Onset of $\eta$-meson binding

Exotic Atoms & Related Topics (EXA2017), Vienna

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$\eta$ nuclear quasibound states

E. Friedman, A. Gal, J. Mareš, PLB 725 (2013) 334
A. Cieplý, E. Friedman, A. Gal, J. Mareš, NPA 925 (2014) 126


Onset of $\eta$ nuclear binding in He

N. Barnea, B. Bazak, E. Friedman, A. Gal, PLB 771 (2017) 297
N. Barnea, E. Friedman, A. Gal, NPA 968 (2017) 35
Background & Motivation

- The $\eta N$ s-wave interaction below $N^*(1535)$ is attractive in a $\pi N - \eta N$ model [Bhalelao–Liu (1985)]. Bound states of $\eta(548)$ in $A \geq 12$ nuclei could exist [Haider–Liu (1986)].

- Chiral $N^*(1535)$ meson-nucleon coupled channel models were introduced by Kaiser, Weise et al (1995-1997) and subsequently by Oset et al (2002). These & other models have been used to calculate $\eta$–nuclear quasibound states.

- Exp. searches for such states with proton, pion or photon induced $\eta$ production reactions are inconclusive. For the onset of binding, Krusche & Wilkin (2015) state: “The most straightforward (but not unique) interpretation of the data is that the $\eta d$ system is unbound, the $\eta^4$He is bound, but that the $\eta^3$He case is ambiguous.”
Hints from $\eta^3$He production

Fitted $dp \rightarrow \eta^3$He x-sections below 2 MeV vs. experiment. Remarkable energy dependence, suggesting a nearby S-matrix pole could be in action. Deduced $a(\eta^3$He) excludes a quasibound state pole.

Xie-Liang-Oset-Moskal-Skurzok-Wilkin, PRC 95 (2017) 015202

$a(\eta^3$He) $= [- (2.23 \pm 1.29) + i (4.89 \pm 0.57)]$ fm

- Would $\eta^4$He be bound? NOT seen in $dd \rightarrow ^3$He+N+$\pi$
  [WASA-at-COSY NPA 959 (2017) 102]. Argued to be more UNBOUND than $\eta^3$He [Fix-Kolesnikov, PLB 772 (2017) 663] owing to a stronger subthreshold suppression in $^4$He.
$\eta$ nuclear quasibound states
\( f_{\eta N}(\sqrt{s}) \) from \( K \)-matrix & \( N^*(1535) \) chiral models

\[
\begin{array}{c|ccccc}
\text{a} & \text{M1} & \text{M2} & \text{GW} & \text{GR} & \text{CS} \\
\hline
\text{Re} & 0.22 & 0.38 & 0.96 & 0.26 & 0.67 \\
\text{Im} & 0.24 & 0.20 & 0.26 & 0.24 & 0.20 \\
\end{array}
\]

Mai et al. PRD 86 (2012) 094033
Green-Wycech PRC 71 (2005) 014001
Garcia-Recio et al. PLB 550 (2002) 47
Cieply-Smejkal, NPA 919 (2013) 46

- Re \( a_{\eta N} \) varies from 0.2 to 1.0 fm, Im \( a_{\eta N} \): 0.2–0.3 fm.
- M1, M2, GW free-space models; GR, CS in-medium.
- Strong subthreshold fall-off in both Re \( f_{\eta N} \) and Im \( f_{\eta N} \).
- In-medium: E dependence, Pauli blocking, self energies.
Self-consistency in mesic-atom & nuclear calculations
Cieplý-Friedman-Gal-Gazda-Mareš, PLB 702 (2011) 402

\[ s_{\eta N} = (\sqrt{s_{\text{th}}} - B_\eta - B_N)^2 - (\vec{p}_\eta + \vec{p}_N)^2 < s_{\text{th}} \]

\[ \sqrt{s_{\eta N}} \rightarrow E_{\text{th}} - B_N - B_\eta - \xi_N \frac{p_N^2}{2m_N} - \xi_\eta \frac{p_\eta^2}{2m_\eta} \]

\[ \xi_N(\eta) = \frac{m_N(\eta)}{(m_N+m_\eta)} \quad \frac{p_\eta^2}{2m_\eta} \sim -V_\eta - B_\eta \]

\( \eta \) is not at rest!

\( E_{\eta N} \) subthreshold shift vs. nuclear density in 1s\( ^{40} \)Ca.

A dominant in-medium effect.
In-medium $\eta N$ amplitudes

Friedman-Gal-Mareš, PLB 725 (2013) 334

Cieplý-Friedman-Gal-Mareš, NPA 925 (2014) 126

• KG equation and self-energies:
\[
\left[ \nabla^2 + \tilde{\omega}_\eta^2 - m_\eta^2 - \Pi_\eta(\omega_\eta, \rho) \right] \psi = 0
\]
\[
\tilde{\omega}_\eta = \omega_\eta - i\Gamma_\eta/2, \quad \omega_\eta = m_\eta - B_\eta
\]
\[
\Pi_\eta(\omega_\eta, \rho) \equiv 2\omega_\eta V_\eta = -4\pi \frac{\sqrt{s}}{m_N} f_{\eta N}(\sqrt{s}, \rho) \rho
\]

• Pauli blocking (Waas-Rho-Weise NPA 617 (1997) 449):
\[
f_{\eta N}^{WRW}(\sqrt{s}, \rho) = \frac{f_{\eta N}(\sqrt{s})}{1 + \xi(\rho)(\sqrt{s}/m_N)f_{\eta N}(\sqrt{s})\rho}, \quad \xi(\rho) = \frac{9\pi}{4p_F^2} I(\tilde{\omega}_\eta)
\]

• $N^*(1535) \Rightarrow$ energy dependent $f_{\eta N}(\sqrt{s})$.

In medium $\Rightarrow$ go subthreshold:
\[
\delta \sqrt{s} = \sqrt{s} - \sqrt{s_{th}}
\]
\[
\delta \sqrt{s} \approx -B_N \frac{\rho}{\rho_0} - \xi_N B_\eta \frac{\rho}{\rho_0} - \xi_N T_N \left( \frac{\rho}{\rho_0} \right)^{2/3} + \xi_\eta \text{Re} V_\eta(\sqrt{s}, \rho)
\]

• A self-consistency cycle in $\delta \sqrt{s}$ for given $\rho$. 
Model dependence I

\begin{itemize}
  \item E dependence treated self consistently.
  \item Larger Re $a_{\eta N} \Rightarrow$ larger $B_\eta$.
  \item Widths are unrelated to Im $a_{\eta N}$.
\end{itemize}
Model dependence II

Sensitivity of calc. $B_{1s\eta}$ & $\Gamma_{1s\eta}$ to self-consistency version

- $\langle \delta \sqrt{s} \rangle$ goes deeper into subthreshold, thereby reducing further $B_{1s\eta}$ & $\Gamma_{1s\eta}$.

- GR’s widths are too large to resolve $\eta$ bound states.

Why $\Gamma_{\eta}(GR) \gg \Gamma_{\eta}(CS)$ for similar Im $a_{\eta N}$?
Model predictions for small widths

- Widths of only a few MeV in each of these models.
- What makes the subthreshold values of $\text{Im } f_{\eta N}$ sufficiently small to generate small widths?
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CM s-wave scattering amplitude $F_{\eta N}(E)$ in two meson-baryon coupled-channel $N^*(1535)$ models.

$a_{\eta N}^{GW} = 0.96 + i0.26$ fm, $a_{\eta N}^{CS} = 0.67 + i0.20$ fm

- Derive local, energy dependent potentials $v_{\eta N}(E;r)$ that reproduce $F_{\eta N}(E)$ below threshold, for use in solving the $\eta NN$, $\eta NNN$, $\eta NNNN$ few-body Schroedinger equations.
\[ F_{\eta N}(E) \Rightarrow v_{\eta N}(E) \text{ in models GW & CS} \]

**Strength \( b(E) \) of effective potential \( v_{\eta N}(E) \) at \( E < 0 \)**

\[
v_{\eta N}(E;r) = -\frac{4\pi}{2\mu_{\eta N}} \ b(E) \left( \frac{\Lambda}{2\sqrt{\pi}} \right)^3 \exp \left( -\frac{\Lambda^2 r^2}{4} \right)
\]

- Scale \( \Lambda \) is inversely proportional to the range of \( v_{\eta N} \).
- \( v_{\eta N} \) is a regulated contact term in \( \pi \)-less EFT which for \( \Lambda \leq m_\rho \approx 4 \text{ fm}^{-1} \) replaces vector-meson exchange.
Energy dependence in $\eta$ nuclear few-body systems

$N^*(1535)$ makes near-threshold $f_{\eta N}(\sqrt{s})$ & input potential $v_{\eta N}(\sqrt{s})$ strongly energy dependent.

$s = (\sqrt{s}_{\text{th}} - B_\eta - B_N)^2 - (\vec{p}_\eta + \vec{p}_N)^2 < s_{\text{th}}$

Expanding NR near $\sqrt{s}_{\text{th}}$ & evaluating $\langle \delta \sqrt{s} \rangle$:

$$\langle \delta \sqrt{s} \rangle = -\frac{B}{A} + \frac{A-1}{A} E_\eta - \xi_N \frac{1}{A} \langle T_A \rangle - \xi_\eta \left( \frac{A-1}{A} \right)^2 \langle T_\eta \rangle,$$

$\delta \sqrt{s} \equiv \sqrt{s} - \sqrt{s}_{\text{th}}, \quad E_\eta = \langle H - H_A \rangle, \quad \xi_N(\eta) \equiv \frac{m_N(\eta)}{m_N + m_\eta}.$

Agrees to $O(1/A)$ with optical-model limit.

Self-consistency: output $\langle \sqrt{s} \rangle =$ input $\sqrt{s}$.

Near threshold $E_\eta \& \langle T_\eta \rangle \to 0$, yet $\langle \delta \sqrt{s} \rangle_{\text{th}} \neq 0$.

Similarly, $\langle \delta \sqrt{s} \rangle_{\text{th}} \neq 0$ in kaonic atoms, starting with $K^- d$: $\langle \delta \sqrt{s} \rangle_{\text{th}} = -\frac{B_d}{2} - \frac{0.655}{2} \langle T_d \rangle = -4.9 \text{ MeV}$. 

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Recent SVM results for $\eta^{3,4}\text{He}$

**Self consistency plot**

$\eta^4\text{He}$ bound-state energy $E$, $\langle \delta \sqrt{s} \rangle$ & $\langle H_N = H_A \rangle$, for AV4' $v_{NN}$ & GW $v_{\eta N}(E)$ with scale $\Lambda = 4$ fm$^{-1}$.

- **Stochastic Variational Method** calculations with correlated Gaussian trial wavefunctions, resulting in:
  - $\eta_d$ is definitely unbound in both GW and CS (2015).
  - $\eta^3\text{He}$ is nearly or just bound in GW & unbound in CS.
  - $\eta^4\text{He}$ is bound in GW and just or nearly bound in CS.
These bindings will decrease by \( \leq 0.3 \) MeV when \( \text{Im v} \) is added. GW just binds \( \eta^3 \text{He} \), & definitely binds \( \eta^4 \text{He} \).

AV4p (Argonne) more realistic than MNC (Minnesota).

CS does not bind \( \eta^3 \text{He} \) & is unlikely to bind \( \eta^4 \text{He} \).
Scale dependence; pionless EFT at LO

$B_{\eta}(\Lambda)$ using $v_{\eta N}(E)$: with & w/o self consistency

- Nuclear dynamics generated from two NN & one NNN contact terms (CT). NNN CT averts $^3$He collapse.
- Add one $\eta N$ & one $\eta NN$ CT; given no $\eta NN$ datum, use $CT(\eta NN) = CT(\text{NNN})$ to start with.
Pionless EFT at LO; $\eta_{\text{NN}}$ CT

Dependence of $B_\eta(\Lambda)$ on choice of $\eta_{\text{NN}}$ CT
from Erratum to PLB 771 (2017) 297

- $\eta_{\text{NN}}=$NNN CTs vs. fitting to assumed $B_\eta(\eta_{\text{NN}})=0$.
- Appreciable model dependence for $\Lambda \leq m_\rho \approx 4 \text{ fm}^{-1}$.
Need data beyond $\eta N$ modeling.
Summary

• Subthreshold behavior of $f_{\eta N}$ is crucial in studies of $\eta$-nuclear bound states to decide whether (i) such states exist, (ii) can they be resolved (i.e. widths), and (iii) which nuclear targets and reactions to try.

• Binding $\eta^3$He requires a minimum value of $\text{Re } a_{\eta N}$ close to 1 fm, yielding then a few MeV $B_{\eta}(\eta^4\text{He})$. Binding $\eta^4$He requires a lower value of $\text{Re } a_{\eta N}$, roughly exceeding 0.7 fm. Calculated widths of near-threshold states are a few MeV.

Thanks to my collaborators N. Barnea, B. Bazak, A. Cieplý, E. Friedman, J. Mareš