# Quasi-bound state in the $\bar{K}NNN$ system

N.V. Shevchenko
*Nuclear Physics Institute, Řež, Czech Republic*

**International conference on exotic atoms and related topics - EXA2017 (September 11-15, 2017, Wien, Austria)**

# On the way to the quasi-bound state in the $\bar{K}NNN$ system

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## $K^-$ pp quasi-bound state – interest to antikaonic nuclei

### Theory

Prediction of the existence of deep and narrow $K^-$ pp bound state

*T. Yamazaki and Y. Akaishi, Phys. Lett. B535 (2002) 70:*

$E_B$ = - 48 MeV, $\Gamma$ = 61 MeV

Many theoretical calculations, different models and inputs (Faddeev, variational calculations, FCA):

$E_B \sim$ -14 - 80 MeV, $\Gamma \sim$ 40 - 110 MeV

- agree only on the fact that the quasi-bound state in $K^-$ pp exists

### Experiment

FINUDA collaboration: $E_B$ = -115 MeV, $\Gamma$ = 67 MeV

*M. Agnello et. al., Phys. Rev. Lett. 94 (2005) 212303*

DISTO collaboration: $E_B$ = -103 MeV, $\Gamma$ = 118 MeV

*T. Yamazaki et al. Phys. Rev. Lett. 104, (2010)*

J-PARC E15 experiment: a broad peak in $^3He(K^-,\Lambda p)n$ spectrum

*H.Tamura, talk at PANIC2017*

## A series of Faddeev calculations with coupled $\bar{K}NN - \pi\Sigma N$ channels

*NVS, J.Révai*

•Three-body pole positions and widths of the quasi-bound states in the $K^-$ pp and $K^-$ $K^-$ p systems were evaluated;

- No quasi-bound state found in the $K^-$ d system (caused by strong interaction)

| |
|---|
| $K^-pp$ pole positions (MeV): |
| $-53.3 - i\,32.4$ (1pole phen) , $-47.4 - i\,24.9$ (2pole phen), $-32.2 - i\,24.3$ (chiral) |

•Near-threshold elastic $K^-$ d amplitudes were calculated (including the $K^-$ d sc.l)
→ and then used for an approximate calculation of the *1s* level shift and width of (anti)kaonic deuterium

| |
|---|
| *1s* level shift and width (eV) of kaonic deuterium: |
| ($-$785,1018) (1pole phen), ($-$797,1025) (2pole phen), ($-$828,1055) (chiral) |

Antikaon-nucleon potentials used:

- Phenomenological $\bar{K}N - \pi\Sigma$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials

- reproducing SIDDHARTA data on kaonic hydrogen *1s* level shift and width together with the scattering $K^- p$ data with the same level of accuracy

## Three-body Faddeev equations in Alt-Grassberger-Sandhas form

$$U_{\alpha\beta}(z) = (1-\delta_{\alpha\beta})(G_0(z))^{-1} + \sum_{\gamma=1}^{3}(1-\delta_{\alpha\gamma})T_\gamma(z)\,G_0(z)\,U_{\gamma\beta}(z), \qquad \alpha,\beta = 1,2,3$$

$U_{\alpha\beta}(z)$ - 3-body transition operators $\beta+(\alpha\gamma)\to\alpha+(\beta\gamma)$
$G_0(z)$ - free Green function
$T_\alpha(z)$ - 2-body $T$-marix

A separable potential leading to a separable $T$-marix

$$V_\alpha = \lambda_\alpha |g_\alpha\rangle\langle g_\alpha| \Rightarrow T_\alpha(z) = |g_\alpha\rangle\,\tau_\alpha(z)\,\langle g_\alpha|$$

allows to write the three-body equations in the form

$$X_{\alpha\beta}(z) = Z_{\alpha\beta}(z) + \sum_{\gamma=1}^{3} Z_{\alpha\gamma}(z)\,\tau_\gamma(z)\,X_{\gamma\beta}(z)$$

with $X_{\alpha\beta}(z) = \langle g_\alpha | G_0(z)\,U_{\alpha\beta}(z)\,G_0(z) | g_\beta\rangle$, $Z_{\alpha\beta}(z) = (1-\delta_{\alpha\beta})\langle g_\alpha | G_0(z) | g_\beta\rangle$

## Four-body Alt-Grassberger-Sandhas equations

*P. Grassberger, W. Sandhas, Nucl. Rev. B2 (1967) 181*

$$U^{\sigma\rho}_{\alpha\beta}(z) = \left(1-\delta_{\sigma\rho}\right)\delta_{\alpha\beta}G_0^{-1}(z)\,T_\alpha^{-1}(z)G_0^{-1}(z) +$$

$$+\sum_{\tau,\gamma}\left(1-\delta_{\sigma\tau}\right)U^{\tau}_{\alpha\gamma}(z)G_0(z)\,T_\gamma(z)G_0(z)U^{\tau\rho}_{\gamma\beta}(z)$$

$U^{\sigma\rho}_{\alpha\beta}(z)$ - 4-body transition operators
$U^{\tau}_{\alpha\beta}(z)$ - 3-body transition operators
$G_0(z)$ - free Green function
$T_\alpha(z)$ - 2-body *T*-marix

A separable potential → separable T-marix → four-body equations:

$$\overline{U}^{\sigma\rho}(z) = \left(1-\delta_{\sigma\rho}\right)\left(\overline{G_0}(z)\right)^{-1} + \sum_{\tau}\left(1-\delta_{\sigma\tau}\right)\overline{T}^{\tau}(z)\,\overline{G_0}(z)\,\overline{U}^{\tau\rho}(\mathrm{z}),$$

with $\overline{U}^{\sigma\rho}_{\alpha\beta}(\mathrm{z}) = \left\langle g_\alpha \,|\, G_0(z)\,U^{\sigma\rho}_{\alpha\beta}(z)\,G_0(z)\,|\, g_\beta\right\rangle$

$\overline{T}^{\tau}_{\alpha\beta}(z) = \left\langle g_\alpha \,|\, G_0(z)\,U^{\tau}_{\alpha\beta}(z)\,G_0(z)\,|\, g_\beta\right\rangle$

and $\overline{(G_0)}_{\alpha\beta}(z) = \delta_{\alpha\beta}\tau_\alpha(z)$

# Four-body AGS equations for separable potentials

*A.Casel, H.Haberzettl, W. Sandhas, Phys. Rev. C25 (1982) 1738*

$$\overline{U}^{\sigma\rho}(z) = (1-\delta_{\sigma\rho})\left(\overline{G_0}(z)\right)^{-1} + \sum_{\tau}(1-\delta_{\sigma\tau})\overline{T}^{\tau}(z)\,\overline{G_0}(z)\,\overline{U}^{\tau\rho}(\mathrm{z})$$

look similar to the three-body AGS equations in the general form.
Separable form of the "effective potentials" → separable "T"-matrix:

$$\overline{T^{\tau}_{\alpha\beta}}(z) = \left|\overline{g}^{\tau}_{\alpha}\right\rangle \overline{\tau}^{\tau}_{\alpha\beta}(\mathrm{z}) \left\langle \overline{g}^{\tau}_{\beta}\right|$$

allows to write the four-body equations in the form

$$\overline{X}^{\sigma\rho}(z) = \overline{Z}^{\sigma\rho}(z) + \sum_{\tau}\overline{Z}^{\sigma\tau}(\mathrm{z})\,\overline{\tau}^{\tau}(\mathrm{z})\,\overline{X}^{\tau\rho}(z)$$

with
$$\overline{X}^{\sigma\rho}(z) = \left\langle \overline{g}^{\sigma} \,|\, \overline{G_0}\,\overline{U}^{\sigma\rho}(z)\,\overline{G_0} \,|\, \overline{g}^{\rho}\right\rangle,$$
$$\overline{Z}^{\sigma\rho}(z) = (1-\delta_{\sigma\rho})\left\langle \overline{g}^{\sigma} \,|\, \overline{G_0}(z) \,|\, \overline{g}^{\rho}\right\rangle$$

## 4-body equations for the $\bar{K}NNN$ system

$$
\begin{aligned}
\bar{X}_1 &= \bar{Z}_{12}\bar{\tau}_2\bar{X}_2 + \bar{Z}_{13}\bar{\tau}_3\bar{X}_3 \\
\bar{X}_2 &= \bar{Z}_{21} + \bar{Z}_{21}\bar{\tau}_1\bar{X}_1 + \bar{Z}_{22}\bar{\tau}_2\bar{X}_2 + \bar{Z}_{23}\bar{\tau}_3\bar{X}_3 \\
\bar{X}_3 &= \bar{Z}_{31} + \bar{Z}_{31}\bar{\tau}_1\bar{X}_1 + \bar{Z}_{32}\bar{\tau}_2\bar{X}_2
\end{aligned}
$$

Two types of partitions: 3+1 and 2+2:

$$
\left|\bar{K} + (NNN)\right\rangle \quad \left|N + (\bar{K}NN)\right\rangle \quad \left|(\bar{K}N) + (NN)\right\rangle
$$

Initial channel $\left|\bar{K} + (NNN)\right\rangle$ is fixed

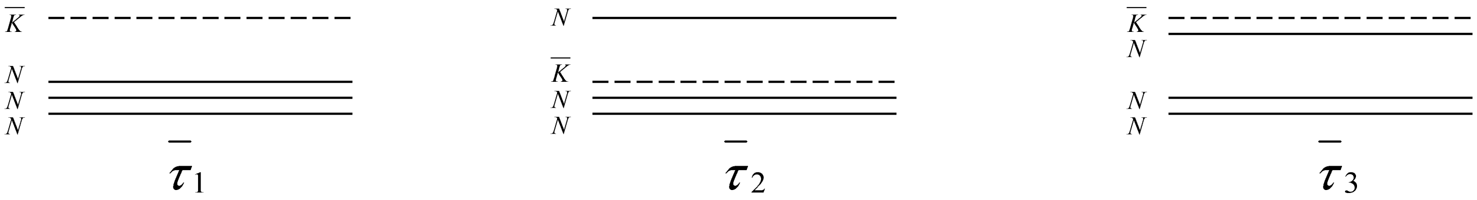
The channels:

channel 1: $\left|\bar{K} + (N_1 N_2 N_3)\right\rangle$

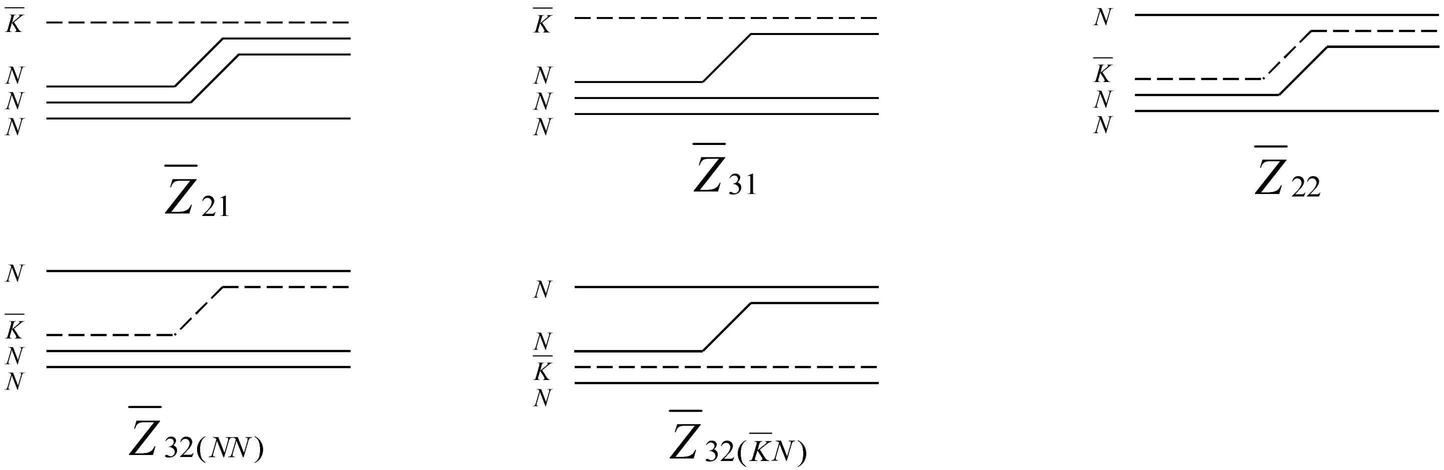
channel $2_1$: $\left|N_1 + (\bar{K}N_2 N_3)\right\rangle$, $2_2$: $\left|N_2 + (\bar{K}N_3 N_1)\right\rangle$, $2_3$: $\left|N_3 + (\bar{K}N_1 N_2)\right\rangle$

channel $3_1$: $\left|(\bar{K}N_1) + (N_2 N_3)\right\rangle$, $3_2$: $\left|(\bar{K}N_2) + (N_3 N_1)\right\rangle$, $3_3$: $\left|(\bar{K}N_3) + (N_1 N_2)\right\rangle$

# Propagators $\tau$:




# $Z$ operators:



# Separabelization of potentials - Hilbert-Schmidt expansion

Lippmann-Schwinger equation:

$$T(p,p';z) = V(p,p') + 4\pi \int_0^\infty \frac{V(p,p'')T(p'',p';z)}{z - p''^2/(2\mu)} p''^2 \, dp''$$

Separable potential leads to the separable *T*-matrix

$$V(p,p') = -\sum_{n=1}^{\infty} \lambda_n g_n(p) g_n(p') \;\Rightarrow\; T(p,p';z) = -\sum_{n=1}^{\infty} \frac{\lambda_n}{1-\lambda_n} g_n(p) g_n(p')$$

were the eigenvalues $\lambda_n$ and eigenfunctions $g_n(p)$ are found from

$$g_n(p) = \frac{1}{\lambda_n} 4\pi \int_0^\infty \frac{V(p,p'')g_n(p'')}{z - p''^2/(2\mu)} p''^2 \, dp''$$

with normalization condition $$4\pi \int_0^\infty \frac{g_n(p'')g_{n'}(p'')}{z - p''^2/(2\mu)} p''^2 \, dp'' = -\delta_{nn'}$$

## Separabelization of "potentials" - Hilbert-Schmidt expansion

Three-body AGS equations:

$$X_{\alpha\beta}(p,p';z) = Z_{\alpha\beta}(p,p';z) + \sum_{\gamma=1}^{3} 4\pi \int_0^\infty Z_{\alpha\gamma}(p,p'';z)\tau_\gamma(p'';z)X_{\gamma\beta}(p'',p';z)p''^2\,dp''$$

Separable "potential" leads to the separable "*T*-matrix"

$$Z_{\alpha\beta}(p,p';z) = -\sum_{n=1}^{\infty}\lambda_n g_{n\alpha}(p)g_{n\beta}(p') \;\Rightarrow\; X_{\alpha\beta}(p,p';z) = -\sum_{n=1}^{\infty}\frac{\lambda_n}{1-\lambda_n}g_{n\alpha}(p)g_{n\beta}(p')$$

were the eigenvalues $\lambda_n$ and eigenfunctions $g_{n\alpha}(p)$ are found from

$$g_{n\alpha}(p) = \frac{1}{\lambda_n}\sum_{\gamma=1}^{3} 4\pi \int_0^\infty Z_{\alpha\gamma}(p,p'';z)\tau_\gamma(p'';z)g_{n\gamma}(p'',p';z)p''^2\,dp''$$

with normalization condition

$$\sum_{\gamma=1}^{3} 4\pi \int_0^\infty g_{n\gamma}(p,p'';z)\tau_\gamma(p'';z)g_{n'\gamma}(p'',p';z)p''^2\,dp'' = -\delta_{nn'}$$

*Z* functions: momentum, isospin and spin parts

$$\overline{Z}_{\alpha\beta} = Z_{\alpha\beta}(p, p'; z)\ {}_I Z_{\alpha\beta, I_\alpha I_\beta}\ {}_S Z_{\alpha\beta, S_\alpha S_\beta}$$

## Quantum numbers

$K^- ppn:\ I^{(4)} = 0, S^{(4)} = 1/2,$ orbital momentum $L^{(4)} = 0$

## Three nucleons - antisymmetrization

$$\overline{X}_1 = \sum_{n2=1}^{3} \overline{Z}_{12_{n2}} \overline{\tau}_{2_{n2}} \overline{X}_{2_{n2}} + \sum_{n3=1}^{3} \overline{Z}_{13_{n3}} \overline{\tau}_{3_{n3}} \overline{X}_{3_{n3}}$$

$$\overline{X}_{2_{n2'}} = \overline{Z}_{21} + \overline{Z}_{2_{n2'}1} \overline{\tau}_1 \overline{X}_1 + \sum_{n2=1}^{3} \overline{Z}_{2_{n2'}2_{n2}} \overline{\tau}_{2_{n2}} \overline{X}_{2_{n2}} + \sum_{n3=1}^{3} \overline{Z}_{2_{n2'}3_{n3}} \overline{\tau}_{3_{n3}} \overline{X}_{3_{n3}}$$

$$\overline{X}_{3_{n3'}} = \overline{Z}_{3_{n3'}1} + \overline{Z}_{3_{n3'}1} \overline{\tau}_1 \overline{X}_1 + \sum_{n2=1}^{3} \overline{Z}_{3_{n3'}2_{n2}} \overline{\tau}_{2_{n2}} \overline{X}_{2_{n2}}$$

where *n2, n3* – indices of the particular nucleons

## $\bar{K}N$ interaction with coupled $\pi\Sigma$ and $\pi\Lambda$ channels

Potentials were fitted to the experimental data

- $1s$ level shift and width of kaonic hydrogen (by SIDDHARTA)

$$\Delta_{1s}^{SIDD} = -283 \pm 36 \pm 6\ \text{eV}, \quad \Gamma_{1s}^{SIDD} = 541 \pm 89 \pm 22\ \text{eV}$$

- Cross-sections of $K^- p \to K^- p$ and $K^- p \to MB$ reactions
- Threshold branching ratios $\gamma$, $R_c$ and $R_n$
- ◦ $\Lambda(1405)$ with one- or two-pole structure

$$M_{\Lambda(1405)}^{PDG} = 1405.1_{-1.0}^{+1.3}\ \text{MeV}, \quad \Gamma_{\Lambda(1405)}^{PDG} = 50.5 \pm 2.0\ \text{MeV}$$

The "exact optical" versions of the:

- Phenomenological $\bar{K}N - \pi\Sigma$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potentials

constructed and used for the three-body calculations

## Two-term *NN* potential

reproduces: Argonne V18 *NN* phase shifts (with sign change)

## Solution of the four-body equations:

$$\bar{X}_{\alpha} = \bar{Z}_{\alpha\beta} + \sum_{\gamma=1}^{3} \bar{Z}_{\alpha\gamma}\, \bar{\tau}_{\gamma}\, \bar{X}_{\gamma}, \;\; \alpha = 1,2,3; \;\; \beta = 1$$

1. Calculate separable 3-body “T-matrices”: evaluate eigenvalues and eigenfunctions for the $\bar{K}NN$ and $NNN$ subsystems
2. Evaluate momentum, isospin and spin parts of the 4-body “potentials” $Z$
3. Solve the homogeneus system of 4-body equations in the complex plane

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<u>Formulas:</u> written 

<u>Numerical computation program:</u> 


<u>Results:</u> will be soon