KAONIC DEUTERIUM
and
Low-Energy Antikaon-Nucleon Interaction

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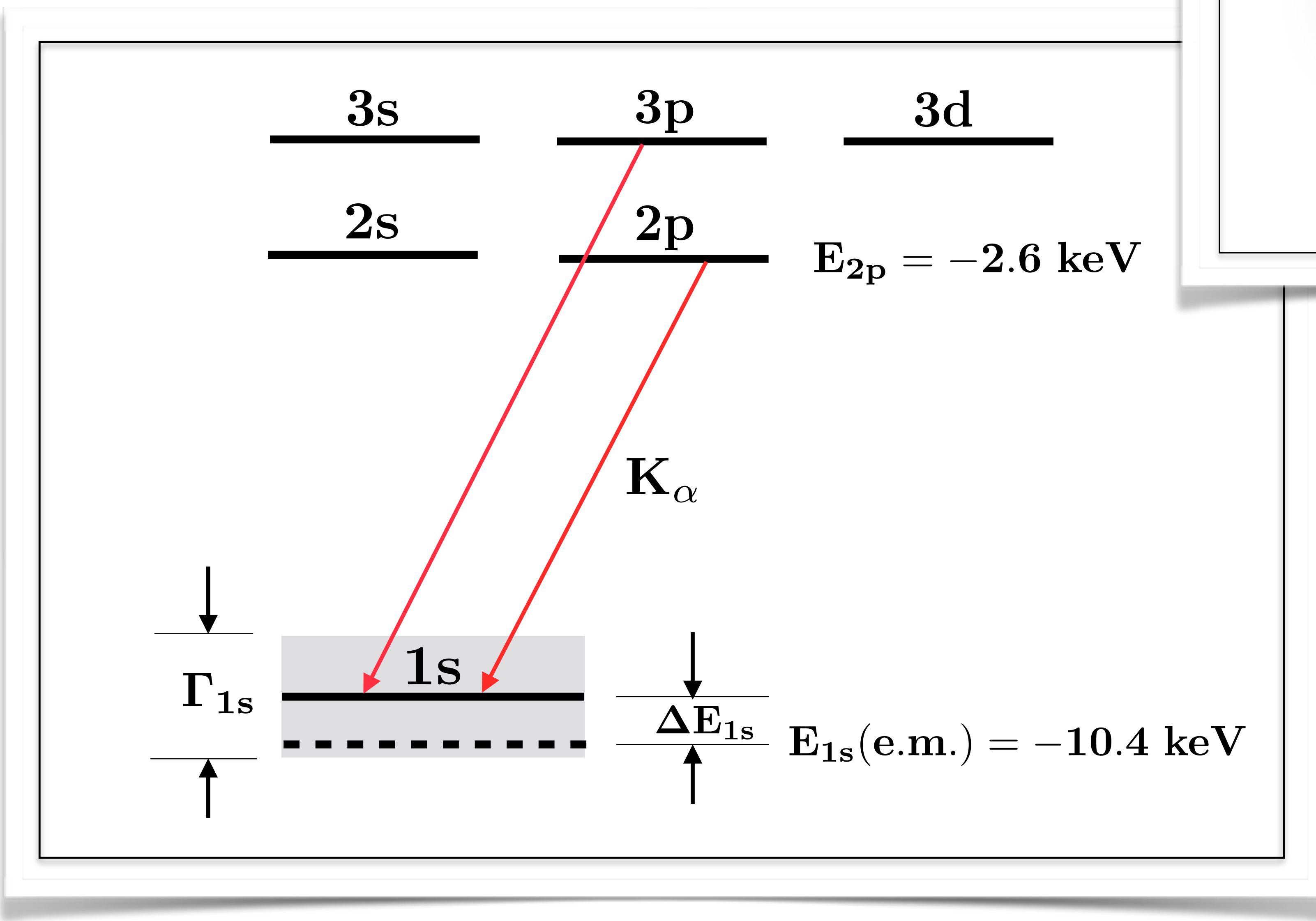
arXiv:1705.06857

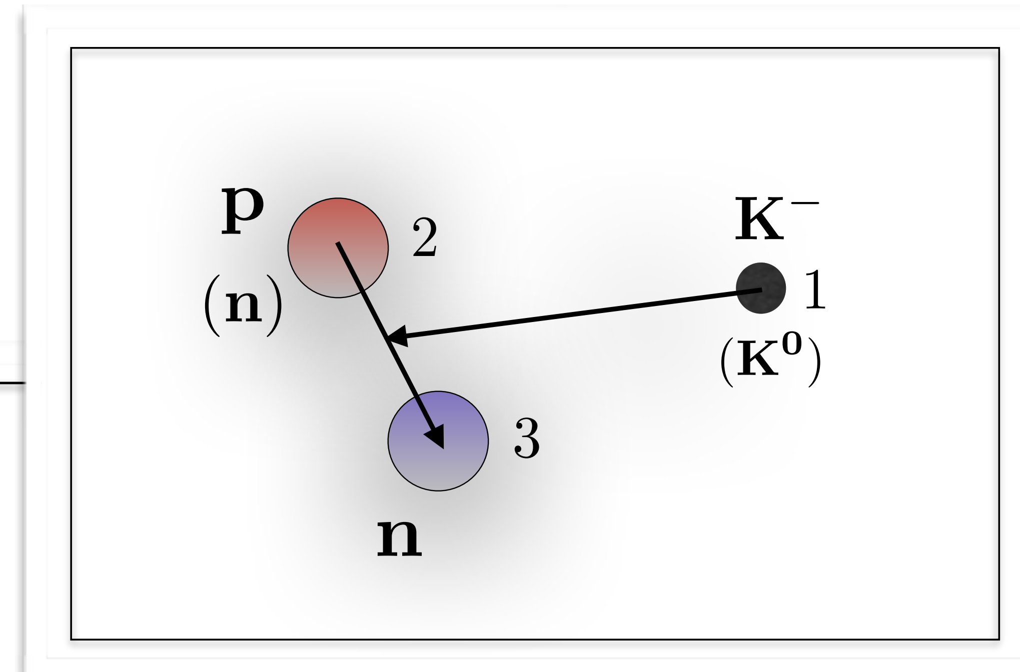
- Reminder of kaonic hydrogen and SIDDHARTA constraints
- Chiral SU(3) coupled-channels dynamics and $\bar{\mathrm{K}}\mathrm{N}$ potential
- Kaonic deuterium and three-body Schrödinger equation
- Generalized Deser-Trueman formula and its limitations
- Constraints for I = 1 $\bar{\mathrm{K}}\mathrm{N}$ interaction from $\mathrm{K}^-\mathrm{d}$ atom
- Outlook: SIDDHARTA2 and J-PARC E57

KAONIC DEUTERIUM

- **Antikaon-NN three-body system**

$$\hat{H} = \sum_{i=1}^{3} \hat{T}_i - \hat{T}_{\mathrm{cm}} + \hat{V}_{23}^{NN} + \sum_{i=2}^{3} (\hat{V}_{1i}^{\bar{K}N} + \hat{V}_{1i}^{EM})$$





Energy shift and width of $1S$ level:

$$\mathbf{\Delta E_{1S} - \frac{i}{2}\Gamma_{1S}}$$

information about strong $\bar{\mathrm{K}}\mathrm{N}$ interaction close to threshold

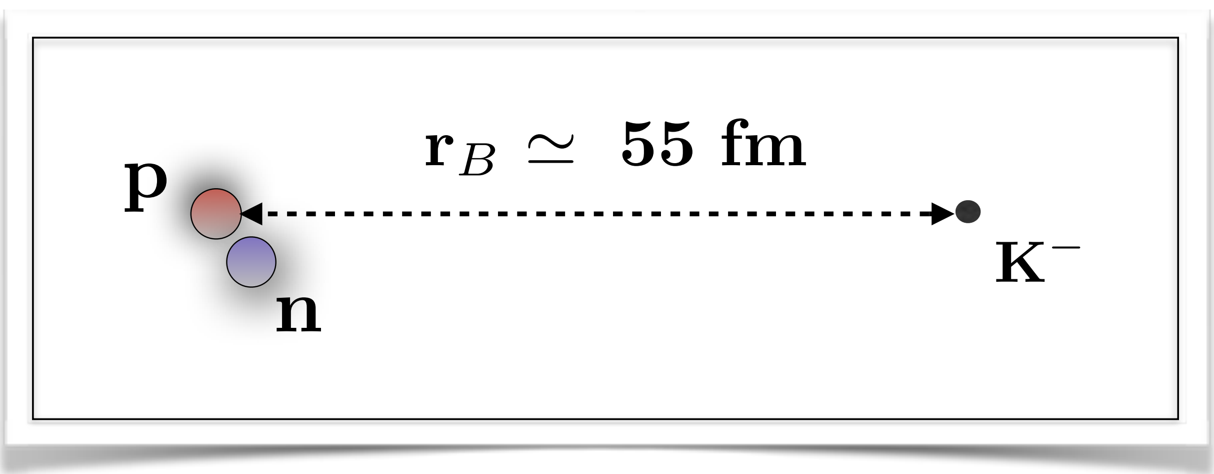
THEORY of KAONIC DEUTERIUM

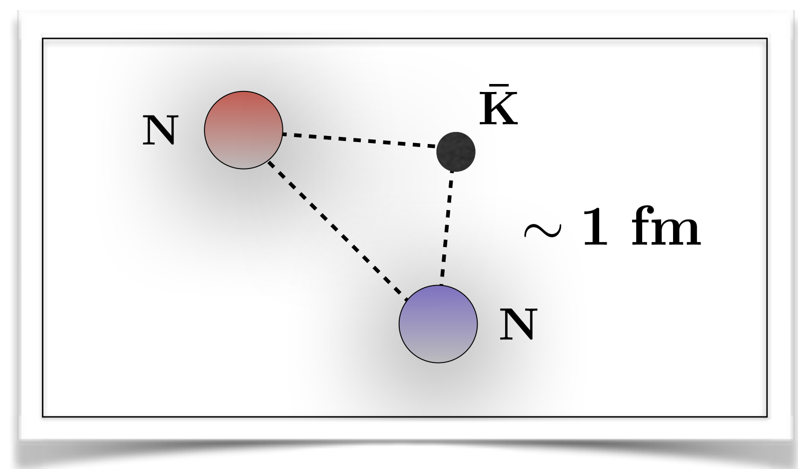
… long history as a challenging three-body problem

- **Faddeev integral equations** (recent advanced results: J. Révai, Phys. Rev. C94 (2016) 054001)
- **Coordinate space methods** (this presentation: T. Hoshino et al., arXiv:1705.06857)

Key issue : NO separation of scales

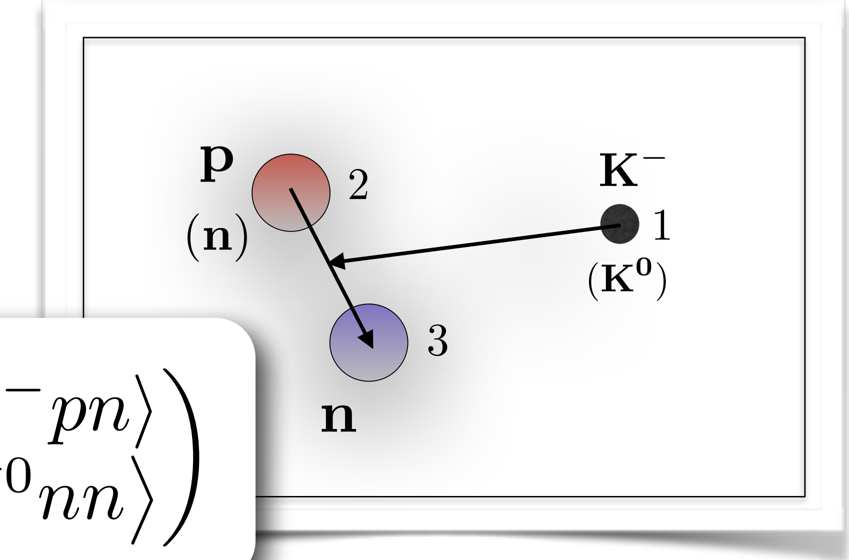
- **COULOMB interaction** $(K^- p)$
  long - distance
- **STRONG interaction** $(\bar{K}N,\ NN)$
  short - distance





- **High computational precision required at ALL length scales**

THEORY of KAONIC DEUTERIUM



- 3-body coupled-channels Schrödinger equation

$$\begin{pmatrix} \hat{H}_{K^-pn} & \hat{V}_{12}^{\bar{K}N} + \hat{V}_{13}^{\bar{K}N} \\ \hat{V}_{12}^{\bar{K}N} + \hat{V}_{13}^{\bar{K}N} & \hat{H}_{\bar{K}^0nn} \end{pmatrix} \begin{pmatrix} |K^-pn\rangle \\ |\bar{K}^0nn\rangle \end{pmatrix} = E \begin{pmatrix} |K^-pn\rangle \\ |\bar{K}^0nn\rangle \end{pmatrix}$$

antikaon-nucleon interaction $\mathbf{K^-p \leftrightarrow \bar{K}^0n}$

nucleon-nucleon interaction

$\mathbf{K^-p}$, $\mathbf{K^-n}$ interaction

Coulomb interaction $\mathbf{K^-p}$

$$\hat{H}_{K^-pn} = \sum_{i=1}^{3} \hat{T}_i - \hat{T}_{\mathrm{cm}} + \hat{V}_{23}^{NN} + \sum_{i=2}^{3} (\hat{V}_{1i}^{\bar{K}N} + \hat{V}_{1i}^{\mathrm{EM}})$$

$$\hat{H}_{\bar{K}^0nn} = \sum_{i=1}^{3} \hat{T}_i - \hat{T}_{\mathrm{cm}} + \hat{V}_{23}^{NN} + \sum_{i=2}^{3} \hat{V}_{1i}^{\bar{K}N} + \Delta M$$

$\mathbf{K^-pn}$ vs. $\mathbf{\bar{K}^0nn}$ mass difference

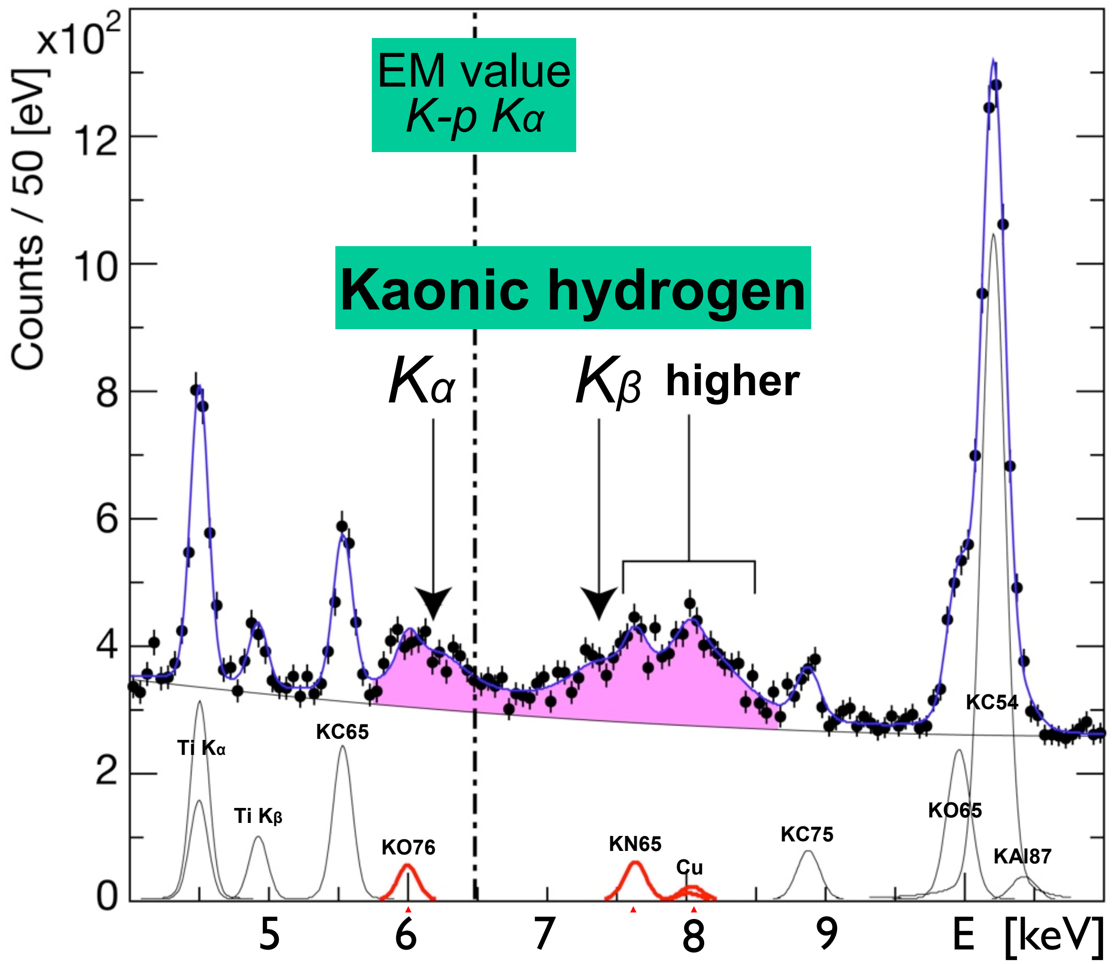
kinetic energies

$\mathbf{\bar{K}^0n}$ interaction

- **Input:**
  physical masses of antikaons and nucleons

CONSTRAINTS from KAONIC HYDROGEN

- **SIDDHARTA**

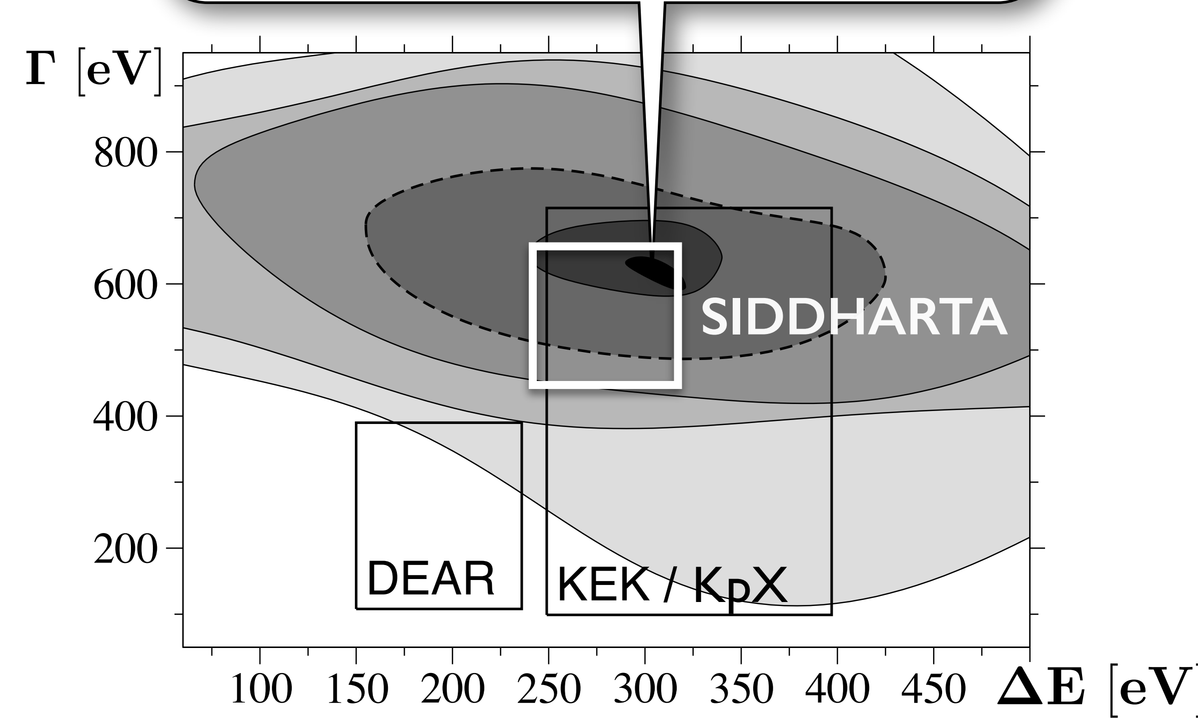


M. Bazzi et al. (SIDDHARTA collaboration)
Phys. Lett. B 704 (2011) 113
Nucl. Phys. A 881 (2012) 88

- **Strong interaction**

1s energy shift and width

Leading order (Weinberg-Tomozawa)
Chiral SU(3) Dynamics



B. Borasoy, R. Nissler, W.W.
Phys. Rev. Lett. 94 (2005) 213401

R. Nissler
Thesis 2008

$$-\varepsilon_{1s} = \mathbf{\Delta E} = \mathbf{283} \pm \mathbf{36}\,(stat) \pm \mathbf{6}\,(syst) \quad \mathbf{eV}$$

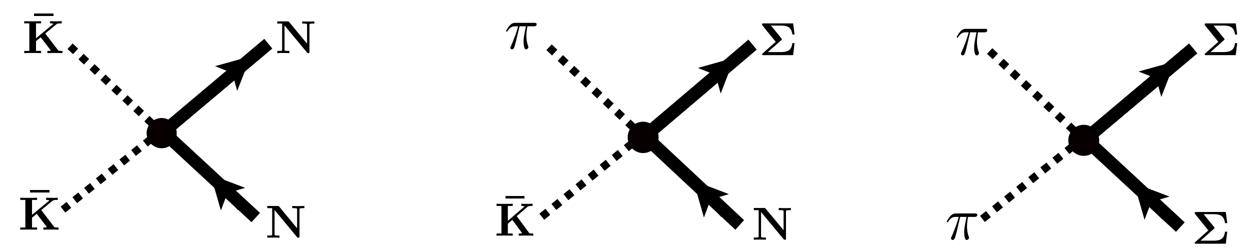
$$\mathbf{\Gamma} = \mathbf{541} \pm \mathbf{89}\,(stat) \pm \mathbf{22}\,(syst) \quad \mathbf{eV}$$

Low-Energy $\bar{\mathrm{K}}\mathrm{N}$ Interactions

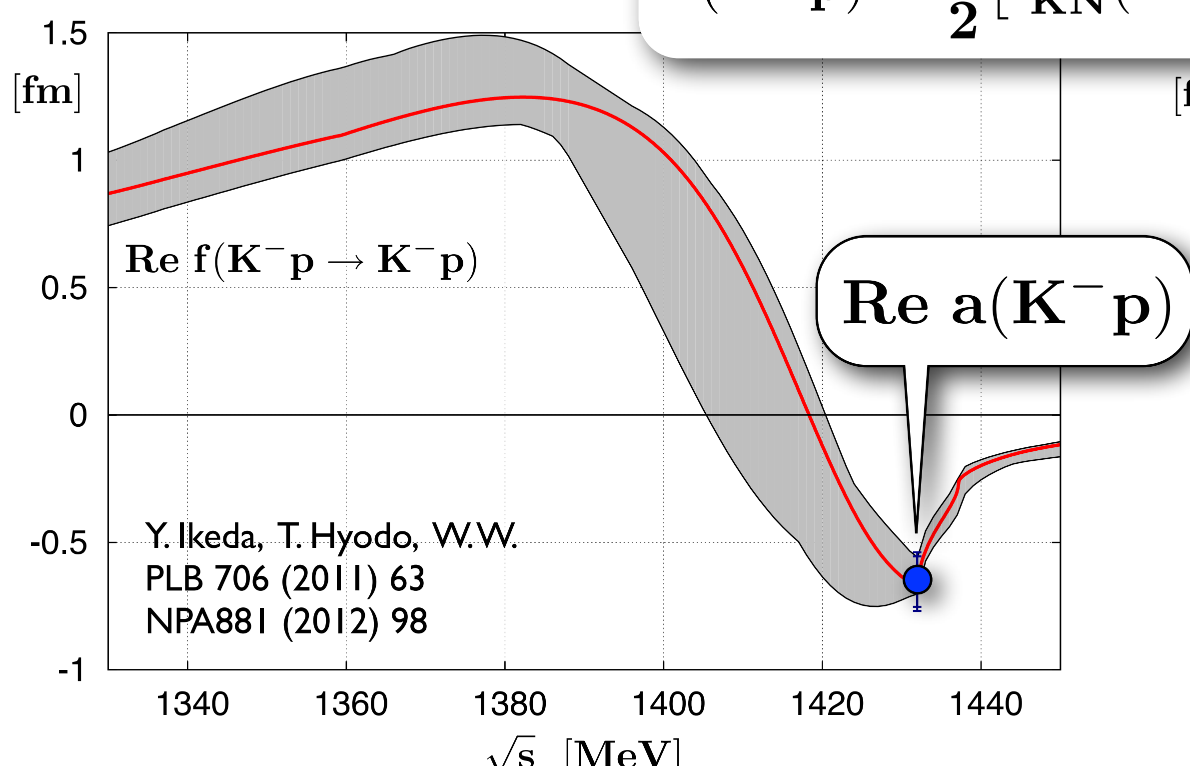
Framework:
Non-perturbative **Coupled Channels** approach based on
Chiral SU(3) Effective Field Theory

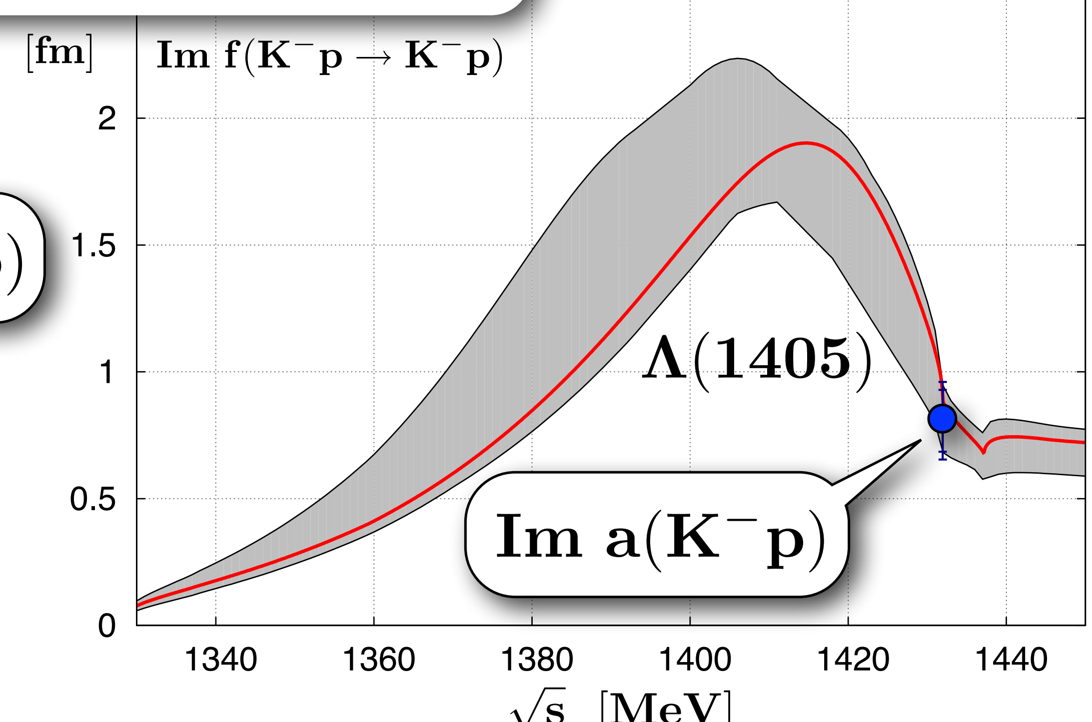
Review: T. Hyodo, D. Jido : Prog. Part. Nucl. Phys. 67 (2012) 55

- Leading s-wave meson-baryon interactions (Weinberg-Tomozawa)



$$\mathbf{f(K^-p) = \frac{1}{2}\left[f_{\bar{K}N}(I=0) + f_{\bar{K}N}(I=1)\right]}$$

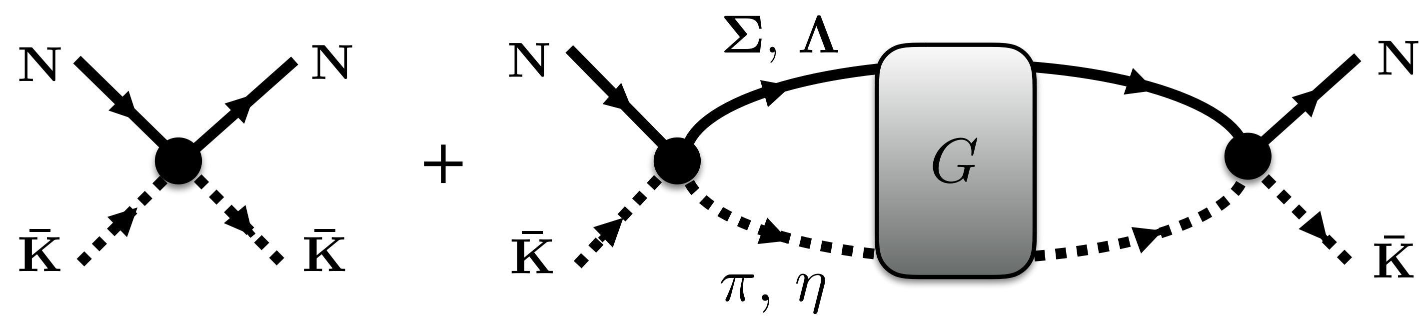




SIDDHARTA - constrained

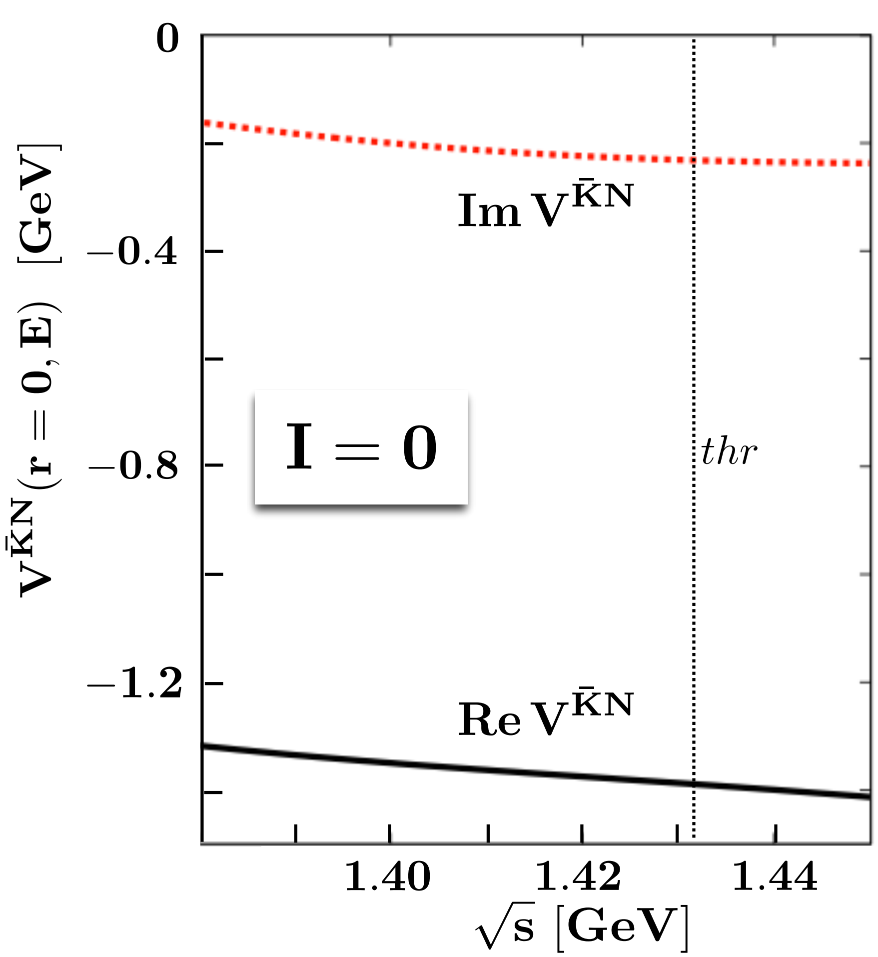
$\overline{\mathrm{K}}\mathrm{N}$ POTENTIAL

$\hat{V}^{\bar{K}N} =$



Input for kaonic deuterium three-body calculation - designed to reproduce :

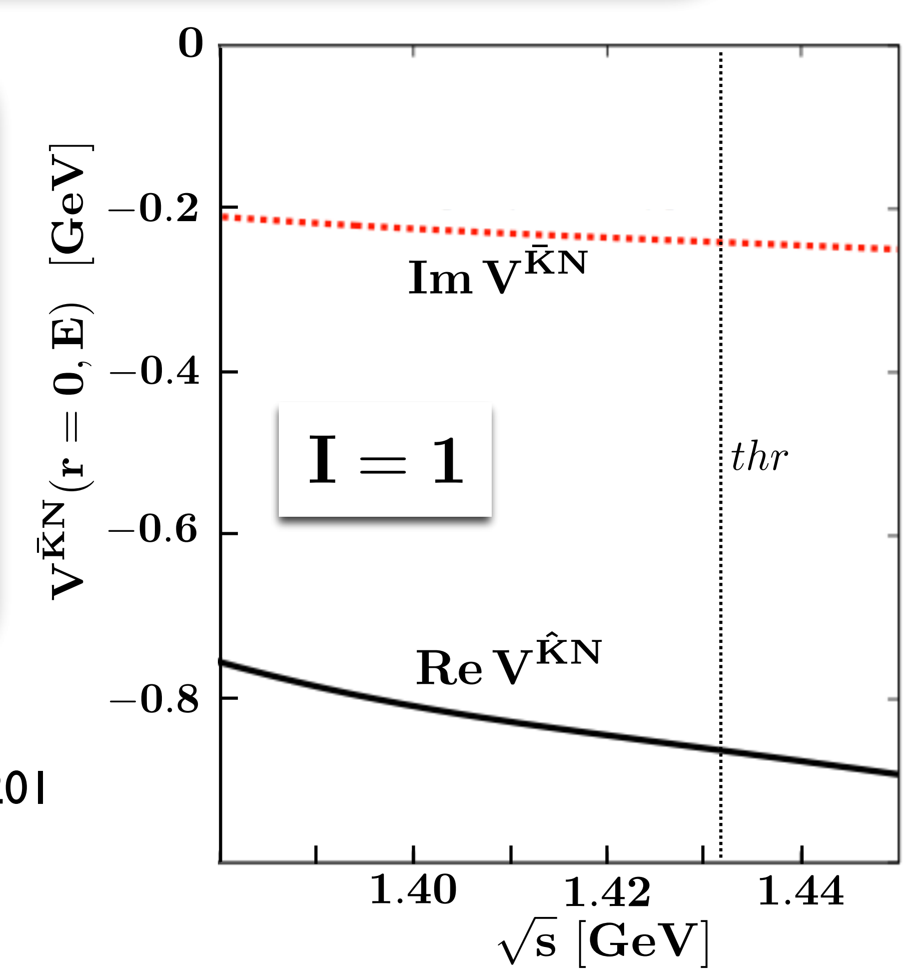
- Two-body scattering data and threshold branching ratios
- Kaonic hydrogen (SIDDHARTA) data
- $\Lambda(1405)$ and $\pi\Sigma$ mass spectra (coupled channels)



$$V^{\bar{K}N}(r,E) = \frac{e^{-r^2/b^2}}{\pi^{3/2}\,b^3}\,U(E)$$

$$b \simeq 0.4\,\mathrm{fm}$$

K. Miyahara, T. Hyodo
Phys. Rev. C93 (2016) 015201



Kyoto $\overline{\mathrm{K}}\mathrm{N}$ POTENTIAL : tests

- **Test for kaonic hydrogen: Solve two-body eqn. including Coulomb**

using **physical** masses

Mass	E-dep.	ΔE (eV)	Γ (eV)
Physical	Self consistent	283	607
Isospin	Self consistent	163	574
Physical	$E_{\bar{K}N} = 0$	283	607
Expt. [31, 32]	(SIDDHARTA)	$283 \pm 36 \pm 6$	$541 \pm 89 \pm 22$

using **isospin-averaged** masses

- **Calculated $\overline{\mathrm{K}}\mathrm{N}$ scattering lengths using physical and isospin-averaged masses for $(\mathbf{K}^-, \bar{\mathbf{K}}^0)$ and $(\mathbf{p}, \mathbf{n})$**

Mass	a_{K^-p} (fm)	$a_{K^-p\text{-}\bar{K}^0n}$ (fm)	$a_{\bar{K}^0n}$ (fm)	a_{K^-n} (fm)
Physical	$-0.66 + i0.89$	$-0.85 + i0.26$	$-0.40 + i1.03$	$0.58 + i0.78$
Isospin	$-0.40 + i0.81$	$-0.99 + i0.04$	$-0.40 + i0.81$	$0.58 + i0.77$

- **Isospin breaking effects important close to threshold(s)**

Solving the Three-Body Schrödinger Equation

(**Hokkaido group:** T. Hoshino, S. Ohnishi, W. Horiuchi)

- **Large set of GAUSSIAN r-space basis functions covering**

$$0.1\,\mathrm{fm} < r < 1000\,\mathrm{fm}$$

$$\sum_{j=1}^{N}(H_{ij} - EB_{ij})C_j = 0 \qquad H_{ij} = \langle\Phi_i|H|\Phi_j\rangle \qquad B_{ij} = \langle\Phi_i|\Phi_j\rangle$$

- **Real part of 1S energy of kaonic deuterium**

N	$\mathrm{Re}E_{1S}$ (keV)
1677	-9.687436
2511	-9.733493
2721	-9.735609
2806	-9.735677
2879	-9.735682

▶ **Convergence at eV precision requires** $N \sim 3000$

▶ **Primary basis sizes used:**

$N \sim 4000$

KAONIC DEUTERIUM : results

- **Energy spectrum of kaonic deuterium (relative to K-d threshold)**
  - **Coulomb** with point charge (2-body) :

$$E_{1S} = -10.406\,\mathrm{keV} \qquad E_{2P} = E_{2S} = -2.602\,\mathrm{keV}$$

  - **Full 3-body computation** :

	$E_{1S}(\mathrm{keV})$	$E_{2P}(\mathrm{keV})$	$E_{2S}(\mathrm{keV})$
Coulomb	-10.398	-2.602	-2.600
Coul. + $\bar{K}N$	$-9.736-i\,0.508$	$-2.602-i\,0.000$	$-2.517-i\,0.067$

- **Energy shift and width of $1\mathrm{S}$ level from $2\mathrm{P} \rightarrow 1\mathrm{S}$ transition**

$$\boldsymbol{\Delta \mathrm{E}_{1\mathrm{S}} = 670\ \mathrm{eV} \qquad \Gamma_{1\mathrm{S}} = 1016\ \mathrm{eV}}$$

- **Uncertainty estimates :**
  - **Deuteron binding effect (subthreshold energy shift) :**
    ~ 5% increase of energy shift, ~ 10% increase of width
  - $\bar{K}NN \rightarrow YN$ **absorption:** less than **2%** increase of width

Test of DESER type FORMULAE

- **Improved Deser-Trueman formula** U.-G. Meißner, U. Raha, A. Rusetsky : EPJ C35 (2004) 349
  relates hadronic atom 1S energy shift and width and hadron-nucleus scattering length a

$$\Delta E - \frac{i\Gamma}{2} = -2\mu^2\alpha^3 a[1 - 2\mu\alpha(\ln\alpha - 1)a]$$

- **Resummed version** V. Baru, J. Epelbaum, A. Rusetsky : EPJ A42 (2009) 111

$$\Delta E - \frac{i\Gamma}{2} = -\frac{2\mu^2\alpha^3 a}{1 + 2\mu\alpha(\ln\alpha - 1)a}$$

- **...works well for kaonic hydrogen** (with Kyoto $\bar{\mathrm{K}}\mathrm{N}$ potential) :

	ΔE (eV)	Γ (eV)
Full Schrödinger equation	283	607
Improved Deser formula	293	596
Resummed formula	284	605

Test of the DESER type formulae for KAONIC DEUTERIUM

$$\Delta E - \frac{i\Gamma}{2} = -2\mu^2\alpha^3 a[1 - 2\mu\alpha(\ln\alpha - 1)a] \quad \text{or} \quad \Delta E - \frac{i\Gamma}{2} = -\frac{2\mu^2\alpha^3 a}{1 + 2\mu\alpha(\ln\alpha - 1)a}$$

- **How does it work for kaonic deuterium ?**

(fixed-center approximation, multiple scattering series)

▶ **K-d scattering length**

$$a_{K^-d} = \frac{\mu_{K^-d}}{m_{K^-}} \int d^3\boldsymbol{r}\, \rho_d(\boldsymbol{r})\, \tilde{a}_{K^-d}(r)$$

$$\tilde{a}_{K^-d}(r) = \frac{\tilde{a}_p + \tilde{a}_n + (2\tilde{a}_p\tilde{a}_n - \tilde{a}_{\text{ex}}^2)/r - 2\tilde{a}_{\text{ex}}^2\tilde{a}_n/r^2}{1 - \tilde{a}_p\tilde{a}_n/r^2 + \tilde{a}_{\text{ex}}^2\tilde{a}_n/r^3}$$

$$\tilde{a}_p \equiv \tilde{a}_{K^-p} \qquad \tilde{a}_n \equiv \tilde{a}_{K^-n}$$

$$\tilde{a}_{\text{ex}}^2 \equiv \tilde{a}_{K^-p\text{-}\bar{K}^0n}^2/(1 + \tilde{a}_{\bar{K}^0n}/r)$$

$$\tilde{a}_{\bar{K}N} \equiv \frac{m_K}{\mu_{\bar{K}N}} a_{\bar{K}N}$$

▶ $a_{K^-d} = (-1.42 + i\,1.60)\ \text{fm}$

	ΔE (eV)	Γ (eV)
Full Schrödinger equation	670	1016
Improved Deser formula	910	989
Resummed formula	818	1188

- **Assumptions behind Deser-Trueman formula NOT reliable for K-d**

OUTLOOK

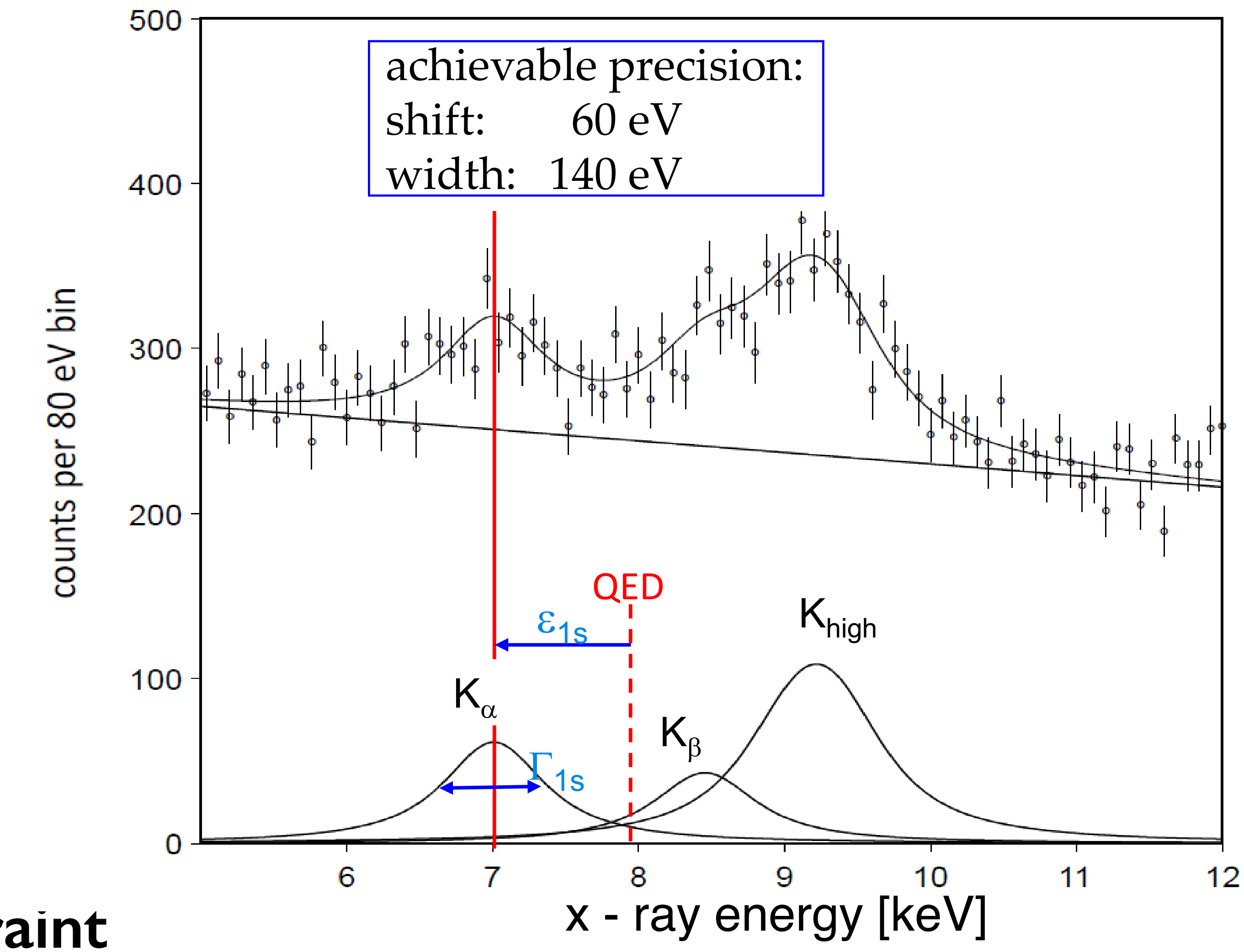
- **SIDDHARTA2**
  in preparation at
  DAΦNE, Frascati

C. Curceanu et al.
Nucl. Phys. A 914 (2013) 251

- **J-PARC E57 proposal**

J. Zmeskal et al., J-PARC PAC 2017

Geant4 simulated K^-d X-ray spectrum



- **Expectation :**
  significantly improved constraint
  on I = 1 component of $\bar{K}N$ interaction
  if deuterium level shift and width can be determined within ~25% accuracy

SUMMARY and CONCLUSIONS

- Progress in KAONIC DEUTERIUM calculations :
  - Chiral SU(3) - based $\bar{\mathrm{K}}\mathrm{N}$ STRONG (short-range) interaction
  - $\mathrm{K^-p}$ COULOMB (long-range) interaction

  Advanced three-body computations (eV precision)

- Solving $\bar{\mathrm{K}}\mathrm{NN}$ Schrödinger equation with complex $\bar{\mathrm{K}}\mathrm{N}$ potential :
  - Strong interaction energy shift and width

    $$\Delta \mathrm{E_{1S}} \simeq 0.7 \ \mathrm{keV} \qquad \Gamma_{1S} \simeq 1.0 \ \mathrm{keV}$$

    (~10% uncertainty)
  - Consistent with advanced Faddeev calculations

    (J. Révai, PRC (2016) , separable interactions)

- Looking forward to SIDDHARTA2 and J-PARC E57
  - … important new constraints on isospin I = 1 $\bar{\mathrm{K}}\mathrm{N}$ interaction

The End

Special thanks to

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Kenta **Miyahara** Shota **Ohnishi**