Magneto-hydrodynamic simulations of Heavy Ion Collisions with ECHO-QGP

5^{th} FAIRNESS workshop

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FIAS Frankfurt Institute for Advanced Studies

H-QM Helmholtz Research School Quark Matter Studies

HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

Outline

Outline of the talk:

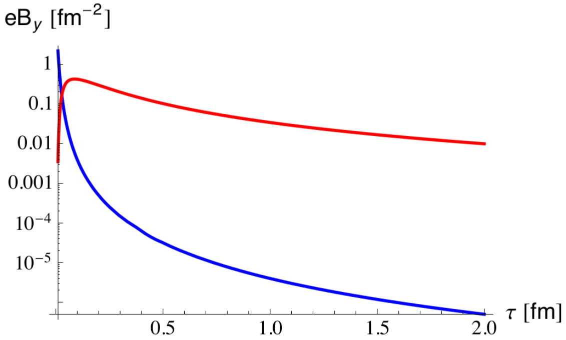
- Motivations
- Estimates of the magnitude of the magnetic field in H.I.C.
- Implementation of ideal magnetohydrodynamics in ECHO-QGP
- Applications to H.I.C.
- Discussions and conclusions

Why to study magnetic fields in HIC?

Strong magnetic fields may produce many interesting effects:

- The Chiral Magnetic Effect
  Kharzeev, McLerran, Warringa - Nuclear Physics A 803 (2008)
- Pressure anisotropy in QGP
  Bali, Bruckmann, Endrődi et al. - Journal of High Energy Physics 08 177 (2014)
- A shift in meson masses
  Andersen - Phys. Rev. D 86, 025020 (2012), Luschevskaya and Larina - JETP Letters 98 (2014)
- Mass shifts in quarkonia states
  Suzuki and Yoshida - arXiv: 1601.02178
- Shift of the Critical Temperature
  Bali, Bruckmann, Endrődi et al. - Journal of High Energy Physics 02 044 (2012)
- Influence on the elliptic flow
  Bali, Bruckmann, Endrődi and Schäfer - Phys. Rev. Lett. 112 (2014)
  Pang, Endrődi and Petersen - arXiv: 1602.06176v1
- Influence on directed flow
  Gürsoy, Kharzeev and Rajagopal - Phys. Rev. C 89 (2014)

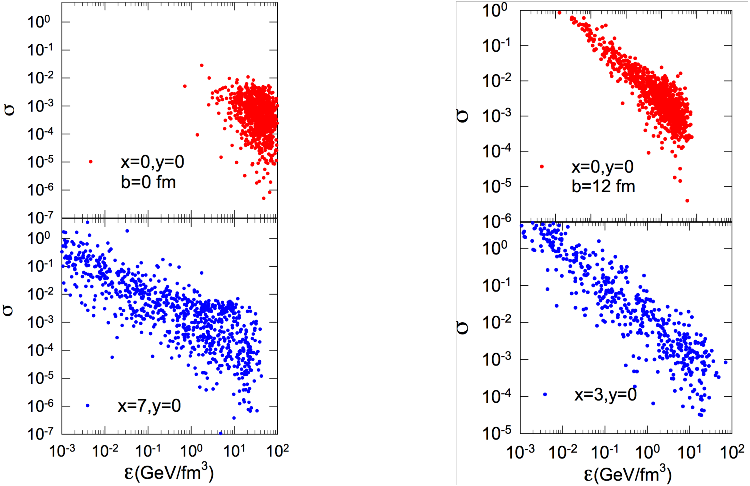
Possible time evolution of $|\vec{B}|$ in HIC



Gürsoy, Kharzeev and Rajagopal - Phys. Rev. C 89, 054905 (2014)

The medium plays a crucial role:
Blue line: $\sigma = 0.\,\mathrm{fm}^{-1}$
Red line: $\sigma = 0.023\,\mathrm{fm}^{-1}$

Event by event estimates by Roy and Pu



$\sigma(x,y,\vec{b}) = \frac{B^2(x,y,\vec{b})}{2\varepsilon(x,y,\vec{b})}$, Au-Au collision at $\sqrt{s}_{\mathrm{NN}}$=200 GeV, Glauber-M.C.

Plots taken from: Roy, Pu - Phys. Rev. C 92 (2015)

Our initial conditions: basic formula for point charge

Reference article: Tuchin, Phys. Rev. C 88 (2013)
For an observer at $\mathbf{r} = z\hat{\boldsymbol{z}} + \mathbf{b}$, $(\mathbf{b} \cdot \hat{\boldsymbol{z}} = 0)$, if $\gamma = 1/\sqrt{1-v^2} \gg 1$:

$$\mathbf{H}(t, \boldsymbol{b}r) = H(t, \mathbf{r})\hat{\boldsymbol{\phi}} =$$

$$\frac{e}{2\pi\sigma}\hat{\boldsymbol{\phi}} \int_0^\infty \frac{J_1(k_\perp b) k_\perp^2}{\sqrt{1 + \frac{4k_\perp^2}{\gamma^2\sigma^2}}} \exp\left\{ \frac{1}{2}\sigma\gamma^2 x_- \left(1 - \sqrt{1 + \frac{4k_\perp^2}{\gamma^2\sigma^2}} \right) \right\} dk_\perp \tag{1}$$

where $x_- = t - z/v$ and $\hat{\boldsymbol{\phi}}$ is the unit vector of the angular polar coordinates in the transverse plane x, y.
Electrical conductivity σ is constant. Ohm law simply: $\vec{J} = \sigma \vec{E}$.
We model the nuclei as uniformly charged spheres which freely propagate into a medium, before and after the collision.

More recent equations which are used at present

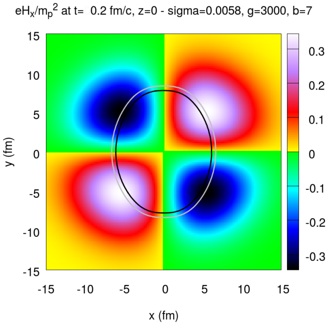
Li, Sheng and Wang, Phys. Rev. C 94, 044903 (2016)

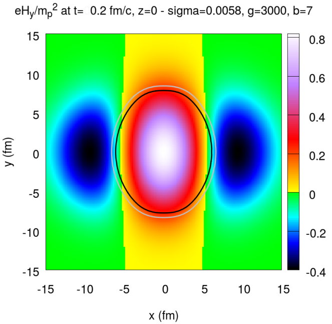
$$
\begin{aligned}
B_\phi(t,\mathbf{x}) &= \frac{Q}{4\pi}\cdot\frac{v\gamma x_T}{\Delta^{3/2}}\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)e^A, \\
B_r(t,\mathbf{x}) &= -\sigma_\chi\frac{Q}{8\pi}\cdot\frac{v\gamma^2 x_T}{\Delta^{3/2}}\cdot\left[\gamma(vt-z)+A\sqrt{\Delta}\right]e^A, \\
B_z(t,\mathbf{x}) &= \sigma_\chi\frac{Q}{8\pi}\cdot\frac{v\gamma}{\Delta^{3/2}}\cdot \\
&\quad \left[\gamma^2(vt-z)^2\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)+\Delta\left(1-\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)\right]e^A
\end{aligned}
$$

where: σ is the electric conductivity, σ_χ the chiral magnetic conductivity, $\Delta \equiv \gamma^2(vt-z)^2+x_T^2$, $A \equiv (\sigma v\gamma/2)[\gamma(vt-z)-\sqrt{\Delta}]$

Estimates with Geometrical Glauber initial conditions

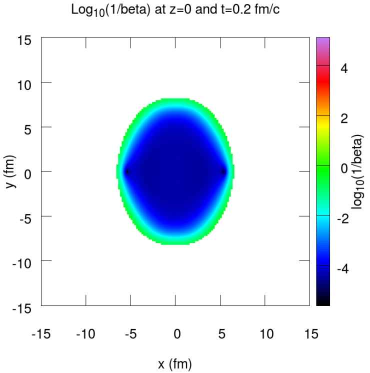
Collision at $\sqrt{s}_{\mathrm{NN}}$=5.5 TeV, b=7 fm:





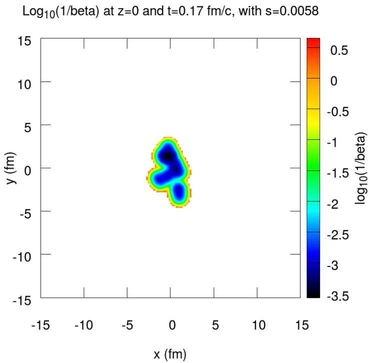
Magnetic permeability $\mu \sim 1$
Black line: $e = 1\mathrm{GeV}/\mathrm{fm}^3$, gray line: $e = 150\mathrm{MeV}/\mathrm{fm}^3 \sim 140\mathrm{MeV}$.

How do magnetic fields compare with thermal pressure?



$\log_{10}\beta^{-1}$, where $\beta = 2\,p/B^2$,

at $\sqrt{s}_{\rm NN}$=5.5 TeV, b=7 fm



$\log_{10}\beta^{-1}$, where $\beta = 2\,p/B^2$,
at $\sqrt{s}_{\rm NN}$=200 GeV

See: Holopainen et. al, Phys. Rev. C 83 (2011)

What is ECHO-QGP

ECHO-QGP derives from the Eulerian Conservative High-Order astrophysical code for general relativistic magnetohydrodynamics, developed by L. Del Zanna.

(Del Zanna, Zanotti, Bucciantini, and Londrillo, A&A 473 (2007))

A collaboration lead by F. Becattini adapted ECHO-QGP to run second order dissipative hydrodynamical simulations of heavy ion collisions, including the computation of particle spectra following the Cooper-Frye prescription.

Del Zanna, Chandra, Inghirami, Rolando, Beraudo, De Pace, Pagliara, Drago, and Becattini, Eur.Phys.J. C73 (2013)

Floerchinger, Wiedemann, Beraudo, Del Zanna, Inghirami, Rolando, PLB 735 (2014)

Becattini, Inghirami, Rolando, Beraudo, Del Zanna, De Pace, Nardi, Pagliara, Chandra, Eur. Phys. J. C 75 (2015)

Inghirami, Del Zanna, Beraudo, M. Haddadi, Becattini, Bleicher
Eur. Phys. J. C (2016) 76: 659

Website: http://theory.fi.infn.it/echoqgp/

The basis

The fundamental equations

Energy and momentum conservation: $d_\mu T^{\mu\nu} = 0$
Baryonic number conservation: $d_\mu N^\mu = 0$
Second law of thermodynamics: $d_\mu s^\mu \geq 0$
Maxwell equations: $d_\mu F^{\mu\nu} = -J^\nu \quad (d_\mu J^\mu = 0) \qquad d_\mu F^{\star\mu\nu} = 0$

The fundamental assumptions

- We neglect all dissipative effects
- We neglect polarization and magnetization effects
- We assume infinite electrical conductivity
- We assume local thermal equilibrium

The ideal RHMD energy-momentum tensor

Polarization and magnetization neglected

$T_f^{\mu\nu} = F^{\mu}{}_{\lambda} F^{\nu\lambda} - \frac{1}{4}(F^{\lambda\kappa} F_{\lambda\kappa}) g^{\mu\nu}$

from Maxwell equations: $d_\mu T_{\mathrm{f}}^{\mu\nu} = J_\mu F^{\mu\nu}$

Dissipative effects neglected:

Eckart frame = Landau frame $\Rightarrow$ single fluid u^μ $(u_\mu u^\mu = -1)$

Infinite electrical conductivity

Ohm's law: $J^\mu = \rho_{\mathrm{e}} u^\mu + j^\mu; \quad j^\mu = \sigma^{\mu\nu} e_\nu \Rightarrow e^\mu = 0$

Energy-momentum tensor $T^{\mu\nu}$

$T^{\mu\nu} = T_m^{\mu\nu} + T_f^{\mu\nu}$

Matter: $T_m^{\mu\nu} = (e+p) u^\mu u^\nu + p g^{\mu\nu}$

Electromagnetic field: $T_f^{\mu\nu} = b^2 u^\mu u^\nu + \frac{1}{2} b^2 g^{\mu\nu} - b^\mu b^\nu$

The energy momentum tensor components

Lorentz transformations from the laboratory to the comoving frame:
$e^\mu = (\gamma v_k E^k, \gamma E^i + \gamma \varepsilon^{ijk} v_j B_k)$
$b^\mu = (\gamma v_k B^k, \gamma B^i - \gamma \varepsilon^{ijk} v_j E_k)$ where:
ε_{ijk} is the Levi-Civita pseudo-tensor of the spatial three-metric
γ = Lorentz factor, $g_{ij} = \mathrm{diag}(1,1,1)$ or $g_{ij} = \mathrm{diag}(1,1,\tau^2))$
e and p are measured in the *comoving fluid frame*,
$\vec{E}$ and $\vec{B}$ are measured in the *laboratory frame*

Components of the energy-momentum tensor

Energy density $\mathcal{E} \equiv -T^0_0 = (e+p)\gamma^2 - p + \frac{1}{2}(E_k E^k + B_k B^k)$
Momentum density $S_i \equiv T^0_i = (e+p)\gamma^2 v_i + \varepsilon_{ijk} E^j B^k$
Stresses $T^i_j = (e+p)\gamma^2 v^i v_j + (p + \frac{1}{2}(E_k E^k + B_k B^k))\delta^i_j - E^i E_j - B^i B_j$

The evolution equations

Ideal Ohm'law in the laboratory frame

$e^\mu = 0 \Rightarrow E_i = -\varepsilon_{ijk} v^j B^k$

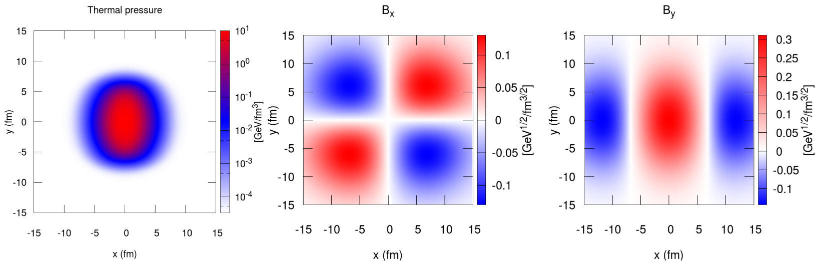
The evolution equations in conservative form

$$\partial_0 \mathbf{U} + \partial_i \mathbf{F}^i = \mathbf{S}$$

where

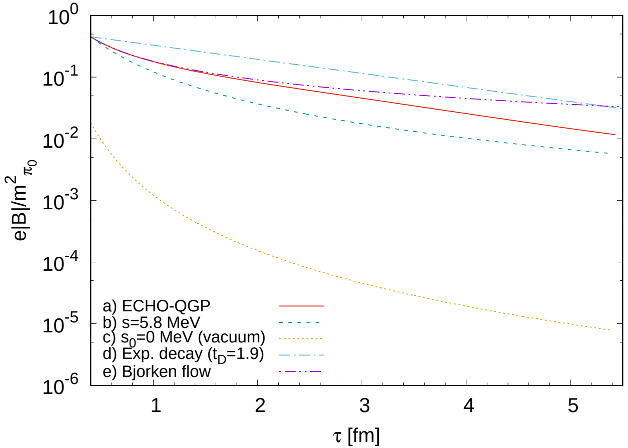
$$\mathbf{U} = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n \\ S_j \equiv T^0_j \\ \mathcal{E} \equiv -T^0_0 \\ B^j \end{pmatrix}, \quad \mathbf{F}^i = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n v^i \\ T^i_j \\ S^i \equiv -T^i_0 \\ v^i B^j - B^i v^j \end{pmatrix}, \quad \mathbf{S} = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2} T^{ik} \partial_j g_{ik} \\ -\frac{1}{2} T^{ik} \partial_0 g_{ik} \\ 0 \end{pmatrix}$$

Example of 2D+1 simulation: initial conditions



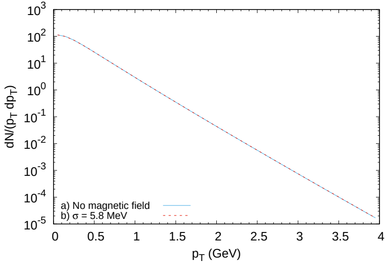
2D+1 simulation in Milne coordinates
Au+Au collision at 200 GeV $\sqrt{s}_{\rm NN}$
Geometrical Glauber initial conditions.
b=10 fm, τ_0=0.4 fm/c, EoS=$p = e/3$
Electromagnetic field computed with Tuchin's model.
Electrical conductivity of the medium ($\tau \leq \tau_0$): $\sigma = 5.8$ MeV, constant.
Electrical conductivity of the QGP ($\tau > \tau_0$): $\sigma = \infty$.

Time evolution of the magnetic field at the center of the grid

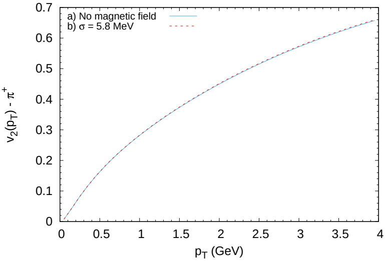


Magnetic field (lab frame) at the center of the grid.

The p_T spectrum and the elliptic flow of charged pions

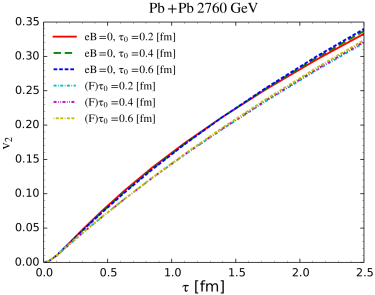


Transverse momentum distribution of π^+, computed with the Cooper-Frye formula.

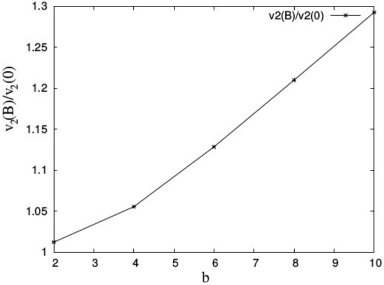


v_2 of π^+, computed with the Cooper-Frye formula

Results from other groups



(Plot from: Pang et al., Phys. Rev. C 93, 044919 (2016))

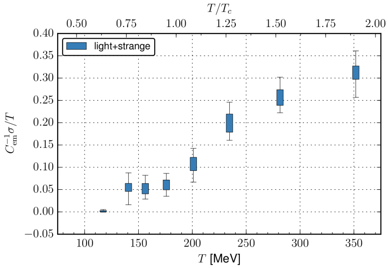


(Plot from: Mohapatra et al., Mod. Phys. Lett. A26, 2477 (2011))

Assuming that magnetic fields may be larger than in our estimates, what is their effect on v_2?

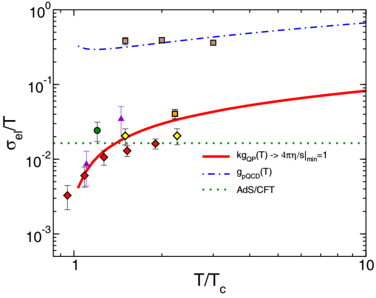
What could be their impact on the shear viscosity?

The electrical conductivity of the Quark Gluon Plasma



$(C_{em} = e^2 \sum_f q_f^2)$

(Plot from: Aarts et al, JHEP **1502** (2015) 186)



(Plot from: Puglisi et al, EPJ Web of Conferences 117 (2016))

Electrical conductivity is finite and temperature dependent!

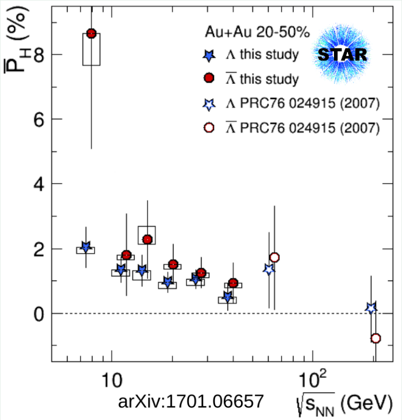
(See also: Greif, Bouras, Xu and Greiner, Phys. Rev. D 90, 094014 (2014))

And it is not isotropic!

(See: Hattori and Satow, Phys. Rev. D 94, 114032)

Beyond flows: other RMHD applications

Λ - $\overline{\Lambda}$ polarization



STAR collaboration, arXiv:1701.06657
For theory, see: Becattini, Karpenko et al, Phys. Rev. C 95, 054902 (2017)

Chiral currents

$$\partial_\mu T^{\mu\nu} = eF^{\nu\lambda} j_\lambda$$
$$\partial_\mu j^\mu = 0$$
$$\partial_\mu j_5^\mu = C E^\mu B_\mu$$

where C is the anomaly coefficient and the electric and the axial vector currents are:

$$j^\mu = n u^\mu + \frac{C\mu_5}{e}\left(1 - \frac{\mu_5 n_5}{\varepsilon + p}\right)$$
$$j_5^\mu = n_5 u^\mu + \frac{C\mu}{e}\left(1 - \frac{\mu n}{\varepsilon + p}\right)$$

(For details, see: Hongo, Hirono and Hirano, arXiv:1309.2823)

Ongoing collaboration with Y.Hirono, M.Mace and D.Kharzeev to couple ECHO-QGP with their anomalous code.

FAIR energies

From a technical point of view, it is possible to study magnetic fields also at FAIR energies, but:

- how good is the hydro model at such low energies for peripheral collisions?
- we start hydro at ≈ 3 fm/c and we run it only for a short time: how large is the impact of this phase compared to the pre-equilibrium and to the hadronic phases?

Conclusions and future perspectives

- Magnetic fields may produce some relevant effects on several observable quantities
- It is uncertain whether they are strong and persistent enough to play a role in HIC at LHC and RHIC energies
- Magnetic fields effects might be detectable at low RHIC BES and FAIR energies, but probably RMHD is not the right tool to investigate them
- However, ECHO-QGP may help to study the influence and the evolution of magnetic fields in the QGP phase
- Future work will involve full 3D+1 simulations with different sets of initial conditions

Thank you!

Conversions

Constants

$\alpha = 0.0072973525664$
$m_{\pi^0} = 139.57018\text{MeV}$
$m^2_{\pi^0} = 0.01932\text{MeV}^2$
$4\pi\alpha = e^2 \;\Rightarrow\; e = \sqrt{4\pi\alpha} = 0.30282212$
$\hbar c = 0.197326\,\text{GeV}\text{fm}$
$(\hbar c)^{\frac{3}{2}} = 0.087655\,(\text{GeV}\text{fm})^{\frac{3}{2}}$