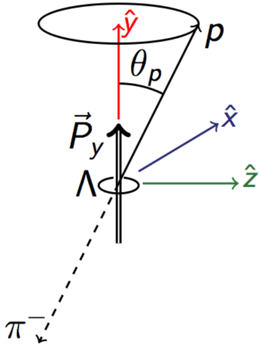
$$\alpha = 2\mathrm{Re}(T_s^* T_p)$$
$$\beta = 2\mathrm{Im}(T_s^* T_p)$$
$$\gamma = |T_s|^2 - |T_p|^2$$

where by construction $\alpha^2 + \beta^2 + \gamma^2 = |T_s|^2 + |T_p|^2 = 1$

The angular distribution is:

$$I(\theta,\phi) = \mathrm{Tr}(\rho(1/2)A(1/2 \to 1/2\ 0)) = \frac{1}{4\pi}(1 + \alpha P_y \sin\theta \sin\phi)$$

or equivalently $$I(\cos\theta_{\mathrm{p}}) = \frac{1}{4\pi}(1 + \alpha P_y \cos\theta_{\mathrm{p}})$$



Polarization of the spin 1/2 hyperon **experimentally accessible** from the angular distribution of the decay products!

spin $1/2 \rightarrow$ *spin* $1/2$ *spin* $0 \rightarrow$ *spin* $1/2$ *spin* 0 *spin* 0

Some hyperons decay to states which also include hyperons, *e.g.*

$$\mathbf{1} : \Xi \rightarrow \Lambda\pi \quad \rightarrow \quad \mathbf{2} : \Lambda \rightarrow \mathrm{p}\pi$$

Using two decay matrices (T_1, T_2) we gain additional information!

The joint angular distribution is obtained as:

$$I = \mathrm{Tr}(T_2 R T_1 \rho T_1^\dagger R^\dagger T_2^\dagger) = \mathrm{Tr}(\rho T_1^\dagger R^\dagger A_2 R T_1)$$

A rotation R is performed to make the spins in the two decays defined with respect to the same axis

$$R = e^{-i\frac{1}{2}\phi_\Lambda \sigma_z} e^{i\frac{1}{2}\theta_\Lambda \sigma_y} e^{i\frac{1}{2}\phi_\Lambda \sigma_z} = \begin{bmatrix} \cos\frac{\theta_\Lambda}{2} & \sin\frac{\theta_\Lambda}{2} e^{-i\phi_\Lambda} \\ -\sin\frac{\theta_\Lambda}{2} e^{i\phi_\Lambda} & \cos\frac{\theta_\Lambda}{2} \end{bmatrix}$$

spin $1/2 \rightarrow$ *spin* $1/2$ *spin* $0 \rightarrow$ *spin* $1/2$ *spin* 0 *spin* 0

By integrating over θ_Λ and ϕ_Λ we get the angular distribution of the second decay:

$$I(\theta_\mathrm{p}, \phi_\mathrm{p}) = \frac{1}{4\pi}\left(1 + \alpha_\Xi \alpha_\Lambda \cos\theta_\mathrm{p} + \frac{1}{2}\alpha_\Lambda P_y \sin\theta_\mathrm{p}(\beta_\Xi \cos\phi_\mathrm{p} + \gamma_\Xi \sin\phi_\mathrm{p})\right)$$

where also β_Ξ and γ_Ξ show up!

spin $3/2 \to$ *spin* $1/2$ *spin* 0

For weak decay both the parity conserving P state and the parity violating D state are allowed $\to T(3/2 \to 1/2\ 0)$ matrix in terms of T_p and T_d

The angular distribution is:

$$\begin{aligned} I(\theta,\phi) &= \mathrm{Tr}(\rho(3/2)A(3/2 \to 1/2\ 0)) \\ &= \frac{1}{4\pi}\Big[1 + \frac{\sqrt{3}}{2}(1 - 3\cos^2\theta)r_0^2 - \frac{3}{2}\sin^2\theta\cos 2\phi\, r_2^2 - \frac{3}{2}\sin 2\theta\cos\phi\, r_1^2 \\ &\quad + \frac{1}{40}\alpha\sin\theta\Big(8\sqrt{15}r^1_{-1}\sin\phi - 9\sqrt{10}r^3_{-1}(3 + 5\cos 2\theta)\sin\phi \\ &\quad - 30(3r^3_{-2}\sin 2\phi\sin 2\theta + \sqrt{6}r^3_{-3}\sin 3\phi\sin^2\theta)\Big)\Big] \end{aligned}$$

For the $\Omega^- \to \Lambda K^-$ decay: $\alpha = 0.0180 \pm 0.0024$ [PDG]

- polarization parameters are accessible using *e.g.* the Method of Moments

  A.Frodesen, O.Skeggestad and H.Tofte, *Probability and Statistics in Particle Physics*, Bergen (1979)

Method of Moments

From the angular distribution of the Λ coming from the $\Omega \rightarrow \Lambda \mathrm{K}$ decay, the polarization parameters $r_0^2,\ r_1^2,\ r_2^2$ can be retrieved:

$$\begin{aligned}\langle \sin\theta_\Lambda \rangle \\ &= \int I(\theta_\Lambda, \phi_\Lambda) \times \sin\theta_\Lambda \, d\Omega_\Lambda \\ &= \frac{\pi}{32}(8 + \sqrt{3} r_0^2)\end{aligned}$$

$$\begin{aligned}\langle \cos\phi_\Lambda \cos\theta_\Lambda \rangle \\ &= \int I(\theta_\Lambda, \phi_\Lambda) \times \cos\phi_\Lambda \cos\theta_\Lambda \, d\Omega_\Lambda \\ &= -\frac{3\pi}{32} r_1^2\end{aligned}$$

$$\begin{aligned}\langle \sin^2\phi_\Lambda \rangle \\ &= \int I(\theta_\Lambda, \phi_\Lambda) \times \sin^2\phi_\Lambda \, d\Omega_\Lambda \\ &= \frac{1}{4}(2 + r_2^2)\end{aligned}$$

Consider the decay chain

$$\Omega^-(\tfrac{3}{2}^+) \to \Lambda(\tfrac{1}{2}^+)\mathrm{K}^-(0^-) \to \mathrm{p}(\tfrac{1}{2}^+)\pi^-(0^-)\mathrm{K}^-(0^-)$$

The joint angular distribution depends on:

- 4 angles: $\theta_\Lambda,\ \phi_\Lambda,\ \theta_\mathrm{p},\ \phi_\mathrm{p}$
- 4 asymmetry parameters: $\alpha_\Omega,\ \beta_\Omega,\ \gamma_\Omega,\ \alpha_\Lambda$
- 7 polarization parameters: $r^1_{-1},\ r^2_0,\ r^2_1,\ r^2_2,\ r^3_{-1},\ r^3_{-2},\ r^3_{-3}$

When the angles of the first decay are integrated out, one gets:

$$I(\theta_\mathrm{p}, \phi_\mathrm{p}) = = \frac{1}{4\pi}(1 + \alpha_\Omega\alpha_\Lambda \cos\theta_\mathrm{p} + \\ + \alpha_\Lambda \left(\frac{1}{2\sqrt{10}} r^3_{-1} - \sqrt{\frac{3}{5}} r^1_{-1} \right) (\beta_\Omega \cos\phi_\mathrm{p} + \gamma_\Omega \sin\phi_\mathrm{p}) \sin\theta_\mathrm{p})$$

- It is possible to extract β_Ω and γ_Ω!
- Using *e.g.* the Method of Moments, all the remaining 4 r_s^l can be determined

The moduli of the remaining 4 polarization parameters r^1_{-1}, r^3_{-1}, r^3_{-2}, r^3_{-3} can be determined:

$$\langle \sin\phi_\Lambda \cos\phi_{\mathrm{p}} \rangle$$
$$= \int I(\theta_\Lambda, \phi_\Lambda, \theta_{\mathrm{p}}, \phi_{\mathrm{p}}) \times \sin\phi_\Lambda \cos\phi_{\mathrm{p}}\, d\Omega_\Lambda\, d\Omega_{\mathrm{p}} =$$
$$= -\frac{3\pi^2 \alpha_\Lambda \gamma_\Omega r^3_{-2}}{1024}$$

$$\langle (3\cos\theta_\Lambda - 1)\sin\phi_{\mathrm{p}} \rangle$$
$$= \int I(\theta_\Lambda, \phi_\Lambda, \theta_{\mathrm{p}}, \phi_{\mathrm{p}}) \times (3\cos\theta_\Lambda - 1)\sin\phi_{\mathrm{p}}\, d\Omega_\Lambda\, d\Omega_{\mathrm{p}} =$$
$$= -\frac{\pi \alpha_\Lambda \gamma_\Omega r^3_{-1}}{4\sqrt{10}}$$

...and...

$$\langle \sin \phi_{\mathrm{p}} \rangle = \int I(\theta_\Lambda, \phi_\Lambda, \theta_{\mathrm{p}}, \phi_{\mathrm{p}}) \times \sin \phi_{\mathrm{p}} \, \mathrm{d}\Omega_\Lambda \, \mathrm{d}\Omega_{\mathrm{p}} = \frac{\pi \alpha_\Lambda \gamma_\Omega}{160} \left(-4\sqrt{15} r^1_{-1} + \sqrt{10} r^3_{-1} \right)$$

$$\langle \sin \phi_\Lambda \cos \phi_\Lambda \cos \phi_{\mathrm{p}} \rangle = \int I(\theta_\Lambda, \phi_\Lambda, \theta_{\mathrm{p}}, \phi_{\mathrm{p}}) \times \sin \phi_\Lambda \cos \phi_\Lambda \cos \phi_{\mathrm{p}} \, \mathrm{d}\Omega_\Lambda \, \mathrm{d}\Omega_{\mathrm{p}} = \frac{\pi \alpha_\Lambda \gamma_\Omega}{640} \left(5\sqrt{6} r^3_{-3} + 4\sqrt{15} r^1_{-1} - \sqrt{10} r^3_{-1} \right)$$

Summary & Outlook

Studying the $\bar{\mathrm{p}}\mathrm{p} \to \bar{\Omega}^{+}\Omega^{-}$ reaction at PANDA, it will be possible to extract for the first time the following parameters:

- β_{Ω}, $\bar{\beta}_{\Omega}$ relevant to the search for baryon CP violation in weak decays
- γ_{Ω}, $\bar{\gamma}_{\Omega}$
- full spin density matrix ρ for the Ω hyperon:
  - r^2_0, r^2_1, r^2_2 from angular distribution of $\Omega \to \Lambda\mathrm{K}$
  - r^1_{-1}, r^3_{-1}, r^3_{-2}, r^3_{-3} from joint angular distribution of $\Omega \to \Lambda\mathrm{K}$ and $\Lambda \to \mathrm{p}\pi$

...Work in progress...

- Follow-up of E. Thomé's work and consistency check
- Simulation studies (W.I.Andersson – Uppsala group)
- Publication containing derivation of spin observables ($\bar{\mathrm{p}}\mathrm{p} \to \bar{\Omega}\Omega$) and simulations

Thank you for your attention!

CP Violation in Hyperon System

- No evidence for baryon CP violation so far
- $\bar{\mathrm{p}}\mathrm{p} \to \bar{\mathrm{Y}}\mathrm{Y}$ process suitable for CP measurements

CP violation parameters:

$$A = \frac{\Gamma\alpha + \bar{\Gamma}\bar{\alpha}}{\Gamma\alpha - \bar{\Gamma}\bar{\alpha}} \simeq \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}}$$

$$B = \frac{\Gamma\beta + \bar{\Gamma}\bar{\beta}}{\Gamma\beta - \bar{\Gamma}\bar{\beta}} \simeq \frac{\beta + \bar{\beta}}{\beta - \bar{\beta}}$$

If CP is conserved, then:

$$\alpha = -\bar{\alpha} \to A \simeq 0$$

$$\beta = -\bar{\beta} \to B \simeq 0$$

where Γ is the partial decay width

- According to experiment A is consistent with 0 for Ω, Λ, Ξ