Weibull Model of Multiplicity Distribution

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Outline

Multiplicity Distribution

Negative Binomial Distribution

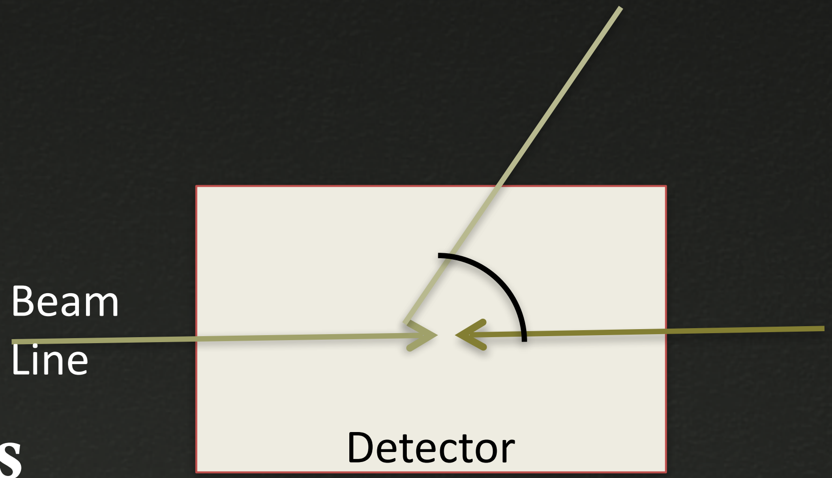
Weibull Distribution

Two component approach

Results (pp, e+e-, heavy-ion)

Summary

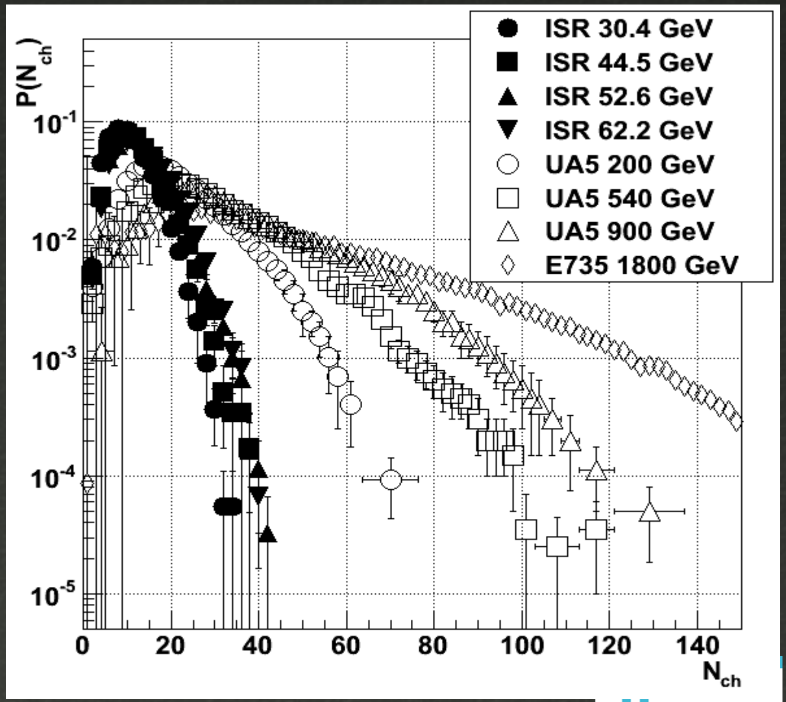
Multiplicity : Primary charged particles per event.



Simple observable in collisions of hadrons

Important ingredient for understanding multi-particle production

Constrain, reject and improve models.

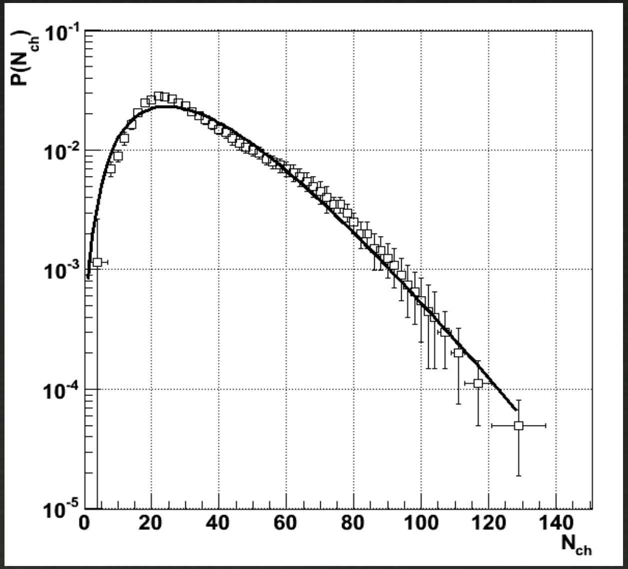


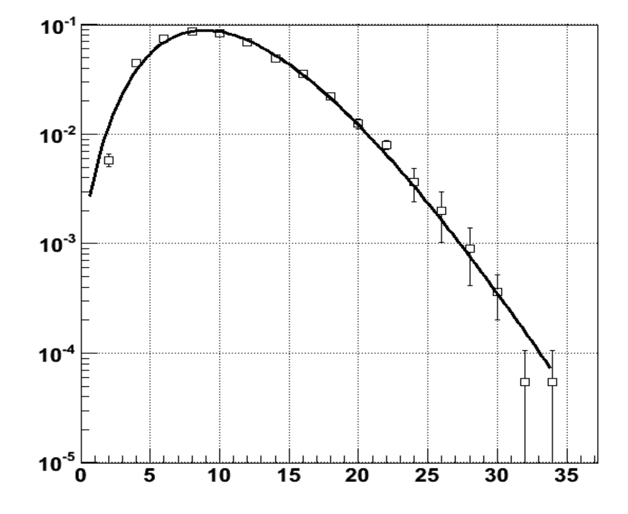
Negative Binomial Distributions

Bernoulli experiment :

Probability for n failures and k success in any order, but the last trial is a success

$$P(n,\langle n\rangle,k)=\frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)}\left[\frac{\langle n\rangle}{k+\langle n\rangle}\right]^{n}\times\left[\frac{k}{k+\langle n\rangle}\right]^{k}$$





Physical interpretation :

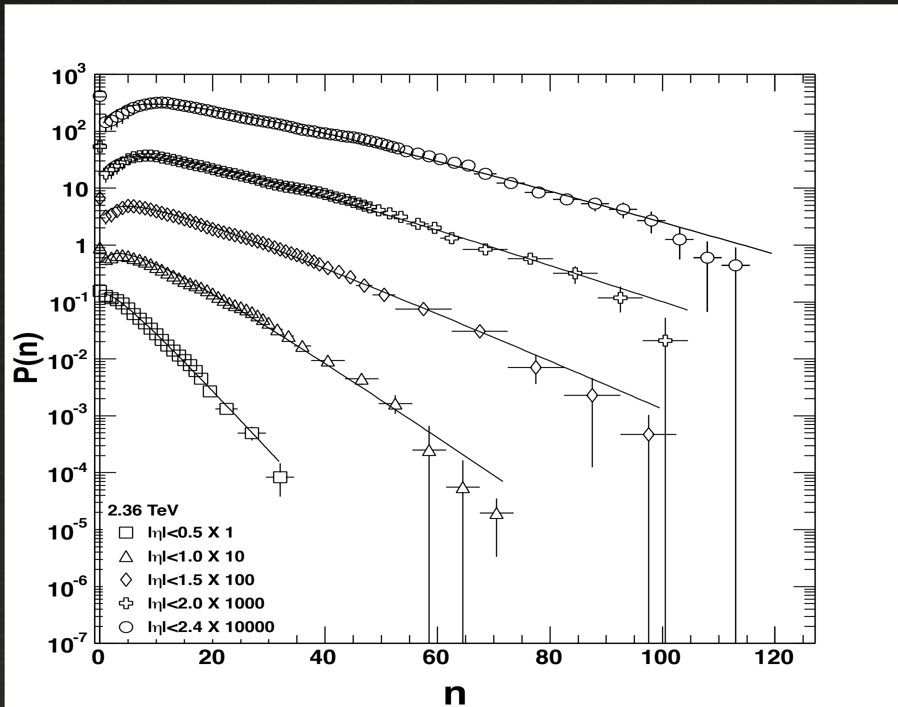
Cascade production (clan model)

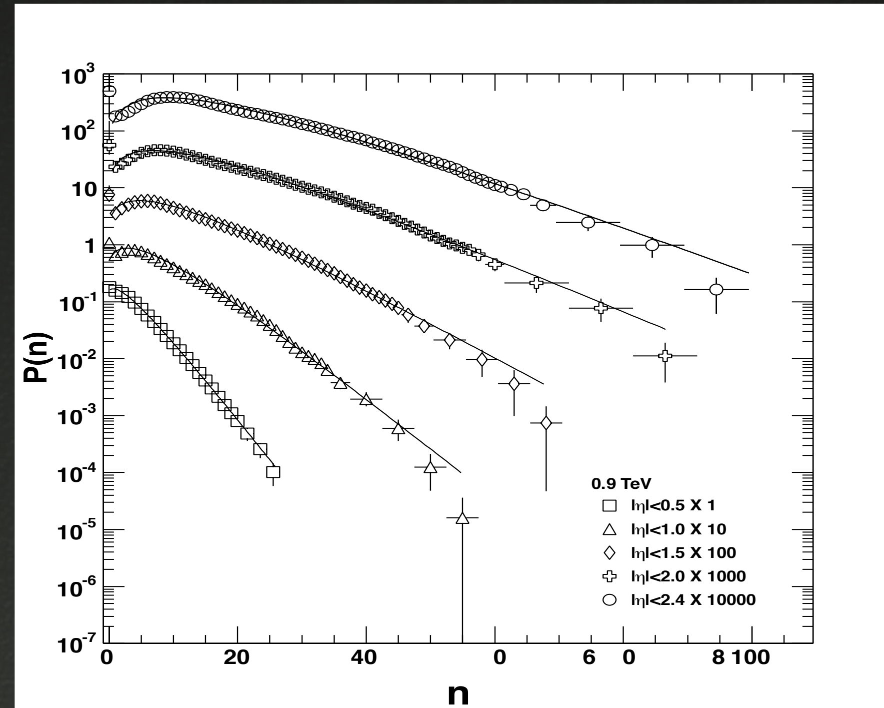
Giovannini, Z. Phys. C30 391 (1986)

Ancestor particle are produced independently (Poisson)

Existing particle can produce additional one with some probability P(n).

Deviations from single NBD





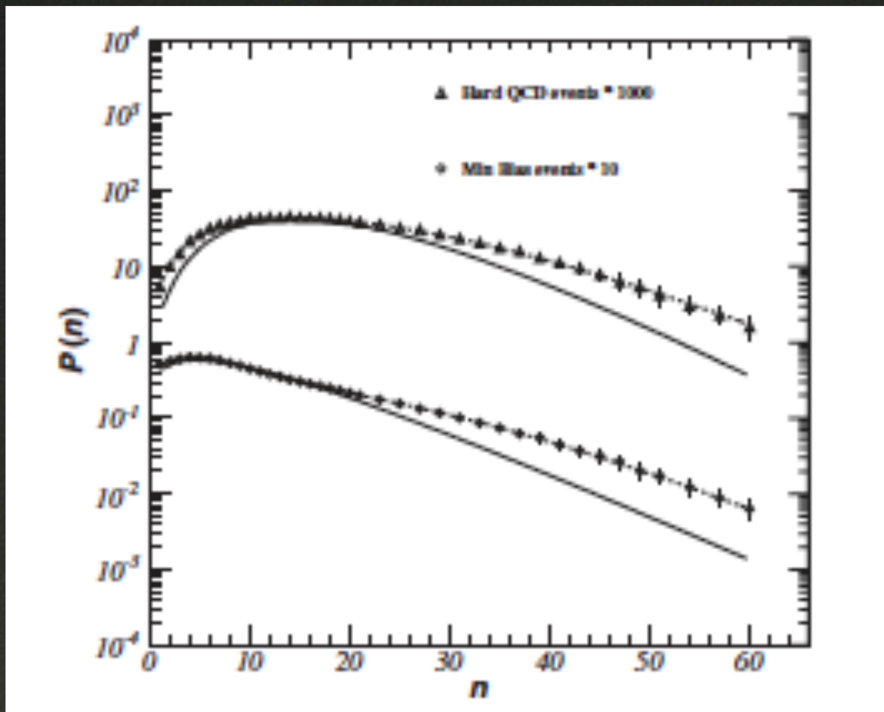
p + p @ 0.9 and 2.36 TeV

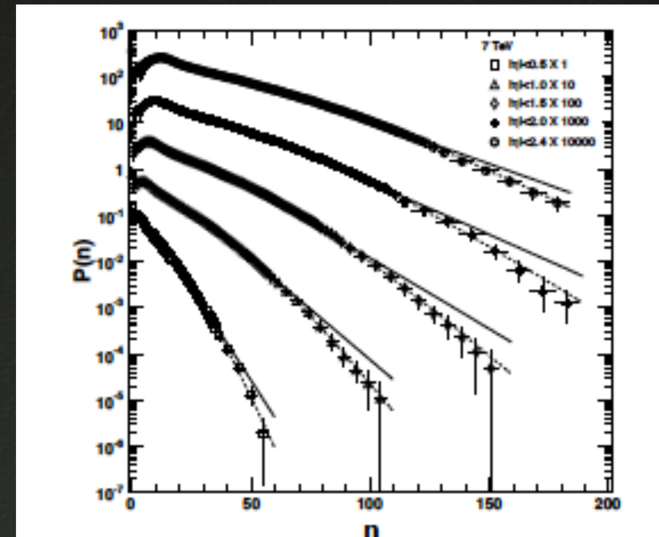
Phys.Rev.D 85, 054017 (2012)

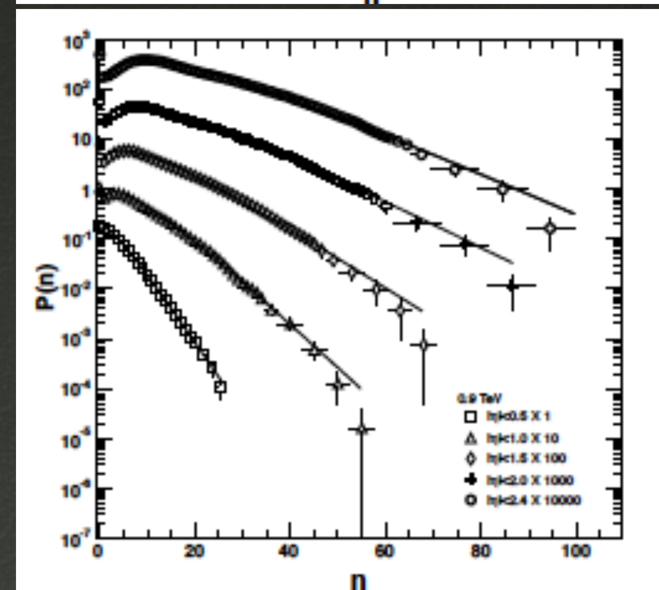
Weighted superposition of soft and semi-hard comp.

$$P_n = \alpha_s P_n(\langle n_s \rangle, k_s) + \alpha_{sh} P_n(\langle n_{sh} \rangle, k_{sh})$$

$$\alpha_s + \alpha_{sh} = 1$$







Phys.Rev.D 87, 094020 (2013)

Weibull Distribution :

The probability distribution of a random variable n in terms of two parameter Weibull Distribution is given by

$$P(n, \lambda, k) = \frac{k}{\lambda} (n/\lambda)^{k-1} e^{-(n/\lambda)^k}$$

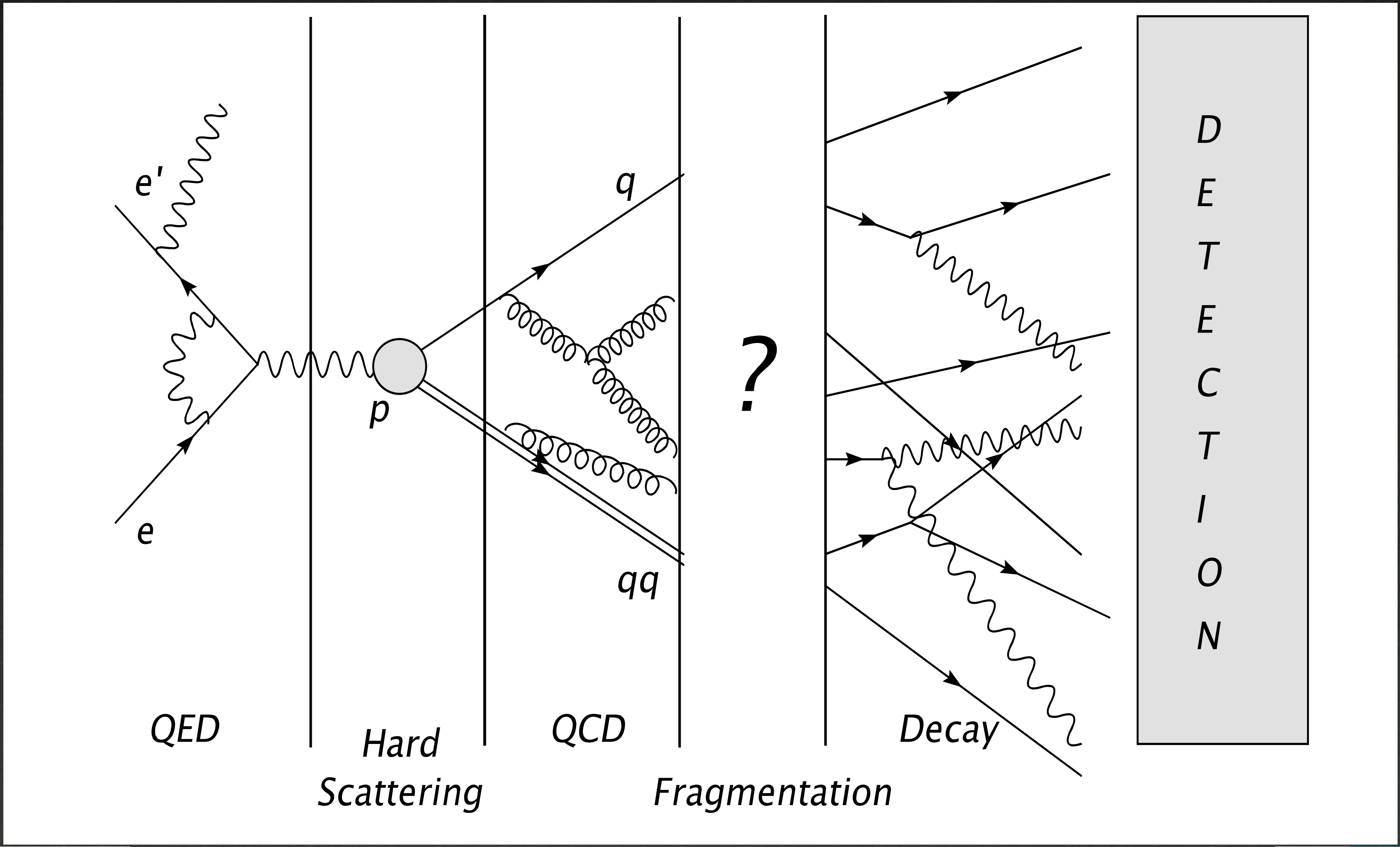
Where k is the shape parameter and lambda is the scale parameter.

The mean of the distribution is given by

$$\langle n \rangle = \lambda \Gamma \left(1 + \frac{1}{k} \right)$$

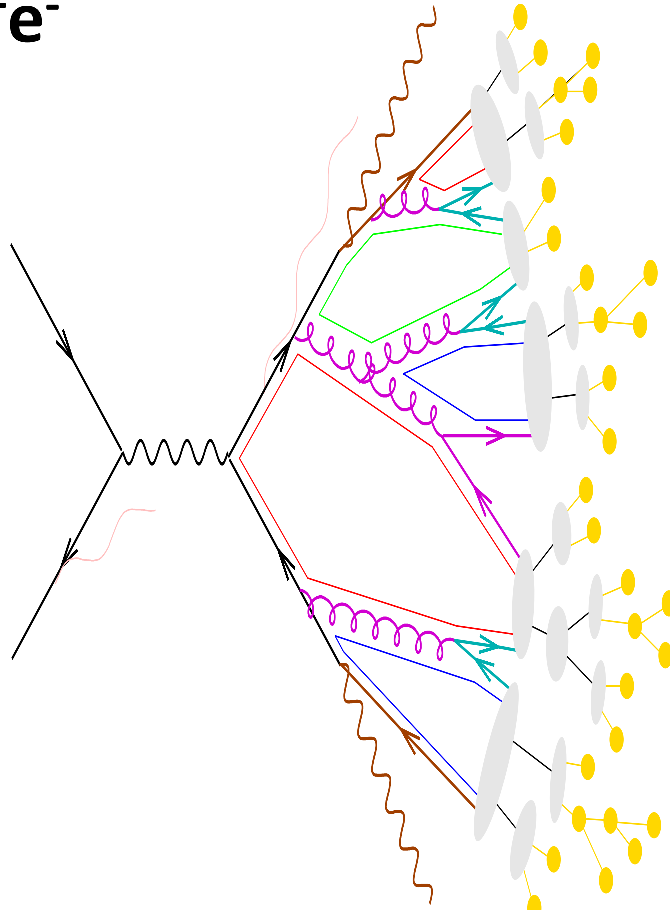
Journal Of Appl. Physics 78 2758-2763 1995

Simple scheme of observed multi-particle state

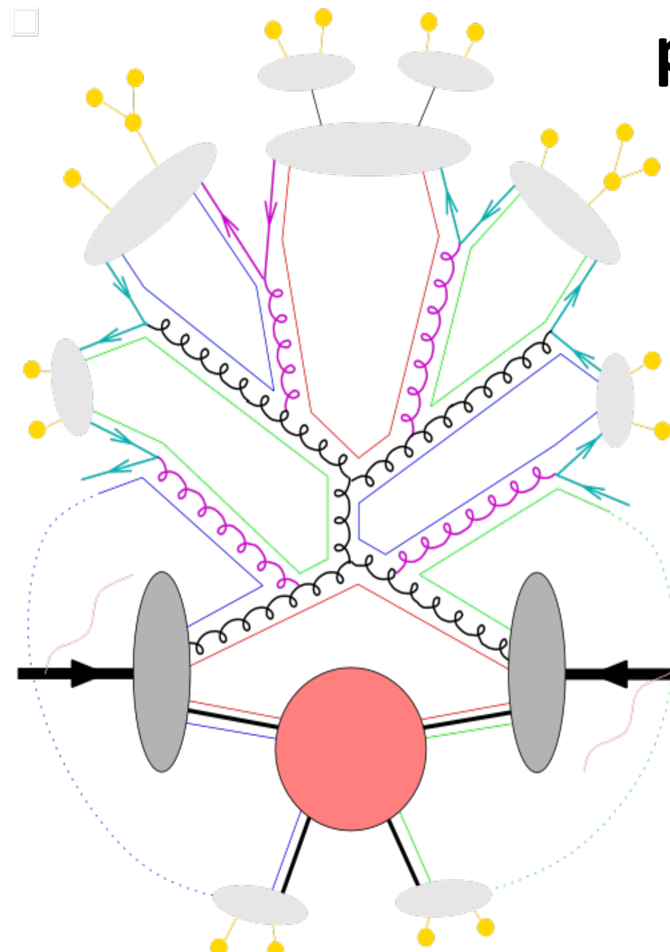


A schematic of parton showers in e^+e^- and pp collisions

e^+e^-



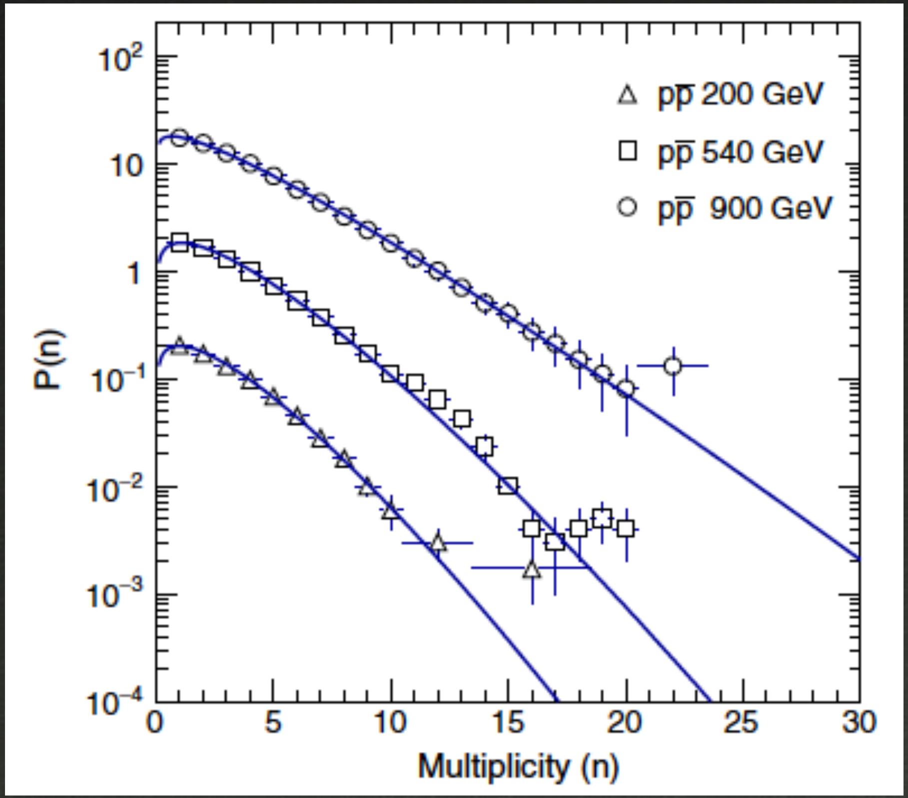
pp

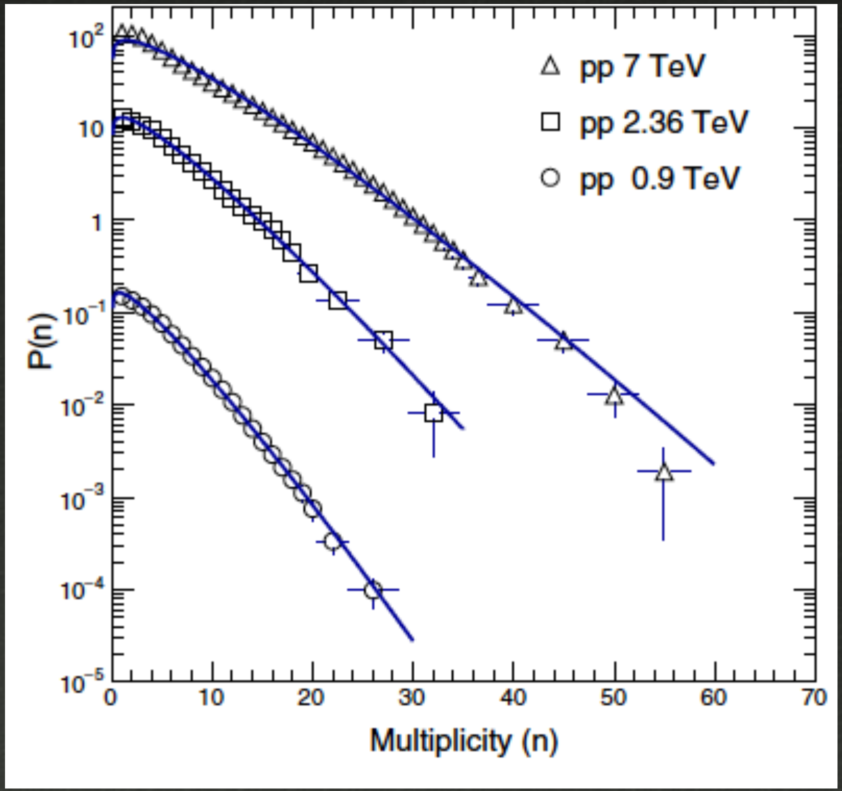


Parton showers refer to cascades of radiation produced from QCD processes and interactions.

Weibull Fit to Data Points – (I)

proton-proton (proton-antiproton) collisions





Phys.Rev D 93, 114022 (2016)

(S.Dash, B.K.Nandi and P. Sett)

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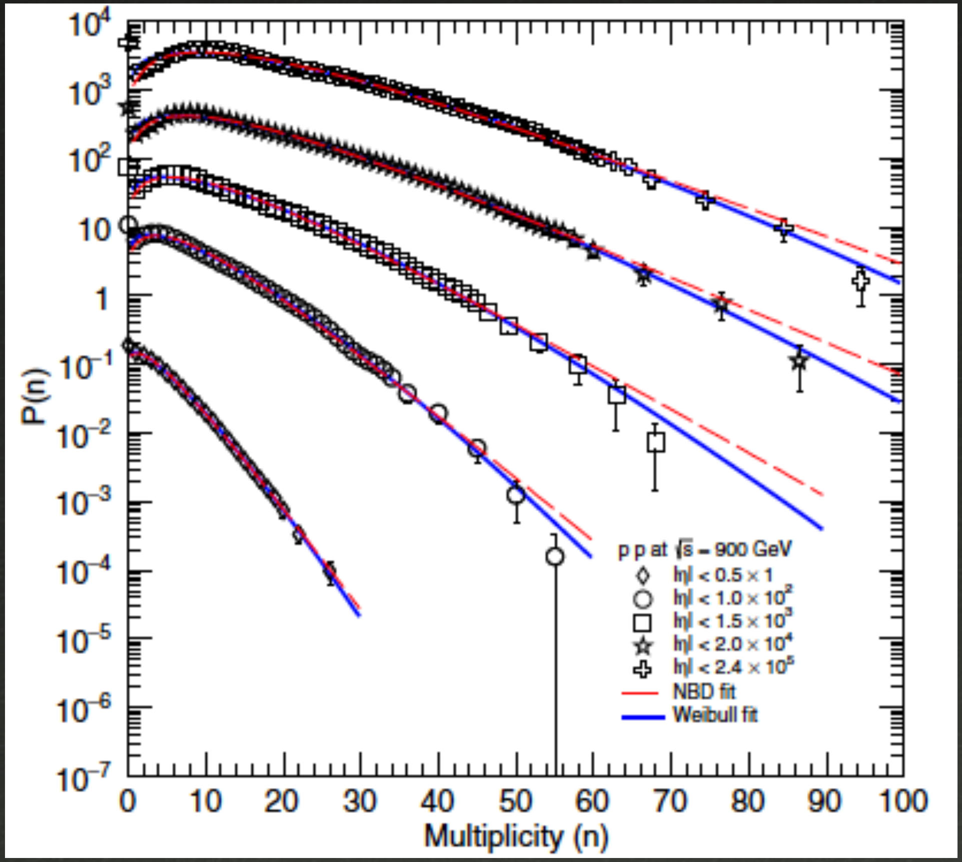
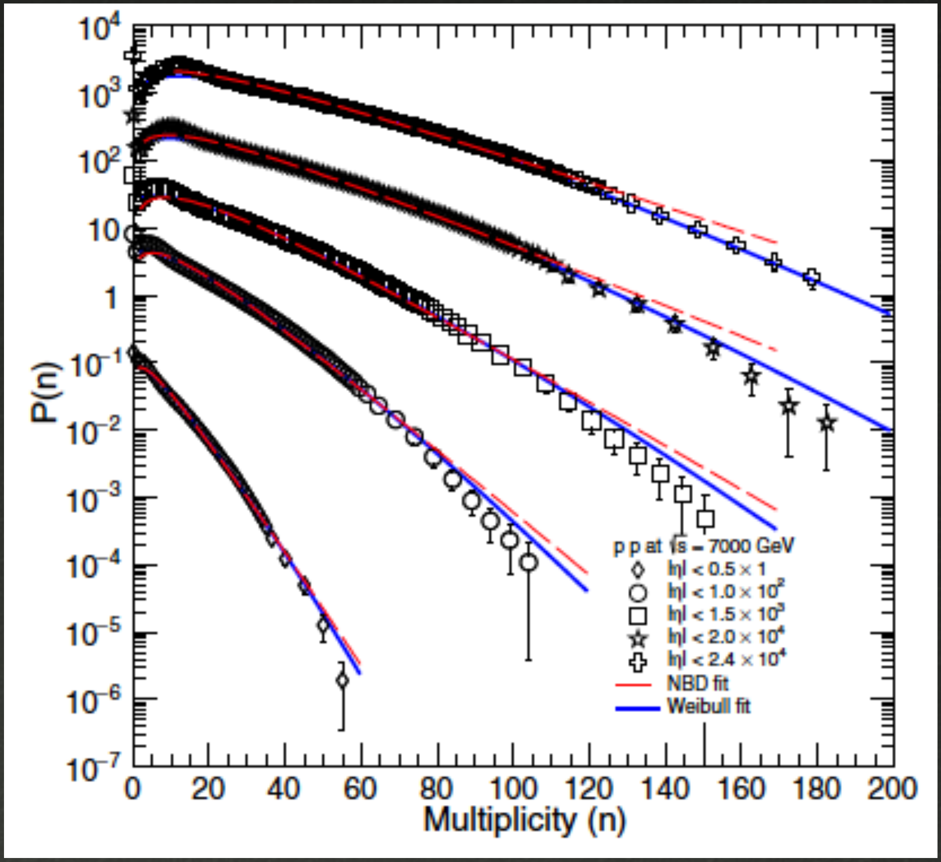
Weibull Fit to Data Points – (II)



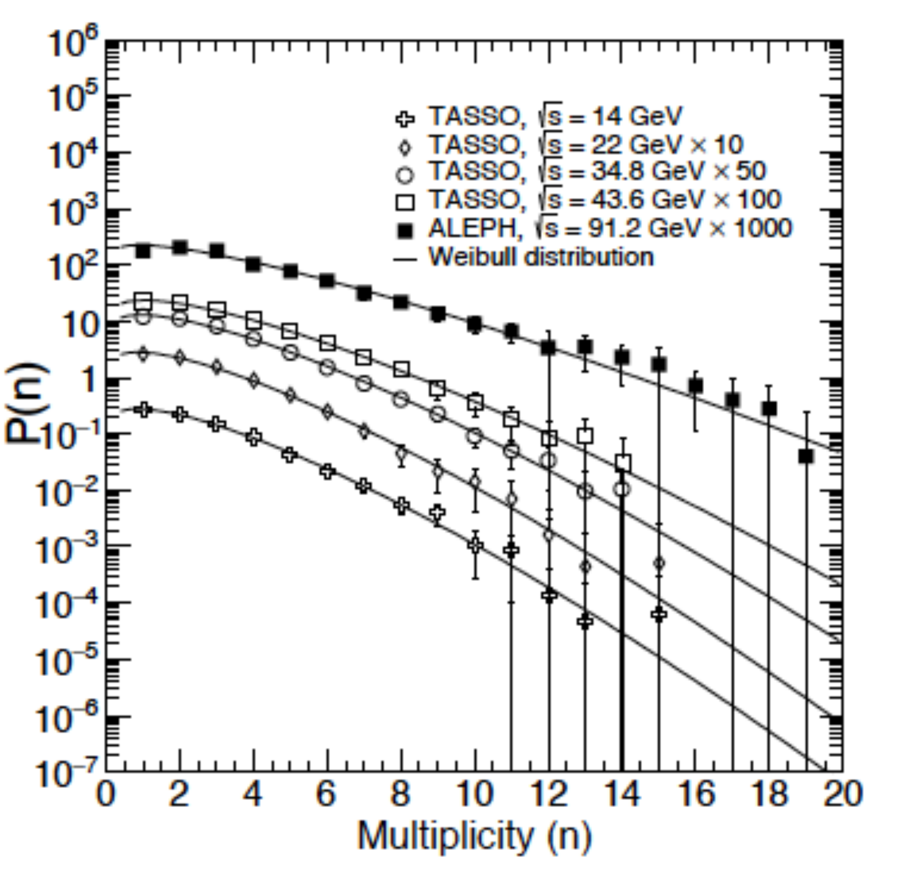


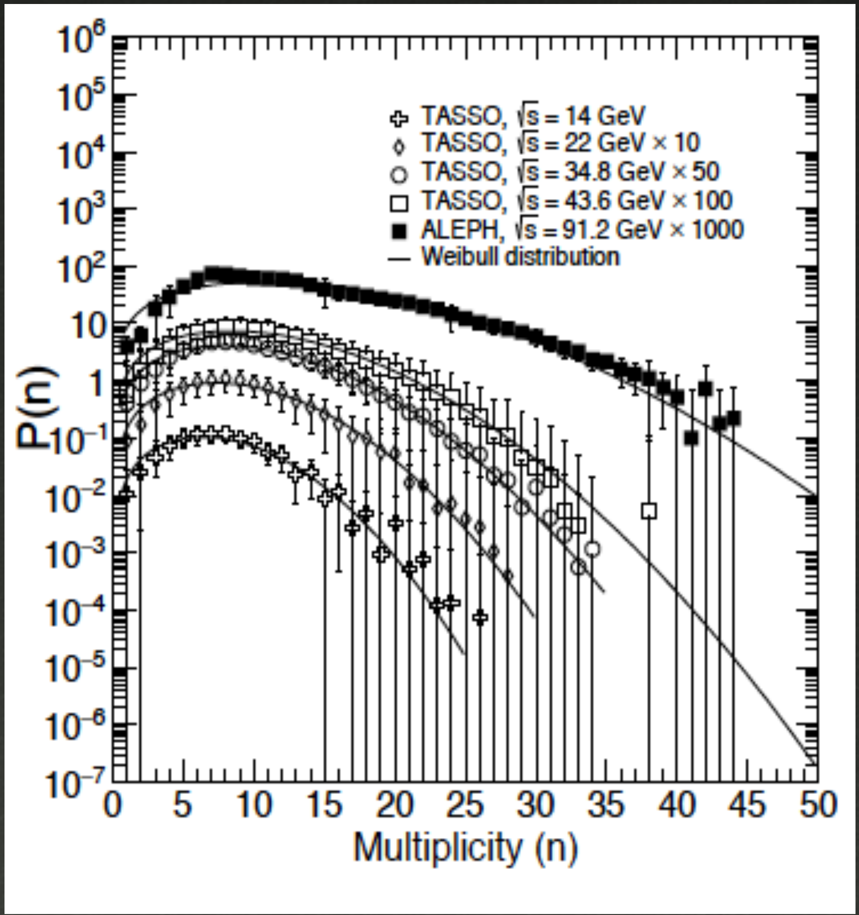
Phys.Rev D 93, 114022 (2016)

(S.Dash, B.K.Nandi and P. Sett)

Weibull Fit to Data Points – (III)

e^+- e^- collisions

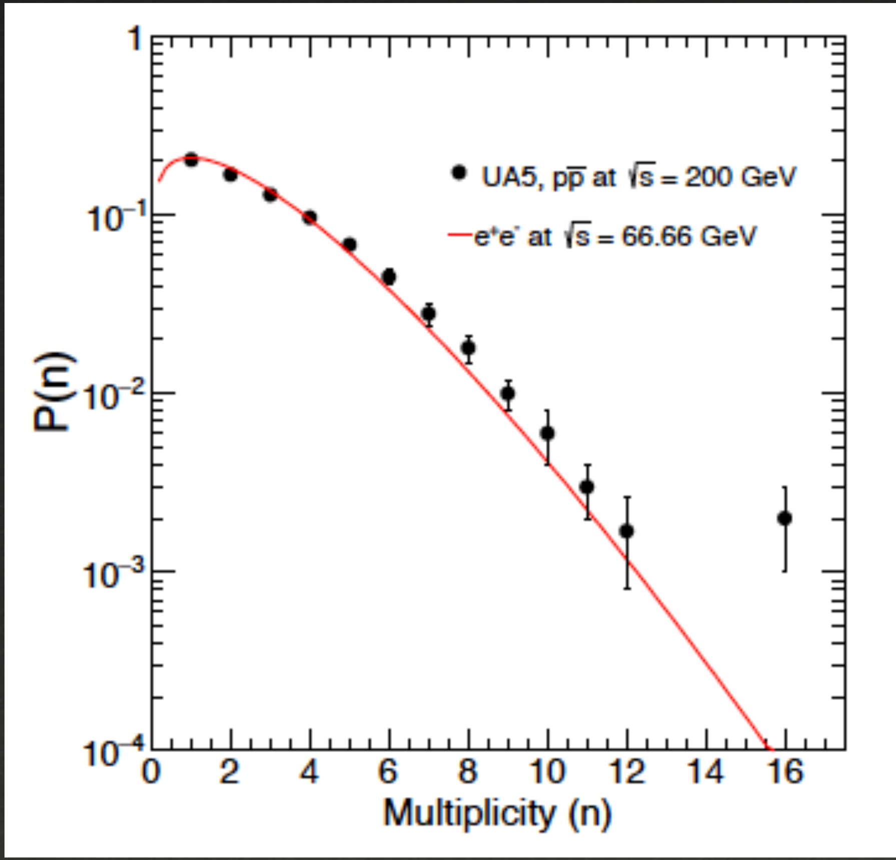


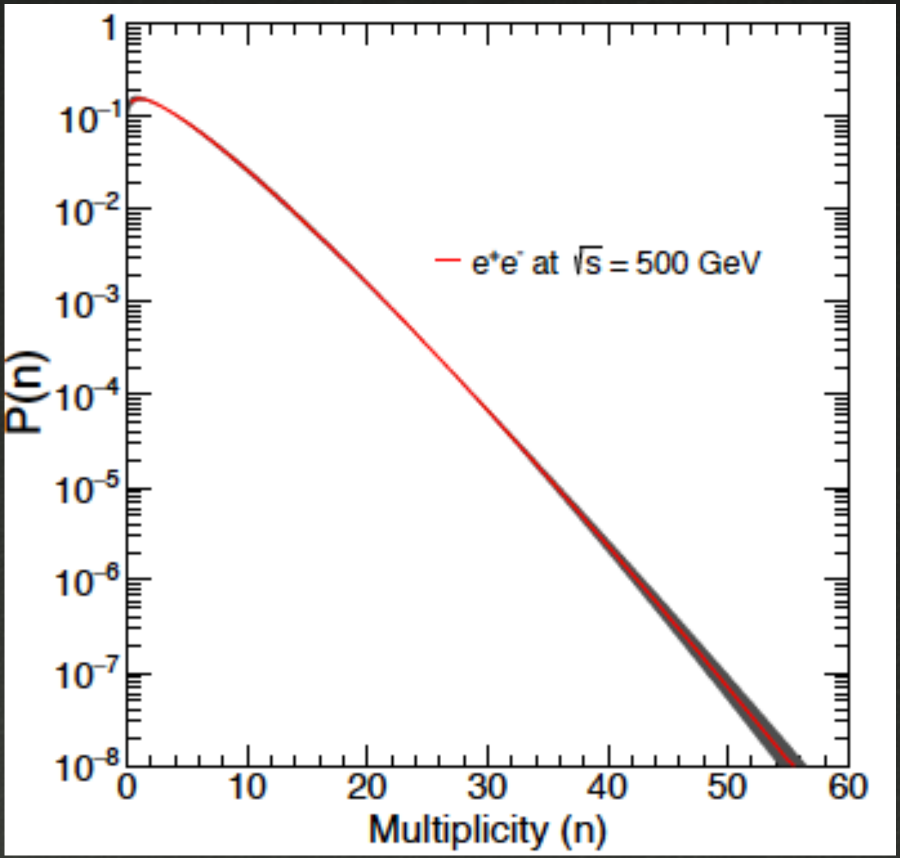


Phys.Rev D 94, 074044 (2016)

(S.Dash, B.K.Nandi and P. Sett)

e^+- e^- collisions





Effective energy scaling observed

Phys.Rev D 94, 074044 (2016)

(S.Dash, B.K.Nandi and P. Sett)

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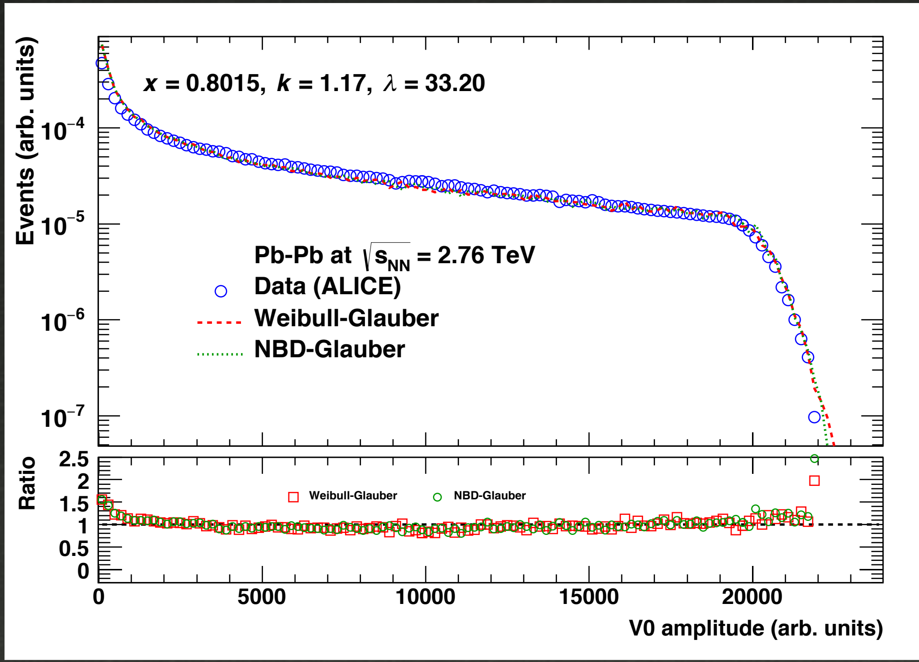
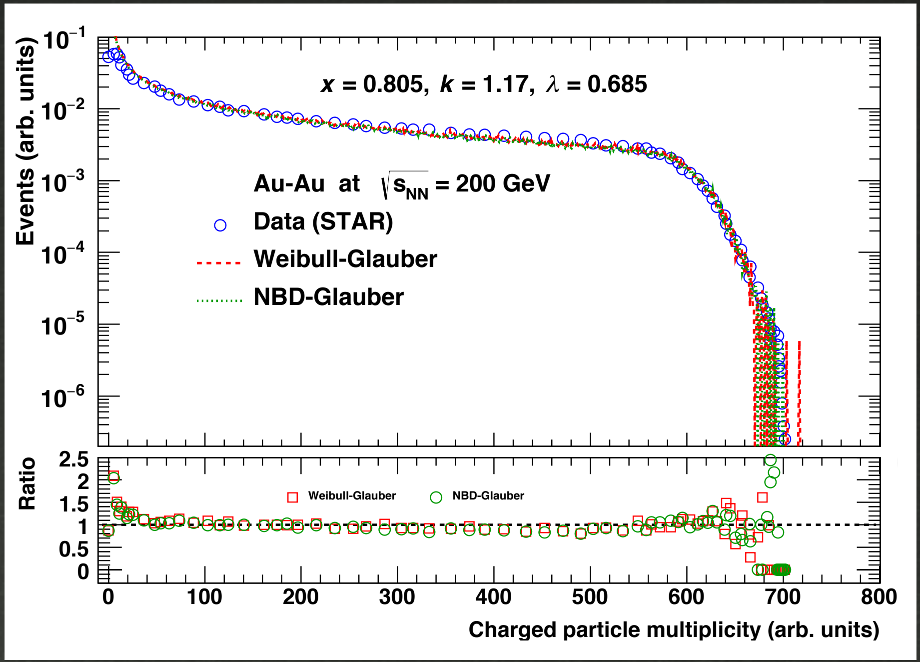
Weibull-Glauber Approach (Heavy ion collisions)

Two component approach

$$N_{ancs} = f(N_{part}) + (1-f)(N_{coll})$$

$$P(n) = P(n)_{pp} \times N$$





S.Dash, N.Behera, B.Naik , B.Nandi and T.Pani : arXiv:1610.02419v2
To appear in Phys.Rev.C

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Normalized Weibull Moments

$$m_n = \lambda^n \Gamma\left(1 + \frac{n}{k}\right)$$

$$\langle n \rangle = \lambda \Gamma\left(1 + \frac{1}{k}\right)$$

$$C_2 = m_2/m_1^2 = \frac{\Gamma(1+\frac{2}{k})}{(\Gamma(1+\frac{1}{k}))^2}$$

$$C_3 = m_3/m_1^3 = \frac{\Gamma(1+\frac{3}{k})}{(\Gamma(1+\frac{1}{k}))^3}$$

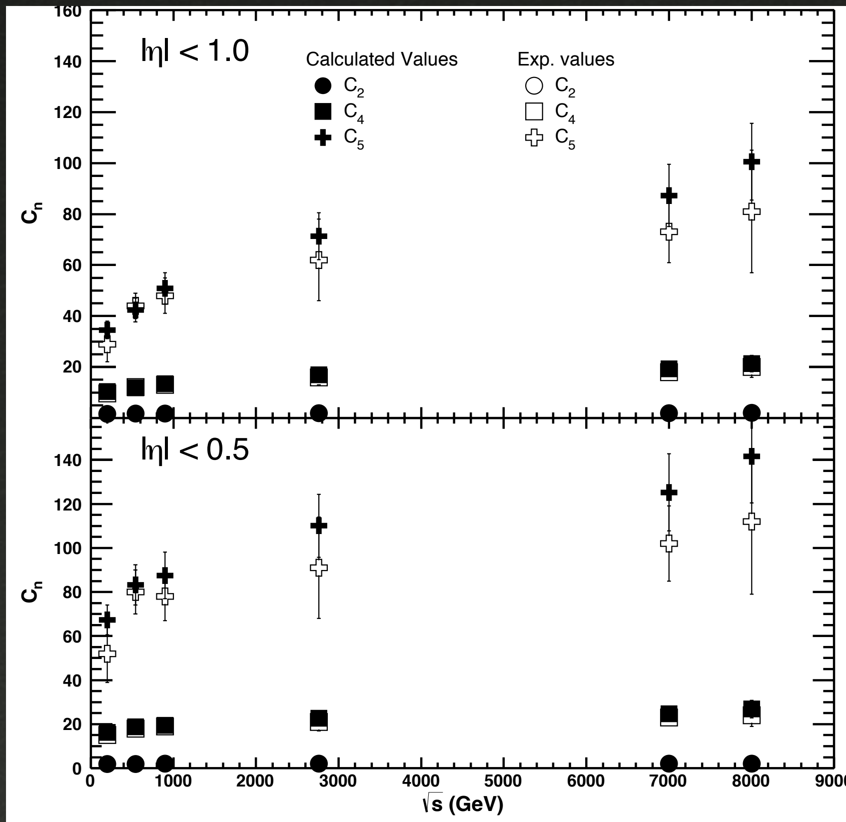
$$C_4 = m_4/m_1^4 = \frac{\Gamma(1+\frac{4}{k})}{(\Gamma(1+\frac{1}{k}))^4}$$

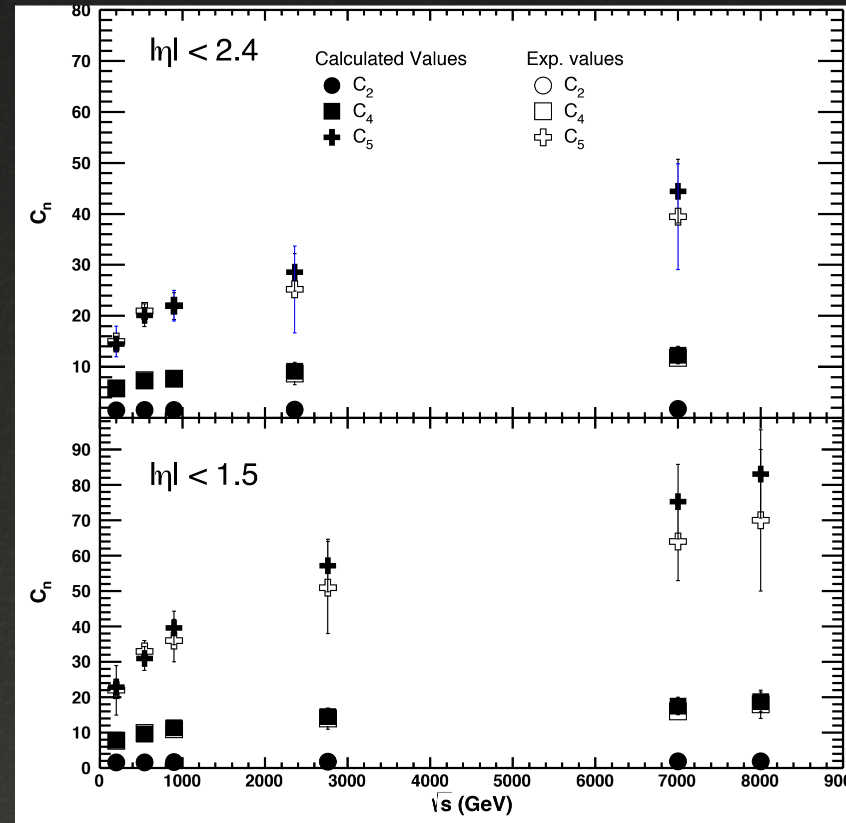
$$C_5 = m_5/m_1^5 = \frac{\Gamma(1+\frac{5}{k})}{(\Gamma(1+\frac{1}{k}))^5}$$

S.Dash, A.Pandey and Sett : arXiv:1 706.07585

To appear in Phys. Rev.D

Normalized Weibull Moments





S.Dash, A.Pandey and Sett : arXiv:1706.07585

To appear in Phys. Rev.D

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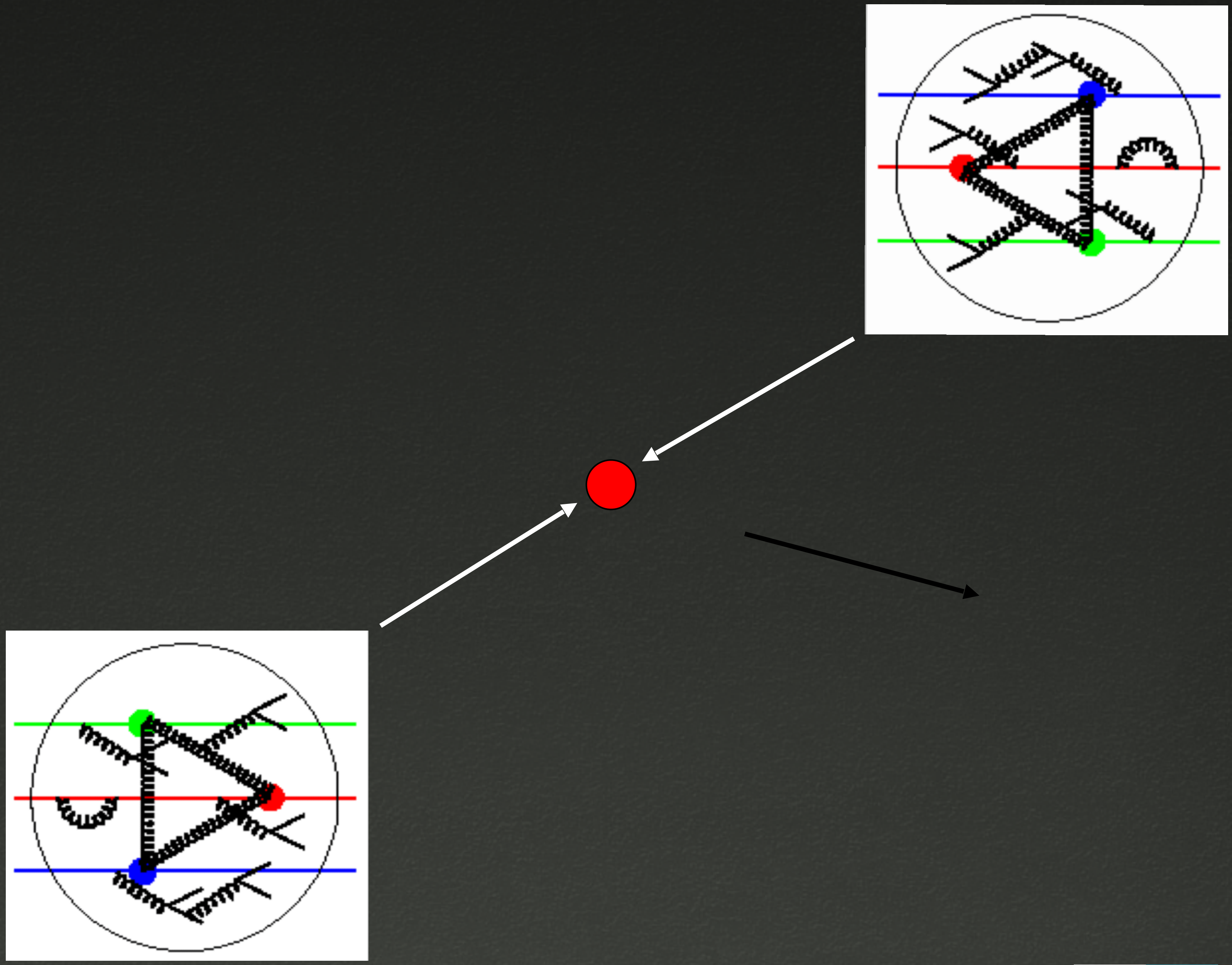
Summary

Weibull distribution is generated in evolving systems involving fragmentation/branching processes.

The evolution of the multi-particle final state results from the fragmentation of color connected partons.

Weibull distribution provides an excellent description of the multiplicity distributions of charged particles in hadronic/leptonic collisions at all energies and pseudo-rapidity intervals.

Reproduced the violation of KNO scaling as observed in measured data at LHC energies.



$$f_n = \left\langle \frac{x!}{(x-n)!} \right\rangle$$

$$F_n = f_n / m_1^n$$

$$F_2 = \frac{\langle x(x-1)\rangle}{m_1^2} = \frac{-\lambda\Gamma(1+\frac{1}{k}) + \lambda^2\Gamma(1+\frac{2}{k})}{(\lambda\Gamma(1+\frac{1}{k}))^2}$$

$$F_3 = \frac{\langle x(x-1)(x-2)\rangle}{m_1^3} = \frac{2\lambda\Gamma(1+\frac{1}{k}) - 3\lambda^2\Gamma(1+\frac{2}{k}) + \lambda^3\Gamma(1+\frac{3}{k})}{(\lambda\Gamma(1+\frac{1}{k}))^3}$$

$$F_4 = \frac{\langle x(x-1)(x-2)(x-3)\rangle}{m_1^4} = \frac{-6\lambda\Gamma(1+\frac{1}{k}) + 11\lambda^2\Gamma(1+\frac{2}{k}) - 6\lambda^3\Gamma(1+\frac{3}{k}) + \lambda^4\Gamma(1+\frac{4}{k})}{(\lambda\Gamma(1+\frac{1}{k}))^4}$$

$$F_5 = \frac{\langle x(x-1)(x-2)(x-3)(x-4)\rangle}{m_1^5}$$

$$= \frac{24\lambda\Gamma(1+\frac{1}{k}) - 50\lambda^2\Gamma(1+\frac{2}{k}) + 35\lambda^3\Gamma(1+\frac{3}{k}) - 10\lambda^4\Gamma(1+\frac{4}{k}) + \lambda^5\Gamma(1+\frac{5}{k})}{(\lambda\Gamma(1+\frac{1}{k}))^5}$$