

Tomography and gravitational radii for hadrons by three-dimensional structure functions

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based on recent work with Qin-Tao Song, Oleg Teryaev

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<https://indico.gsi.de/event/5012/>
<https://www.oeaw.ac.at/smi/talks-and-events/exa/exa2017/>

September 13, 2107

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- **Introduction to hadron tomography**
- **GPDs and possible J-PARC project**
GPD (Generalized Parton Distribution)
- **GDAs and possible KEKB/ILC/LHC project**
GDA (Generalized Distribution Amplitude)
Gravitational radii for hadrons
- **Summary**

Motivations

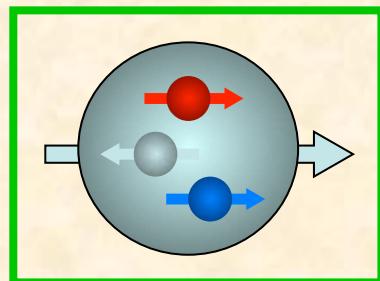
- **3D structure of hadrons**
- **Nucleon spin structure**
- **Exotic hadrons**
- **Gravitational properties of quarks and gluons
(hadrons)?**

...

Hadron tomography: 3D structure functions are (can be) investigated at high-energy lepton and hadron facilities (BNL, JLab, Fermilab, CERN, J-PARC, KEKB, GSI, IHEP@China & Russia, EIC, LHeC, ILC, ...).

Recent progress on origin of nucleon spin

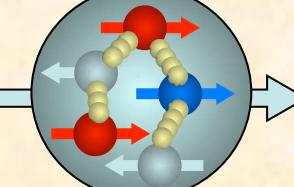
“old” standard model



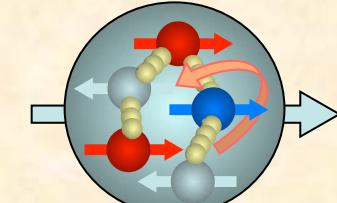
$$p_\uparrow = \frac{1}{3\sqrt{2}} (uud [2 \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow] + \text{permutations})$$

$$\Delta q(x) \equiv q_\uparrow(x) - q_\downarrow(x)$$

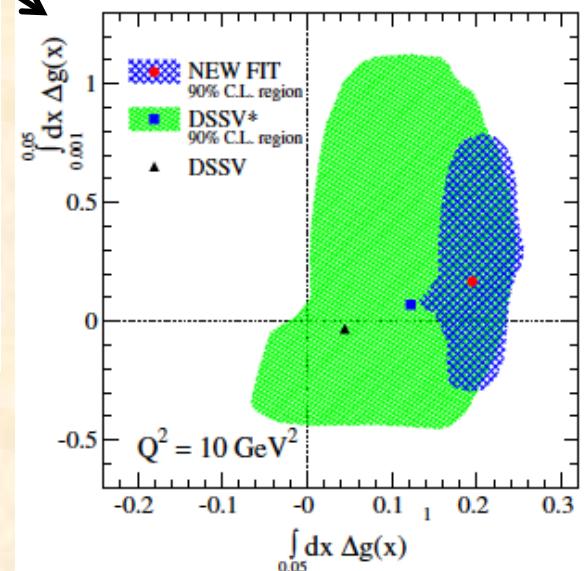
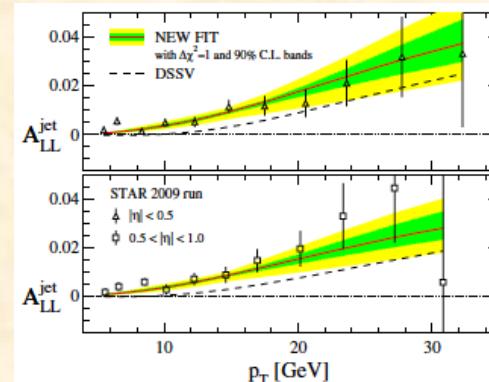
$$\Delta\Sigma = \sum_i \int dx [\Delta q_i(x) + \Delta \bar{q}_i(x)] \rightarrow 1 \text{ (100%)}$$



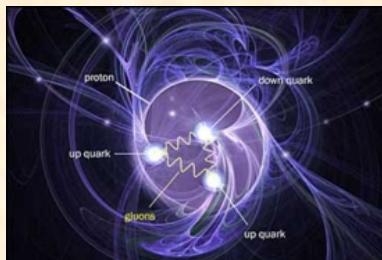
gluon spin



angular momentum



CNN (2014)

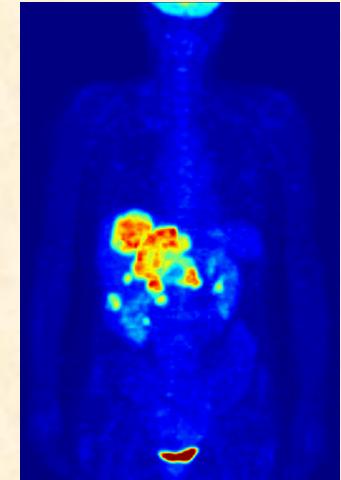


Scientific American (2014)

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta g + L_{q,g}$$

Nucleon (hadron) tomography

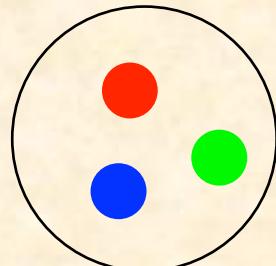
PET (Positron Emission Tomography)



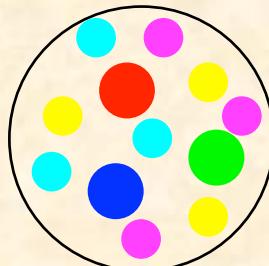
Classical density distribution

3D picture of nucleon
(Density distribution of quantum system:
Quantum tomography)

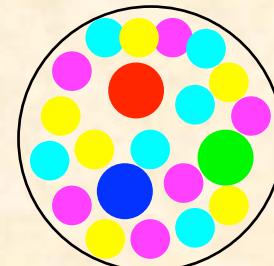
Low energy



Intermediate energy



High energy



1D(Bjorken- x) picutre@HERA



© DESY

Progress in exotic hadrons

$q\bar{q}$ Meson
 q^3 Baryon

$q^2\bar{q}^2$ Tetraquark
 $q^4\bar{q}$ Pentaquark
 q^6 Dibaryon

...
 $q^{10}\bar{q}$ e.g. Strange tribaryon

...
 gg Glueball

...

- $\Theta^+(1540)???$: LEPS
Pentaquark?

$uudd\bar{s}$?

- **Kaonic nuclei?**: KEK-PS, ...
Strange tribaryons, ...

$K^- pnn, K^- ppn$?
 $K^- pp$?

- **X (3872), Y(3940)**: Belle
Tetraquark, $D\bar{D}$ molecule

$c\bar{c}$
 $D^0(c\bar{u})\bar{D}^0(\bar{c}u)$
 $D^+(c\bar{d})D^-(\bar{c}d)$?

- **$D_{sJ}(2317), D_{sJ}(2460)$** : BaBar, CLEO, Belle
Tetraquark, DK molecule

$c\bar{s}$
 $D^0(c\bar{u})K^+(u\bar{s})$
 $D^+(c\bar{d})K^0(d\bar{s})$?

- **Z (4430)**: Belle
Tetraquark, ...

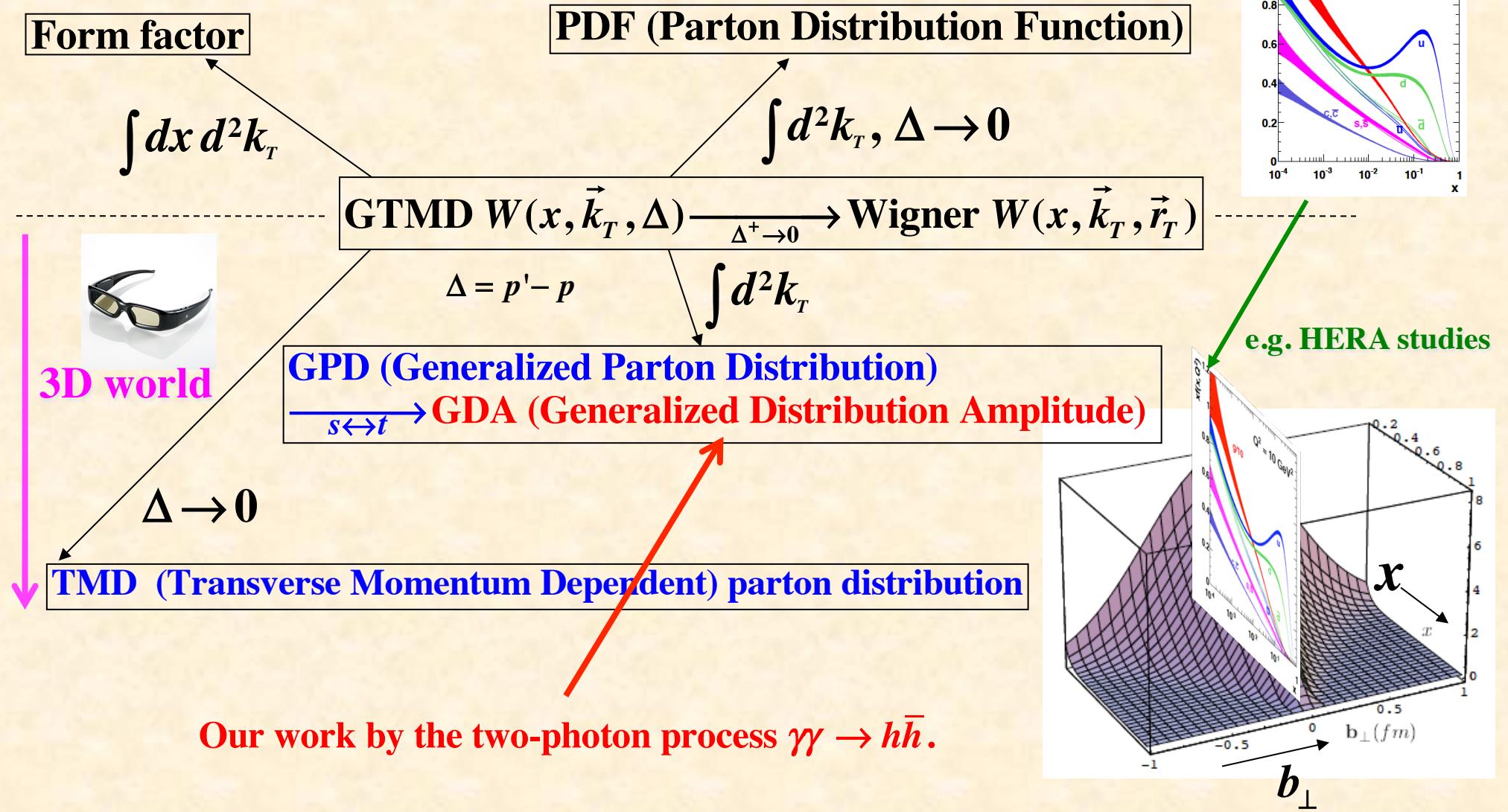
$c\bar{c}u\bar{d}$, D molecule?

- **$P_c(4380), P_c(4450)$** : LHCb

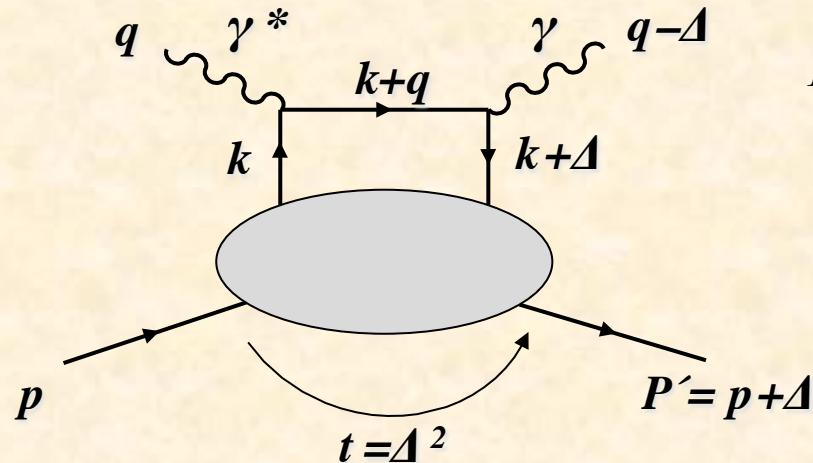
$u\bar{c}udc, \bar{D}(u\bar{c})\Sigma_c^*(udc), \bar{D}^*(u\bar{c})\Sigma_c(udc)$ molecule?

- ...

Wigner distribution and various structure functions



Generalized Parton Distributions (GPDs)



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \frac{Q^2}{2 p \cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewness parameter $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\textcolor{violet}{H}(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + \textcolor{violet}{E}(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

Forward limit: PDFs $H(x, \xi, t) \Big|_{\xi=t=0} = f(x), \quad \tilde{H}(x, \xi, t) \Big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2

$$\int_{-1}^1 dx H(x, \xi, t) = F_1(t), \quad \int_{-1}^1 dx E(x, \xi, t) = F_2(t)$$

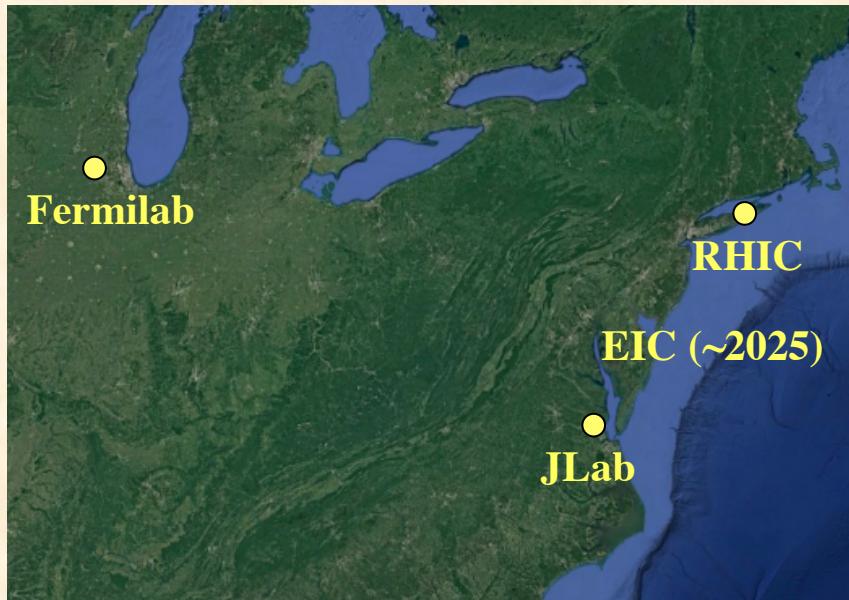
Axial and Pseudoscalar form factors G_A, G_P

$$\int_{-1}^1 dx \tilde{H}(x, \xi, t) = g_A(t), \quad \int_{-1}^1 dx \tilde{E}(x, \xi, t) = g_P(t)$$

Second moments: Angular momenta

Sum rule: $J_q = \frac{1}{2} \int_{-1}^1 dx x [H_q(x, \xi, t=0) + E_q(x, \xi, t=0)], \quad J_q = \frac{1}{2} \Delta q + L_q$

Hadron-tomography studies in US and Europe



Fermilab: Main Injector (120 GeV proton),
Neutrino (Minerva, several GeV)

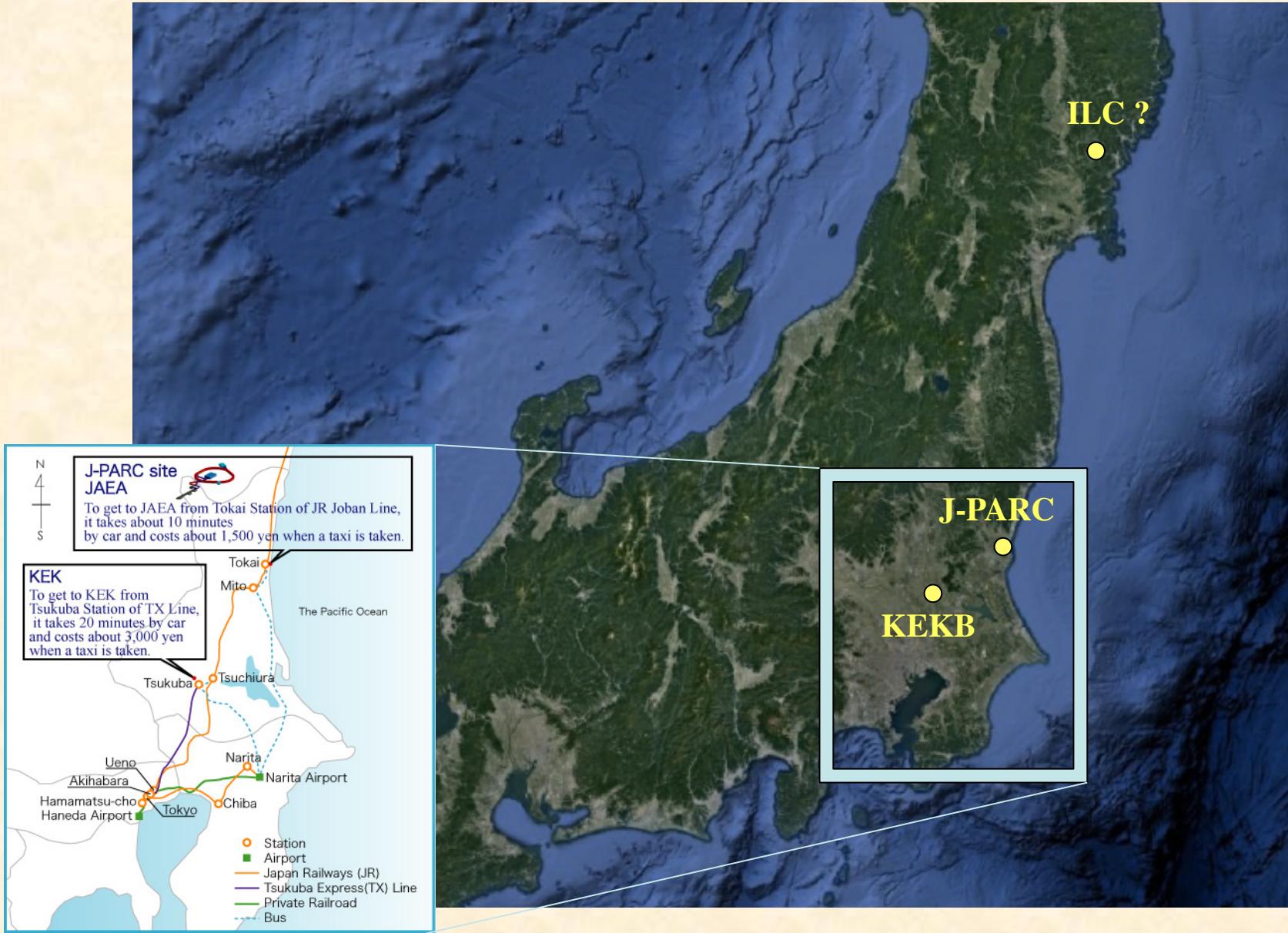
RHIC: Spin (polarized p + polarized p)
Heavy ion (*e.g.* UPC: Ultra-Peripheral Collision)

EIC (Electron Ion Collider, ~2025)

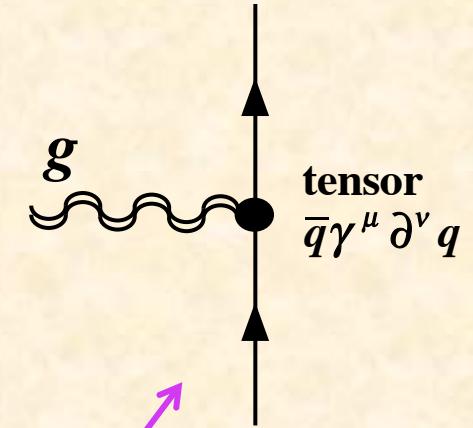
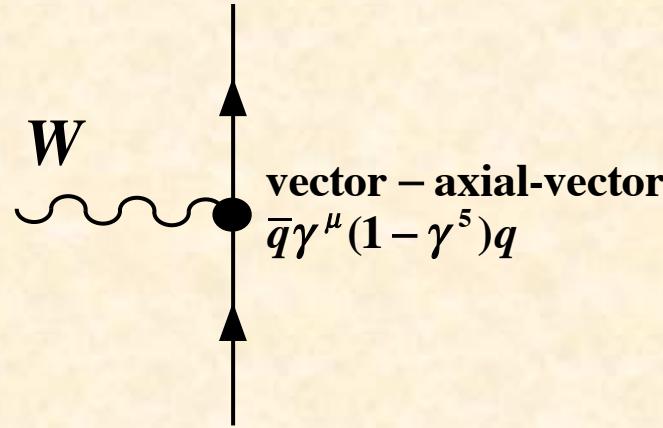
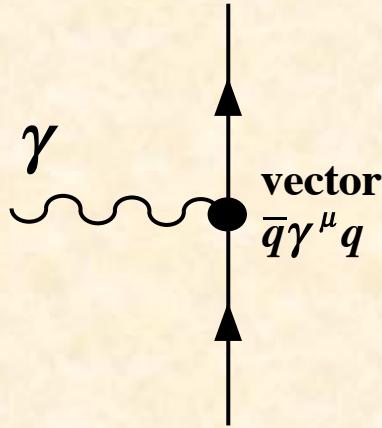


CERN: COMPASS (μ , π beams)
LHC Heavy ion
(*e.g.* UPC: Ultra-Peripheral Collision)

Possible hadron-tomography studies at J-PARC, KEKB, ILC?



Why gravitational interactions with hadrons ?



Electron-proton elastic scattering cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_f \cos^2 \frac{\theta}{2}}{4E_i^3 \sin^4(\theta/2)} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \right], \quad \tau = -\frac{q^2}{4M^2}$$

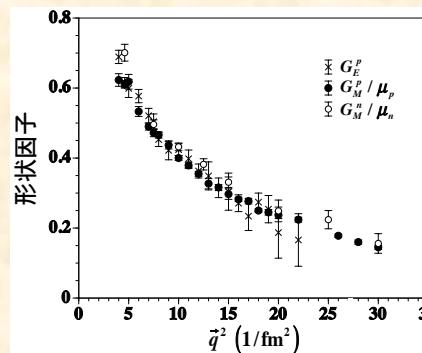
$$F(\vec{q}) = \int d^3x e^{i\vec{q} \cdot \vec{x}} \rho(\vec{x}) = \int d^3x \left[1 - \frac{1}{2} (\vec{q} \cdot \vec{x})^2 + \dots \right] \rho(\vec{x})$$

$$\langle r^2 \rangle = \int d^3x r^2 \rho(\vec{x}), \quad r = |\vec{x}|$$

$\sqrt{\langle r^2 \rangle}$ = root-mean-square (rms) radius

$$F(\vec{q}) = 1 - \frac{1}{6} \vec{q}^2 \langle r^2 \rangle + \dots, \quad \langle r^2 \rangle = -6 \frac{dF(\vec{q})}{d\vec{q}^2} \Big|_{\vec{q}^2 \rightarrow 0}$$

$$\rho(r) = \frac{\Lambda^3}{8\pi} e^{-\Lambda r} \Leftrightarrow \text{Dipole form: } F(q) = \frac{1}{\left(1 + |\vec{q}|^2 / \Lambda^2\right)^2}, \quad \Lambda^2 \approx 0.71 \text{ GeV}^2$$



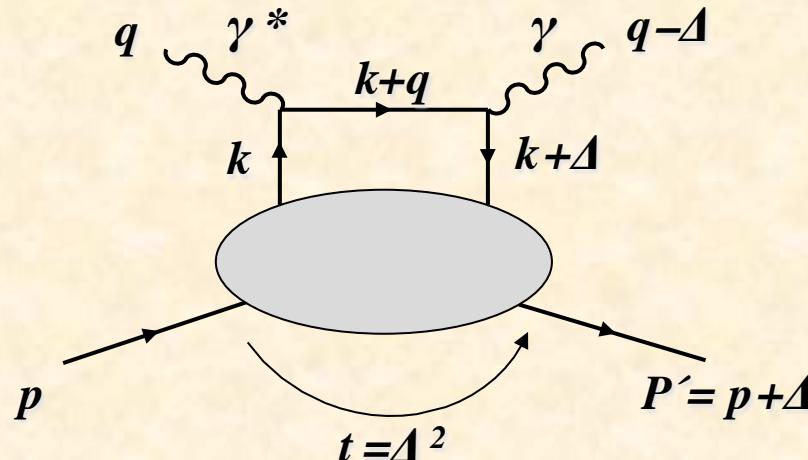
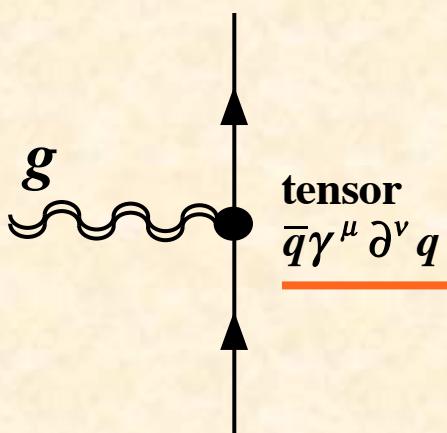
How about gravitational radius?

Proton-charge-radius puzzle:

$$R_{\text{electron scattering}} = 0.8775 \text{ fm} \quad \Updownarrow \quad R_{\text{muonic atom}} = 0.8418 \text{ fm}$$



Gravitational interactions and 3D structure functions



GPDs:
$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

Angular momentum: $J_q = \frac{1}{2} \int_{-1}^1 dx x \left[H_q(x, \xi, t=0) + E_q(x, \xi, t=0) \right], \quad J_q = \frac{1}{2} \Delta q + L_q$

Non-local operator of GPDs/GDAs:

$$\begin{aligned} & \left(P^+ \right)^n \int dx x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{q}(-z/2) \gamma^+ q(z/2) \right]_{z^+=0, \vec{z}_\perp=0} = \left(i \frac{d}{dz^-} \right)^{n-1} \left[\bar{q}(-z/2) \gamma^+ q(z/2) \right]_{z=0} \\ & = \bar{q}(0) \gamma^+ \left(i \tilde{\partial}^+ \right)^{n-1} q(0) \end{aligned}$$

= energy-momentum tensor of a quark for $n=2$ (electromagnetic for $n=1$)

= source of gravity

References

GPDs at J-PARC

SK, M. Strikman, K. Sudoh, PRD 80 (2009) 074003.

**T. Sawada, Wen-Chen Chang, S. Kumano, Jen-Chieh Peng,
S. Sawada, K. Tanaka, PRD 93 (2016) 114034.**

GPDs and GDAs (including exotic hadrons)

H. Kawamura, SK, PRD 89 (2014) 054007.

SK, Q.-T. Song, O. Teryaev, KEK-TH-1959/J-PARC-TH-0086.

Related topics: Constituent counting rule:

H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010.

W.-C. Chang, SK, T. Sekihara, PRD 93 (2016) 034006.

Generalized Parton Distributions (GPDs)

and J-PARC project

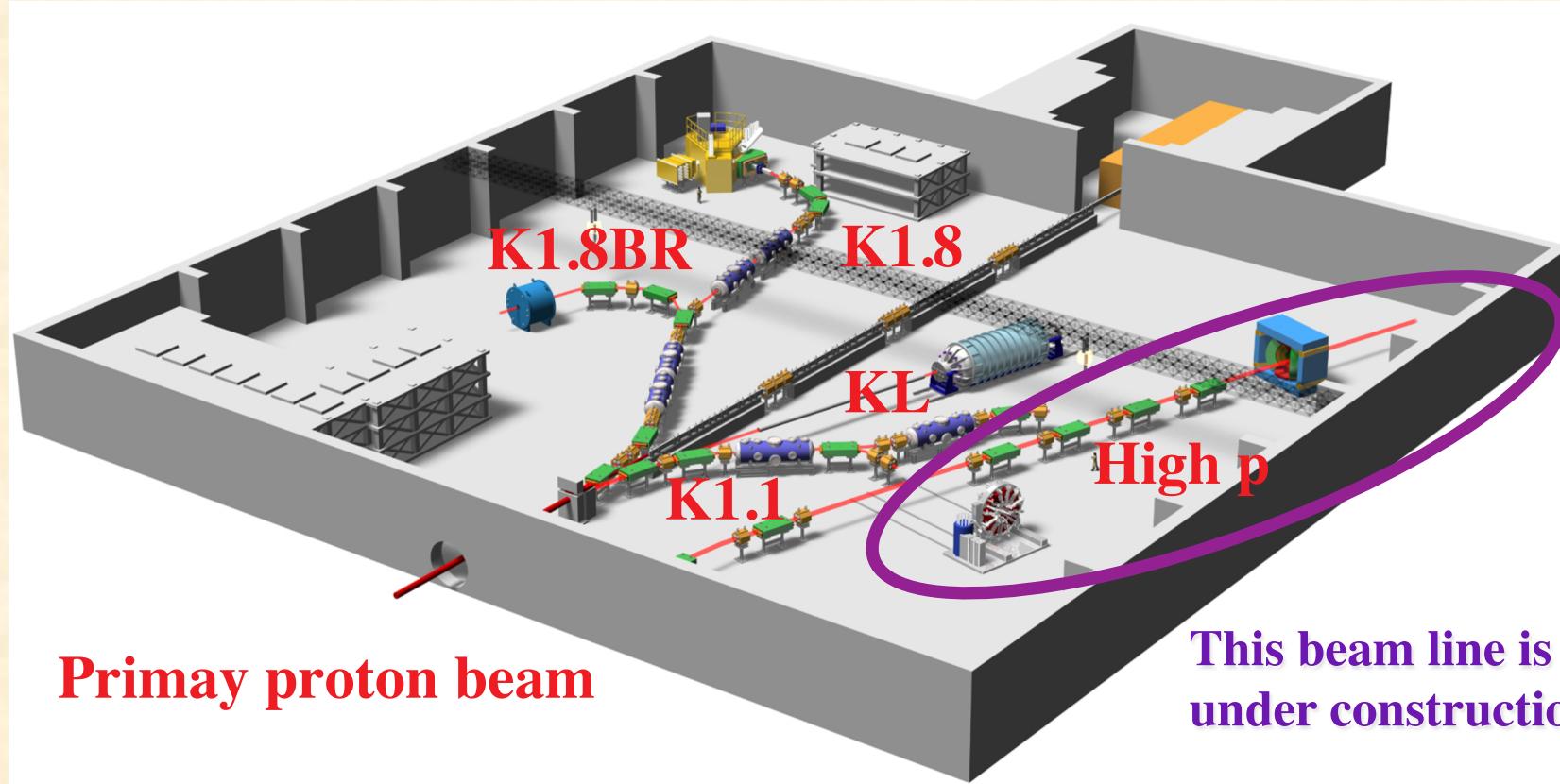
Comments on J-PARC project

**T. Sawada, W.-C. Chang, S. Kumano,
J.-C. Peng, S. Sawada, and K. Tanaka,
Phys. Rev. D93 (2016) 114034.**

**S. Kumano, Int. J. Mod. Phys.:
Conf. Series, 40 (2016) 1660009.**

Hadron facility

Workshops on high-momentum beamline physics,
<http://www-conf.kek.jp/hadron1/j-parc-hm-2013/>
<http://research.kek.jp/group/hadron10/j-parc-hm-2015/>.



- Proton beam up to 30 GeV
- Unseparated hadron (pion, ...) beam up to 15~20 GeV

Toward a new proposal at J-PARC

**T. Sawada, W.-C. Chang, S. Kumano, J.-C. Peng,
S. Sawada, and K. Tanaka, PRD93 (2016) 114034.**

Exclusive Drell-Yan: $\pi^- + p \rightarrow \mu^+ \mu^- + n$

PHYSICAL REVIEW D 93, 114034 (2016)

Accessing proton generalized parton distributions and pion distribution amplitudes with the exclusive pion-induced Drell-Yan process at J-PARC

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Generalized parton distributions (GPDs) encoding multidimensional information of hadron partonic structure appear as the building blocks in a factorized description of hard exclusive reactions. The nucleon GPDs have been accessed by deeply virtual Compton scattering and deeply virtual meson production with lepton beam. A complementary probe with hadron beam is the exclusive pion-induced Drell-Yan process. In this paper, we discuss recent theoretical advances on describing this process in terms of nucleon GPDs and pion distribution amplitudes. Furthermore, we address the feasibility of measuring the exclusive pion-induced Drell-Yan process $\pi^- p \rightarrow \mu^+ \mu^- n$ via a spectrometer at the High Momentum Beamline being constructed at J-PARC in Japan. Realization of such measurement at J-PARC will provide a new test of perturbative QCD descriptions of a novel class of hard exclusive reactions. It will also offer the possibility of experimentally accessing nucleon GPDs at large timelike virtuality.

Exclusive Drell-Yan $\pi^- + p \rightarrow \mu^+ \mu^- + n$ and GPDs

$$\boxed{\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \operatorname{Re} \left\{ \tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t) \right\} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]}$$

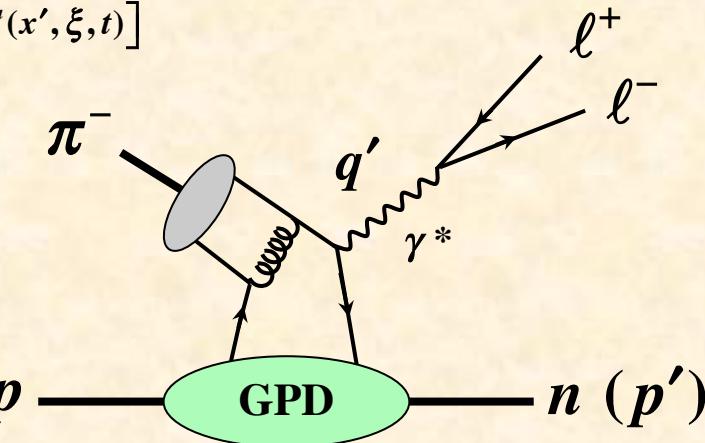
$$Q'^2=q'^2,\;\;t=(p-p')^2,\;\;\tau=\frac{Q'^2}{2p\cdot q_\pi}\simeq\frac{Q'^2}{s-m_N^2}$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p(p') | \bar{q}(-z/2) \gamma^+ \gamma_5 q(z/2) | p(p) \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_p^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_p^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle n(p') | \bar{q}_d(-z/2) \gamma^+ \gamma_5 q_u(z/2) | p(p) \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\tilde{H}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon} \right] \left[\tilde{H}^d(x', \xi, t) - \tilde{H}^u(x', \xi, t) \right]$$

$$\tilde{E}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\varepsilon} - \frac{e_u}{x+x'-i\varepsilon} \right] \left[\tilde{E}^d(x', \xi, t) - \tilde{E}^u(x', \xi, t) \right]$$

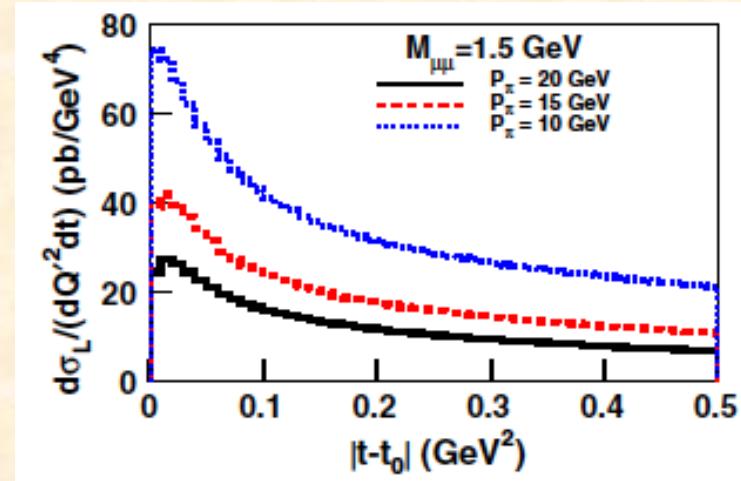
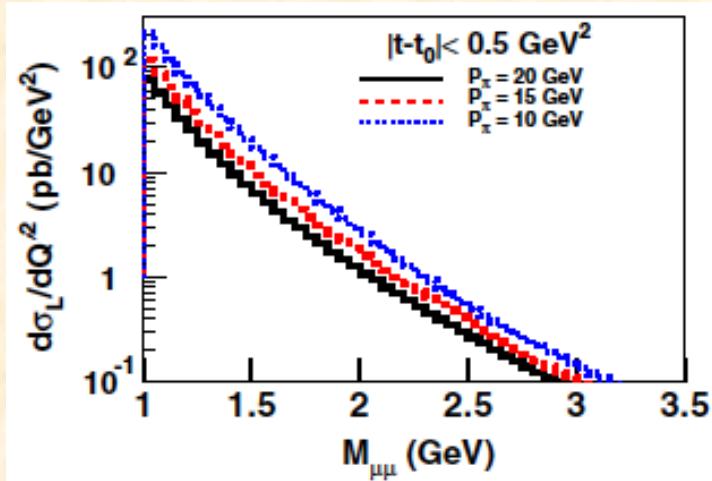


$$\pi^-(\bar{u}d) + p(uud) \rightarrow n(udd) + \gamma^*(\rightarrow \ell^+ \ell^-)$$

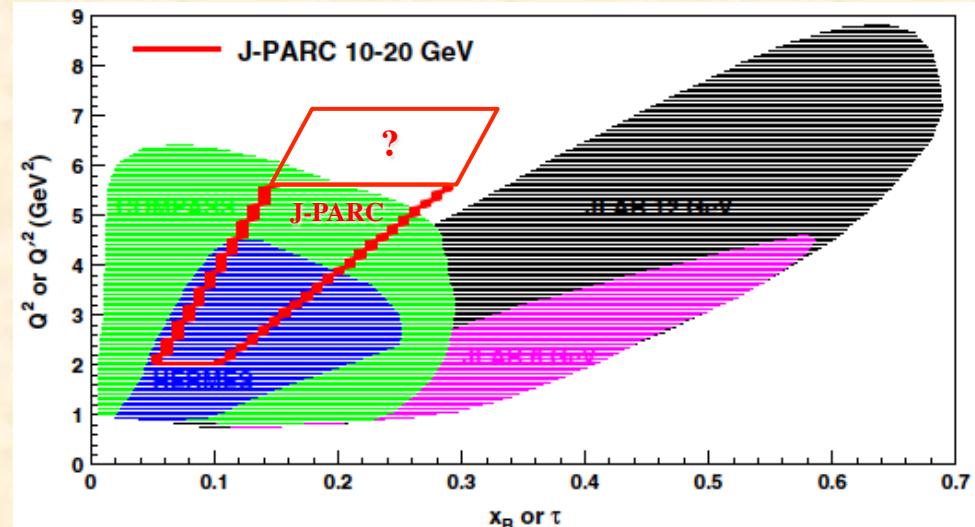
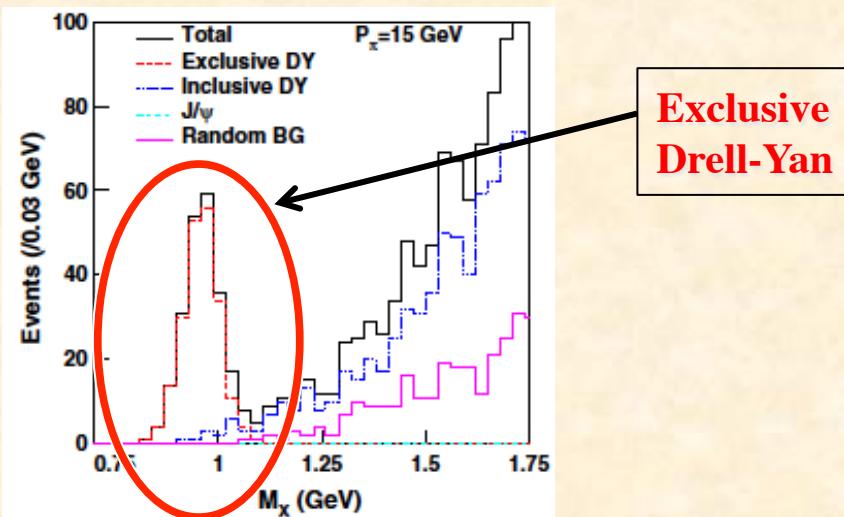
Expected Drell-Yan events at J-PARC

$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_N^2}$$

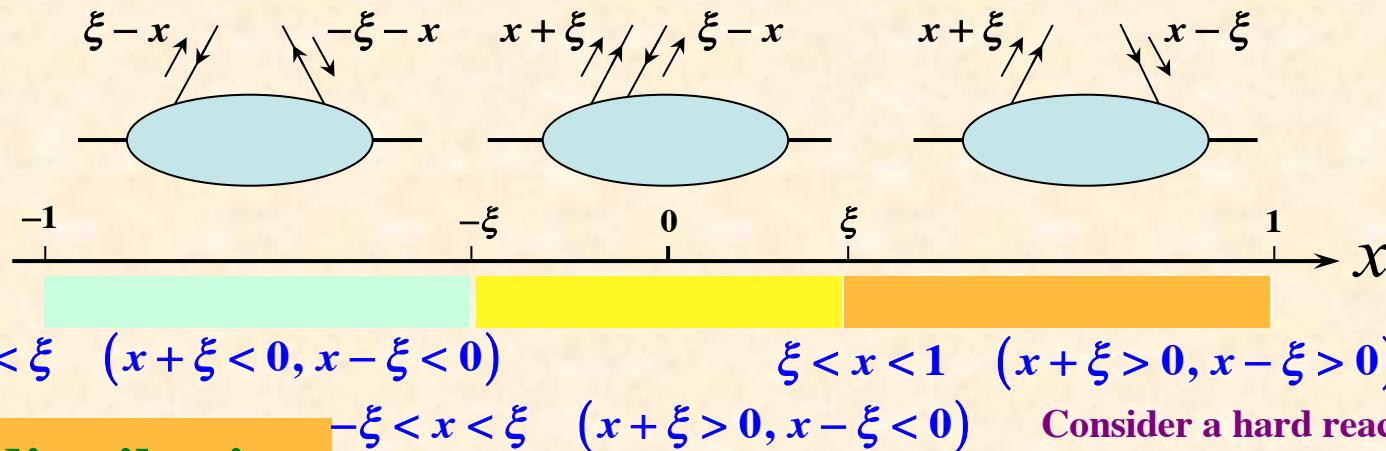
$$\boxed{\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \operatorname{Re}\{\tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t)\} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]}$$



Missing mass



GPDs in different x regions and GPDs at hadron facilities



Quark distribution

Emission of quark with momentum fraction $x+\xi$
Absorption of quark with momentum fraction $x-\xi$

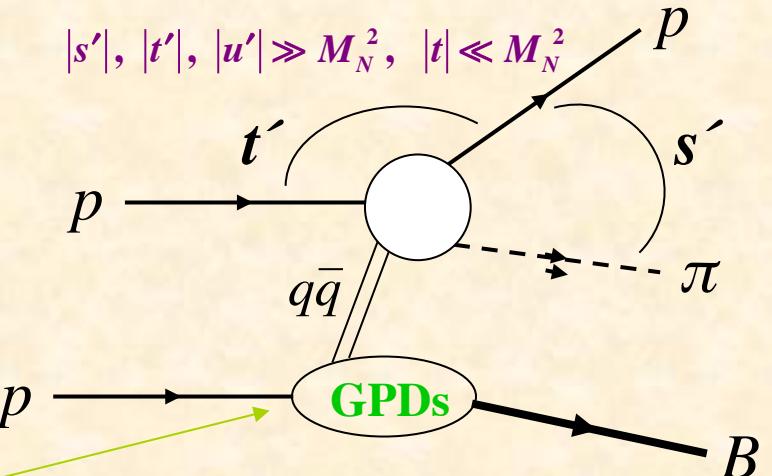
$q\bar{q}$ (meson)-like distribution amplitude

Emission of quark with momentum fraction $x+\xi$
Emission of antiquark with momentum fraction $\xi-x$

Antiquark distribution

Emission of antiquark with momentum fraction $\xi-x$
Absorption of antiquark with momentum fraction $-\xi-x$

Consider a hard reaction with
 $|s'|, |t'|, |u'| \gg M_N^2, |t| \ll M_N^2$



GPDs at J-PARC: S. Kumano, M. Strikman,
and K. Sudoh, PRD 80 (2009) 074003.

Efremov-Radyushkin
-Brodsky-Lepage (ERBL) region

GPDs for exotic hadrons at hadron facilities

**H. Kawamura and S. Kumano
Phys. Rev. D 89 (2014) 054007.**

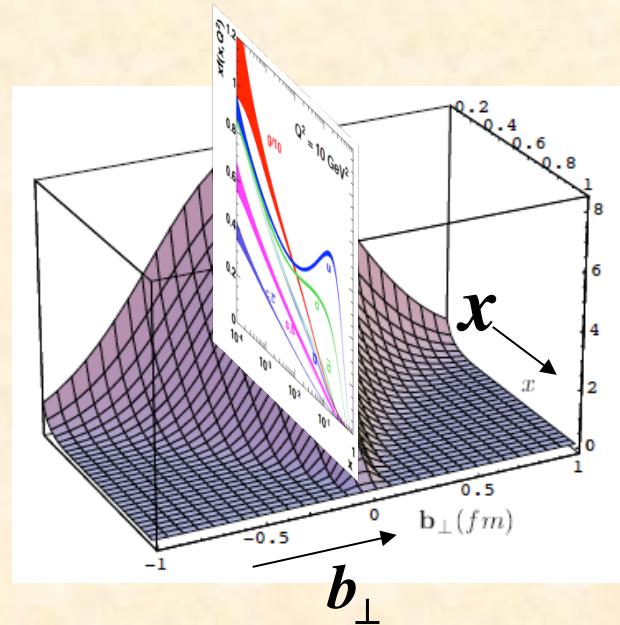
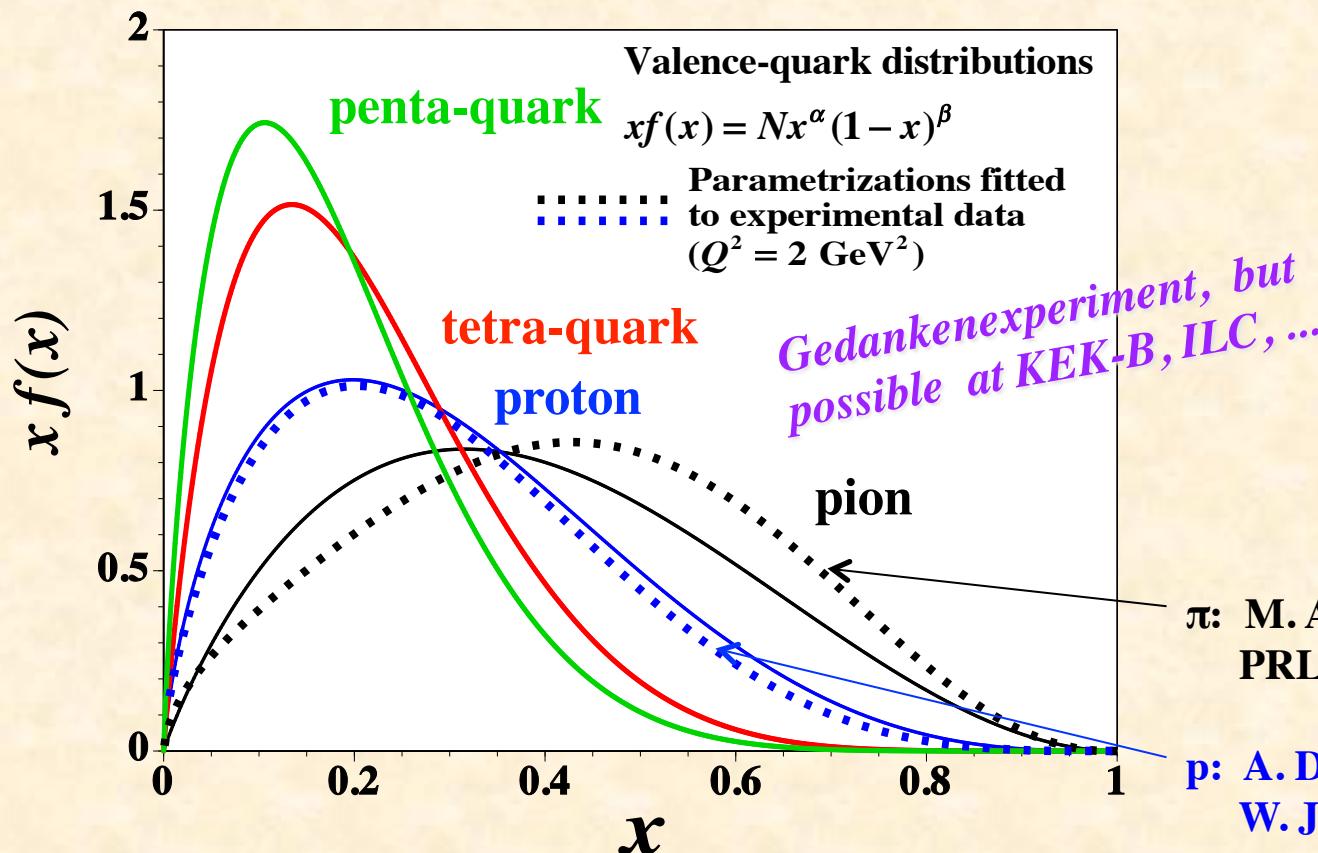
Simple function of GPDs

$$H_q^h(x,t) = f(x)F(t,x)$$

M. Guidal, M.V. Polyakov,
A.V. Radyushkin, M. Vanderhaeghen,
PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks: $f(x) = q_v(x) = c_n x^{\alpha_n} (1-x)^{\beta_n}$

- Valence-quark number sum rule (charge and baryon numbers): $\int_0^1 dx f(x) = n$
- Constituent counting rule at $x \rightarrow 1$: $\beta_n = 2n - 3 + 2\Delta S$ (n = number of constituents)
- Momentum carried by quarks $\langle x \rangle_q \simeq \int_0^1 dx x f(x)$

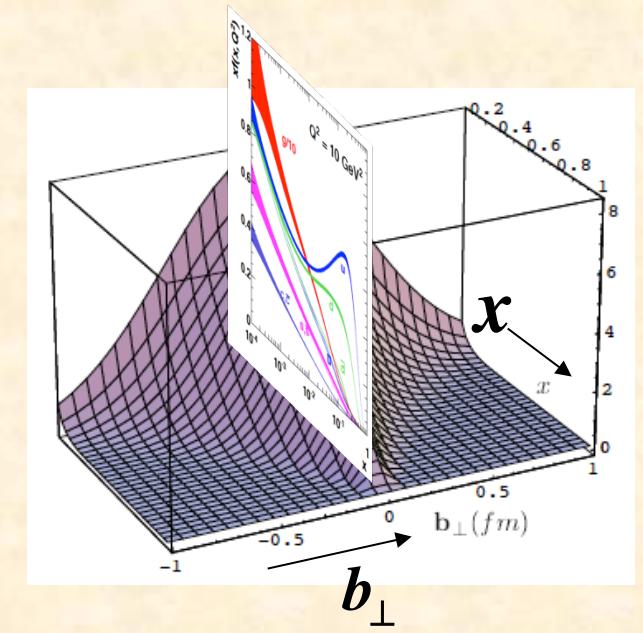
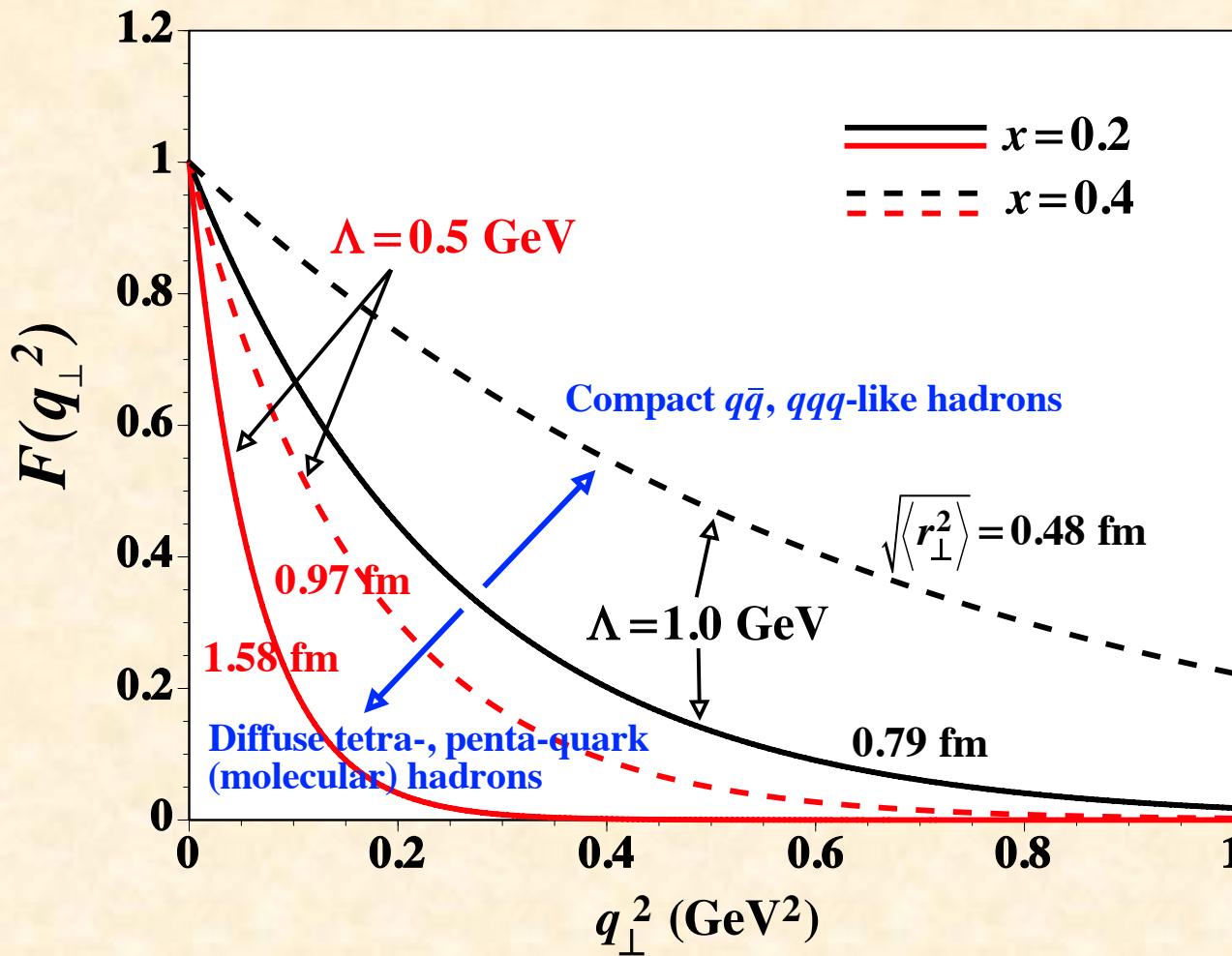


π : M. Aicher, A. Schafer, W. Vogelsang,
PRL 105 (2010) 252003.

p : A. D. Martin, R. G. Roberts,
W. J. Stirling, PLB 636, 259 (2006)

Two-dimensional form factor

$$H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$

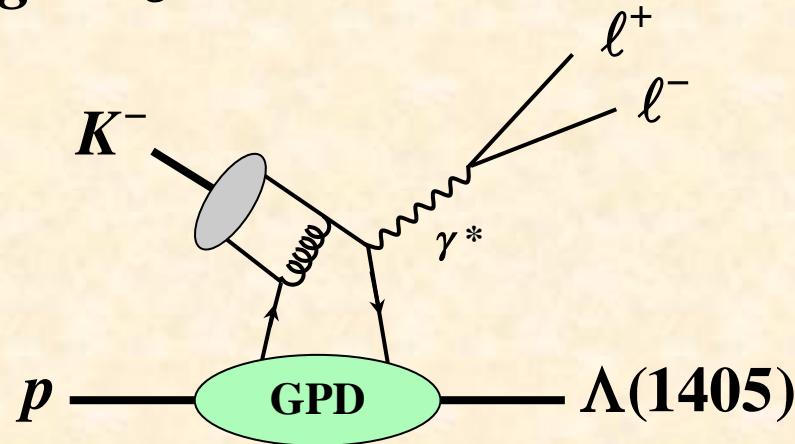


GPDs for exotic hadrons

Because stable targets do not exist for exotic hadrons,
it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

e.g. at J-PARC



Generalized Distribution Amplitudes (GDAs)

and KEKB/ILC project

**H. Kawamura and S. Kumano,
Phys. Rev. D 89 (2014) 054007.**

**S. Kumano, Q.-T. Song, O. Teryaev,
KEK-TH-1959, J-PARC-TH-0086.**

GPDs for exotic hadrons !?

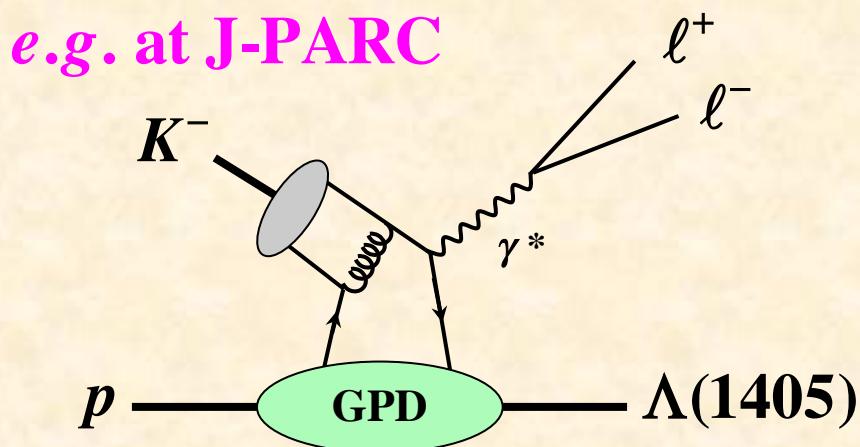
Because stable targets do not exist for exotic hadrons,
it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

or

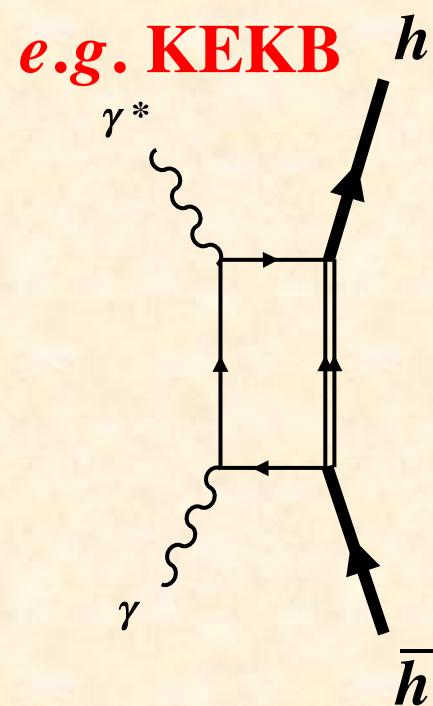
→ $s \leftrightarrow t$ crossed quantity = GDAs at KEKB, Linear Collider

e.g. at J-PARC



$$K^-(\bar{u}s) + p(uud) \rightarrow \Lambda_{1405}(uud\bar{u}s) + \gamma^*$$

Λ_{1405} = pentaquark ($\bar{K}N$ molecule) candidate



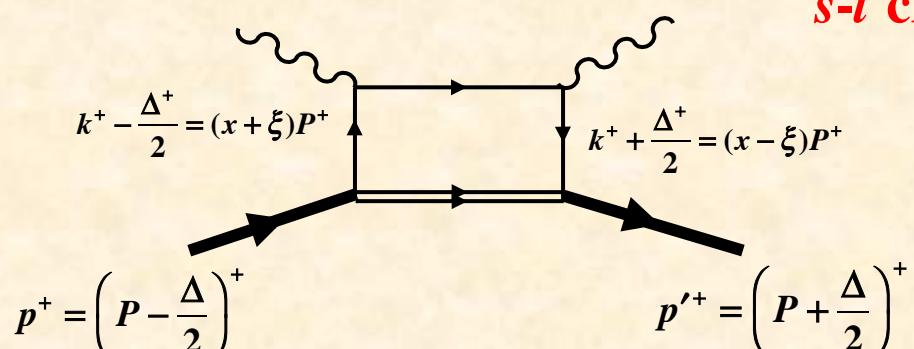
GPD $H_q^h(x, \xi, t)$ and GDA $\Phi_q^{hh}(z, \zeta, W^2)$

GPD: $H_q(x, \xi, t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | h(p) \rangle \Big|_{y^+=0, \vec{y}_\perp=0}, \quad P^+ = \frac{(p+p')^+}{2}$

GDA: $\Phi_q(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p) \bar{h}(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | \mathbf{0} \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

DA: $\Phi_q^\pi(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2) \gamma^+ \gamma_5 \psi(y/2) | \mathbf{0} \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

$H_q^h(x, \xi, t)$



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

Bjorken variable:

$$x = \frac{Q^2}{2p \cdot q}$$

Momentum transfer squared: $t = \Delta^2$

Skewness parameter: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

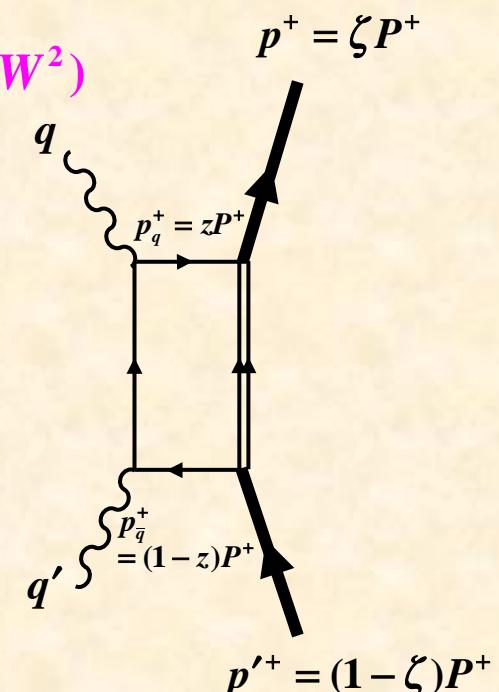
s-t crossing

$\Phi_q^{hh}(z, \zeta, W^2)$

$$\boxed{z \Leftrightarrow \frac{1-x/\xi}{2}}$$

$$\zeta \Leftrightarrow \frac{1-1/\xi}{2}$$

$$W^2 \Leftrightarrow t$$



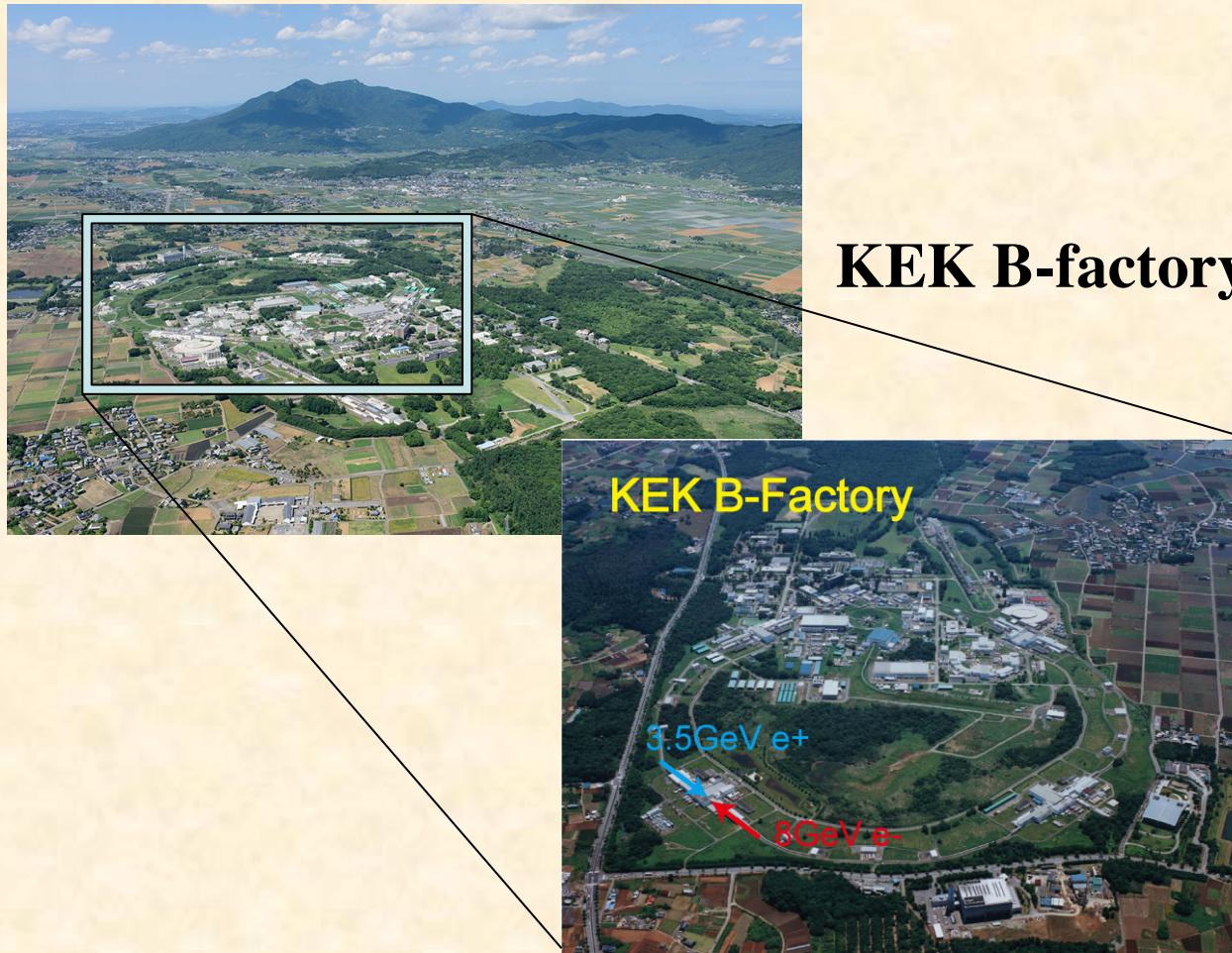
Bjorken variable for $\gamma\gamma^*$: $x = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \frac{p^+}{P^+} = \frac{1 + \beta \cos \theta}{2}$

Invariant mass of $h\bar{h}$: $W^2 = (p + p')^2$

Experimental studies of GDAs in future

$\gamma\gamma \rightarrow h\bar{h}$ for internal structure of exotic hadron candidate h



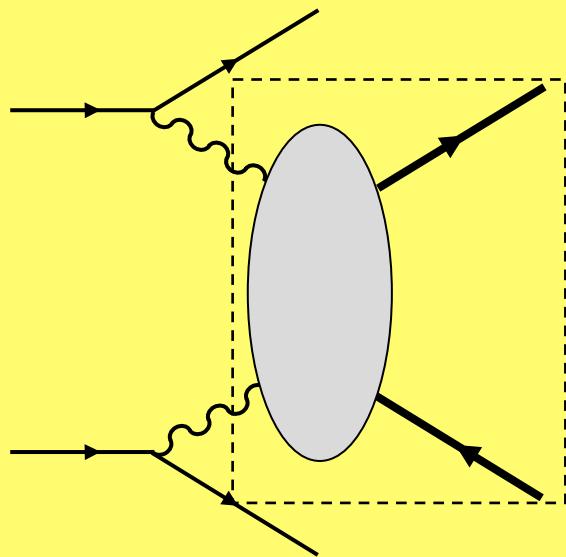
KEK B-factory

Linear Collider ?



Generalized Distribution Amplitudes (GDAs) for pion

from KEKB measurements



$$\gamma \rightarrow h\bar{h}$$

SK, Q.-T. Song, O. Teryaev,
KEK-TH-1959, J-PARC-TH-0086

Cross section for $\gamma\gamma^* \rightarrow \pi^0\pi^0$

$$d\sigma = \frac{1}{4\sqrt{(q \cdot q')^2 - q^2 q'^2}} (2\pi)^4 \delta^4(q + q' - p - p') \sum_{\lambda, \lambda'} |\mathcal{M}|^2 \frac{d^3 p}{(2\pi)^3 2E} \frac{d^3 p'}{(2\pi)^3 2E'}$$

$$q = (q^0, 0, 0, |\vec{q}|), \quad q' = (|\vec{q}|, 0, 0, -|\vec{q}|), \quad q'^2 = 0 \text{ (real photon)}$$

$$p = (p^0, |\vec{p}| \sin \theta, 0, |\vec{p}| \cos \theta), \quad p' = (p^0, -|\vec{p}| \sin \theta, 0, -|\vec{p}| \cos \theta)$$

$$\beta = \frac{|\vec{p}|}{p^0} = \sqrt{1 - \frac{4m_\pi^2}{W^2}}$$

$$\frac{d\sigma}{d(\cos \theta)} = \frac{1}{16\pi(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} \sum_{\lambda, \lambda'} |\mathcal{M}|^2$$

$$\mathcal{M} = \epsilon_\mu^\lambda(q) \epsilon_\nu^{\lambda'}(q') T^{\mu\nu}, \quad T^{\mu\nu} = i \int d^4 \xi e^{-i\xi \cdot q} \langle \pi(p) \pi(p') | T J_{em}^\mu(\xi) J_{em}^\nu(0) | 0 \rangle$$

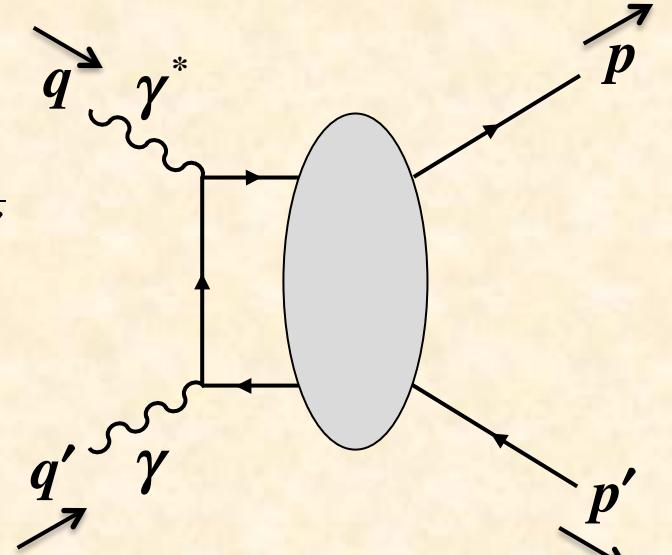
$$\mathcal{M} = e^2 A_{\lambda\lambda'} = 4\pi\alpha A_{\lambda\lambda'}$$

$$A_{\lambda\lambda'} = \frac{1}{e^2} \epsilon_\mu^\lambda(q) \epsilon_\nu^{\lambda'}(q') T^{\mu\nu} = -\epsilon_\mu^\lambda(q) \epsilon_\nu^{\lambda'}(q') g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

$$\text{GDA: } \Phi_q^{\pi\pi}(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) \pi(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$$

$$A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2), \quad \epsilon_\mu^+(q) \epsilon_\nu^+(q') g_T^{\mu\nu} = -1$$

$$\frac{d\sigma}{d(\cos \theta)} \simeq \frac{\pi\alpha^2}{4(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} |A_{++}|^2$$



GDA parametrization for pion

$$\frac{d\sigma}{d(\cos \theta)} = \frac{\pi \alpha^2}{4(s+Q^2)} \sqrt{1 - \frac{4m^2}{s}} |A_{++}|^2$$

$$A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

- Continuum: GDAs without intermediate-resonance contribution

$$\Phi_q^{\pi\pi}(z, \zeta, W^2) = N_\pi z^\alpha (1-z)^\beta (2z-1) \zeta (1-\zeta) F_q^\pi(s)$$

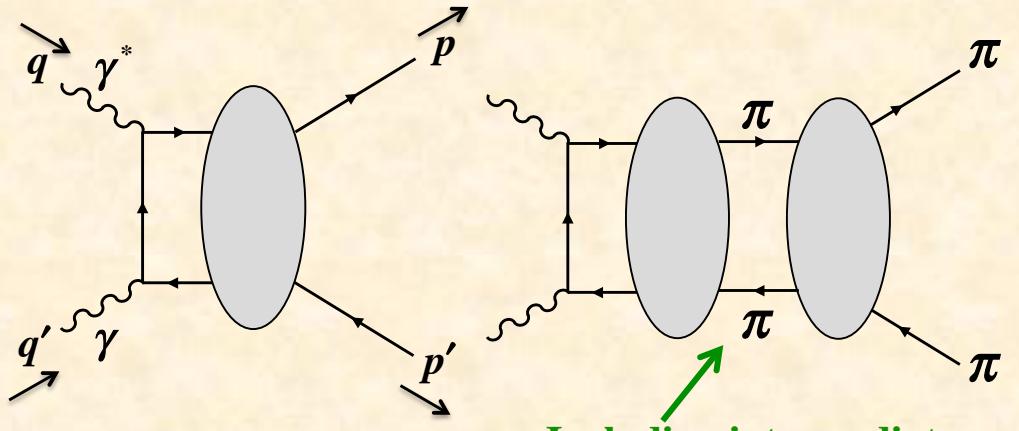
- Resonances: There exist resonance contributions to the cross section.

$$\sum_q \Phi_q^{\pi\pi}(z, \zeta, W^2) = 18N_f z^\alpha (1-z)^\alpha (2z-1) \left[\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos \theta) \right]$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$\tilde{B}_{10}(W)$ = resonance $[f_0(500) \equiv \sigma, f_0(980) \equiv f_0]$ + continuum

$\tilde{B}_{12}(W)$ = resonance $[f_2(1270)]$ + continuum



Including intermediate resonance contributions

$f_0(500)$ or σ [g]
was $f_0(600)$

$J^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400-550)$ MeV
Full width $\Gamma = (400-700)$ MeV

$f_0(980)$ [J]

$J^G(J^{PC}) = 0^+(0^{++})$

Mass $m = 990 \pm 20$ MeV
Full width $\Gamma = 10$ to 100 MeV

$f_2(1270)$

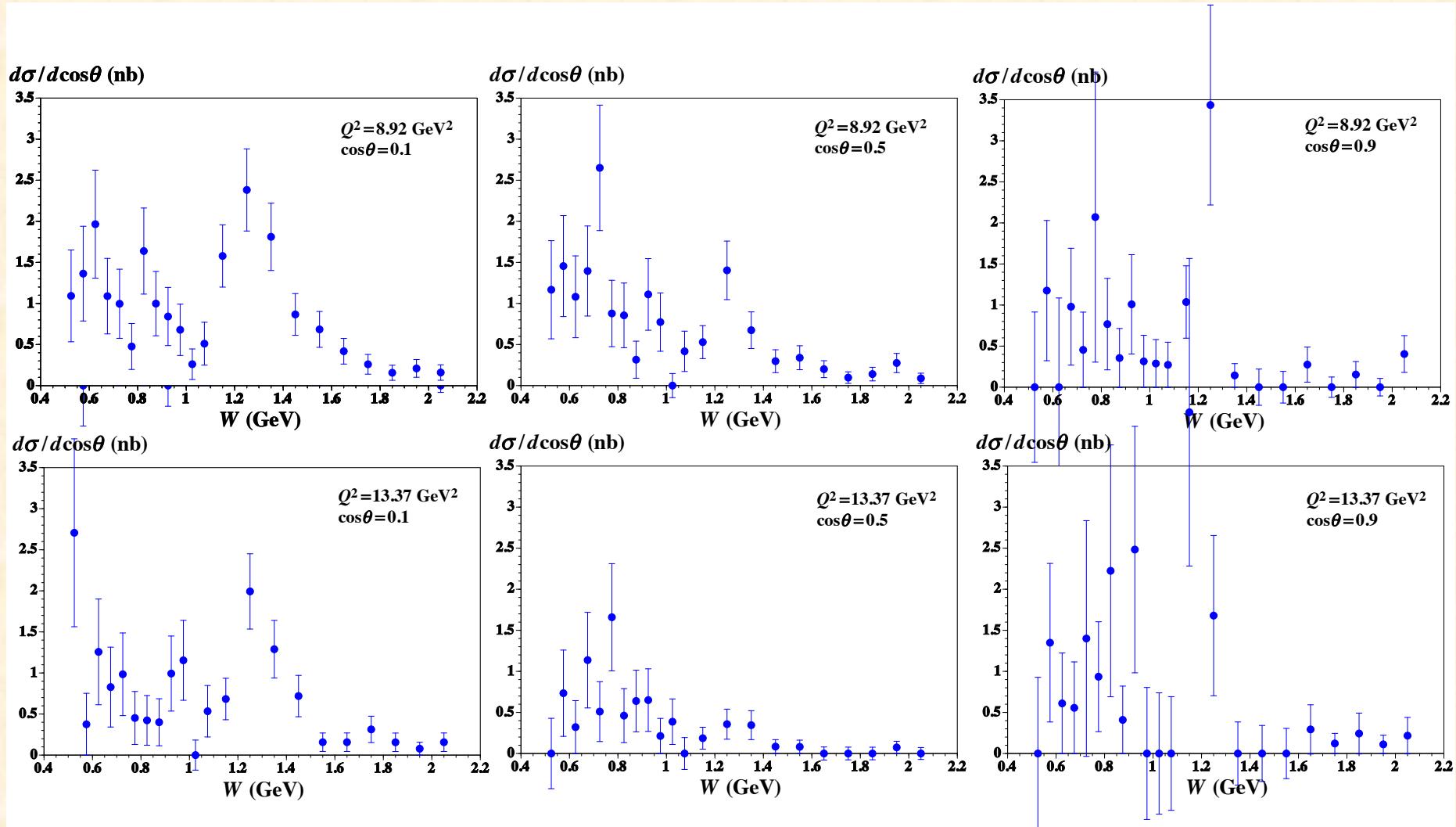
$J^G(J^{PC}) = 0^+(2^{++})$

Mass $m = 1275.5 \pm 0.8$ MeV
Full width $\Gamma = 186.7^{+2.2}_{-2.5}$ MeV (S = 1.4)

Analysis of Belle data on $\gamma\gamma^* \rightarrow \pi^0\pi^0$

$Q^2 = 8.92, 13.37 \text{ GeV}^2$

Belle measurements:
 M. Masuda *et al.*,
 PRD93 (2016) 032003.



Generalized Distribution Amplitudes (GDAs) and gravitational radius for pion

**S. Kumano, Q.-T. Song, O. Teryaev,
KEK-TH-1959, J-PARC-TH-0086,
to be submitted for publication.**

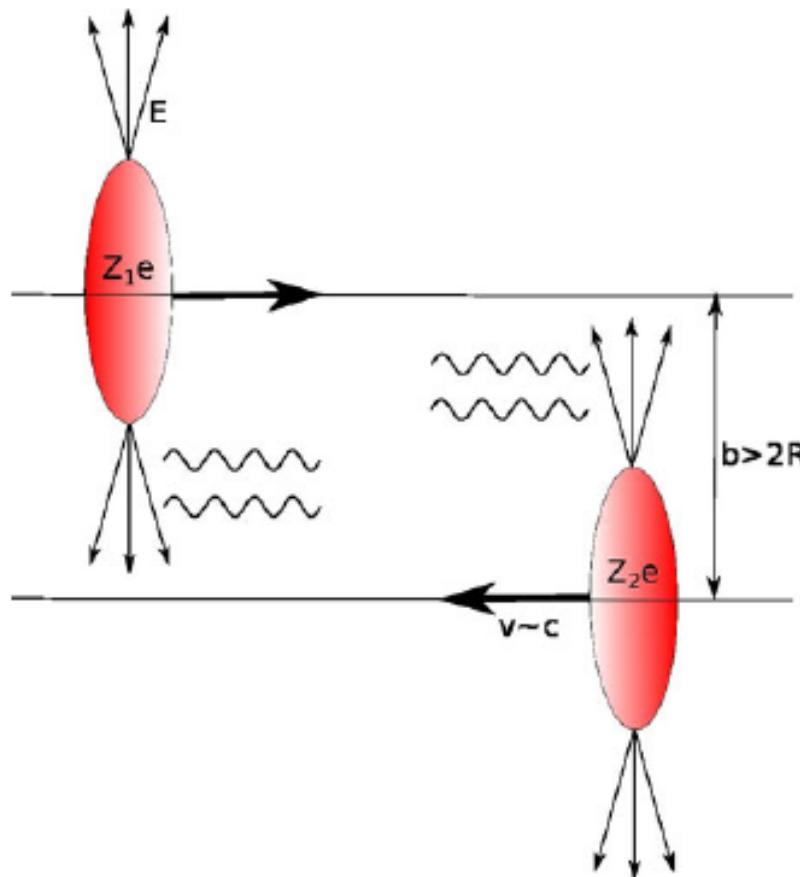
Prospects & Summary

Ultra-Peripheral Collision (UPC) @ LHC/RHIC

INT Workshop INT-17-65W

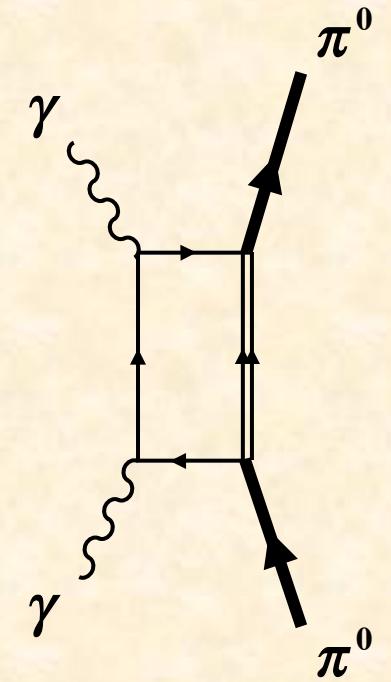
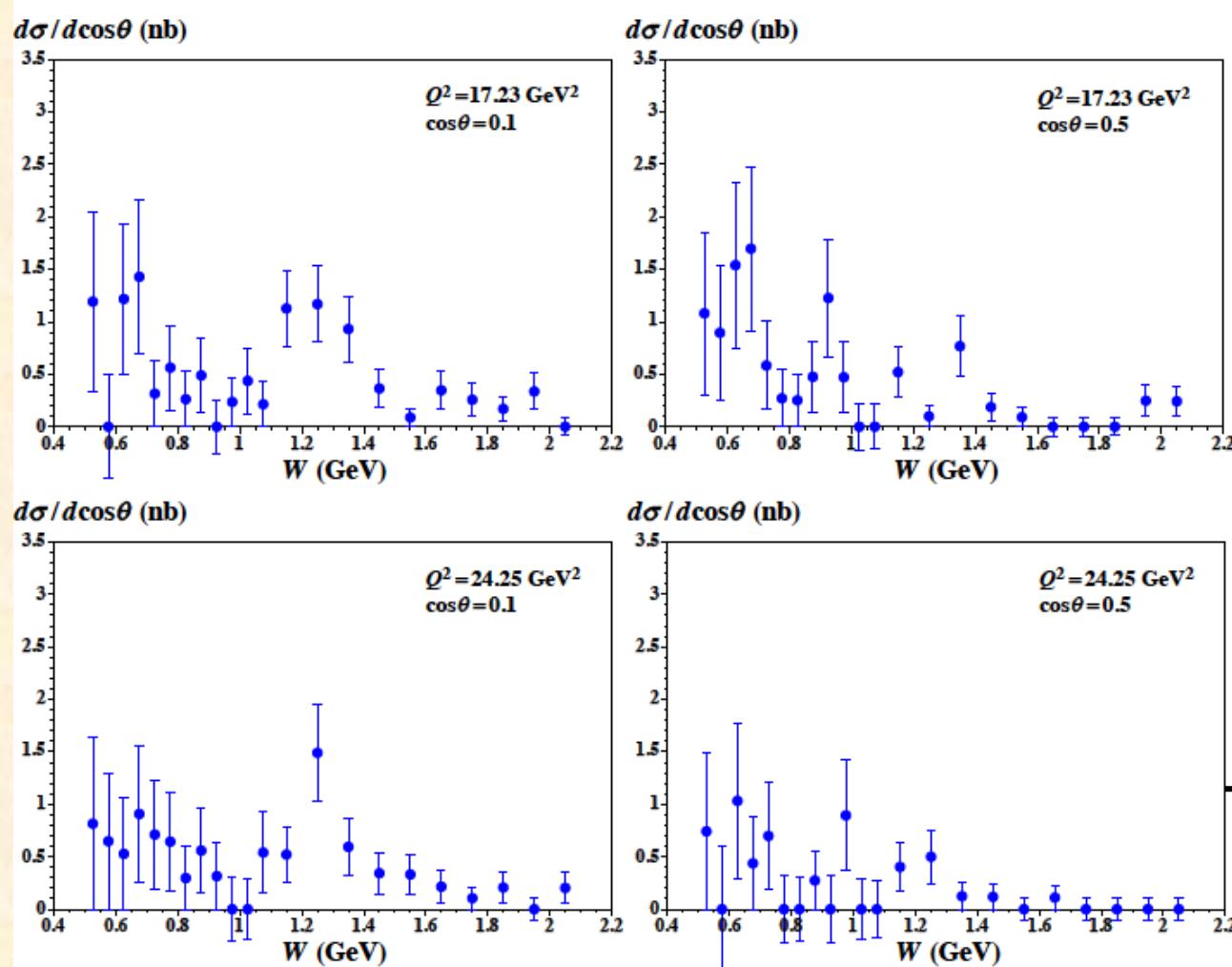
Probing QCD in Photon-Nucleus Interactions at RHIC and LHC: the Path to EIC

February 13 - 17, 2017



KEKB-Belle measurement (2016)

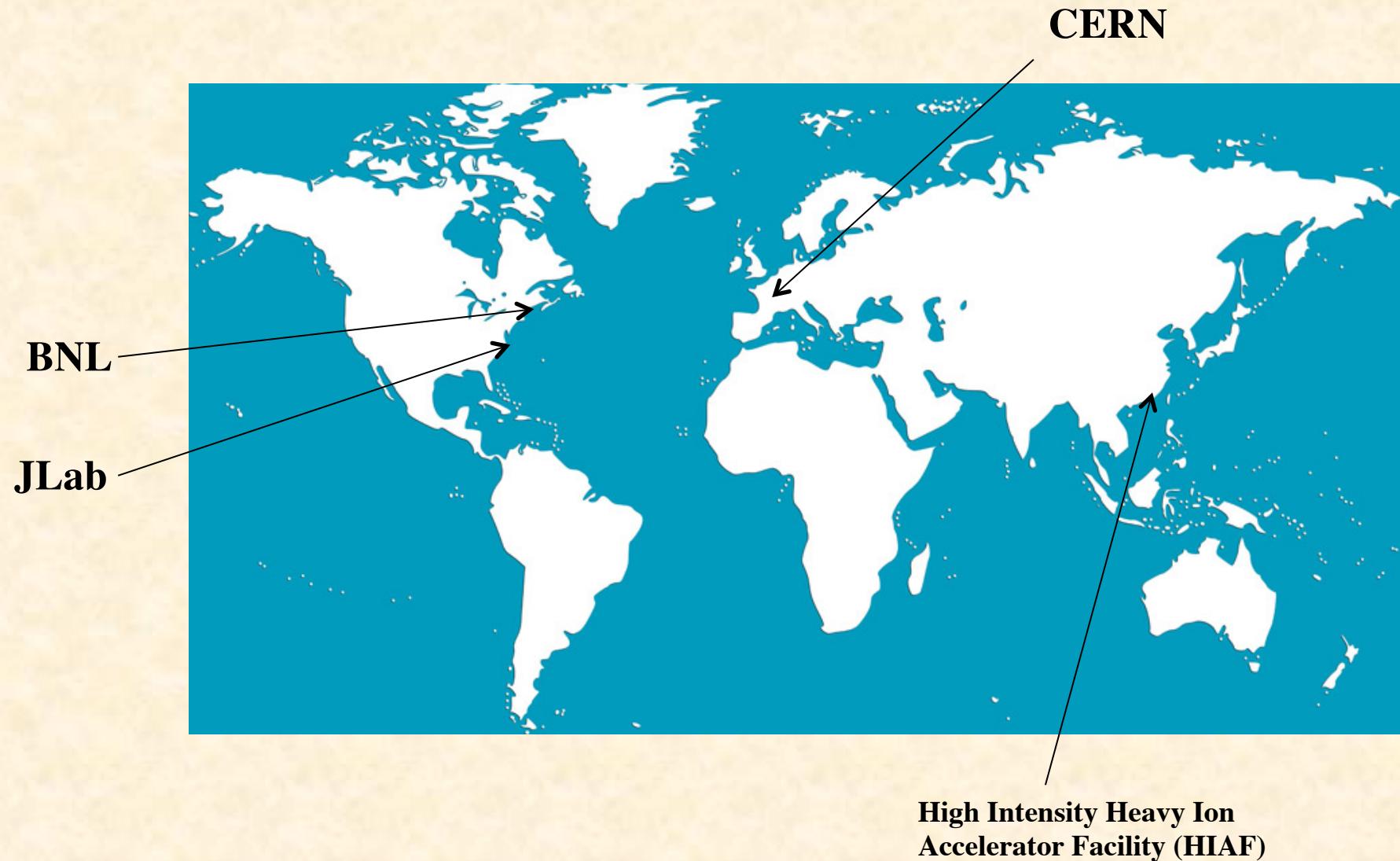
M. Masuda *et al.*, Phys. Rev. D 93 (2016) 032003.



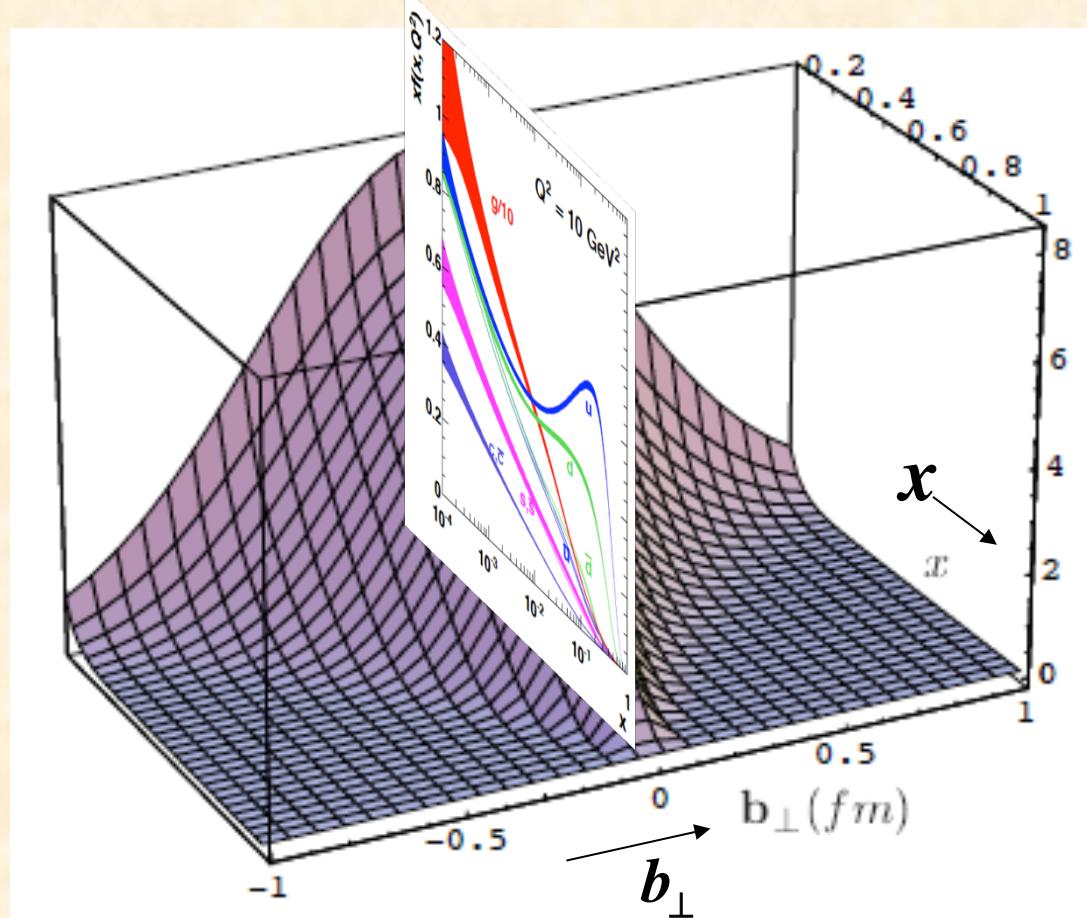
- Very Large Q^2
- Large W^2
- for extracting GDAs



Electron-ion collider



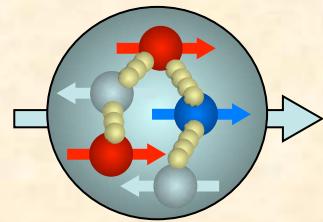
3D view of hadrons



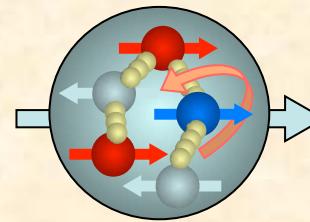
Origin of nucleon spin ...



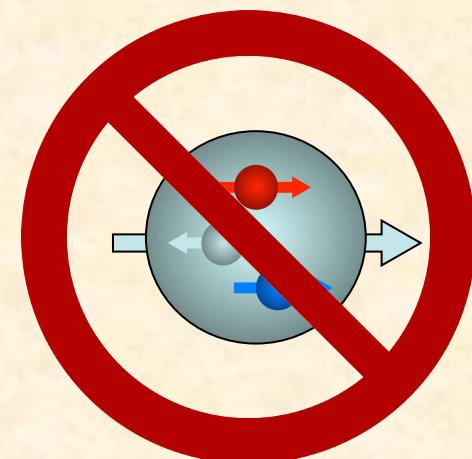
By the tomography, we determine



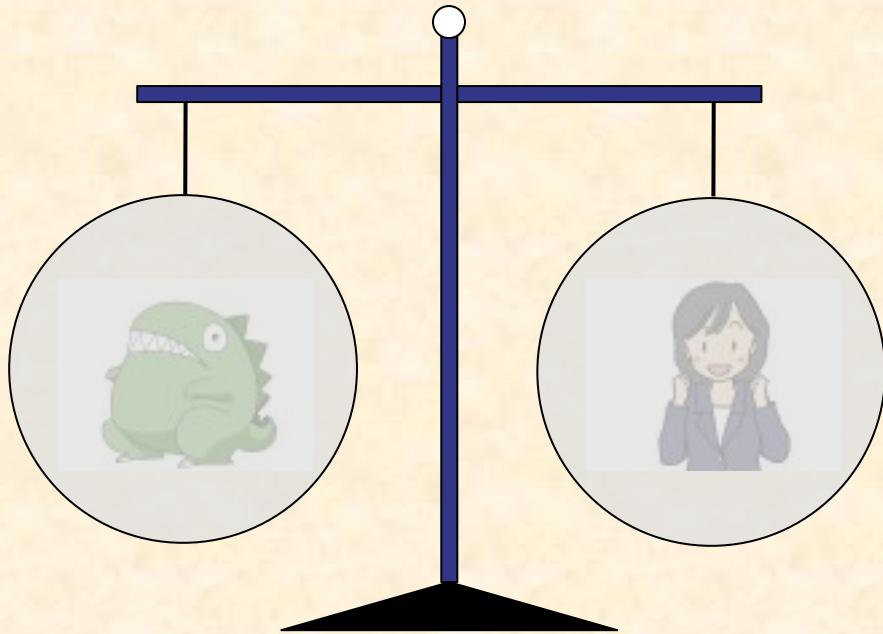
or



.



Search for exotic hadrons ...



It is difficult to determine whether or not a hadron is exotic by low-energy observables, masses, decay widths, ...

(Already, history of a half century)



By the tomography, we determine

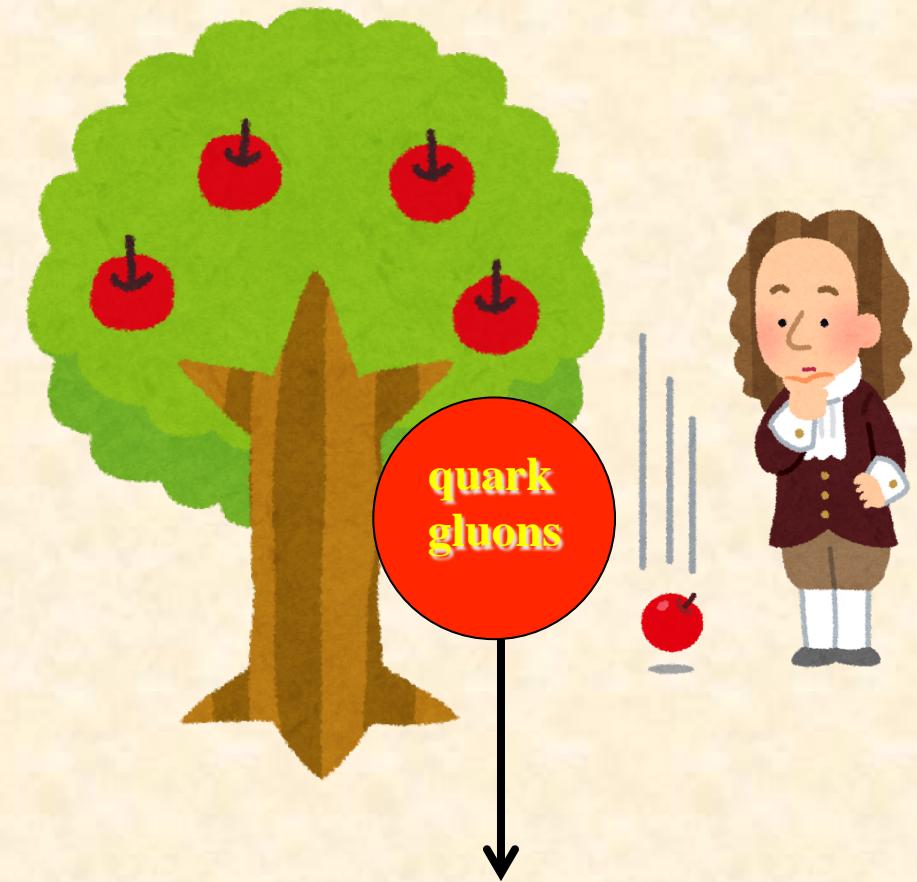


or



.

Origin of gravity in terms of quarks and gluons...



By the tomography, we determine gravitational sources (interactions) with quarks and gluons.

Workshop on Gravitational physics with particle accelerators 2017

Nov.30, 2017, KEK Tokai, Japan

* hadron tomography topics

<http://j-parc-th.kek.jp/workshops/2017/11-30/>

----- Theory -----

Hadron tomography and quark-gluon energy-momentum tensor as a source of gravity

Shunzo Kumano (KEK/J-PARC)

Introduction to theory of gravity for novices, Shunya Mizoguchi (KEK)

Gravitational radius for pion by analysis of KEKB measurements, Qin-Tao Song (SOKENDAI/KEK)

Energy momentum tensor on lattice, Hiroshi Suzuki (Kyushu University)

Generalized parton distribution function studies at J-PARC, Kazuhiro Tanaka (Juntendo Univ/KEK)

----- Experiment -----

Ultracold neutron project at TRIUMF, Shinsuke Kawasaki (KEK)

Meson-pair production in two-photon processes at KEKB, Masataka Masuda (Tokyo University)

Probing strong gravity using geodetic precession in nuclear scale, Jiro Murata (Rikkyo University)

gamma-gamma collisions at ILC, Tomoyuki Sanuki (Tohoku University)

Ultracold neutron project at J-PARC, Hirohiko Shimizu (Nagoya University)

Gravitational effects in muon experiments, Tamaki Yoshioka (Kyushu University)

8th International Conference on Quarks and Nuclear Physics

November 13-17, 2018, Tsukuba, Japan

<http://www-conf.kek.jp/qnp2018/>

Quark and gluon structure of hadrons:

- parton distribution functions, generalized parton distributions,
- transverse momentum distributions, high-energy hadron reactions, ...

Hadron spectroscopy:

- heavy quark physics, exotics, N^* , ...

Hadron interactions and nuclear structure:

- hypernuclear physics, kaonic nuclei, baryon interactions, ...

Hot and cold dense matter:

- quark-gluon plasma, color glass condensate, dense stars,
- strong magnetic field, mesons in nuclear medium, hadronization, ...



Summary

**Hadron tomography studies are important
for solving the origin of the nucleon spin,
for probing internal structure of exotic hadrons,
for probing gravitational sources in quark/gluon level.**

GPDs at J-PARC

GPDs can be investigated by not only DVCS at lepton facilities
but also exclusive reactions at hadron facilities.

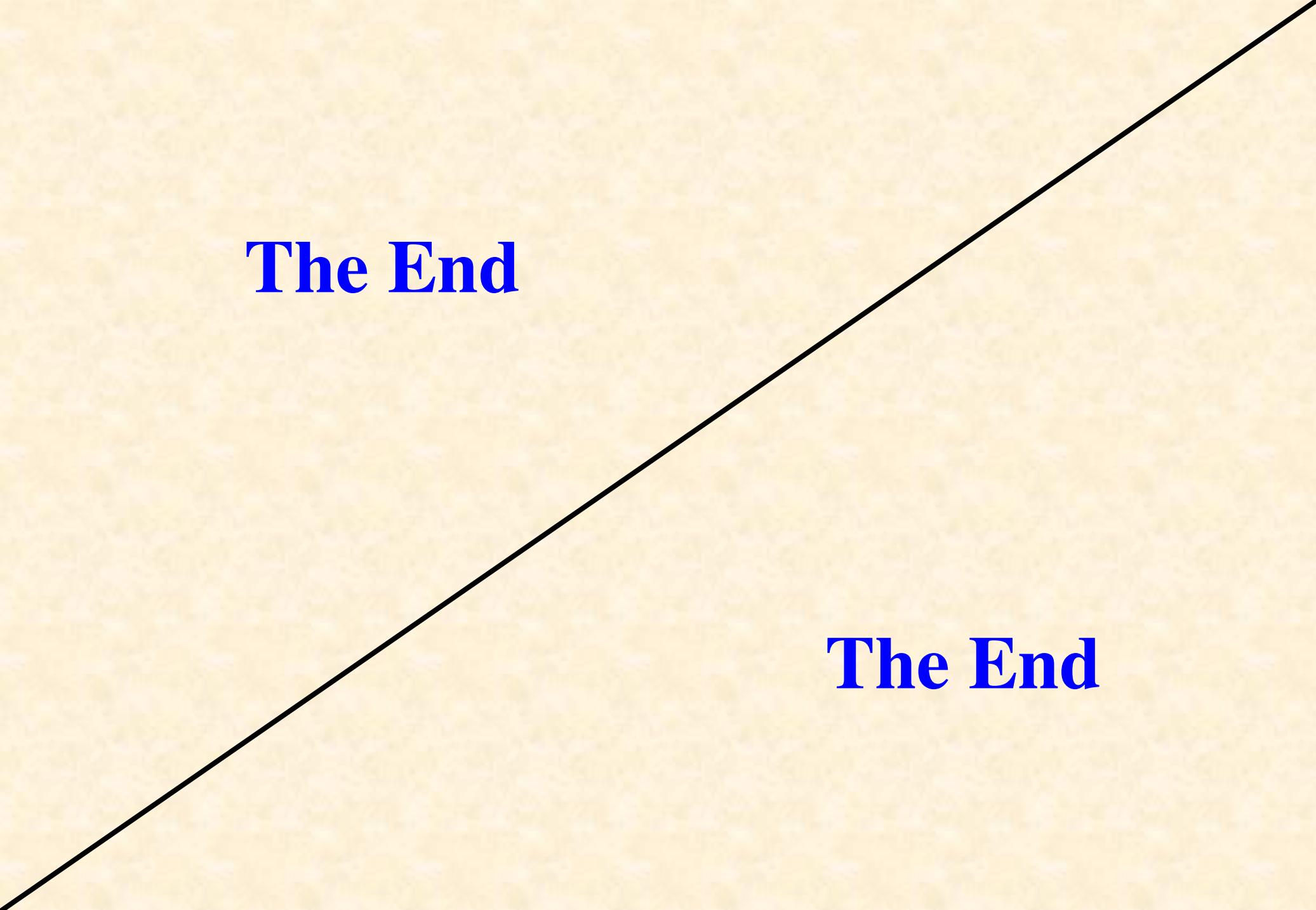
GDAs at KEKB

3D structure of hadrons can be investigated by GDAs ($s \leftrightarrow t$).

Related experimental projects

RHIC, Fermilab, CERN-COMPASS, JLab, BES, ILC,
LHC (UPC), GSI, EIC, LHeC, ...

Gravitational radii can be obtained for hadrons!



The End

The End