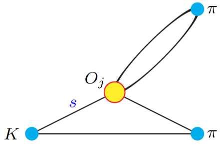
Review of lattice results for low energy particle physics

Urs Wenger
Albert Einstein Center for Fundamental Physics
University of Bern

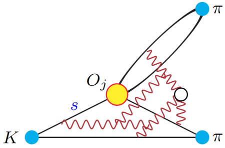
EXA 2017, 13/09/2017, Vienna, Austria

The role of (precision) flavour physics

- (Precision) Flavour physics is a key tool in exploring the Standard Model of particle physics.
- It is complementary to high-energy precision experiments
  - necessary to understand the underlying theoretical framework
  - important for the discovery of new physics beyond the SM
- Precision flavour physics requires control of hadronic effects:

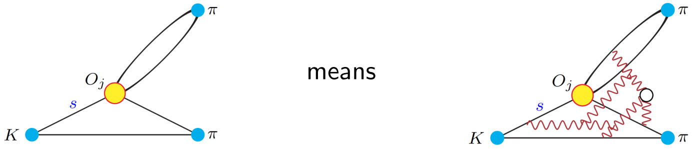


means



The role of (precision) flavour physics

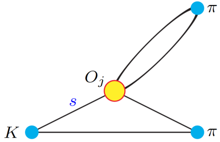
- (Precision) Flavour physics is a key tool in exploring the Standard Model of particle physics.
- It is complementary to high-energy precision experiments
  - necessary to understand the underlying theoretical framework
  - important for the discovery of new physics beyond the SM
- Precision flavour physics requires control of hadronic effects:



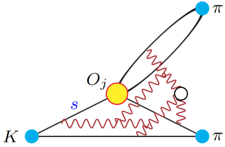
- Systematic, theoretical tools include χPT, sum rules, dispersion relations, lattice QCD

The role of (precision) flavour physics

- (Precision) Flavour physics is a key tool in exploring the Standard Model of particle physics.
- It is complementary to high-energy precision experiments
  - necessary to understand the underlying theoretical framework
  - important for the discovery of new physics beyond the SM
- Precision flavour physics requires control of hadronic effects:



means



Lattice QCD is *the* nonperturbative method
for hadronic ab-initio calculations

Quantum Chromodynamics

QCD – the theory of strong interactions

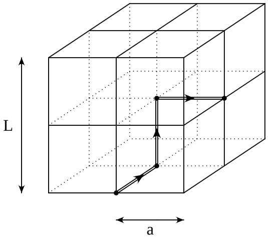
$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}(i\not{D} - m_q)\psi - \frac{1}{4} G_{\mu\nu} G^{\mu\nu}$$

- describes the interactions between the quarks and gluons,
- parameters are the quark masses m_q and the dimensionless gauge coupling α_s,
- in the chiral limit $m_q \to 0$, a scale is generated through *dimensional transmutation*,
- all dimensionful quantities can be expressed in units of *one characteristic scale*, e.g. the proton mass.

QCD on the lattice

Gauge invariant lattice regularization:

- discretize Euclidean space-time



Challenges for QCD on the lattice:

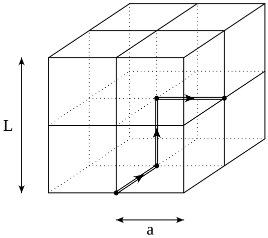
- Thermodynamic limit $\Rightarrow$ $L \to \infty$
- Continuum limit $\Rightarrow$ $a \to 0$
  - discretisation errors become negligible
- Chiral limit $\Rightarrow$ $m_q \to m_q^{\text{phys}} \sim 0$

We need at least $L > 3$ fm, $a < 0.1$ fm and $m_\pi < 200$ MeV.

QCD on the lattice

Gauge invariant lattice regularization:

- discretize Euclidean space-time



Challenges for QCD on the lattice:

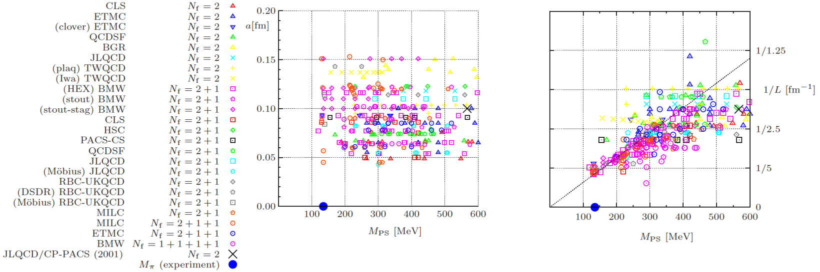
- Thermodynamic limit ⇒ $L \to \infty$
- Continuum limit ⇒ $a \to 0$
  - discretisation errors become negligible
- Chiral limit ⇒ $m_q \to m_q^{\text{phys}} \sim 0$

We need at least $L > 3$ fm, $a < 0.1$ fm and $m_\pi < 200$ MeV.

- Isospin breaking and electro-magnetic (QED) corrections

Huge progress over the last decade. . .

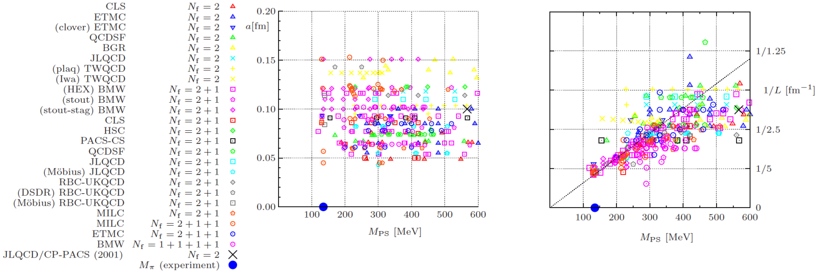
. . . thanks to algorithmic and theoretical breakthroughs & increase in computational power:



[courtesy of G. Herdoiza]

Huge progress over the last decade. . .

. . . thanks to algorithmic and theoretical breakthroughs & increase in computational power:



[courtesy of G. Herdoiza]

It's not easy for a non-expert to keep the overview:

- what is the 'best' current lattice value for a given quantity?

For phenomenological applications

- What is the 'best' current lattice value?
  - digging through lattice literature not easy for non-experts
  - solution: compilation of results ready-to-use
  - examples: PDG, HFAG, etc.

FLAG 2016
Flavour Lattice Averaging Group

Collaboration	Ref.	N_f	publication status	renormalization scale	perturbative behaviour	continuum extrapolation	$\alpha_{\overline{\rm MS}}(M_Z)$	Method	Table
HPQCD 14A	[5]	2+1+1	A	○	★	○	0.11822(74)	current two points	46
ETM 13D	[648]	2+1+1	A	○	○	■	0.1196(4)(8)(16)	gluon-ghost vertex	47
ETM 12C	[649]	2+1+1	A	○	○	■	0.1200(14)	gluon-ghost vertex	47
ETM 11D	[650]	2+1+1	A	○	○	■	$0.1198(9)(5)(^{+0}_{-5})$	gluon-ghost vertex	47
Bazavov 14	[61]	2+1	A	○	★	○	$0.1166(^{+12}_{-8})$	Q-$\bar{Q}$ potential	43
Bazavov 12	[603]	2+1	A	○	○	○	$0.1156(^{+21}_{-22})$	Q-$\bar{Q}$ potential	43
HPQCD 10	[9]	2+1	A	○	★	○	0.1183(7)	current two points	46
HPQCD 10	[9]	2+1	A	○	★	★	0.1184(6)	Wilson loops	45
JLQCD 10	[612]	2+1	A	■	■	■	$0.1118(3)(^{+16}_{-17})$	vacuum polarization	44
PACS-CS 09A	[62]	2+1	A	★	★	○	$0.118(3)^{\#}$	Schrödinger functional	42
Maltman 08	[63]	2+1	A	○	○	★	0.1192(11)	Wilson loops	45
HPQCD 08B	[152]	2+1	A	■	■	■	0.1174(12)	current two points	46
HPQCD 08A	[616]	2+1	A	○	★	★	0.1183(8)	Wilson loops	45
HPQCD 05A	[615]	2+1	A	○	○	○	0.1170(12)	Wilson loops	45

$^{\#}$ Result with a linear continuum extrapolation in a.

What is FLAG?

FLAG 2016 — Flavour Lattice Averaging Group

FLAG: Flavour Lattice Averaging Group

- Worldwide collaboration to provide answers to
  - What is the current best lattice value for quantity X?
  - How reliable is the estimated systematic error?
- Collection of all results in a user-friendly format:

Review of lattice results concerning low-energy particle physics

August 1, 2017

Flavour Lattice Averaging Group (FLAG)

S. Aoki,[1] Y. Aoki,[2,3*] D. Bečirević,[4] C. Bernard,[5] T. Blum,[6,3] G. Colangelo,[7] M. Della Morte,[8]
P. Dimopoulos,[9] S. Dürr,[10] H. Fukaya,[11] M. Golterman,[12] Steven Gottlieb,[13] S. Hashimoto,[14,15]
U. M. Heller,[16] R. Horsley,[17] A. Jüttner,[18] T. Kaneko,[14,15] L. Lellouch,[19] H. Leutwyler,[7]
C.-J. D. Lin,[20,19] V. Lubicz,[21,22] E. Lunghi,[13] R. Mawhinney,[23] T. Onogi,[11] C. Pena,[24]
C. T. Sachrajda,[18] S. R. Sharpe,[25] S. Simula,[22] R. Sommer,[26] A. Vladikas,[27] U. Wenger,[7] H. Wittig[28]

What is FLAG?

FLAG: Flavour **L**attice **A**veraging **G**roup

- Worldwide collaboration to provide answers to
  - What is the current best lattice value for quantity X?
  - How reliable is the estimated systematic error?
- Collection of all results in a user-friendly format:

…view of lattice results concerning low-energy particle physics

August 1, 2017

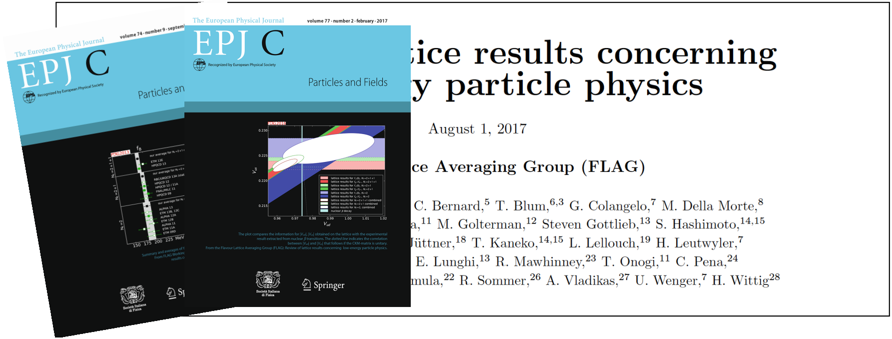
Flavour Lattice Averaging Group (FLAG)

…Aoki,[2,3*] D. Bečirević,[4] C. Bernard,[5] T. Blum,[6,3] G. Colangelo,[7] M. Della Morte,[8]
…os,[9] S. Dürr,[10] H. Fukaya,[11] M. Golterman,[12] Steven Gottlieb,[13] S. Hashimoto,[14,15]
…ler,[16] R. Horsley,[17] A. Jüttner,[18] T. Kaneko,[14,15] L. Lellouch,[19] H. Leutwyler,[7]
…in,[20,19] V. Lubicz,[21,22] E. Lunghi,[13] R. Mawhinney,[23] T. Onogi,[11] C. Pena,[24]
…[18] S. R. Sharpe,[25] S. Simula,[22] R. Sommer,[26] A. Vladikas,[27] U. Wenger,[7] H. Wittig[28]

What is FLAG?

FLAG: Flavour Lattice Averaging Group

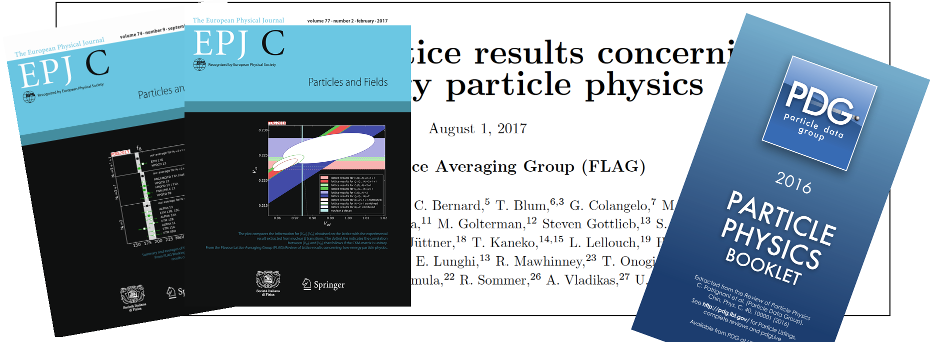
- Worldwide collaboration to provide answers to
  - What is the current best lattice value for quantity X?
  - How reliable is the estimated systematic error?
- Collection of all results in a user-friendly format:



What is FLAG?

FLAG: Flavour **L**attice **A**veraging **G**roup

- Worldwide collaboration to provide answers to
  - What is the current best lattice value for quantity X?
  - How reliable is the estimated systematic error?
- Collection of all results in a user-friendly format:



Similar to the efforts of the PDG...

What is FLAG?

- Advisory Board:

  S. Aoki, C. Bernard, M. Golterman, H. Leutwyler, C. Sachrajda

- Editorial Board:

  G. Colangelo, S. Hashimoto, A. Jüttner, S. Sharpe, A. Vladikas, UW

- Working Groups:
  - Quark masses: L. Lellouch, T. Blum, V. Lubicz
  - V_{us}, V_{ud}: T. Kaneko, S. Simula, (P. Boyle)
  - LEC: S. Dürr, H. Fukaya, U. Heller
  - B_K: H. Wittig, P. Dimopoulos, B. Mawhinney
  - α_s: R. Sommer, R. Horsley, T. Onogi
  - f_B, f_D, B_B: Y. Aoki, M. Della Morte, D. Lin
  - $B, D \to H\ell\nu$: D. Becirevic, S. Gottlieb, E. Lunghi, C. Pena

- 3rd ed. of review appeared July 2016 [EPJC **77** (2017) 2, arXiv:1607.00299]

What is FLAG?

- Advisory Board:

  S. Aoki, C. Bernard, M. Golterman, H. Leutwyler, C. Sachrajda

- Editorial Board:

  G. Colangelo, S. Hashimoto, A. Jüttner, S. Sharpe, A. Vladikas, UW

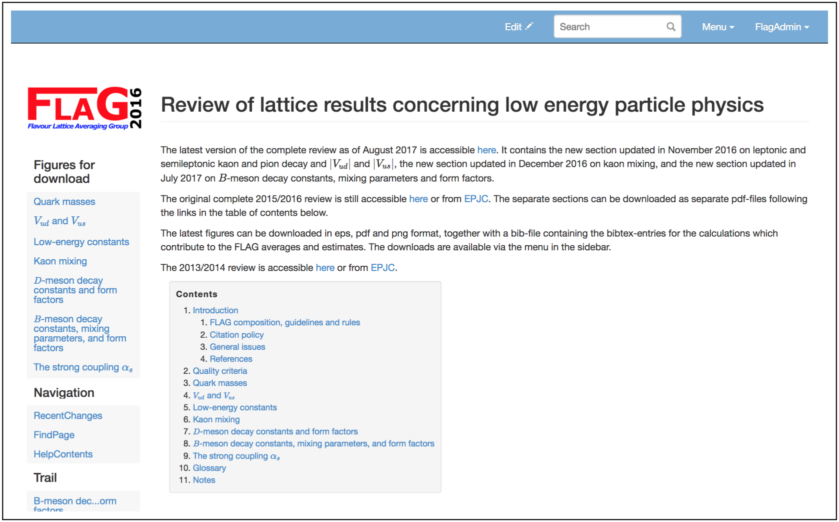
- Working Groups:
  - Quark masses: L. Lellouch, T. Blum, V. Lubicz
  - V_{us}, V_{ud}: T. Kaneko, S. Simula, (P. Boyle)
  - LEC: S. Dürr, H. Fukaya, U. Heller
  - B_K: H. Wittig, P. Dimopoulos, B. Mawhinney
  - α_s: R. Sommer, R. Horsley, T. Onogi
  - f_B, f_D, B_B: Y. Aoki, M. Della Morte, D. Lin
  - $B, D \to H\ell\nu$: D. Becirevic, S. Gottlieb, E. Lunghi, C. Pena

- 3rd ed. of review appeared July 2016 [EPJC **77** (2017) 2, arXiv:1607.00299]

  $\to$ partly updated as of July 2017 [http://flag.unibe.ch]

What is FLAG?

Most recent updates available under http://flag.unibe.ch:


Edit Search Menu FlagAdmin

FLAG 2016 Flavour Lattice Averaging Group

Figures for download

Quark masses

V_{ud} and V_{us}

Low-energy constants

Kaon mixing

D-meson decay constants and form factors

B-meson decay constants, mixing parameters, and form factors

The strong coupling α_s

Navigation

RecentChanges

FindPage

HelpContents

Trail

B-meson dec...orm factors

Review of lattice results concerning low energy particle physics

The latest version of the complete review as of August 2017 is accessible here. It contains the new section updated in November 2016 on leptonic and semileptonic kaon and pion decay and $|V_{ud}|$ and $|V_{us}|$, the new section updated in December 2016 on kaon mixing, and the new section updated in July 2017 on B-meson decay constants, mixing parameters and form factors.

The original complete 2015/2016 review is still accessible here or from EPJC. The separate sections can be downloaded as separate pdf-files following the links in the table of contents below.

The latest figures can be downloaded in eps, pdf and png format, together with a bib-file containing the bibtex-entries for the calculations which contribute to the FLAG averages and estimates. The downloads are available via the menu in the sidebar.

The 2013/2014 review is accessible here or from EPJC.

Contents

What exactly does FLAG offer?

- Complete list of references
- Summary of relevant formulae and notation
- Summary of essential aspects of each calculation:
  - lattice action and number of dynamical quarks (N_f)
  - minimal value and range of quark masses
  - minimal value and range of lattice spacings
  - maximal value and range of lattice volumes
  - renormalization method (where applicable)

  in a unified and easy to read manner (color code)
- Averages or estimates (if sensible)
- Lattice dictionary for non-experts (details of lattice actions, etc.)

What exactly does FLAG offer?

Some original contributions:

- thorough discussion and parametrization of electromagnetic contributions to meson masses
  (and their role in the determination of quark masses)

- some new χPT two-loop formulae
  (either completely new or written in a user-friendly way)

- a thorough consistency test of lattice calculations of $f_+(0)$ and f_K/f_π assuming unitarity of the CKM matrix

In the future: discussion of isospin breaking/e.m. corrections where necessary

FLAG colour codes, averages and estimates

- Quality criteria rate *possibility* to control a systematic error:
  - ★ data allow for satisfactory control
  - ○ data allow for reasonable control, but could be improved
  - ■ unlikely to allow for reasonable control (result is dropped!)

FLAG colour codes, averages and estimates

- Chiral extrapolation:
  - ★ $M_{\pi,\mathrm{min}} < 200$ MeV
  - ○ 200 MeV $\leq M_{\pi,\mathrm{min}} \leq$ 400 MeV
  - ■ 400 MeV $< M_{\pi,\mathrm{min}}$

FLAG colour codes, averages and estimates

- Continuum extrapolation:
  - ★ 3 or more lattice spacings, at least 2 below 0.1 fm
  - ○ 2 or more lattice spacings, at least 1 below 0.1 fm
  - ■ otherwise

FLAG colour codes, averages and estimates

- Finite-volume effects:
  - ★ $M_{\pi,\mathrm{min}} L > 4$ or at least 3 volumes
  - ○ $M_{\pi,\mathrm{min}} L > 3$ and at least 2 volumes
  - ■ otherwise

FLAG colour codes, averages and estimates

- Renormalization (where applicable):
  - ★ non-perturbative
  - ○ 1-loop perturbation theory or higher
  - ■ otherwise

FLAG colour codes, averages and estimates

- Quality criteria rate *possibility* to control a systematic error:
  - ★ data allow for satisfactory control
  - ○ data allow for reasonable control, but could be improved
  - ■ unlikely to allow for reasonable control (result is dropped!)

- Different lattice results will be averaged, if and only if
  - no red tags
  - published [lattice proceedings not enough]
  - same number of flavours N_f

    [no average of $N_f = 2$, $2+1$ and $2+1+1$ calculations]

FLAG colour codes, averages and estimates

- Quality criteria rate *possibility* to control a systematic error:
  - ★ data allow for satisfactory control
  - ○ data allow for reasonable control, but could be improved
  - ■ unlikely to allow for reasonable control (result is dropped!)
- Different lattice results will be averaged, if and only if
  - no red tags
  - published [lattice proceedings not enough]
  - same number of flavours N_f [no average of $N_f = 2$, $2+1$ and $2+1+1$ calculations]
- Estimate is given if average fails to cover all uncertainties

FLAG colour codes, averages and estimates

- Quality criteria rate *possibility* to control a systematic error:
  - ★ data allow for satisfactory control
  - ○ data allow for reasonable control, but could be improved
  - ■ unlikely to allow for reasonable control (result is dropped!)

- Different lattice results will be averaged, if and only if
  - no red tags
  - published [lattice proceedings not enough]
  - same number of flavours N_f

    [no average of $N_f = 2$, $2+1$ and $2+1+1$ calculations]

- Estimate is given if average fails to cover all uncertainties

- Colour code in figures:
  - ■ results included in the average or estimate
  - □ results not included in the average but passing all criteria
  - □ all other results
  - ■ FLAG average or estimate

The connection between the lattice and the continuum

- lattice QCD contains only the dimensionless coupling g and implicitly the lattice spacing a as parameters
- for a physical mass m or a length ξ one has

$$m = f(g) \cdot \frac{1}{a} \qquad \xi = h(g) \cdot a$$

- continuum limit reached when $1/m$ or $\xi \gg a$:
  - system approaches continuous PT (statistical physics)
  - in asymptotically free theories: $a \to 0$ for $g \to 0$
- physical quantities should become independent of a in the continuum limit:

$$\frac{d}{da} m = 0 \quad (a \to 0) \qquad \Longleftrightarrow \qquad \text{renormalizability}$$

The connection between the lattice and the continuum

- this yields a differential equation for $f(g)$:

$$-f(g) + f'(g)\left(a\frac{d}{da}g\right) = 0$$

where

$$\beta(g) \equiv a\frac{d}{da}g = -b_0 g^3 - b_1 g^5 - \dots$$

- every physical quantity can be expressed in terms of a single, RG-invariant mass parameter Λ^{latt}, e.g. $m = c_m \cdot \Lambda^{\mathrm{latt}}$

$$\Lambda^{\mathrm{latt}} = \frac{1}{a}\, e^{-1/2b_0 g^2} \left(b_0 g^2\right)^{-b_1/2b_0^2} \cdot [1 + \mathcal{O}(g^2)]$$

- analogously in a continuum ren. scheme one has

$$\Lambda = M\, e^{-1/2b_0 g(M)^2} \left(b_0 g(M)^2\right)^{-b_1/2b_0^2} \cdot [1 + \mathcal{O}(g(M)^2)]$$

The connection between the lattice and the continuum

- of course, the Λ parameter is nonperturbatively defined:

$$\Lambda = M\, e^{-1/2b_0 g(M)^2} \left(b_0 g(M)^2\right)^{-b_1/2b_0^2} \times \left[1 + \mathcal{O}(g(M)^2)\right]$$

- lattice QCD relates it nonperturbatively to the low-energy properties of QCD

The connection between the lattice and the continuum

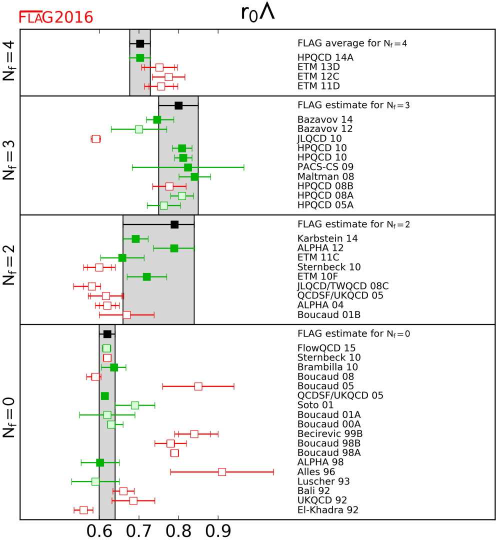
- of course, the Λ parameter is nonperturbatively defined:

$$\Lambda = M\, e^{-1/2b_0 g(M)^2} \left(b_0 g(M)^2\right)^{-b_1/2b_0^2} \times \exp\left[-\int_0^{g(M)} dx \left(\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right)\right]$$

- lattice QCD relates it nonperturbatively to the low-energy properties of QCD

The connection between the lattice and the continuum

- Λ parameter in the $\overline{\mathrm{MS}}$-scheme in units of r_0:



The connection between the lattice and the continuum

- of course, the Λ parameter is nonperturbatively defined:

$$\Lambda = M\, e^{-1/2b_0 g(M)^2} \left(b_0 g(M)^2\right)^{-b_1/2b_0^2} \times \exp\left[-\int_0^{g(M)} dx \left(\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right)\right]$$

- lattice QCD relates it nonperturbatively to the low-energy properties of QCD

The connection between the lattice and the continuum

- of course, the Λ parameter is nonperturbatively defined:

$$\Lambda = M\, e^{-1/2b_0 g(M)^2} \left(b_0 g(M)^2\right)^{-b_1/2b_0^2} \times \exp\left[-\int_0^{g(M)} dx \left(\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right)\right]$$

- lattice QCD relates it nonperturbatively to the low-energy properties of QCD
- closely related is the running coupling α_s at scale M

$$\alpha_s(M) = \frac{g^2(M)}{4\pi}$$

- measure a short distance quantity $\mathcal{Q}$ at scale M and match with perturbative expansion

$$\mathcal{Q}(M) = c_1 \alpha_{\overline{\mathrm{MS}}}(M) + c_2 \alpha_{\overline{\mathrm{MS}}}(M)^2 + \ldots$$

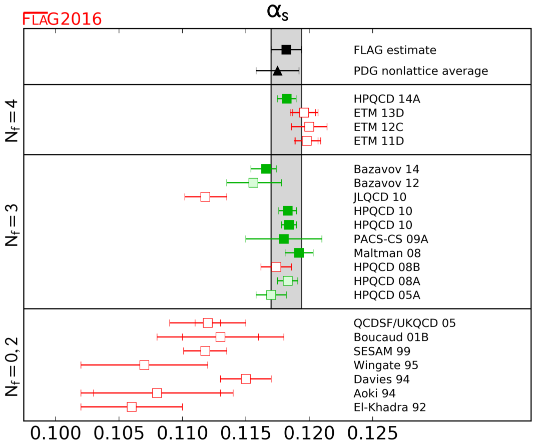
The strong coupling α_s

Collaboration	Ref.	N_f	publication status	renormalization scale	perturbative behaviour	continuum extrapolation	$\alpha_{\overline{\rm MS}}(M_Z)$	Method	Table
HPQCD 14A	[5]	2+1+1	A	○	★	○	0.11822(74)	current two points	46
ETM 13D	[648]	2+1+1	A	○	○	■	0.1196(4)(8)(16)	gluon-ghost vertex	47
ETM 12C	[649]	2+1+1	A	○	○	■	0.1200(14)	gluon-ghost vertex	47
ETM 11D	[650]	2+1+1	A	○	○	■	$0.1198(9)(5)(^{+0}_{-5})$	gluon-ghost vertex	47
Bazavov 14	[61]	2+1	A	○	★	○	$0.1166(^{+12}_{-8})$	Q-$\bar{Q}$ potential	43
Bazavov 12	[603]	2+1	A	○	○	○	$0.1156(^{+21}_{-22})$	Q-$\bar{Q}$ potential	43
HPQCD 10	[9]	2+1	A	○	★	○	0.1183(7)	current two points	46
HPQCD 10	[9]	2+1	A	○	★	★	0.1184(6)	Wilson loops	45
JLQCD 10	[612]	2+1	A	■	■	■	$0.1118(3)(^{+16}_{-17})$	vacuum polarization	44
PACS-CS 09A	[62]	2+1	A	★	★	○	$0.118(3)^{\#}$	Schrödinger functional	42
Maltman 08	[63]	2+1	A	○	○	★	0.1192(11)	Wilson loops	45
HPQCD 08B	[152]	2+1	A	■	■	■	0.1174(12)	current two points	46
HPQCD 08A	[616]	2+1	A	○	★	★	0.1183(8)	Wilson loops	45
HPQCD 05A	[615]	2+1	A	○	○	○	0.1170(12)	Wilson loops	45

Result with a linear continuum extrapolation in a.

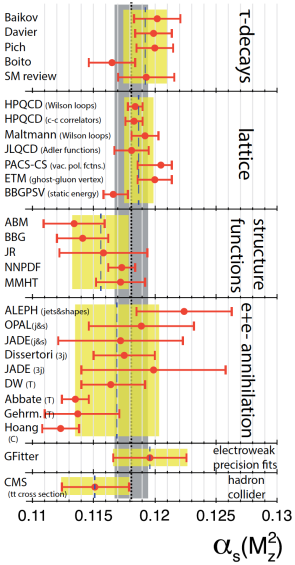
- critical assessment of the situation is necessary
- dominant source of uncertainty from discretization errors and truncation of continuum/lattice PT

The strong coupling α_s



- critical assessment of the situation is necessary
- dominant source of uncertainty from discretization errors and truncation of continuum/lattice PT

The strong coupling α_s



- FLAG 16 estimate yields [arXiv:1607.00299]

$$\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.1182(12)$$

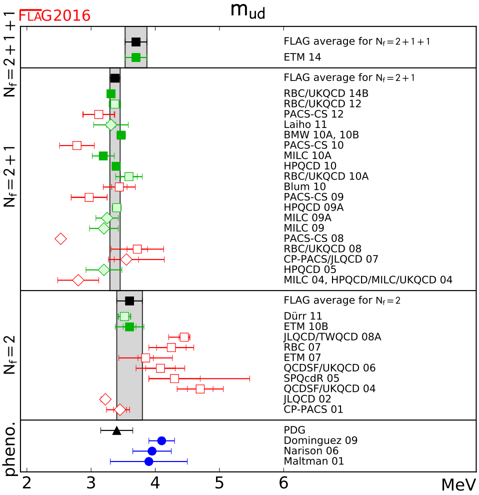
- to be compared with PDG 16 values

$$\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.1175(17) \quad (\textit{phen. only})$$
$$\alpha^{(5)}_{\overline{\mathrm{MS}}}(M_Z) = 0.1181(13)$$

- still room for systematic improvement (smaller lattice spacing,. . .)

- in the long term, it pays off to be conservative

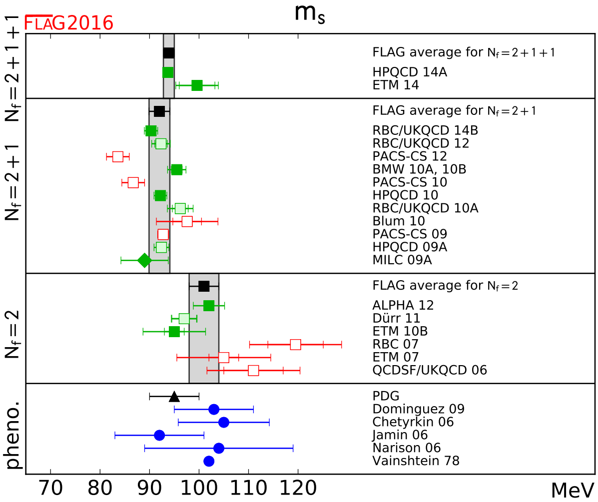
Light quark masses



$$N_f = 2+1: \qquad m_{ud}^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 3.37(8)\,\mathrm{MeV} \quad (\sim 2.4\%)$$

more precise than PDG

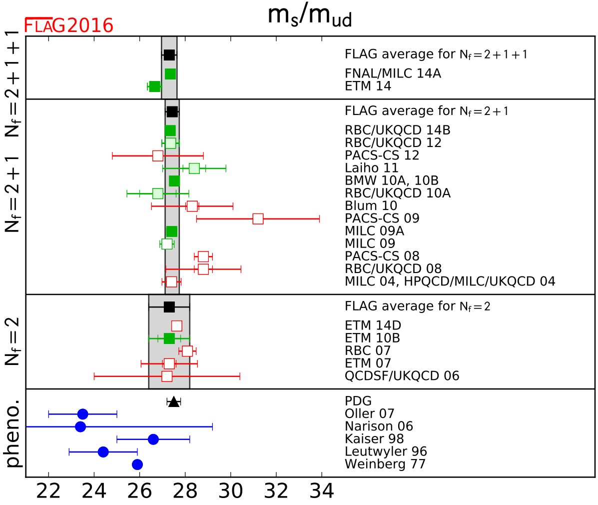
Light quark masses



$N_f = 2+1$: $\quad m_s^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 92.0(2.1)\,\mathrm{MeV} \quad (\sim 2.3\%)$

more precise than PDG

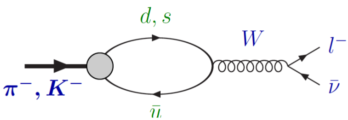
Light quark masses

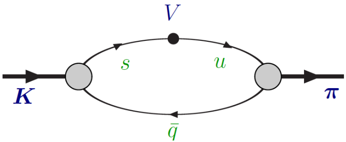


$N_f = 2+1$: $\qquad m_s/m_{ud} = 27.43(31) \quad (\sim 1.1\%)$

more precise than PDG

Kaon and pion decay constants and form factors





QCD effects contained in lept. decay constants $f_{\pi^\pm}$ and $f_{K^\pm}$:

$$\langle 0|\bar{d}\gamma_\mu\gamma_5 u|\pi^+(p)\rangle = ip_\mu \cdot f_{\pi^+}$$
$$\langle 0|\bar{s}\gamma_\mu\gamma_5 u|K^+(p)\rangle = ip_\mu \cdot f_{K^+}$$

and form factors $f_{0,+}(q)$ for the semi-leptonic decay $K^0 \to \pi^-\ell\nu$

$$\langle \pi^-(p_\pi)|\bar{s}\gamma_\mu u|K^0(p_K)\rangle = f_0(q^2)\frac{m_{K^0}^2 - m_{\pi^-}^2}{q^2}q_\mu + f_+(q^2)\left[(p_\pi + p_K)_\mu - \frac{m_{K^0}^2 - m_{\pi^-}^2}{q^2}q_\mu\right]$$

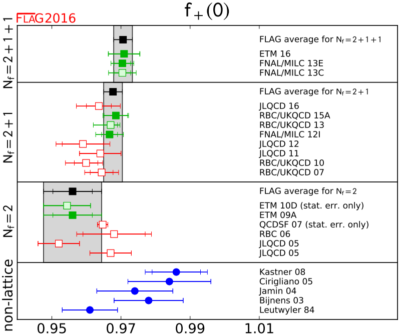
where $q = p_K - p_\pi$ and $f_+(0) = \lim_{q\to 0} f_+^{K^0\pi^-}(q)$.

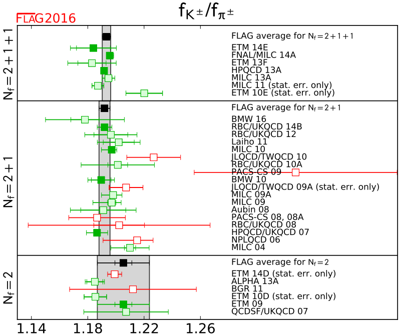
Kaon and pion decay constants and form factors

- Precision experimental data on kaon decays yields

$$|V_{us}|\, f_+(0) = 0.2165(4), \qquad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2760(4)$$

- Lattice calculations of $f_+(0)$ or $f_{K^\pm}/f_{\pi^\pm}$ determine V_{ud}, V_{us}

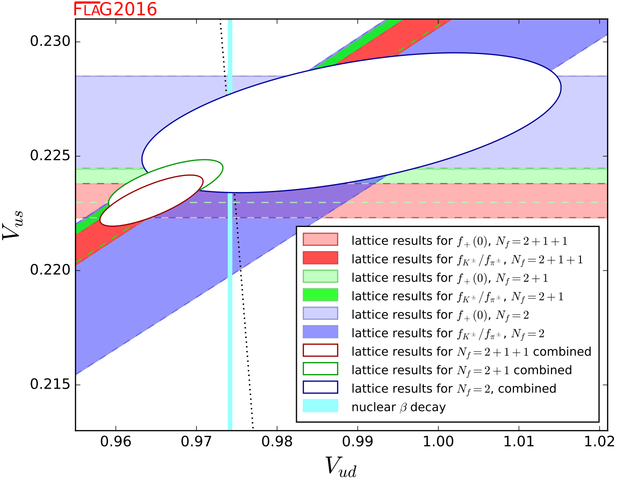




Kaon and pion decay constants and form factors

Provides a test of the SM via unitarity of the CKM matrix, e.g.

$$|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1, \qquad |V_{ub}| = 4.13(49) \cdot 10^{-3}$$



For $N_f = 2+1+1$:
$|V_u|^2 = 0.9798(82)$

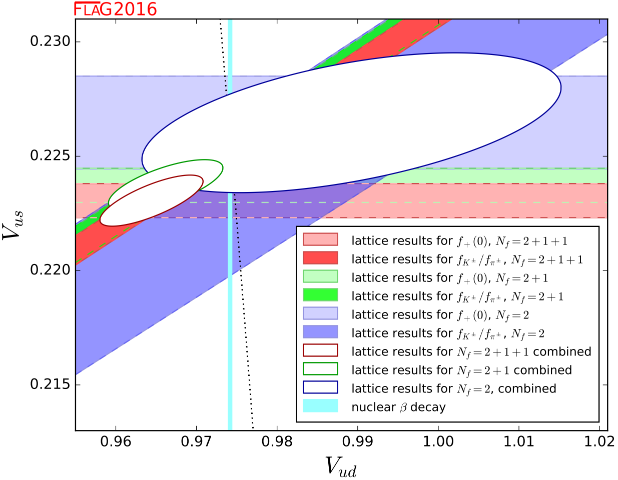
$\simeq 2.5\sigma$ deviation

SM passes nontrivial test using only lattice data and kaon decay branching ratios!

Kaon and pion decay constants and form factors

Provides a test of the SM via unitarity of the CKM matrix, e.g.

$$|V_u|^2 \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1, \qquad |V_{ub}| = 4.13(49)\cdot 10^{-3}$$



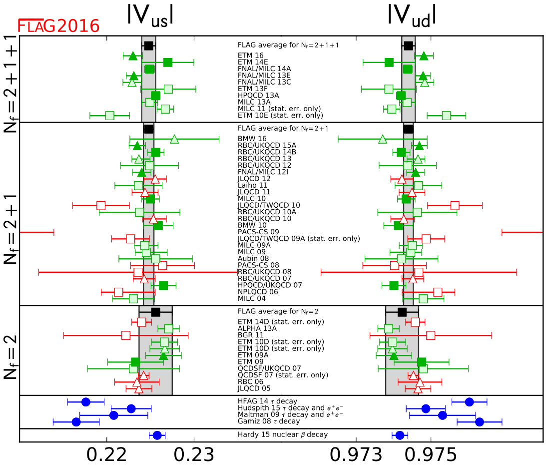
Using β-decay value for $|V_{ud}|^2$:

$f_+(0)$ leads to
$|V_u|^2 = 0.9988(5)$
$\simeq 2.5\sigma$ deviation

$f_{K^\pm}/f_{\pi^\pm}$ implies
$|V_u|^2 = 0.9998(5)$
$\rightarrow$ below ‰ level

Analysis within the SM

- The SM implies CKM matrix unitarity:
  - precise exp. data and unitarity condition reduce $V_{ud}, V_{us}, f_+(0), f_{K^\pm}/f_{\pi^\pm}$ to one unknown



squares from $f_{K^\pm}/f_{\pi^\pm}$

triangles from $f_+(0)$

agreement between lattice and β-decay

Summary

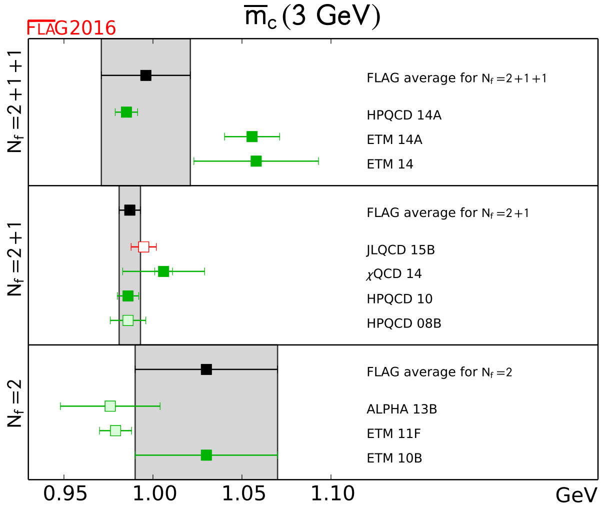
- Similar analysis available for decay constants and form factors involving the c and b quarks
  $\Rightarrow$ determination of $|V_{cd}|, |V_{cs}|$ and $|V_{ub}|$

- Lattice QCD plays an essential role in fully exploiting the potential of flavour physics
  - reached the era of $\mathcal{O}(1\%)$ accuracy for many quantities
  - improvement in precision will continue
  - range of computed quantities continue to be extended

- FLAG aims to review lattice determinations of phenomenologically relevant quantities for non-experts

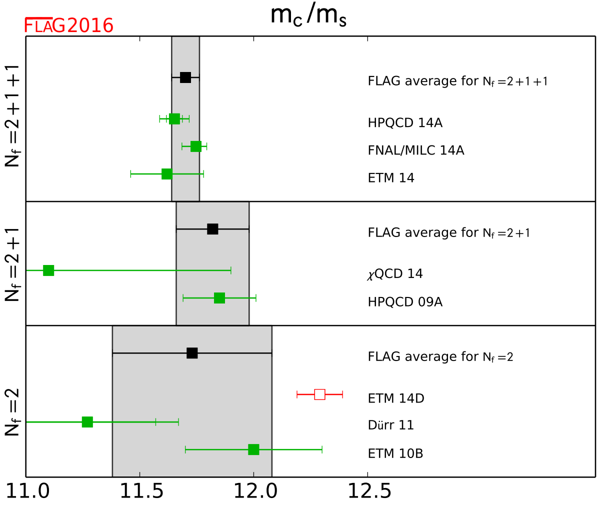
- FLAG is now entering stage 4, new review expected in 2019

Heavy quark masses



$$N_f = 2+1: \qquad m_c^{\overline{\mathrm{MS}}}(3\,\mathrm{GeV}) = 0.987(6)\,\mathrm{GeV} \qquad (\sim 0.6\%)$$

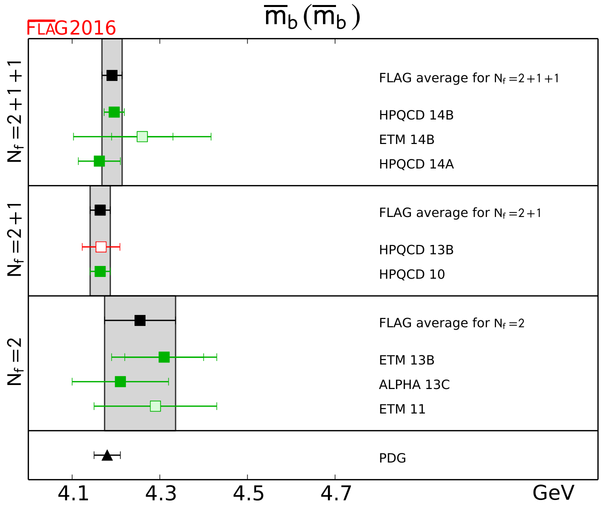
Heavy quark masses



$N_f = 2+1+1$: $\quad m_c/m_s = 11.70(6) \quad (\sim 0.5\%)$

more precise than PDG

Heavy quark masses



$$N_f = 2+1: \qquad \overline{m_b}(\overline{m_b}) = 4.190(21)\,\text{GeV} \qquad (\sim 0.5\%)$$

more precise than PDG