Single- and multi-nucleon K^- interactions with nuclei near threshold

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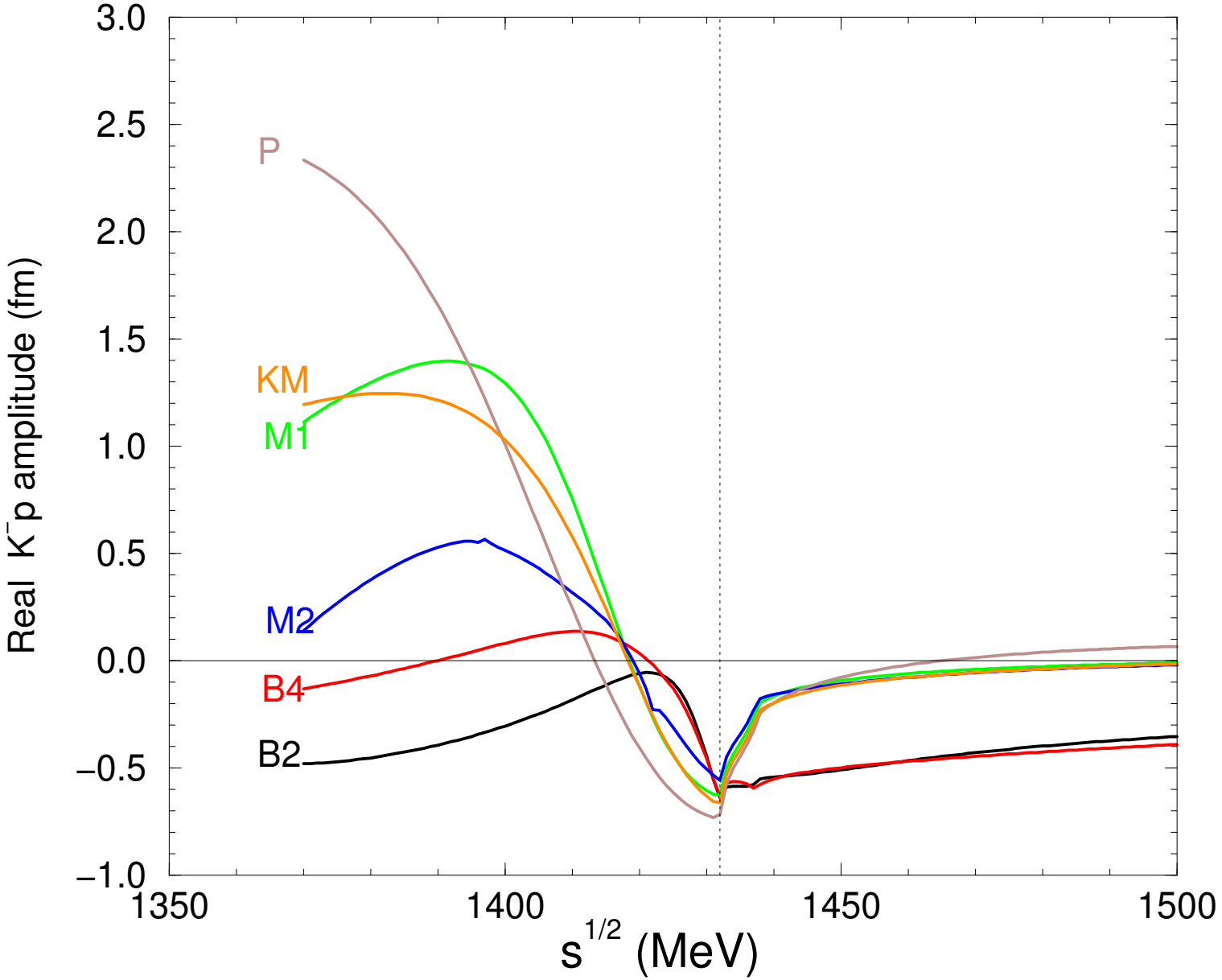
Six K^-N s-wave amplitudes taken from chiral SU(3) meson-baryon coupled-channel dynamical models

- B1 and B2 for Bonn, Eur. Phys. J. A 51 (2015) 30
- M1 and M2 for Murcia, Phys. Rev. C 87 (2013) 035202
- P for Prague, Nucl. Phys. A 881 (2012) 115
- KM for Kyoto-Munich, Nucl. Phys. A 881 (2012) 98

Fits to low energy reaction cross sections, threshold branching ratios and K^-H atoms from the SIDDHARTA experiment.

See Cieplý et al. Nucl. Phys. A 954 (2016) 17 for details and references.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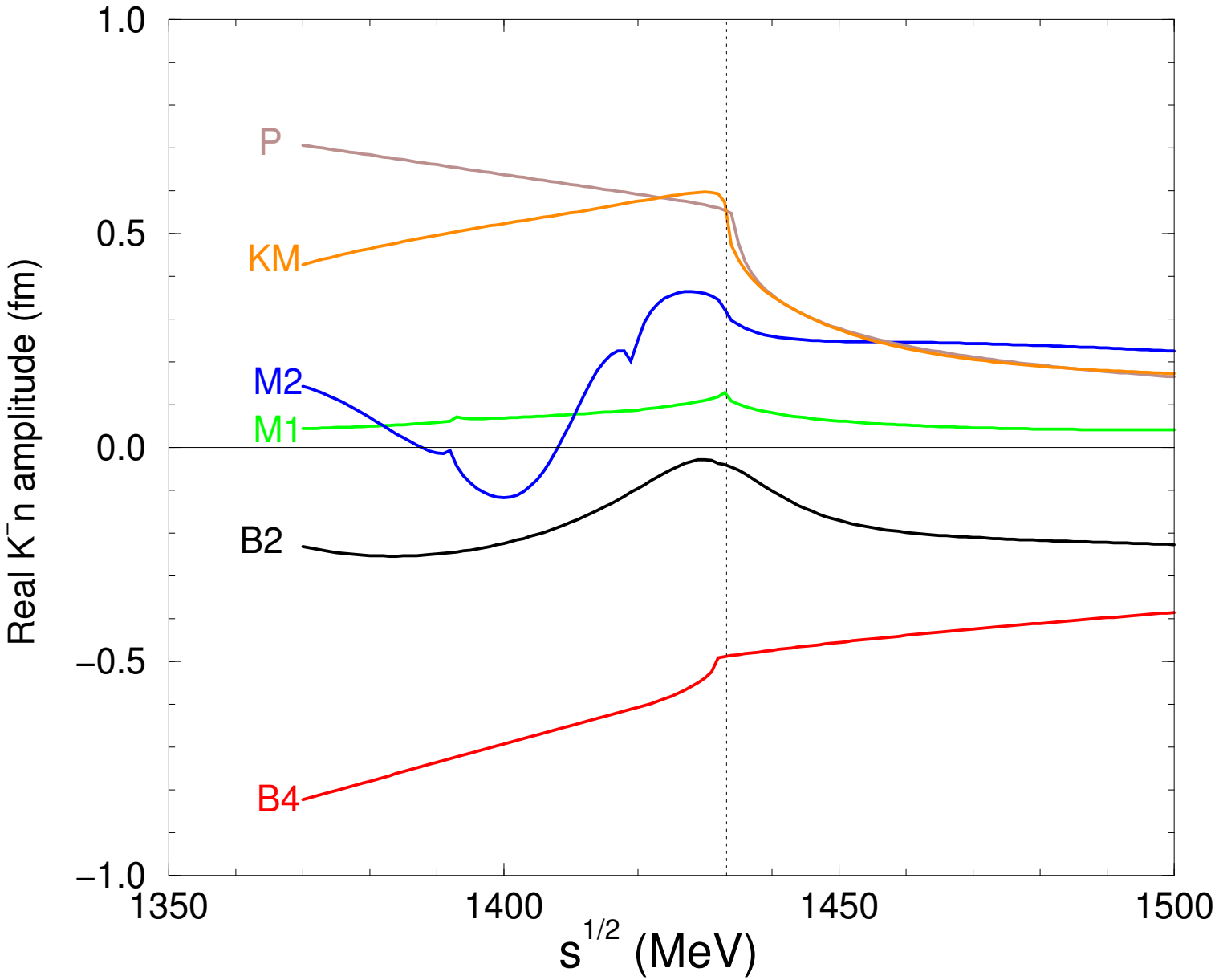
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
1350
1400
1450
1500
Real K̄p amplitude (fm)
s^1/2 (MeV)
P
KM
M1
M2
B4
B2


1.0
0.5
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1350
1400
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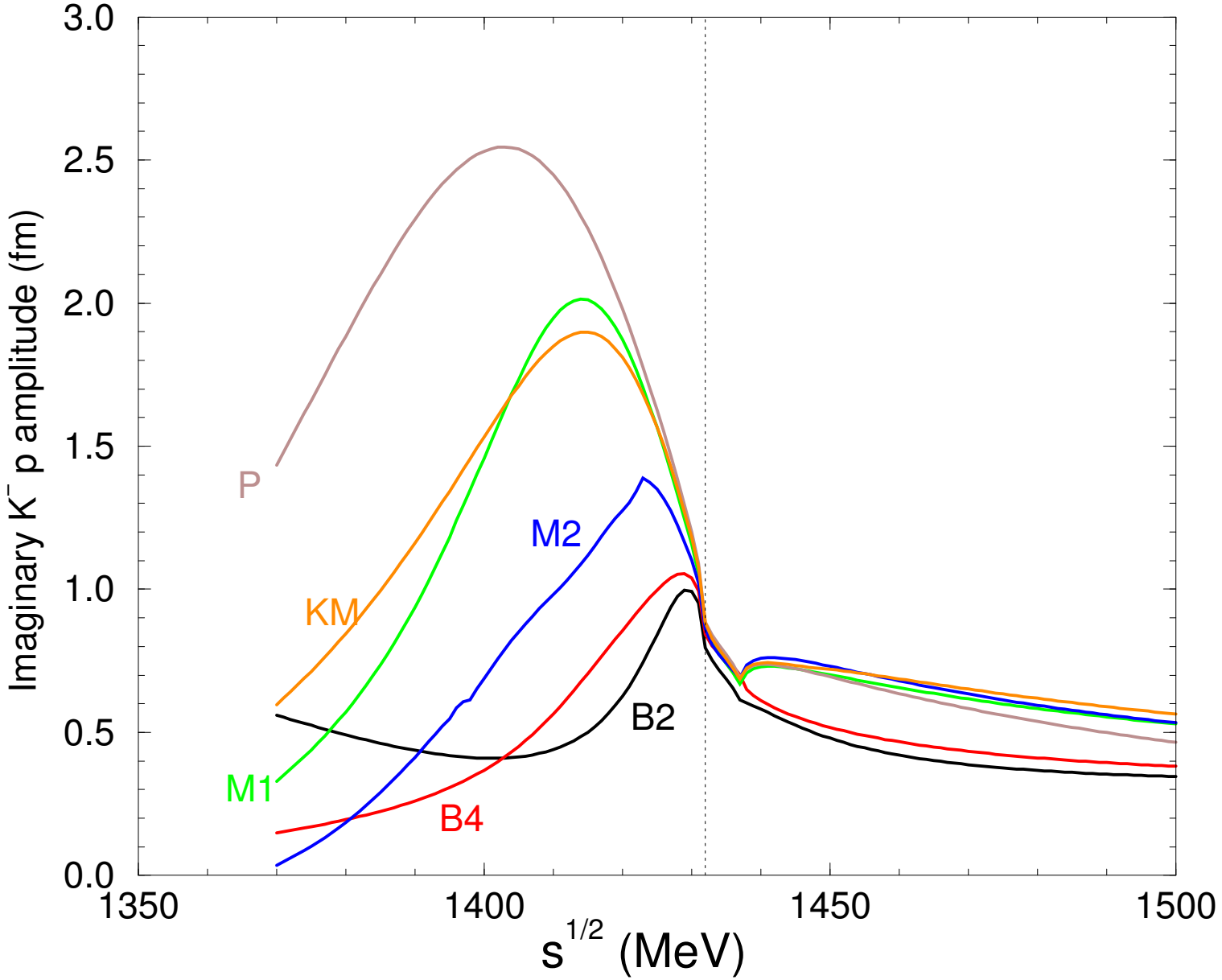
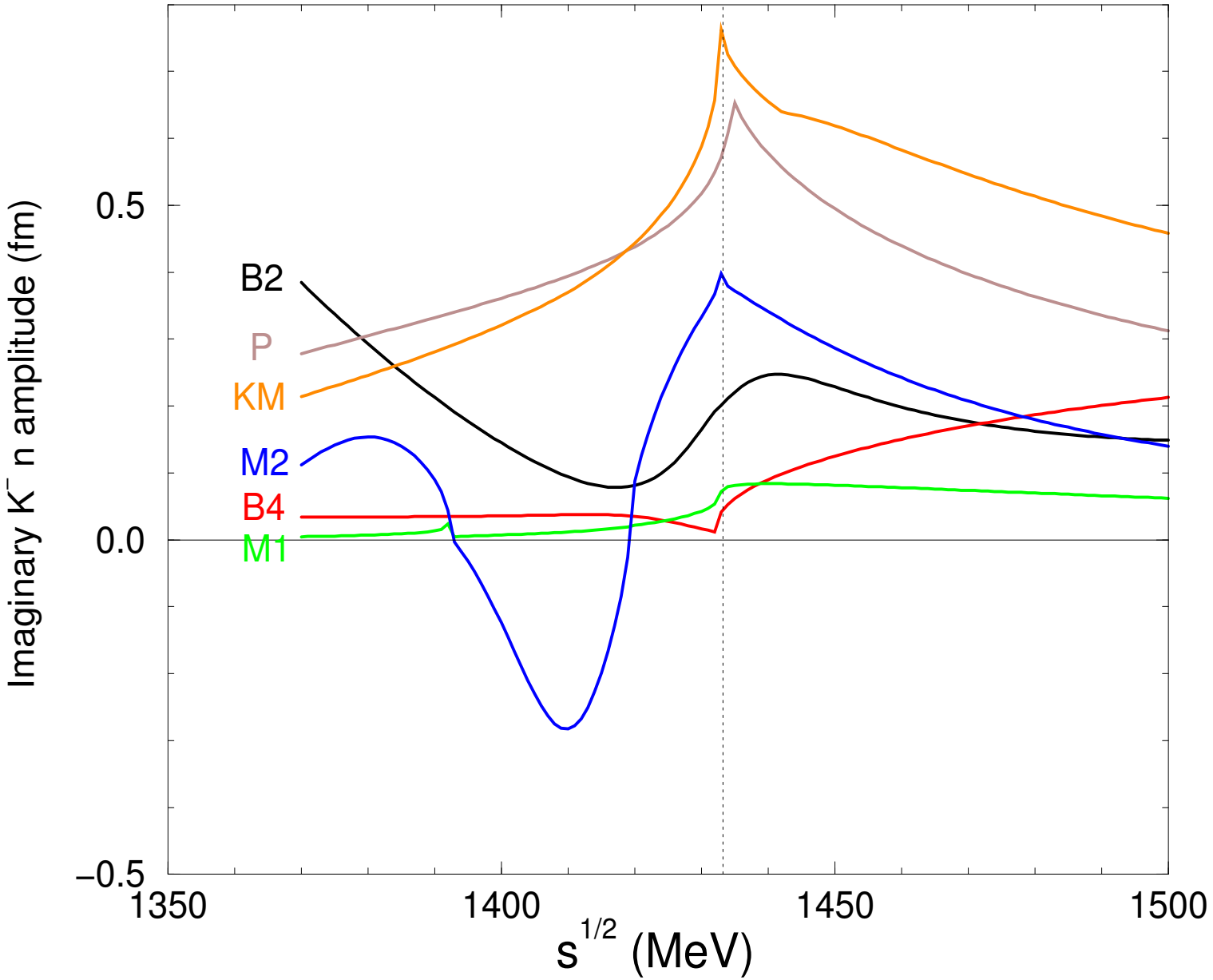
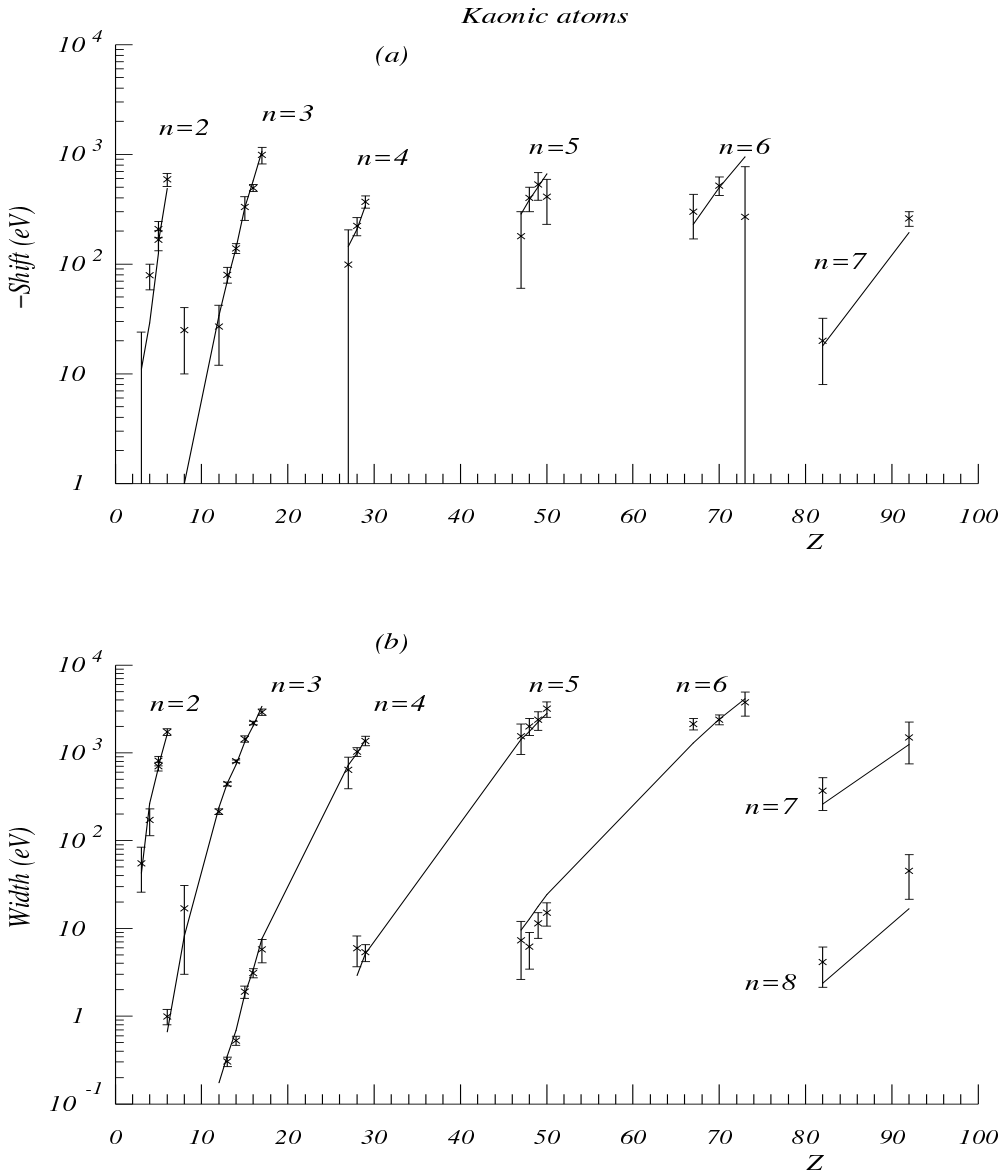
1500
Real K¯n amplitude (fm)
s^1/2 (MeV)
P
KM
M2
M1
B2
B4


3.0
2.5
2.0
1.5
1.0
0.5
0.0
1350
1400
1450
1500
Imaginary K⁻ p amplitude (fm)
s^1/2 (MeV)
P
KM
M1
M2
B2
B4


Imaginary K⁻ n amplitude (fm)
0.5
0.0
−0.5
1350
1400
1450
1500
s^1/2 (MeV)
B2
P
KM
M2
B4
M1

Outline

- Near threshold K^--*nucleus* interactions from kaonic atoms
- Ambiguities with state-of-the-art K^-N amplitudes
- Multi-N interaction essential for fits to data
- Ambiguities remain; need additional experimental results
- Single-N to multiple-N absorption ratios from bubble chambers experiments remove ambiguiteis
- Relative strength of single-N and multi-N amplitudes determined for the first time
- Sensitivities and relevant densities



Fits shown are for phenomenological optical potentials

Reminder of 'In-medium kinematics'

The simplest optical potential at threshold is $V_{K^-}(\rho) = \frac{-4\pi}{2\mu_{K^-N}} f_{K^-N}\rho$.

Adopt the invariant Mandelstam variable $s = (E_{K^-} + E_N)^2 - (\vec{p}_{K^-} + \vec{p}_N)^2$ as the argument transforming free-space to in-medium K^-N amplitudes.

In exotic atoms $\vec{p}_{K^-} + \vec{p}_N \neq 0$.

$(\vec{p}_{K^-})^2$ determined by the atomic environment, typical size of 30-100 fm, $(\vec{p}_N)^2$ is determined by the nuclear environment, typical size of 5 fm.

On average $(\vec{p}_{K^-} + \vec{p}_N)^2 = (\vec{p}_{K^-})^2 + (\vec{p}_N)^2$.

PLB 702 (2011) 402; NPA 899 (2013) 60; EPJ Web of Conferences 81 (2014) 01018; NPA 959 (2017); (partial list).

Reminder of 'in-medium kinematics'

Defining $\delta\sqrt{s} = \sqrt{s} - E_{\rm th}$ with $E_{\rm th} = m_{K^-} + m_N$, then to first order in $B/E_{\rm th}$ one gets

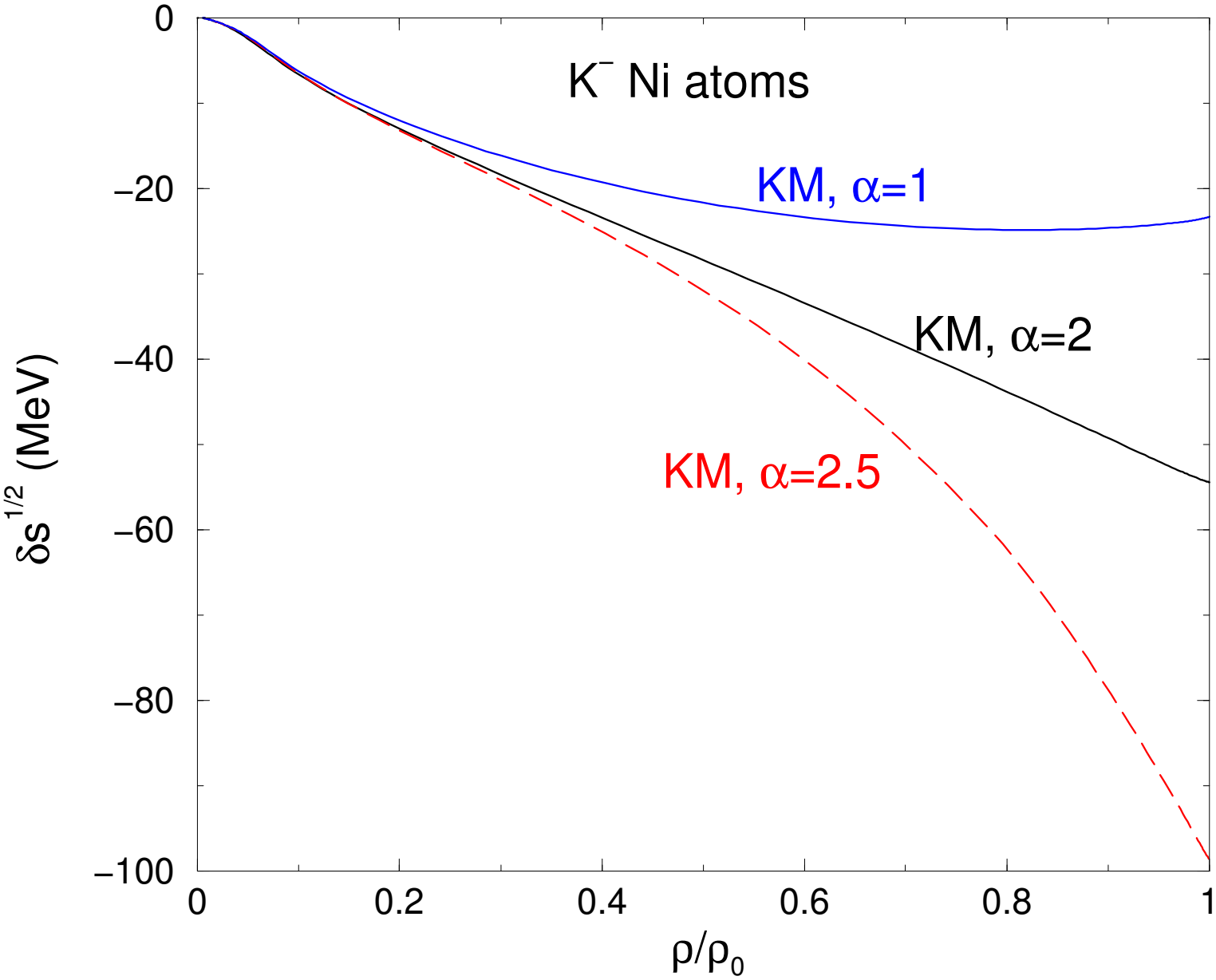
$$\delta\sqrt{s} = -B_N\rho/\bar{\rho} - \beta_N[T_N(\rho/\bar{\rho})^{2/3} + B_{K^-}\rho/\rho_0] + \beta_{K^-}[\mathrm{Re}\ V_{K^-} + V_c(\rho/\rho_0)^{1/3}],$$

with $\beta_N = m_N/(m_N + m_{K^-})$, $\beta_{K^-} = m_{K^-}/(m_N + m_{K^-})$, and $\rho_0 = 0.17$ fm^{-3}. An average binding energy value of $B_N = 8.5$ MeV is used. T_N=23 MeV in the Fermi gas model.
The specific ρ/ρ_0 and $\rho/\bar{\rho}$ forms of density dependence ensure that $\delta\sqrt{s} \to 0$ when $\rho \to 0$ in accordance with the low-density limit.

Solving by iterations, $\sqrt{s}$ and hence amplitudes become functions of ρ, essentially *averaging over subthreshold energies.*

Accepting 'Minimal Substitution' (MS), $V_c(r)$ is subtracted from $\delta\sqrt{s}$, (as supported by analyses of pion-nucleus experiments).

Examples of relevant sub-threshold energies



The in-medium K^-N scattering amplitudes in the K^--nuclear c.m. frame, $\tilde{f}_{K^-N}(\rho)$, are related kinematically to the in-medium K^-N c.m. amplitudes $f_{K^-N}(\rho)$ by $\tilde{f}_{K^-N}(\rho) = (1 + \frac{A-1}{A}\frac{\mu_K}{m_N}) f_{K^-N}(\sqrt{s})$. The in-medium potential is (Wass et al., NPA 617 (1997) 449)

$$2\mu_K V^{(1)}_{K^-}(\rho) = -4\pi \left[\frac{(2\tilde{f}_{K^-p} - \tilde{f}_{K^-n})\,\frac{1}{2}\rho_p}{1 + \frac{1}{4}\xi_k(\rho)\tilde{f}_0\rho(r)} + \frac{\tilde{f}_{K^-n}(\frac{1}{2}\rho_p + \rho_n)}{1 + \frac{1}{4}\xi_k(\rho)\tilde{f}_1\rho(r)} \right] ,$$

with the Pauli-correlations determined by

$$\xi_k(\rho) = \frac{9\pi}{k_F^2}\left(4\int_0^\infty \frac{dr}{r}\,\exp(\mathrm{i}kr)\,j_1^2(k_F r)\right) ,$$

with $k = [(E_{K^-} - i\Gamma/2)^2 - m_k^2]^{1/2}$ and where Γ is the width of the particular kaonic atom state. The Fermi momentum is given by $k_F = (3\pi^2\rho/2)^{1/3}$; $\tilde{f}_0$ and $\tilde{f}_1$ are the isospin 0 and 1 combinations, respectively, of the K^-N amplitudes; and ρ_p and ρ_n are proton and neutron densities, respectively.

For attractive potentials the energy $\sqrt{s}$ is below threshold within the nuclear medium.

In addition there are corrections due to Pauli correlations.

The algorithm performs averaging over subthreshold energies.

χ^2 for 65 kaonic atoms data points from optical potentials based only on one-nucleon amplitudes.

model	B2	B4	M1	M2	P	KM
$\chi^2(65)$	1174	2358	2544	3548	2300	1806

χ^2 for 18 high quality data points (P, S, Cl, Cu, Ag, Pb)

model	B2	B4	M1	M2	P	KM
$\chi^2(18)$	364	733	949	1232	480	449

Not fits!

Fits to 65 kaonic atoms data points when single-nucleon amplitudes are supplemented by a $B(\rho/\rho_0)^\alpha$ term. B in units of fm. Correlations between α and B cause some of the errors of B to be large.

model	B2	B4	M1	M2	P	KM
α	0.29(10)	0.24(8)	0.24(16)	0.85(13)	1.4(1.2)	1.8(8)
ReB	2.3±0.8	2.8±0.3	0.2±0.2	1.9±0.4	−1.2±0.3	0.1±1.3
ImB	0.8±0.2	1.0±0.1	0.7±0.1	1.3±0.2	2.4±3.1	3.3±1.1
$\chi^2(65)$	111	105	121	108	125	122

Similar qualities of fits are possible with purely phenomenological potentials.

For 18 high quality data points (P, S, Cl, Cu, Ag, Pb)

model	B2	B4	M1	M2	P	KM
α	0.1(5)	0.24(7)	0.6(2)	1.2(3)	0.8(1.3)	0.6(1.9)
ReB	1.1±2.8	2.6±0.3	1.6±0.2	1.9±0.9	−1.3±0.3	-0.9±0.3
ImB	0.6±0.7	0.9±0.2	0.7±0.1	1.2±0.3	1.0±0.2	1.3±0.1
$\chi^2(18)$	16	15	17	16	16	16

Fits to 65 kaonic atoms data points when single-nucleon amplitudes are supplemented by a $B(\rho/\rho_0)^\alpha$ term with fixed α compatible with its best-fit value. B in units of fm.

model	B2	B4	M1	M2	P	KM
α	0.3	0.3	0.3	1.0	1.0	1.0
ReB	2.4±0.2	3.1±0.1	0.3±0.1	2.1±0.2	−1.3±0.2	−0.9±0.2
ImB	0.8±0.1	0.8±0.1	0.8±0.1	1.2±0.2	1.5±0.2	1.4±0.2
$\chi^2(65)$	111	105	121	109	125	123

α=1 means ρ^2 dependence of the potential.

For P and KM $\alpha = 2$ may be acceptable;
P: Re$B = -0.5 \pm 0.6$, Im$B = 4.6 \pm 0.7$, $\chi^2(65)$ =125
KM: Re$B = 0.3 \pm 0.7$, Im$B = 3.8 \pm 0.7$, $\chi^2(65)$ =122

Can one accept all six models?

Fractions of multinucleon absorptions at rest

H. Davis et al., Nuovo Cimento 53 (1968) 313
J.W. Moulder et al., Nucl. Phys. B 35 (1971) 332
C. Vander Velde-Wilquet et al., Nuovo Cimento 39A (1977) 538

$$K^- + N \rightarrow Y + \pi$$
$$K^- + N + N \rightarrow Y + N$$

Fraction of multinucleon absorptions at rest

Relative capture at rest from bubble-chamber experiments:

0.26 ± 0.03 on a mixture of C, F and Br (Berkeley)
0.28 ± 0.03 on Ne (BNL)
0.19 ± 0.03 on C (CERN)
Results from nuclear emulsions quote larger uncertainties.

We therefore adopt as a best estimate of experimental K^- multinucleon absorption-at-rest fraction an average value of 0.25 ± 0.05 for C and heavier nuclei.

Apply fraction of *single*-nucleon absorptions 0.75 ± 0.05 as an additional constraint.

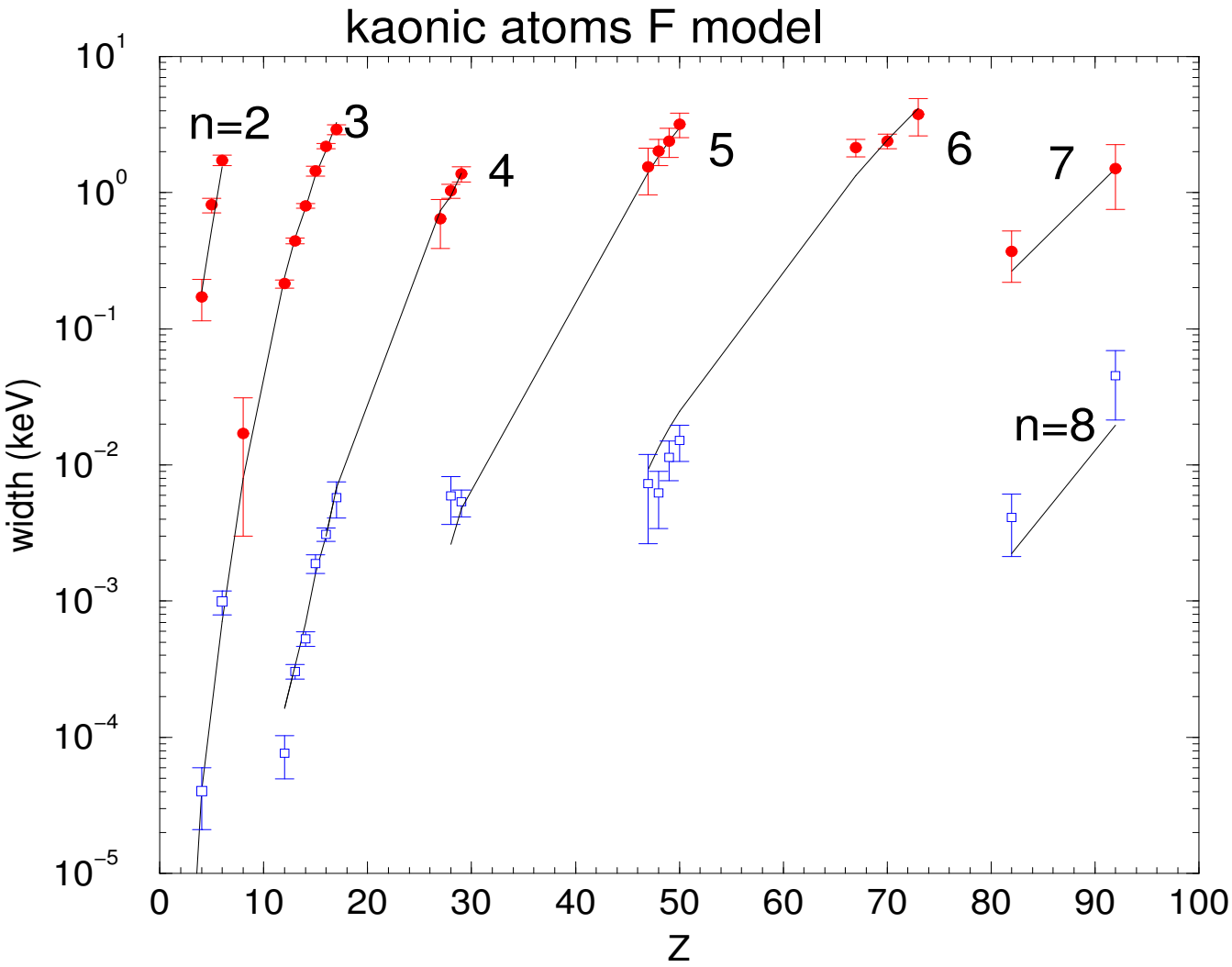
The level width Γ is obtained from the eigenvalue $E_{K^-} - i\Gamma/2$ when solving the Klein-Gordon equation with an optical potential, ($E_{K^-} = m_{K^-} - B_{K^-}$). It is also related to the imaginary part of the potential by the overlap integral of $\mathrm{Im}\, V_{K^-}$ and $|\psi|^2$,

$$\Gamma = -2 \frac{\int \mathrm{Im}\, V_{K^-} \, |\psi|^2 \, d\vec{r}}{\int [1 - (B_{K^-} + V_{\mathrm{C}})/\mu_K] \, |\psi|^2 \, d\vec{r}}$$

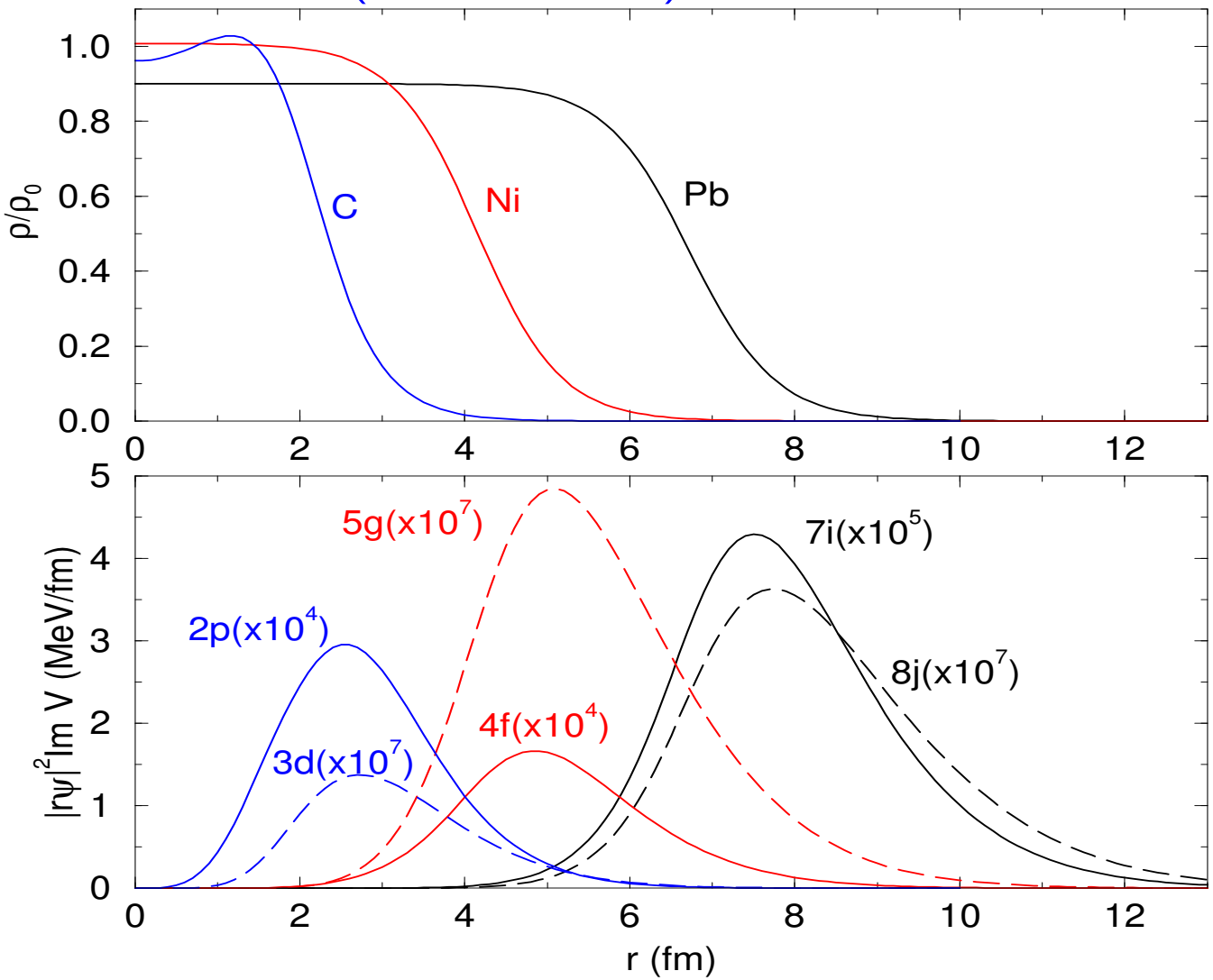
where B_{K^-}, V_{C} and μ_K are the K^- binding energy, Coulomb potential and reduced mass, respectively, and ψ is the K^- wave function of the particular state concerned.

When the *best fit* optical potential is $V^{(1)}_{K^-} + V^{(2)}_{K^-}$, the sum of a single-nucleon part and a multinucleon part, it is possible to calculate the fraction of single-nucleon absorptions, separately for any nucleus and for any specific kaonic atom state.

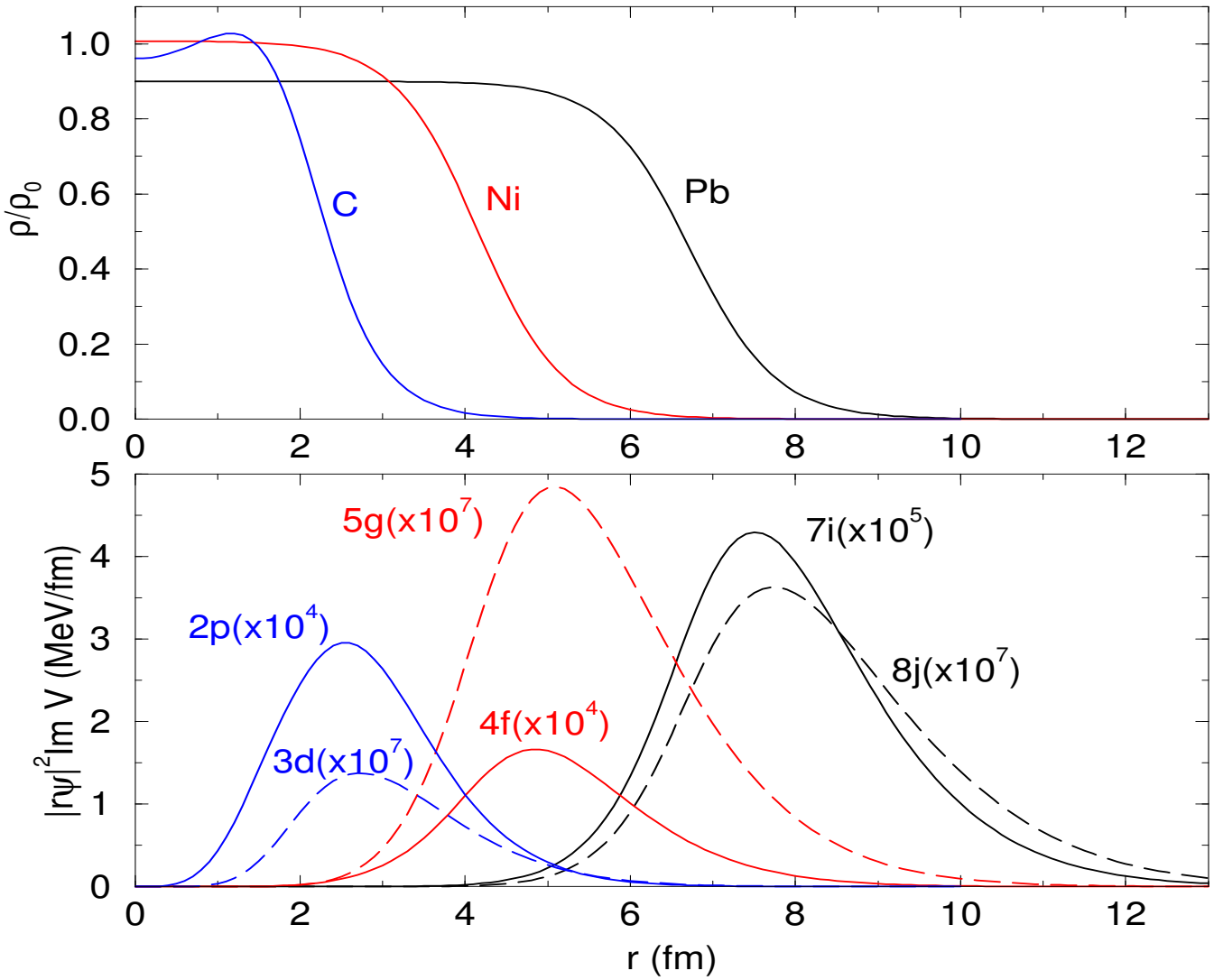
Upper and lower strong interaction widths



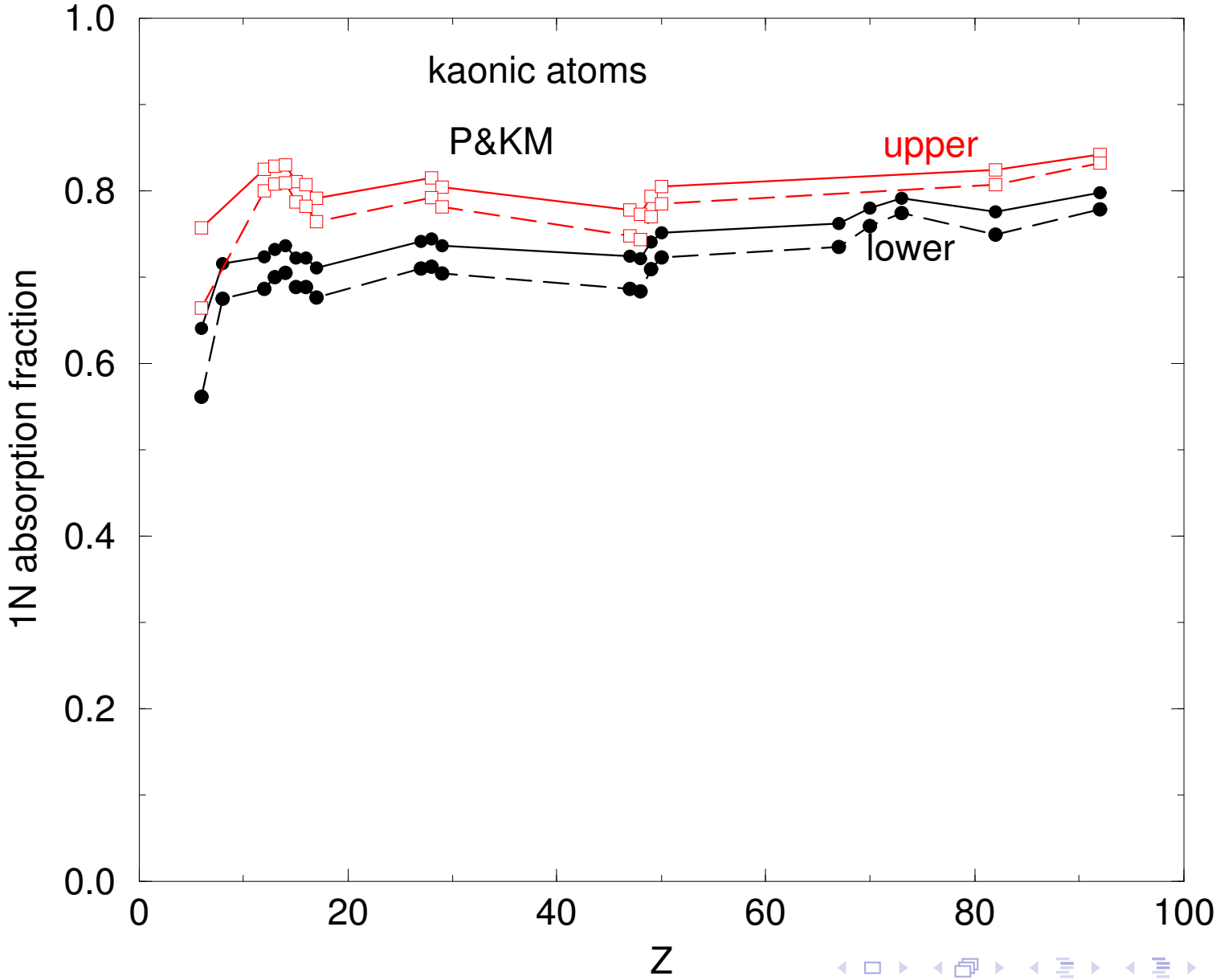
Kaonic atoms overlaps for 'lower' (solid curves) and 'upper' (dashed curve) states.



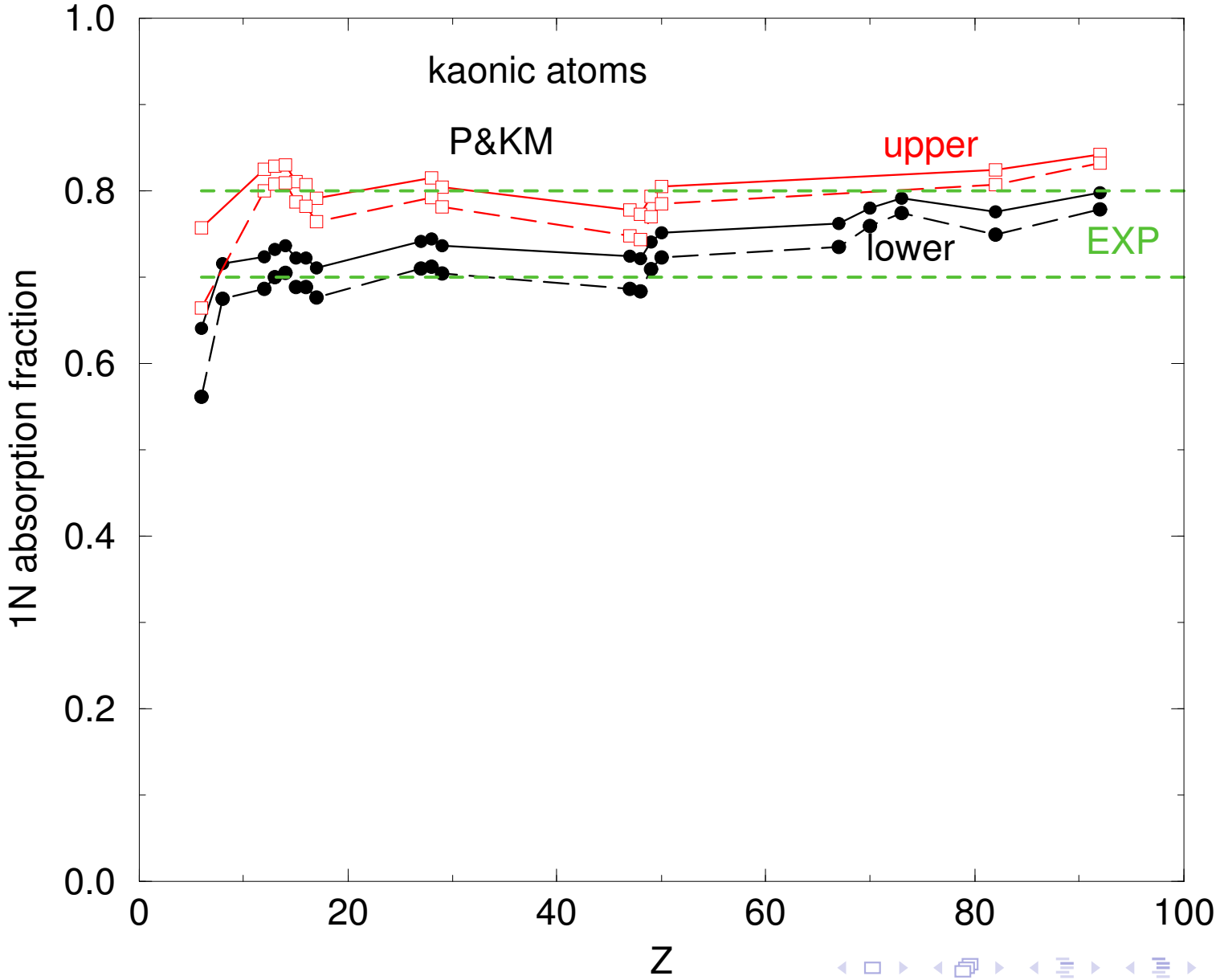
Bohr radii= 31.8 and 85.8 fm for C
31.5 and 49.3 fm for Ni
32.7 and 42.8 fm for Pb



Fraction of single-nucleon absorption for amplitudes P and KM.
Solid circles for lower states, open squares for upper states.



Fraction of single-nucleon absorption for amplitudes P and KM.
Solid circles for lower states, open squares for upper states.

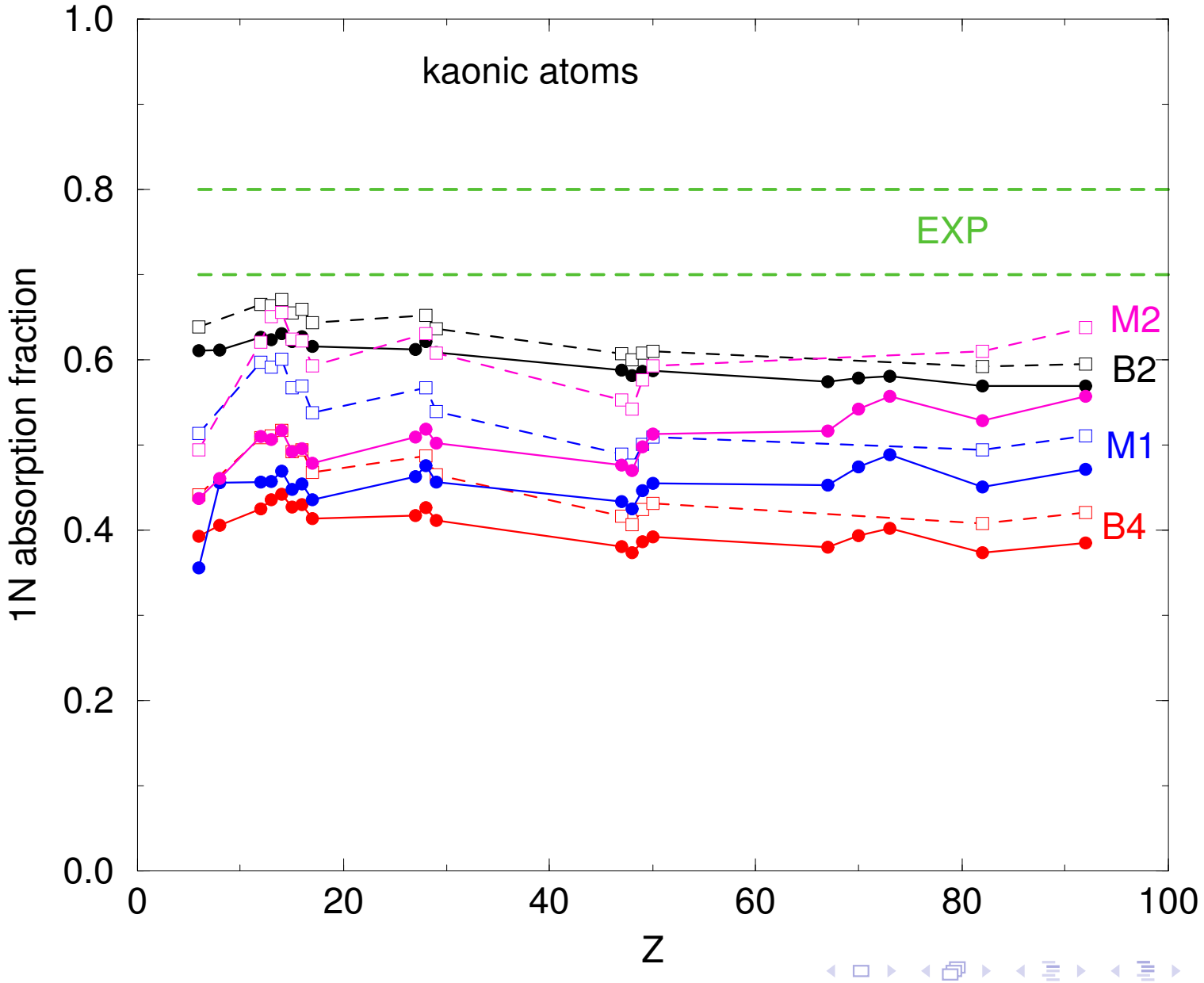


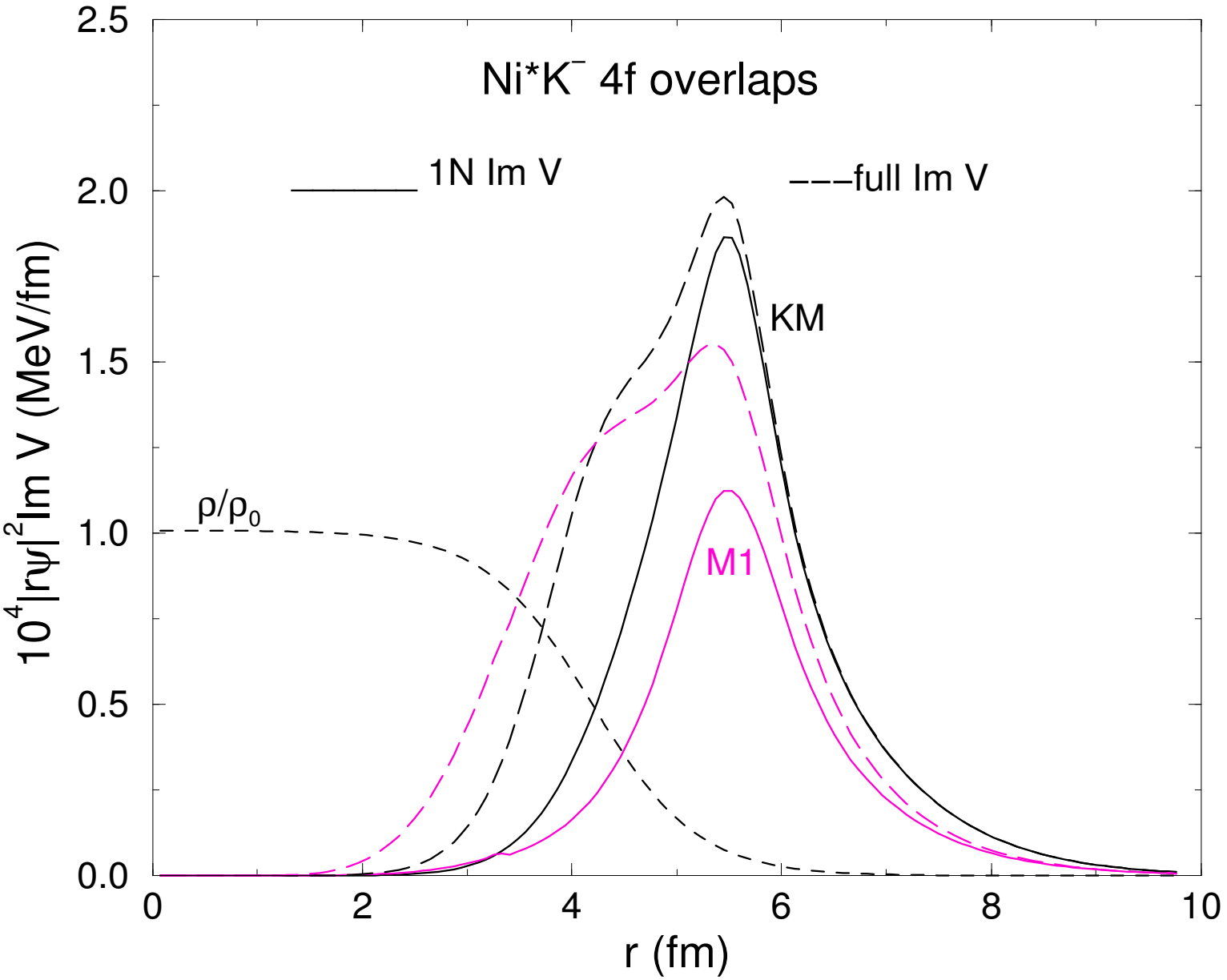
Note that for almost all species of kaonic atoms in the data base, the absorptions from the upper state are of the order of 10-15% of all absorptions, as deduced from the measured upper level to lower level radiation yields.

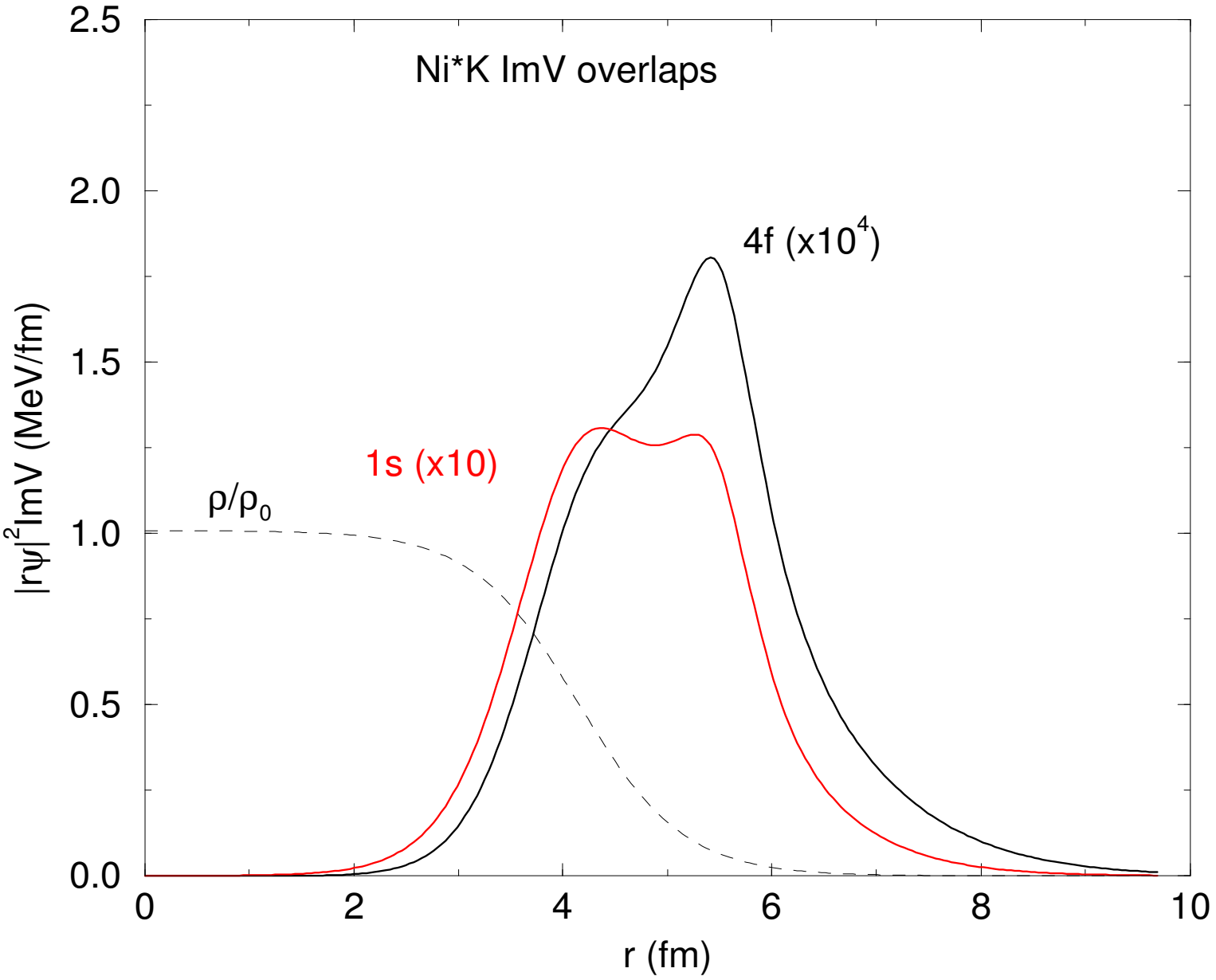
Carbon is an exception with the lowest *radiation* yield (of 0.07 ± 0.013). Atomic cascade calculations show that for C about 75% of the absorptions take place from the upper state.

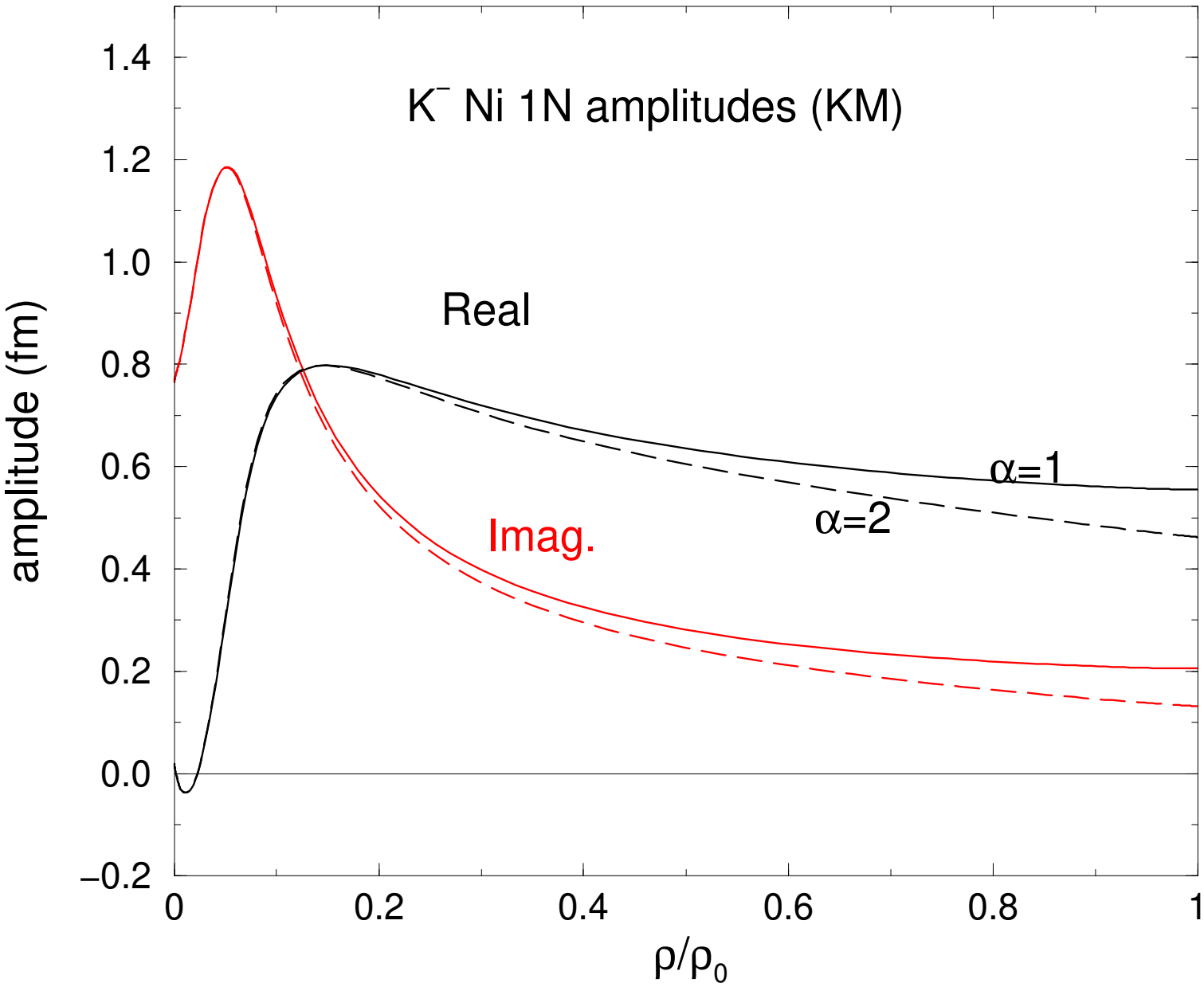
Therefore the calculated single-nucleon absorption fractions should be close to the upper level points for C and very close to the lower level points for all the other species.

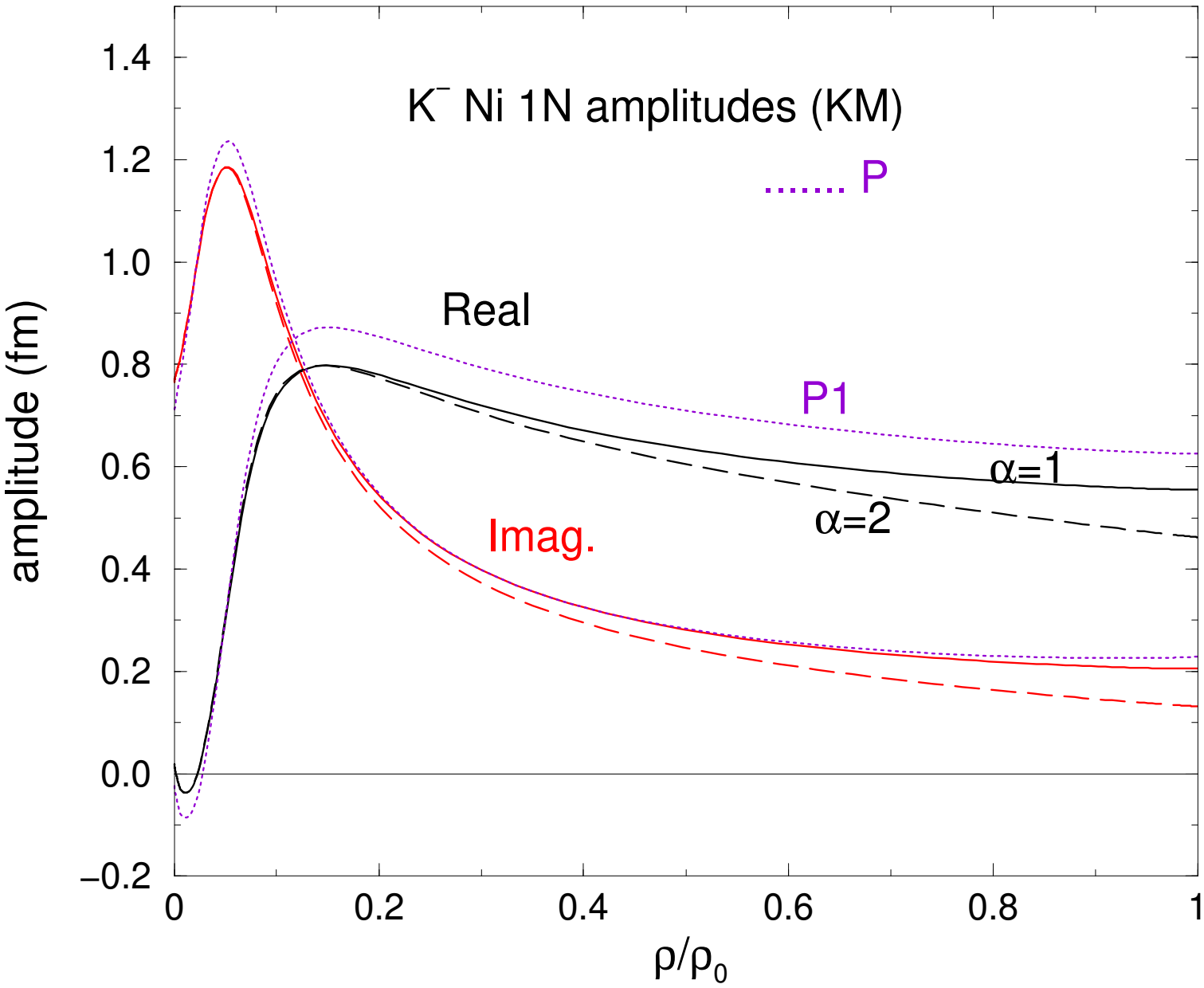
Fraction of single-nucleon absorption for amplitudes B2, B4, M1 and M2.
Solid circles for lower states, open squares for upper states.




Ni*K⁻ 4f overlaps
1N Im V
full Im V
KM
M1
ρ/ρ0
10^4|rψ|^2Im V (MeV/fm)
r (fm)
0.0
0.5
1.0
1.5
2.0
2.5
0
2
4
6
8
10


Ni*K ImV overlaps
4f (x10^4)
1s (x10)
ρ/ρ0
|rψ|^2ImV (MeV/fm)
r (fm)
0.0
0.5
1.0
1.5
2.0
2.5
0
2
4
6
8
10


K⁻ Ni 1N amplitudes (KM)
Real
Imag.
α=1
α=2
amplitude (fm)
ρ/ρ0
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
0.2
0.4
0.6
0.8
1


K⁻ Ni 1N amplitudes (KM)
..... P
Real
P1
α=1
α=2
Imag.
amplitude (fm)
ρ/ρ0
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
0.2
0.4
0.6
0.8
1

Reminder of 'subthreshold in-medium kinematics'

$$2\mu_K V^{(1)}_{K^-}(\rho) = -4\pi \left[\frac{(2\tilde{f}_{K^-p} - \tilde{f}_{K^-n})\,\frac{1}{2}\rho_p}{1+\frac{1}{4}\xi_k(\rho)\tilde{f}_0\rho(r)} + \frac{\tilde{f}_{K^-n}(\frac{1}{2}\rho_p+\rho_n)}{1+\frac{1}{4}\xi_k(\rho)\tilde{f}_1\rho(r)} \right], \quad (1)$$

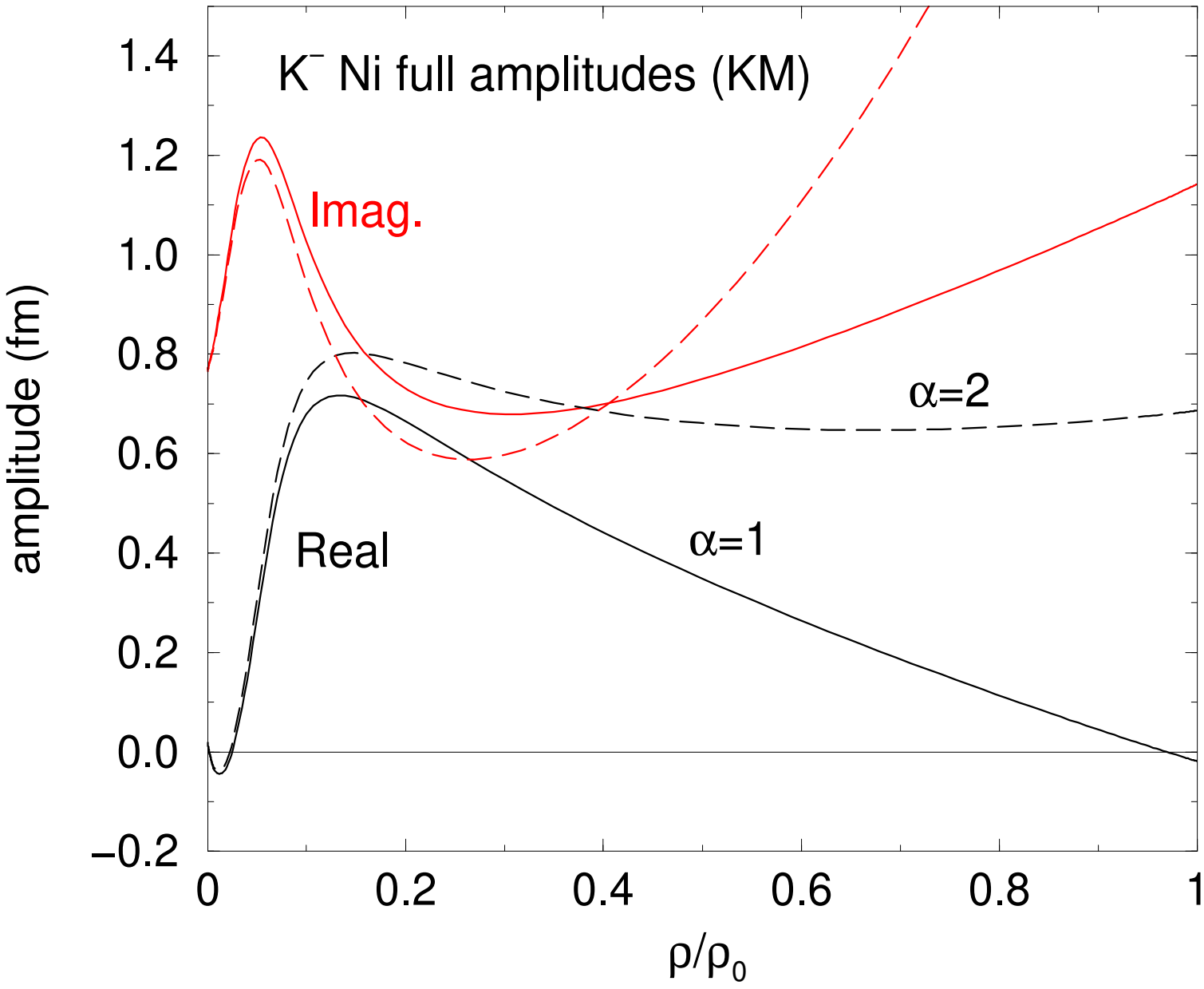
with $k = [(E_{K^-} - i\Gamma/2)^2 - m_k^2]^{1/2}$ and where Γ is the width of the particular kaonic atom state. $\tilde{f}_0$ and $\tilde{f}_1$ are the isospin 0 and 1 combinations, respectively, of the K^-N amplitudes; and ρ_p and ρ_n are proton and neutron densities, respectively.

The energy argument $\sqrt{s} = m_N + m_{K^-} + \delta\sqrt{s}$

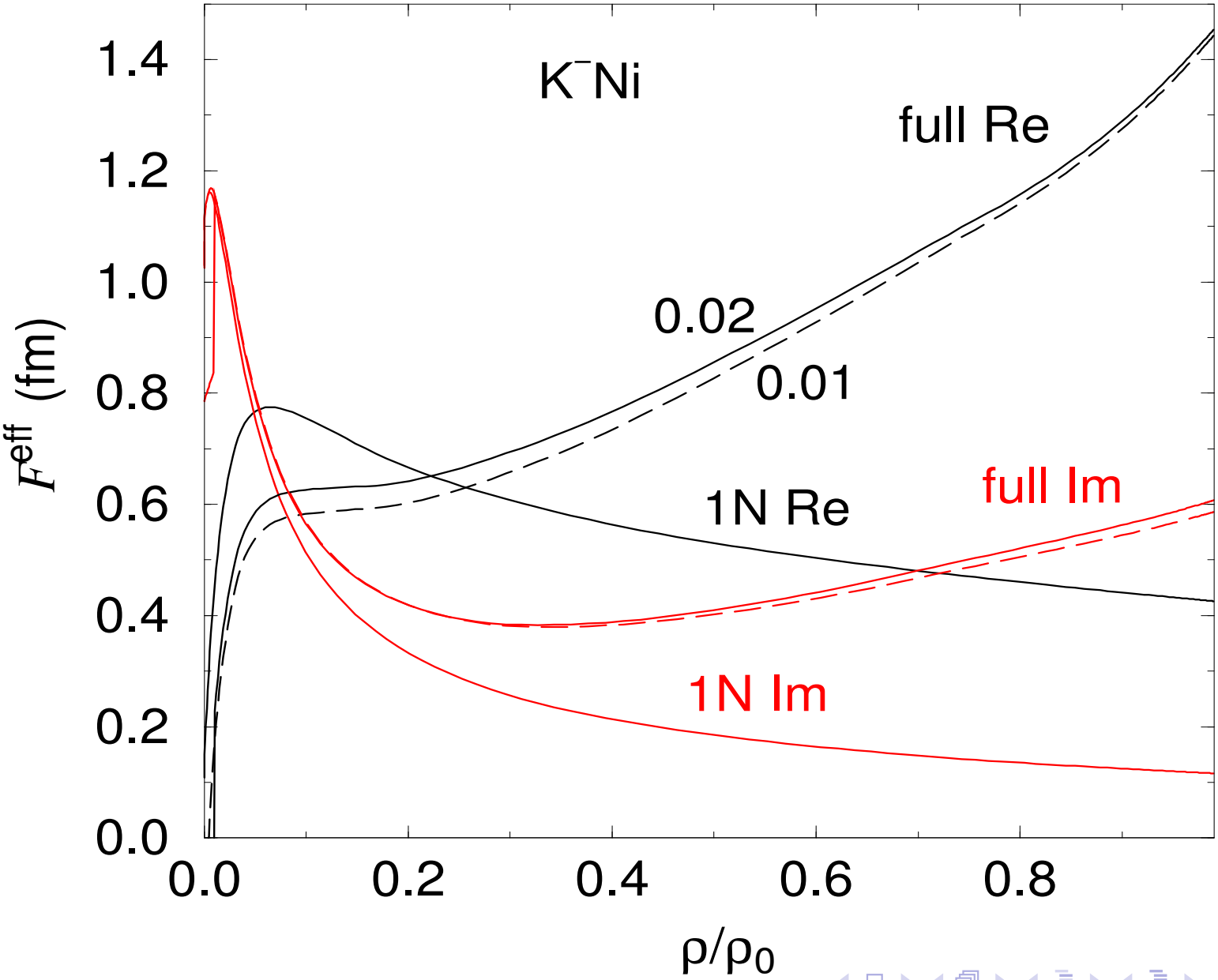
$$\delta\sqrt{s} = -B_N\rho/\bar{\rho} - \beta_N[T_N(\rho/\bar{\rho})^{2/3} + B_{K^-}\rho/\rho_0 + V_c(\rho/\rho_0)^{1/3}] + \beta_{K^-}\,\mathrm{Re}\,V_{K^-}, \quad (2)$$

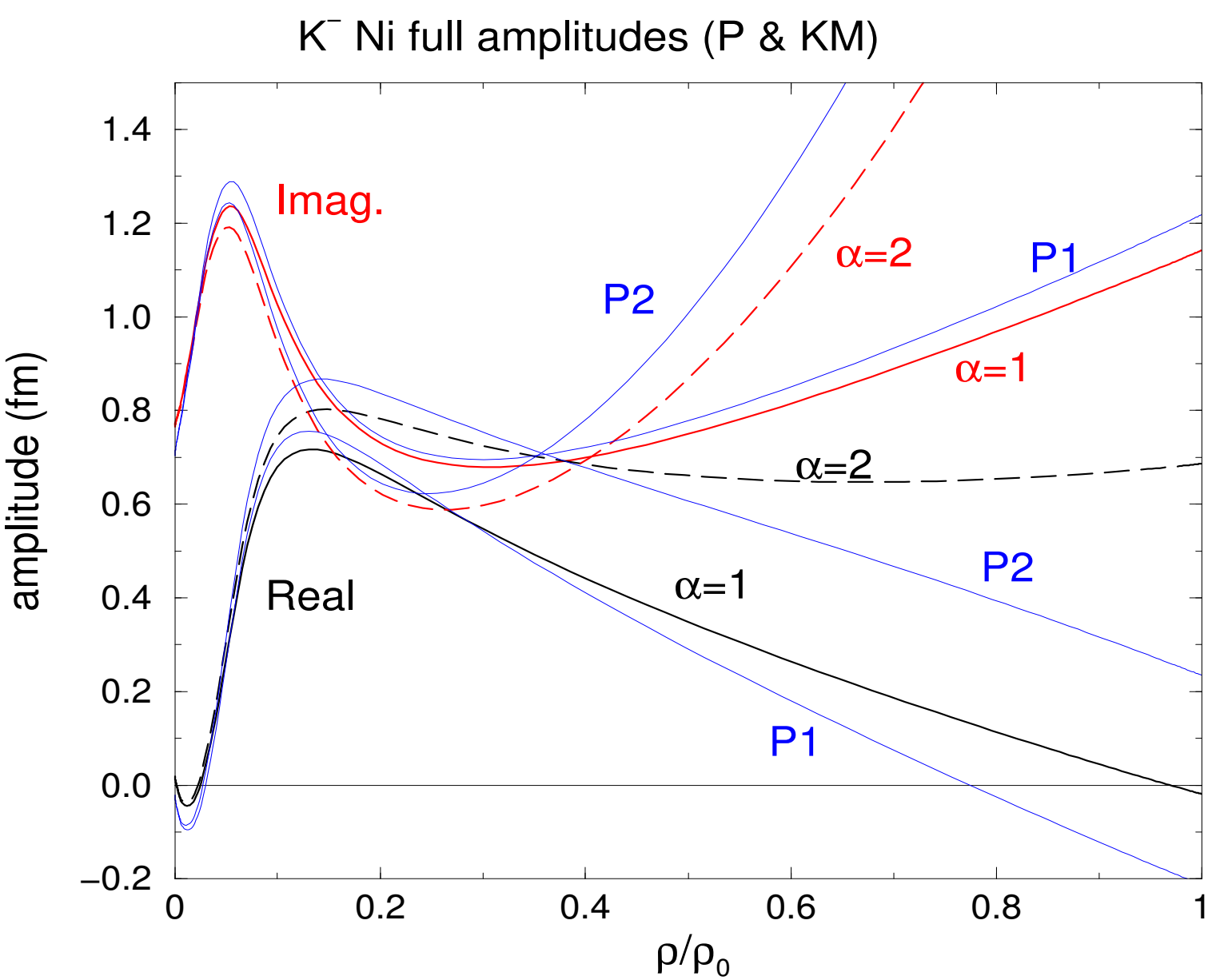
with $\beta_N = m_N/(m_N+m_{K^-})$, $\beta_{K^-} = m_{K^-}/(m_N+m_{K^-})$, and $\rho_0 = 0.17$ fm^{-3}. 'Minimal substitution' $E \to E - V_c$ included.

The expression $-(2\mu_{K^-}/4\pi)V_{K^-}(\rho)$ may be defined as the full in-medium K^-N amplitude times ρ. It varies very little with the nuclear species and the kaonic-atom state.

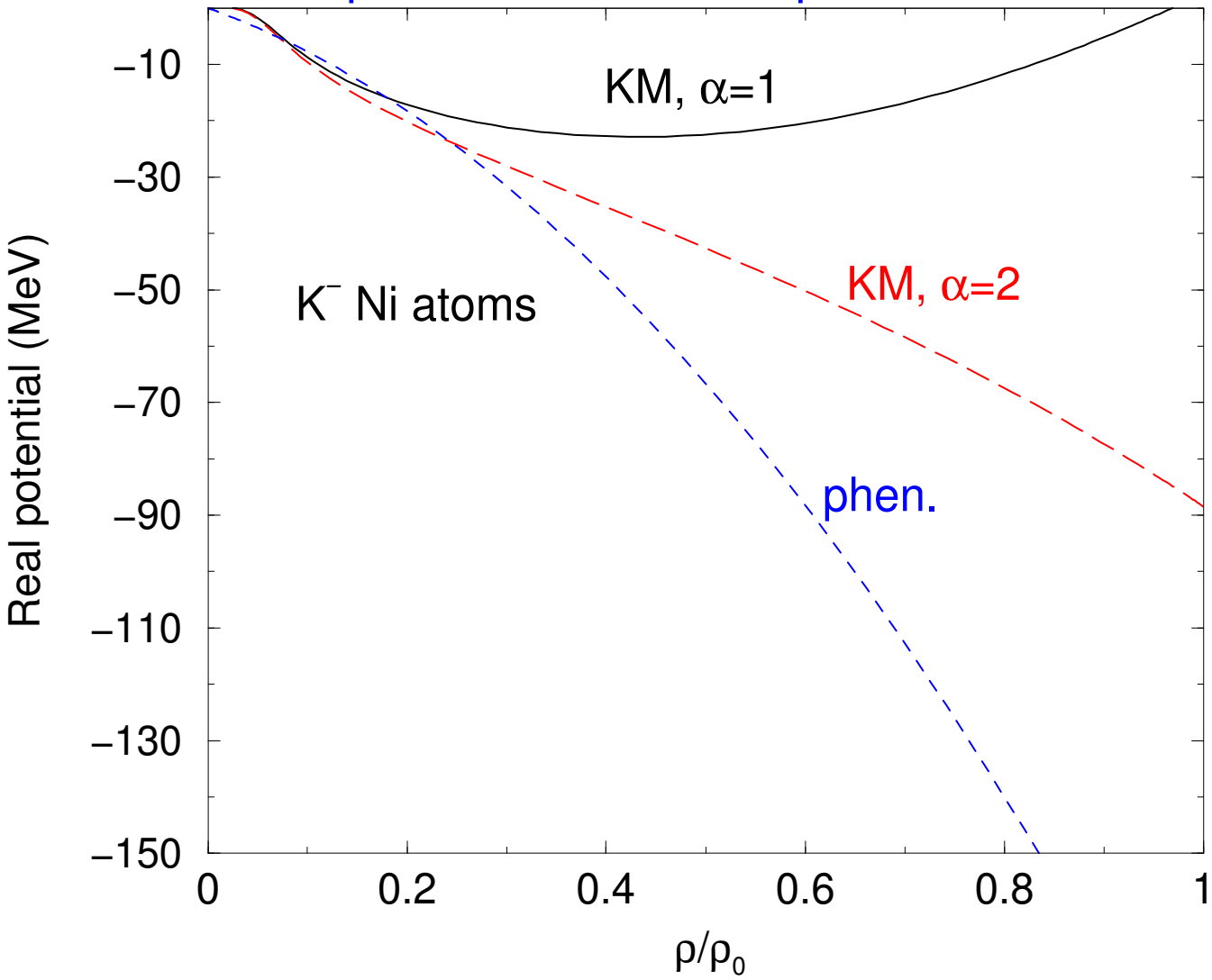

K⁻ Ni full amplitudes (KM)
Imag.
Real
α=2
α=1
amplitude (fm)
ρ/ρ0
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
0.2
0.4
0.6
0.8
1

2011 IHW basis, plus a ρ and ρ^2 phenomenological term.

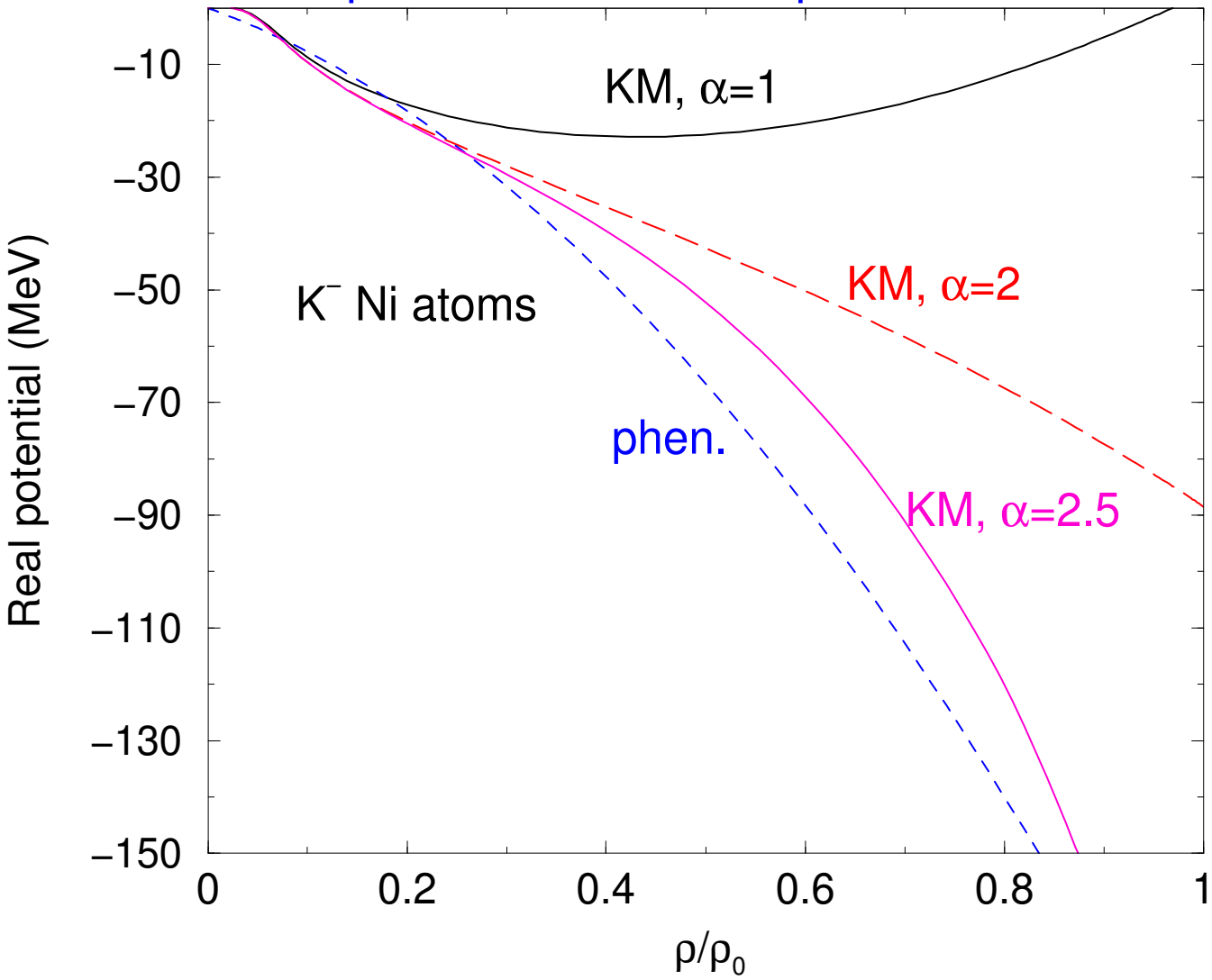



K⁻ Ni full amplitudes (P & KM)
amplitude (fm)
ρ/ρ0
Imag.
Real
α=2
α=1
α=2
α=1
P2
P1
P2
P1
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
0.2
0.4
0.6
0.8
1

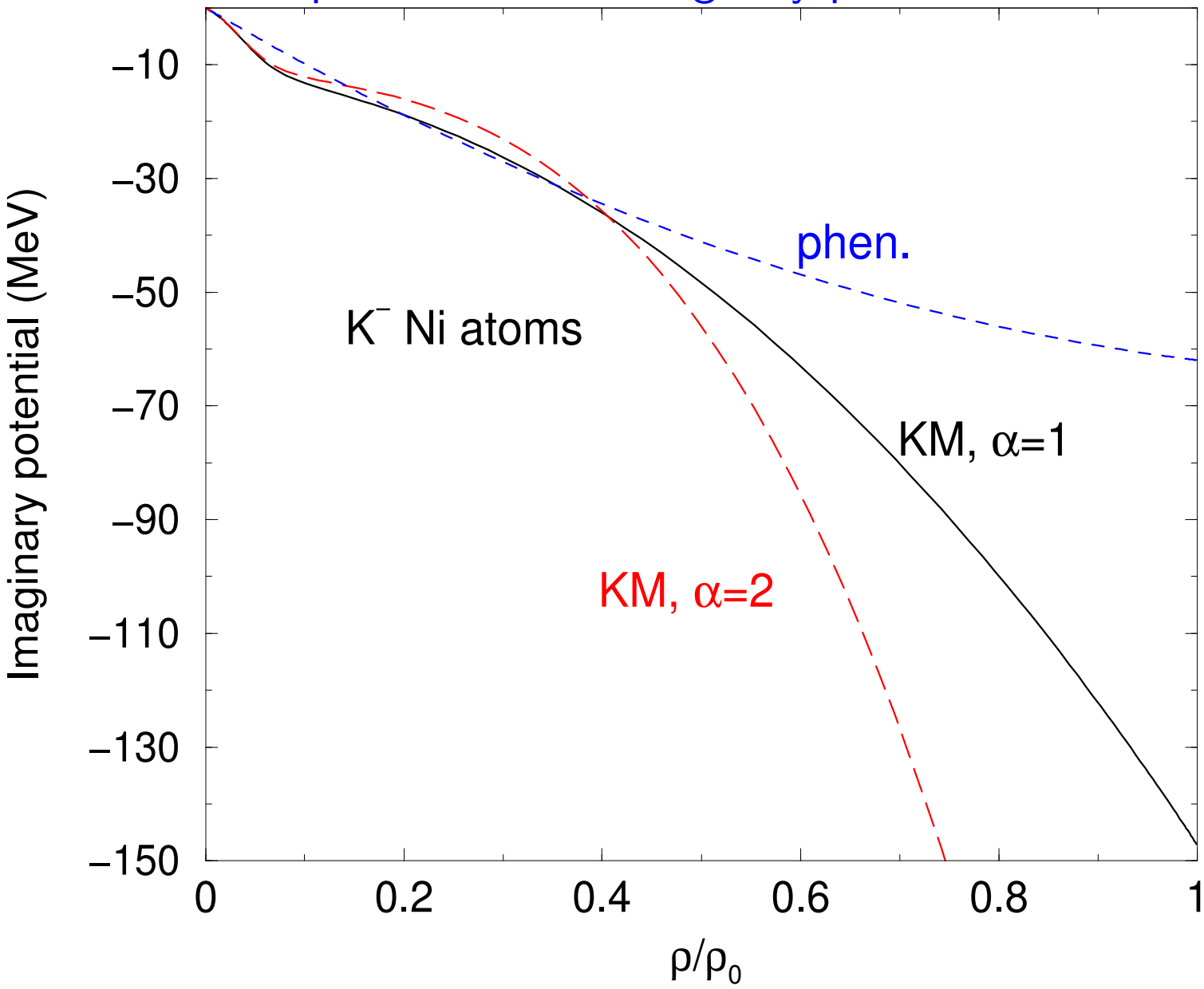
Examples for K^- Ni real potential



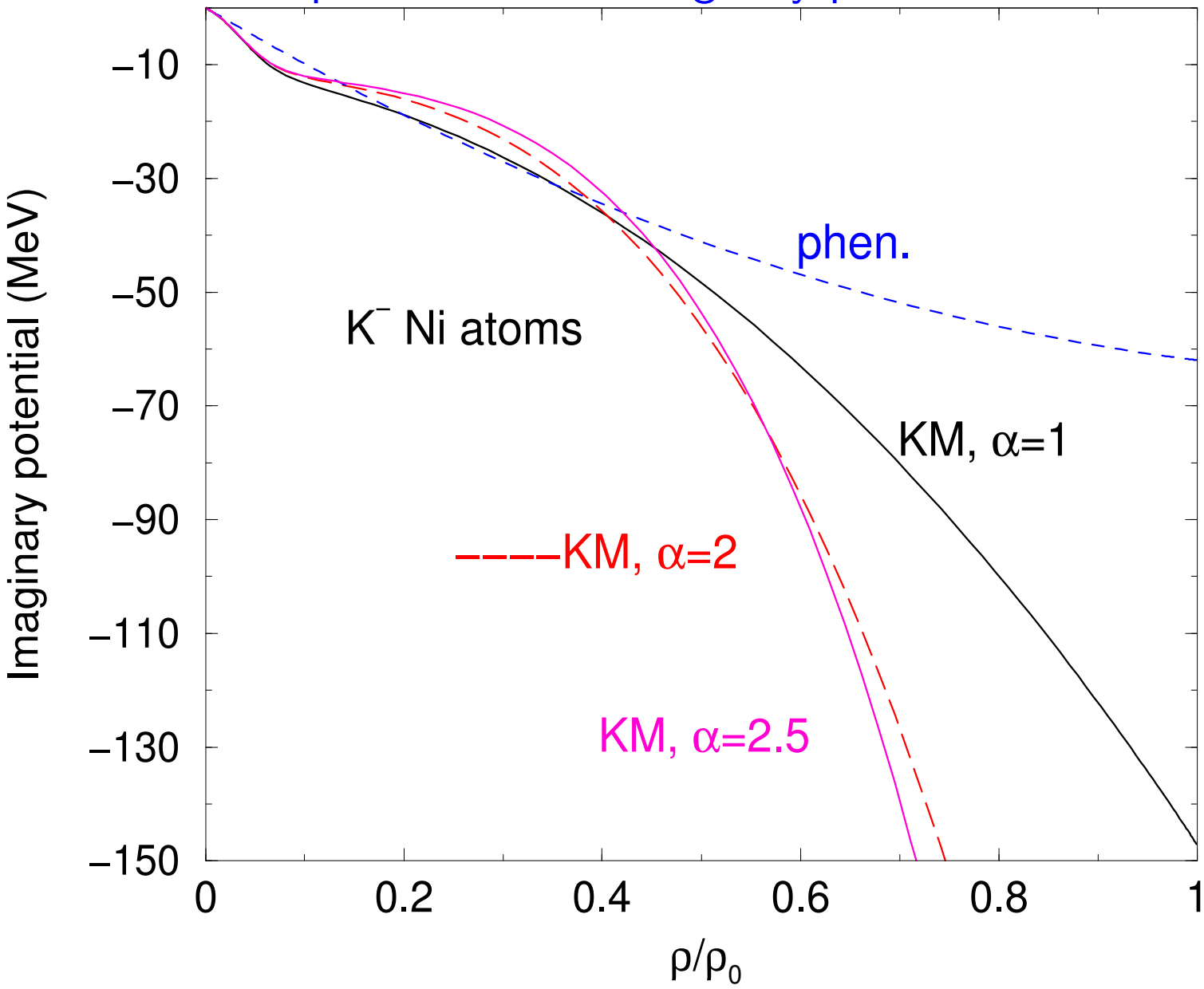
Examples for K^- Ni real potential



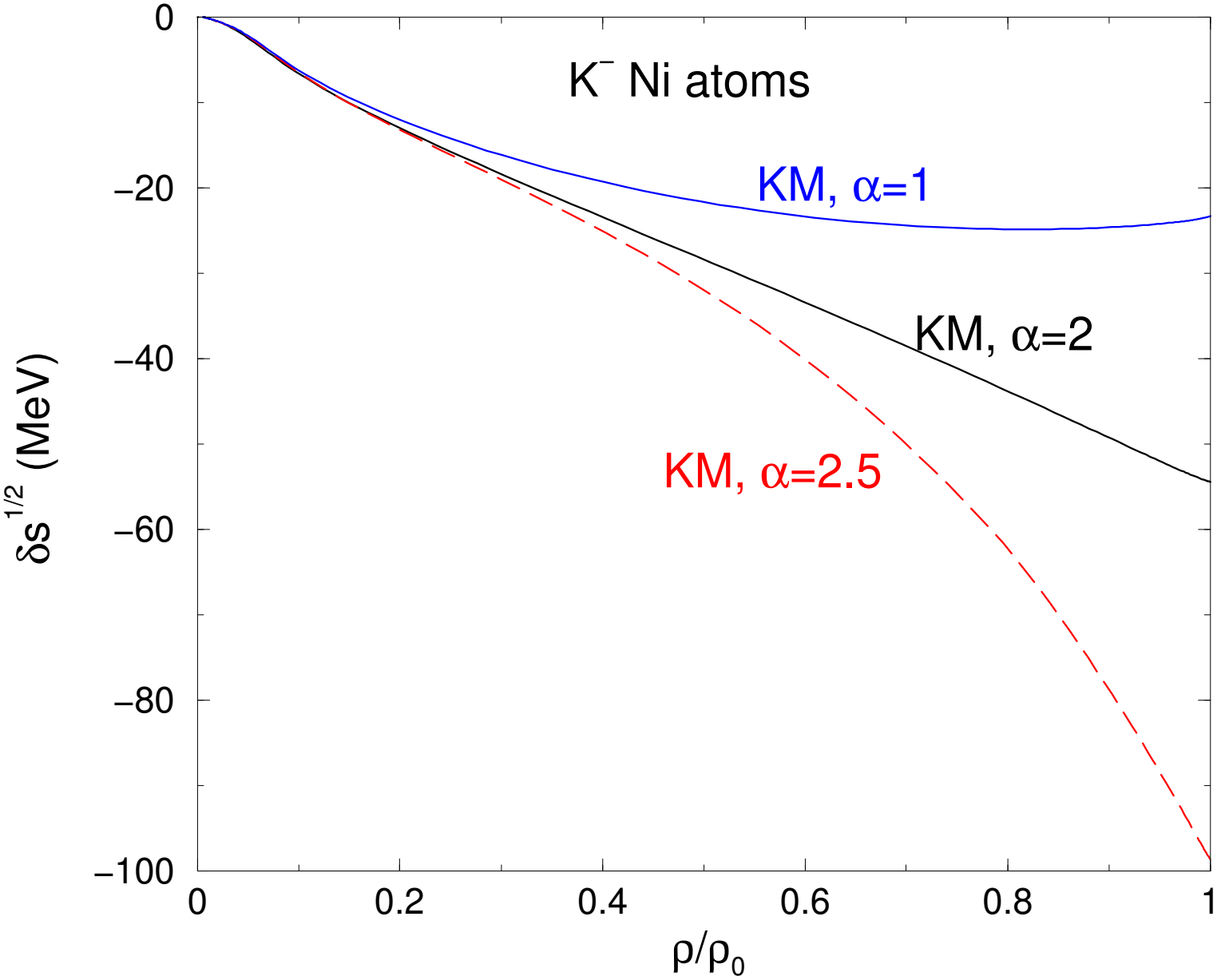
Examples for K^- Ni imaginary potential



Examples for K^- Ni imaginary potential



Strong-interaction effects in kaonic atoms are dominated by the imaginary part of the potential. Typical level widths are factor of 3-4 larger than the level shifts.


K⁻ Ni atoms
KM, α=1
KM, α=2
KM, α=2.5
δs^1/2 (MeV)
ρ/ρ0
0
−20
−40
−60
−80
−100
0
0.2
0.4
0.6
0.8
1

Summary

- Fractions of single-nucleon absorption favor the P and the KM models. (B2 and B4 may require further attention).
- All six models predict that these fractions depend very little on nuclear species and atomic state.
- Real potential known only up to 25% of central density.
- Imaginary potential known up to 50% of central density.
- Relevant energies: threshold to 30 MeV below (Minimal substitution included. Without MS χ^2 increases by 5 units.)
- Models for multi-nucleon interactions must fit kaonic atoms data and be tested against the above conclusions.

We thank Aleš Cieplý for providing us with K^-N amplitudes in tabulated form.

Thank you for your attention!

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