

# Onset of $\eta$ -meson binding

Exotic Atoms & Related Topics (EXA2017), Vienna

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## $\eta$ nuclear quasibound states

E. Friedman, A. Gal, J. Mareš, PLB 725 (2013) 334

A. Cieplý, E. Friedman, A. Gal, J. Mareš, NPA 925 (2014) 126

Review: A. Gal et al., Acta Phys. Polon. B 45 (2014) 673

## Onset of $\eta$ nuclear binding in He

N. Barnea, E. Friedman, A. Gal, PLB 747 (2015) 345

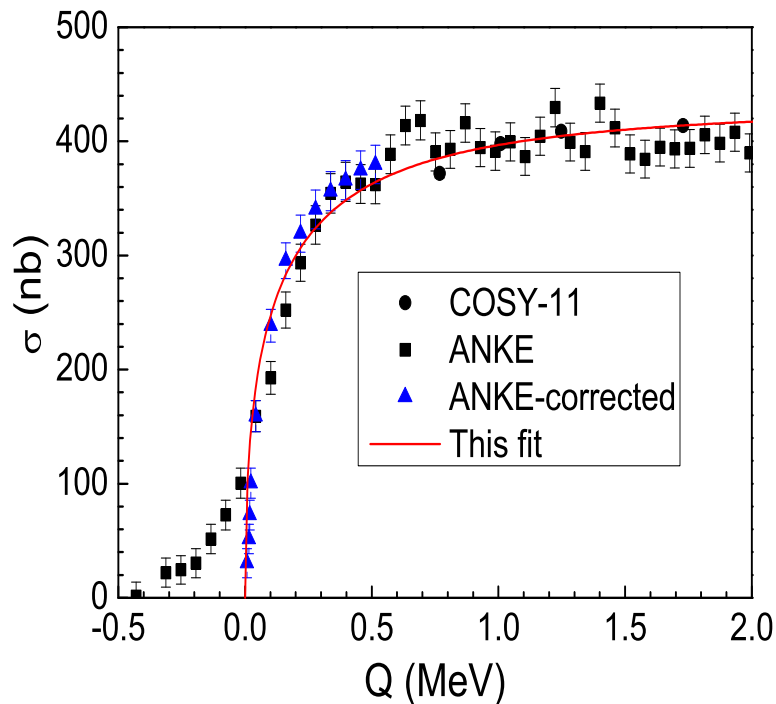
N. Barnea, B. Bazak, E. Friedman, A. Gal, PLB 771 (2017) 297

N. Barnea, E. Friedman, A. Gal, NPA 968 (2017) 35

# Background & Motivation

- The  $\eta N$  s-wave interaction below  $N^*(1535)$  is attractive in a  $\pi N - \eta N$  model [Bhalelao–Liu (1985)]. Bound states of  $\eta(548)$  in  $A \geq 12$  nuclei could exist [Haider–Liu (1986)].
- Chiral  $N^*(1535)$  meson-nucleon coupled channel models were introduced by Kaiser, Weise et al (1995-1997) and subsequently by Oset et al (2002). These & other models have been used to calculate  $\eta$ -nuclear quasibound states.
- Exp. searches for such states with proton, pion or photon induced  $\eta$  production reactions are inconclusive.  
For the onset of binding, Krusche & Wilkin (2015) state:  
“The most straightforward (but not unique) interpretation of the data is that the  $\eta d$  system is unbound, the  $\eta^4\text{He}$  is bound, but that the  $\eta^3\text{He}$  case is ambiguous.”

# Hints from $\eta^3\text{He}$ production



Fitted  $dp \rightarrow \eta^3\text{He}$  x-sections below 2 MeV vs. experiment. Remarkable energy dependence, suggesting a nearby S-matrix pole could be in action.

**Deduced  $a(\eta^3\text{He})$  excludes a quasibound state pole.**

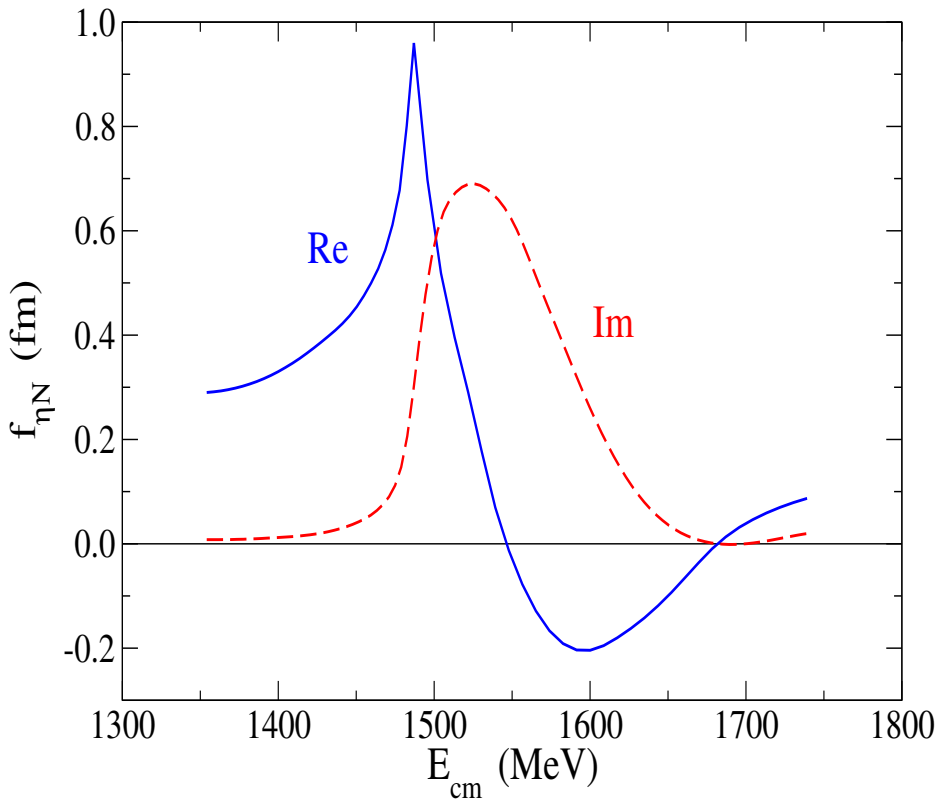
Xie-Liang-Oset-Moskal-Skurzok-Wilkin, PRC 95 (2017) 015202

$$a(\eta^3\text{He}) = [-(2.23 \pm 1.29) + i(4.89 \pm 0.57)] \text{ fm}$$

- Would  $\eta^4\text{He}$  be bound? **NOT** seen in  $dd \rightarrow ^3\text{He} + \text{N} + \pi$  [WASA-at-COSY NPA 959 (2017) 102]. **Argued** to be more **UNBOUND** than  $\eta^3\text{He}$  [Fix-Kolesnikov, PLB 772 (2017) 663] owing to a stronger subthreshold suppression in  $^4\text{He}$ .

# $\eta$ nuclear quasibound states

## $f_{\eta N}(\sqrt{s})$ from $K$ -matrix & $N^*(1535)$ chiral models



### $a_{\eta N}$ (fm) model dependence

a	M1	M2	GW	GR	CS
Re	0.22	0.38	0.96	0.26	0.67
Im	0.24	0.20	0.26	0.24	0.20

Mai et al. PRD 86 (2012) 094033

Green-Wycech PRC 71 (2005) 014001

Garcia-Recio et al. PLB 550 (2002) 47

Cieply-Smejkal, NPA 919 (2013) 46

- **Re  $a_{\eta N}$  varies from 0.2 to 1.0 fm, Im  $a_{\eta N}$ : 0.2–0.3 fm.**
- **M1, M2, GW free-space models; GR, CS in-medium.**
- **Strong subthreshold fall-off in both Re  $f_{\eta N}$  and Im  $f_{\eta N}$ .**
- **In-medium: E dependence, Pauli blocking, self energies.**

# Self-consistency in mesic-atom & nuclear calculations

Cieplý-Friedman-Gal-Gazda-Mareš, PLB 702 (2011) 402

$$s_{\eta N} = (\sqrt{s_{\text{th}}} - B_{\eta} - B_N)^2 - (\vec{p}_{\eta} + \vec{p}_N)^2 < s_{\text{th}}$$

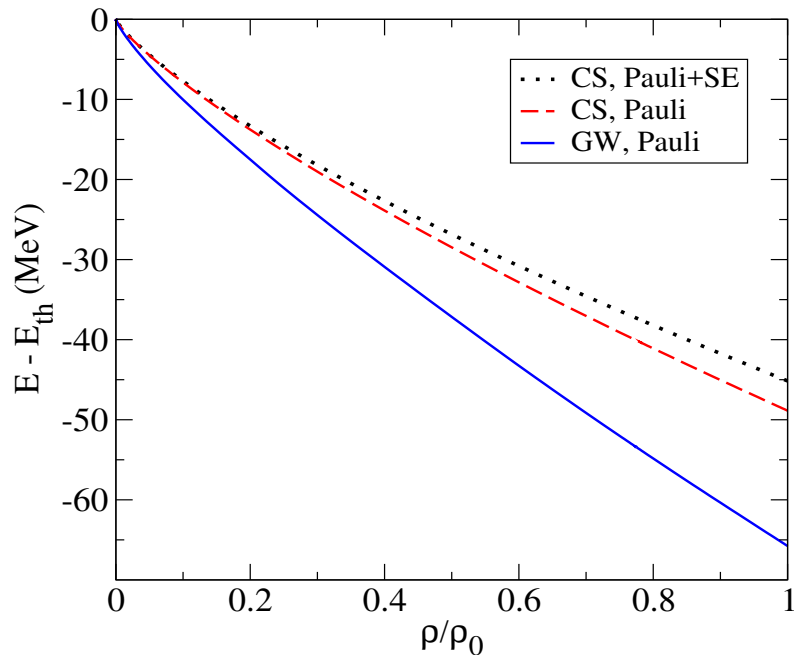
$$\sqrt{s_{\eta N}} \rightarrow E_{\text{th}} - B_N - B_{\eta} - \xi_N \frac{p_N^2}{2m_N} - \xi_{\eta} \frac{p_{\eta}^2}{2m_{\eta}}$$

$$\xi_{N(\eta)} = \frac{m_{N(\eta)}}{(m_N + m_{\eta})} \quad \frac{p_{\eta}^2}{2m_{\eta}} \sim -V_{\eta} - B_{\eta}$$

$\eta$  is not at rest!

$E_{\eta N}$  subthreshold shift vs. nuclear density in  $1s_{\eta}^{40}\text{Ca}$ .

A dominant in-medium effect.



Cieplý-Friedman-Gal-Mareš, NPA 925 (2014) 126

# In-medium $\eta N$ amplitudes

Friedman-Gal-Mareš, PLB 725 (2013) 334

Cieplý-Friedman-Gal-Mareš, NPA 925 (2014) 126

- KG equation and self-energies:

$$[ \nabla^2 + \tilde{\omega}_\eta^2 - m_\eta^2 - \Pi_\eta(\omega_\eta, \rho) ] \psi = 0$$

$$\tilde{\omega}_\eta = \omega_\eta - i\Gamma_\eta/2, \quad \omega_\eta = m_\eta - B_\eta$$

$$\Pi_\eta(\omega_\eta, \rho) \equiv 2\omega_\eta V_\eta = -4\pi \frac{\sqrt{s}}{m_N} f_{\eta N}(\sqrt{s}, \rho) \rho$$

- Pauli blocking (Waas-Rho-Weise NPA 617 (1997) 449):

$$f_{\eta N}^{\text{WRW}}(\sqrt{s}, \rho) = \frac{f_{\eta N}(\sqrt{s})}{1 + \xi(\rho)(\sqrt{s}/m_N)f_{\eta N}(\sqrt{s})\rho}, \quad \xi(\rho) = \frac{9\pi}{4p_F^2} I(\tilde{\omega}_\eta)$$

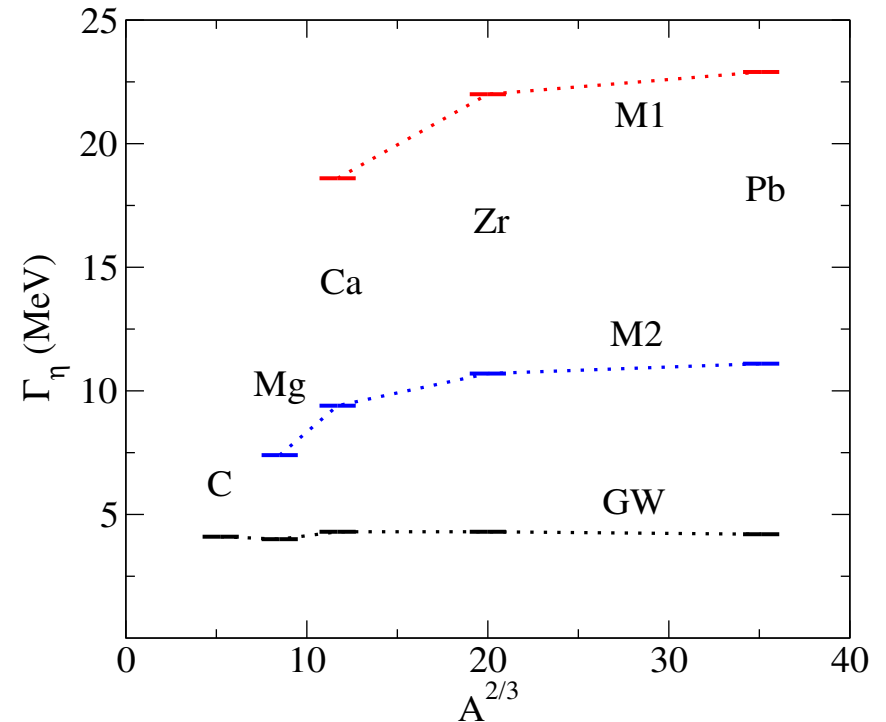
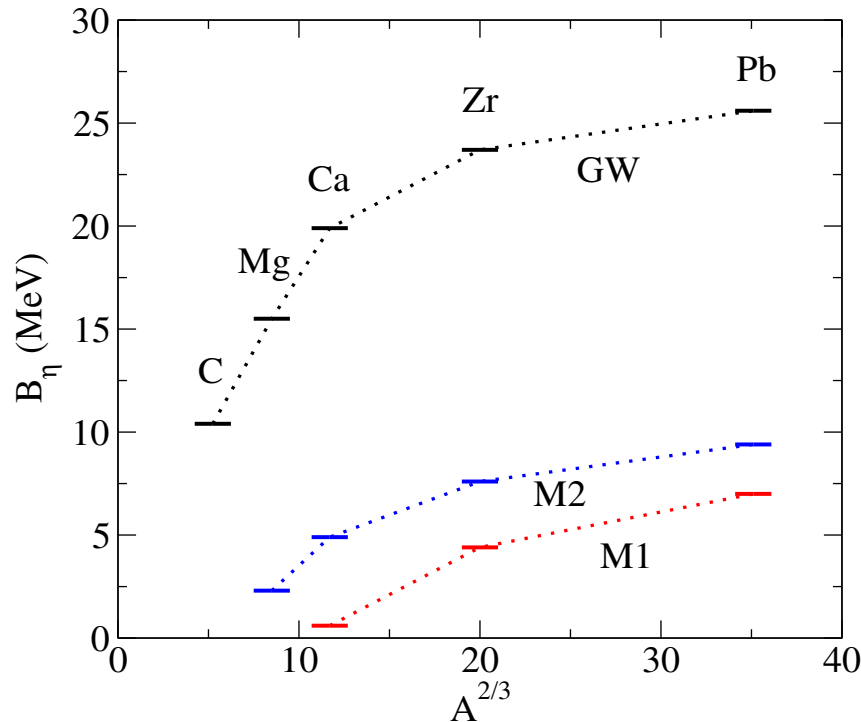
- $N^*(1535) \Rightarrow$  energy dependent  $f_{\eta N}(\sqrt{s})$ .

$$\text{In medium} \Rightarrow \text{go subthreshold: } \delta\sqrt{s} = \sqrt{s} - \sqrt{s_{\text{th}}}$$

$$\delta\sqrt{s} \approx -B_N \frac{\rho}{\rho_0} - \xi_N B_\eta \frac{\rho}{\rho_0} - \xi_N T_N \left(\frac{\rho}{\rho_0}\right)^{2/3} + \xi_\eta \text{Re } V_\eta(\sqrt{s}, \rho)$$

- **A self-consistency cycle in  $\delta\sqrt{s}$  for given  $\rho$ .**

# Model dependence I

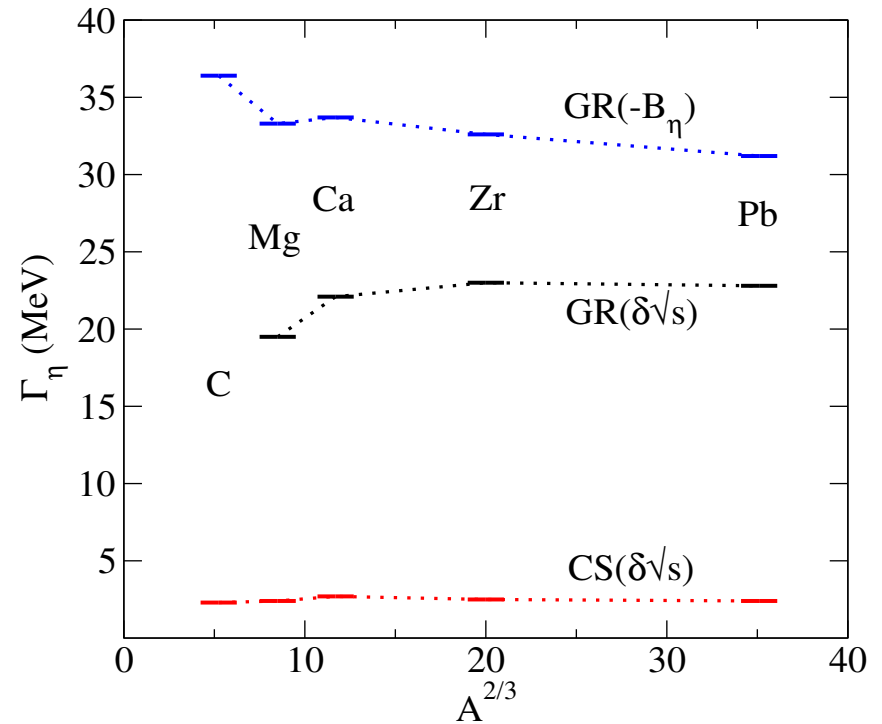
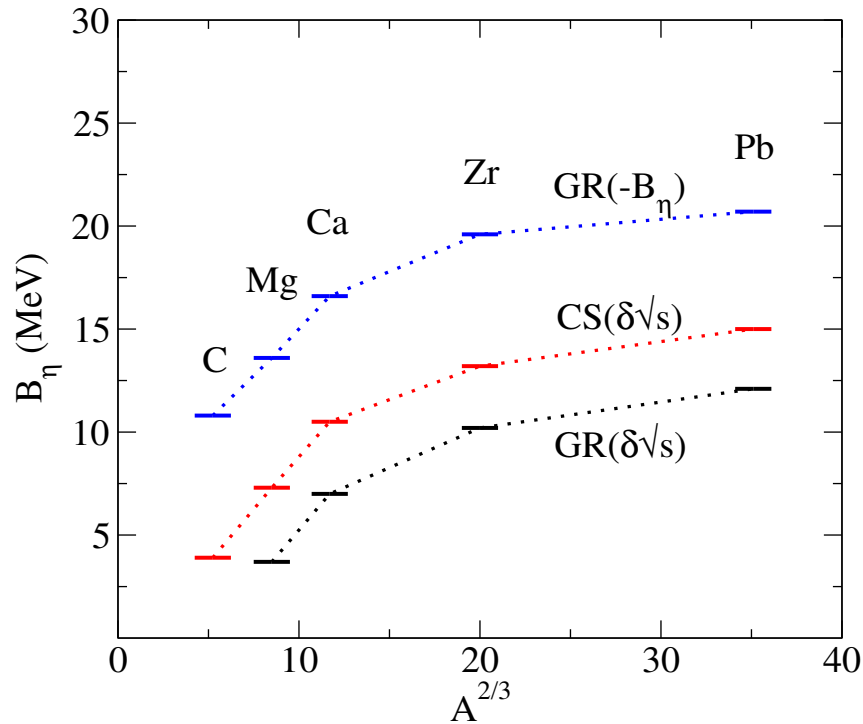


$1s_\eta$  B &  $\Gamma$  using WRW Pauli-blocked  $f_{\eta N}$

- **E dependence treated self consistently.**
- Larger  $\text{Re } a_{\eta N} \Rightarrow$  larger  $B_\eta$ .
- Widths are unrelated to  $\text{Im } a_{\eta N}$ .



# Model dependence II

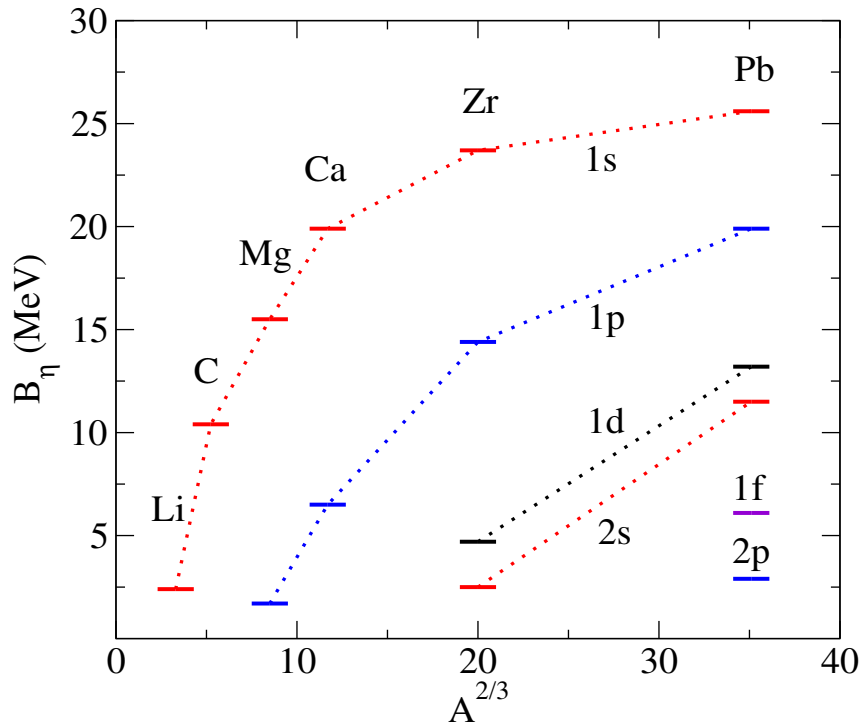


Sensitivity of calc.  $B_{1s_{\eta}}$  &  $\Gamma_{1s_{\eta}}$  to self-consistency version

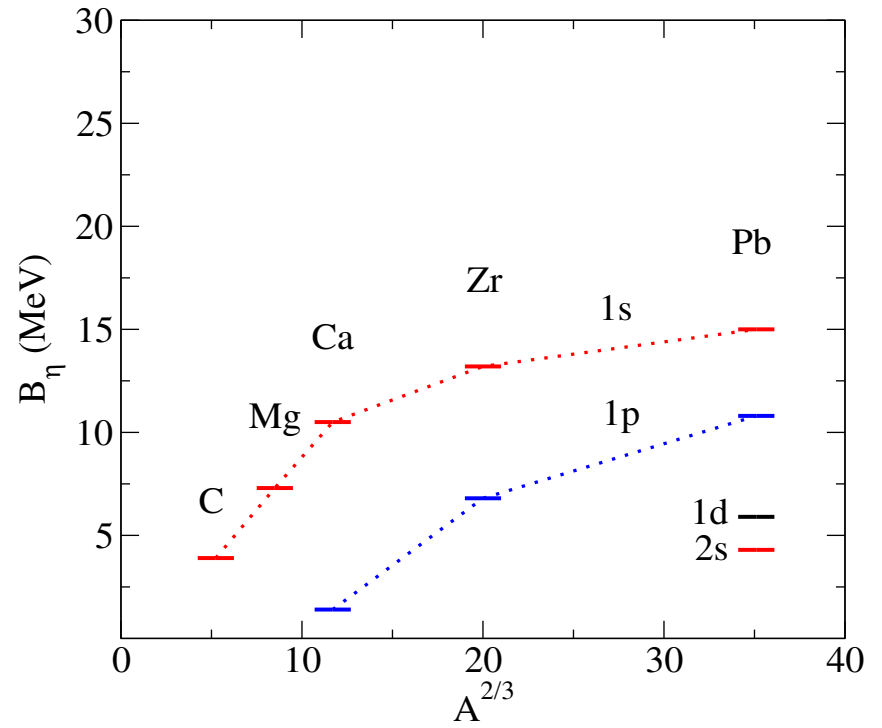
- $\langle \delta\sqrt{s} \rangle$  goes deeper into subthreshold, thereby reducing further  $B_{1s_{\eta}}$  &  $\Gamma_{1s_{\eta}}$ .
- GR's widths are too large to resolve  $\eta$  bound states.

Why  $\Gamma_{\eta}(\text{GR}) \gg \Gamma_{\eta}(\text{CS})$  for similar  $\text{Im } a_{\eta N}$ ?

# Model predictions for small widths



GW model



CS model

- Widths of only a few MeV in each of these models.
- What makes the subthreshold values of  $\text{Im } f_{\eta N}$  sufficiently small to generate small widths?

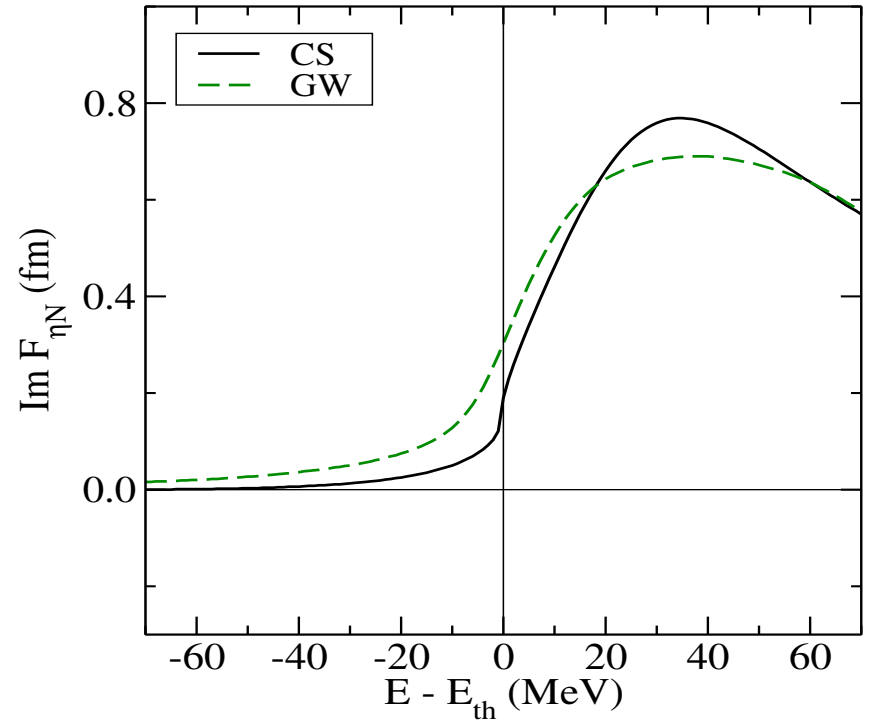
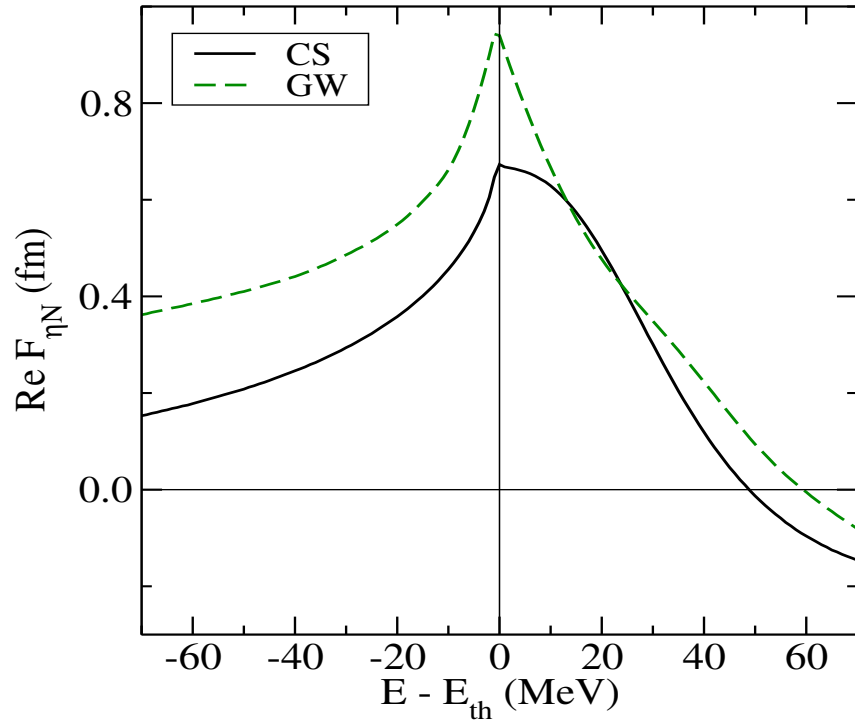
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# $\eta N$ model input

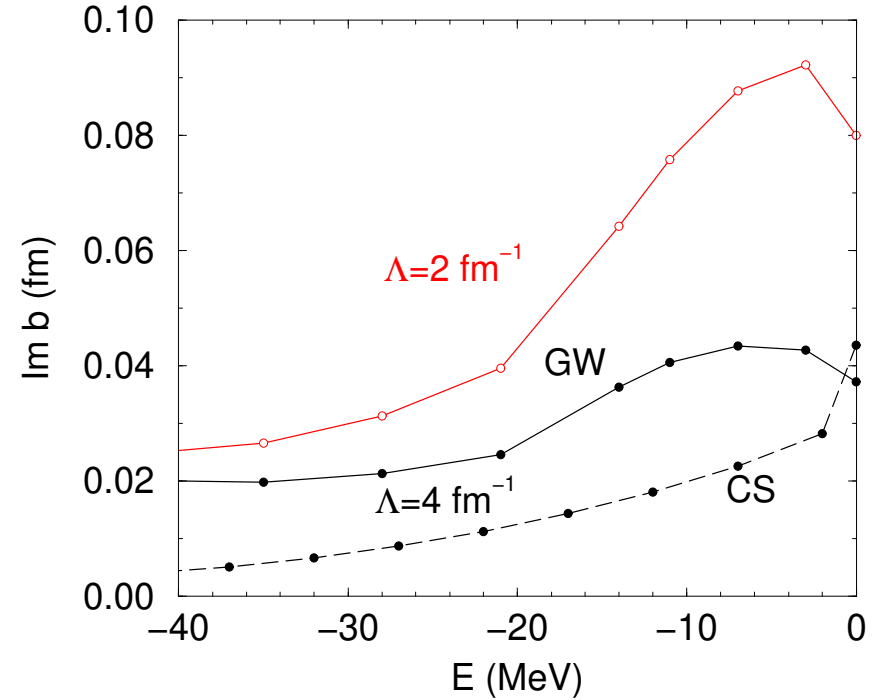
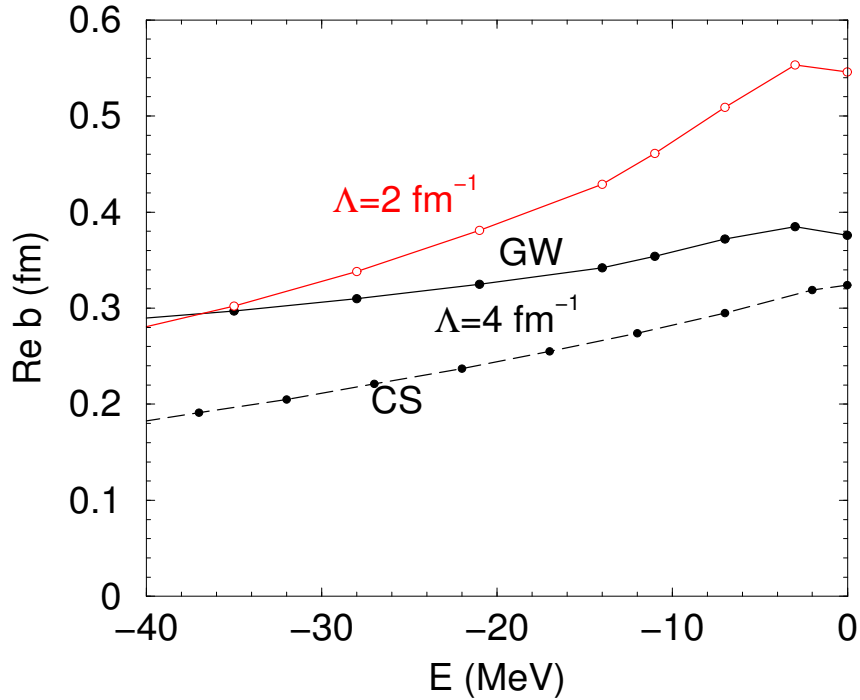


CM s-wave scattering amplitude  $F_{\eta N}(E)$  in two meson-baryon coupled-channel  $N^*(1535)$  models.

$$\mathbf{a_{\eta N}^{GW} = 0.96 + i0.26 \text{ fm}, \quad a_{\eta N}^{CS} = 0.67 + i0.20 \text{ fm}}$$

- Derive local, energy dependent potentials  $v_{\eta N}(E; r)$  that reproduce  $F_{\eta N}(E)$  below threshold, for use in solving the  $\eta NN$ ,  $\eta NNN$ ,  $\eta NNNN$  few-body Schroedinger equations.

# $F_{\eta N}(\mathbf{E}) \Rightarrow v_{\eta N}(\mathbf{E})$ in models GW & CS



Strength  $b(E)$  of effective potential  $v_{\eta N}(\mathbf{E})$  at  $E < 0$

$$v_{\eta N}(\mathbf{E}; \mathbf{r}) = -\frac{4\pi}{2\mu_{\eta N}} b(\mathbf{E}) \left( \frac{\Lambda}{2\sqrt{\pi}} \right)^3 \exp\left(-\frac{\Lambda^2 r^2}{4}\right)$$

- Scale  $\Lambda$  is inversely proportional to the range of  $v_{\eta N}$ .
- $v_{\eta N}$  is a regulated contact term in  $\pi$ -less EFT which for  $\Lambda \leq m_\rho \approx 4 \text{ fm}^{-1}$  replaces vector-meson exchange.

# Energy dependence in $\eta$ nuclear few-body systems

- $N^*(1535)$  makes near-threshold  $f_{\eta N}(\sqrt{s})$  & input potential  $v_{\eta N}(\sqrt{s})$  strongly energy dependent.

$$s = (\sqrt{s_{\text{th}}} - B_\eta - B_N)^2 - (\vec{p}_\eta + \vec{p}_N)^2 < s_{\text{th}}$$

- Expanding NR near  $\sqrt{s_{\text{th}}}$  & evaluating  $\langle \delta\sqrt{s} \rangle$ :

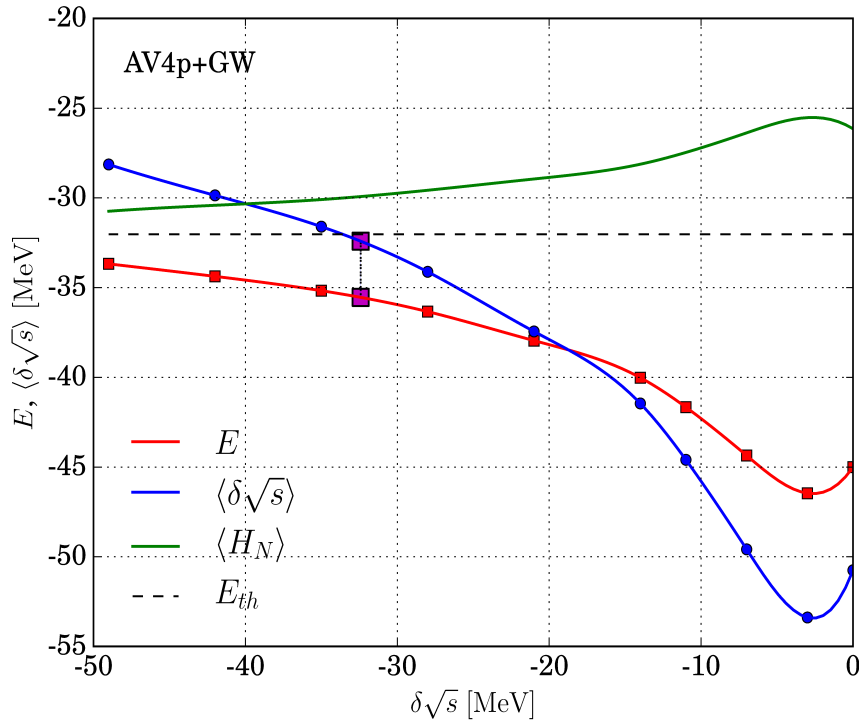
$$\langle \delta\sqrt{s} \rangle = -\frac{B}{A} + \frac{A-1}{A} E_\eta - \xi_N \frac{1}{A} \langle T_A \rangle - \xi_\eta \left(\frac{A-1}{A}\right)^2 \langle T_\eta \rangle,$$

$$\delta\sqrt{s} \equiv \sqrt{s} - \sqrt{s_{\text{th}}}, \quad E_\eta = \langle H - H_A \rangle, \quad \xi_{N(\eta)} \equiv \frac{m_{N(\eta)}}{(m_N + m_\eta)}.$$

Agrees to  $O(1/A)$  with optical-model limit.

- Self-consistency: output  $\langle \sqrt{s} \rangle =$  input  $\sqrt{s}$ .
- Near threshold  $E_\eta$  &  $\langle T_\eta \rangle \rightarrow 0$ , yet  $\langle \delta\sqrt{s} \rangle_{\text{th}} \neq 0$ .  
Similarly,  $\langle \delta\sqrt{s} \rangle_{\text{th}} \neq 0$  in kaonic atoms, starting with  $K^- d$ :  $\langle \delta\sqrt{s} \rangle_{\text{th}} = -\frac{B_d}{2} - \frac{0.655}{2} \langle T_d \rangle = -4.9$  MeV.

# Recent SVM results for $\eta^{3,4}\text{He}$

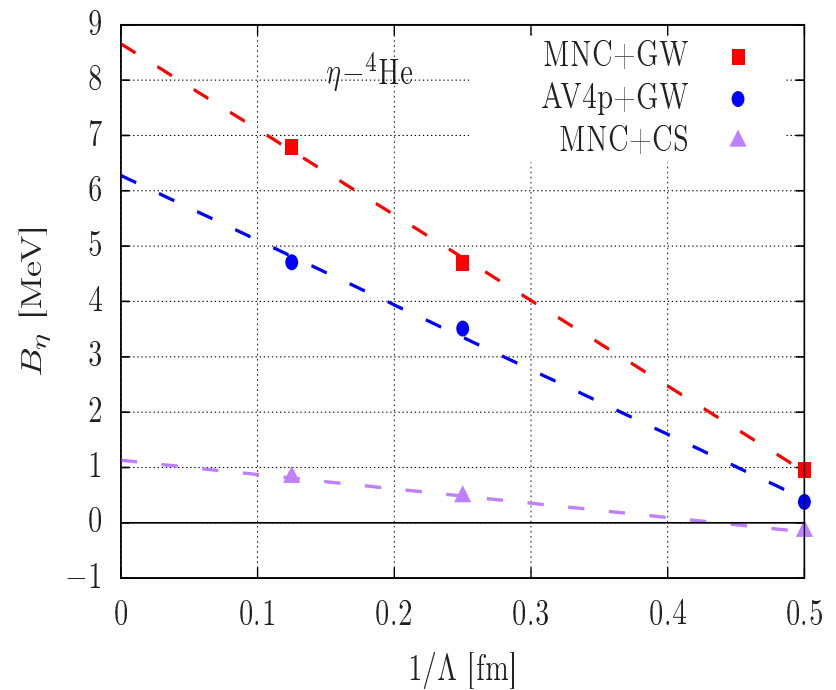
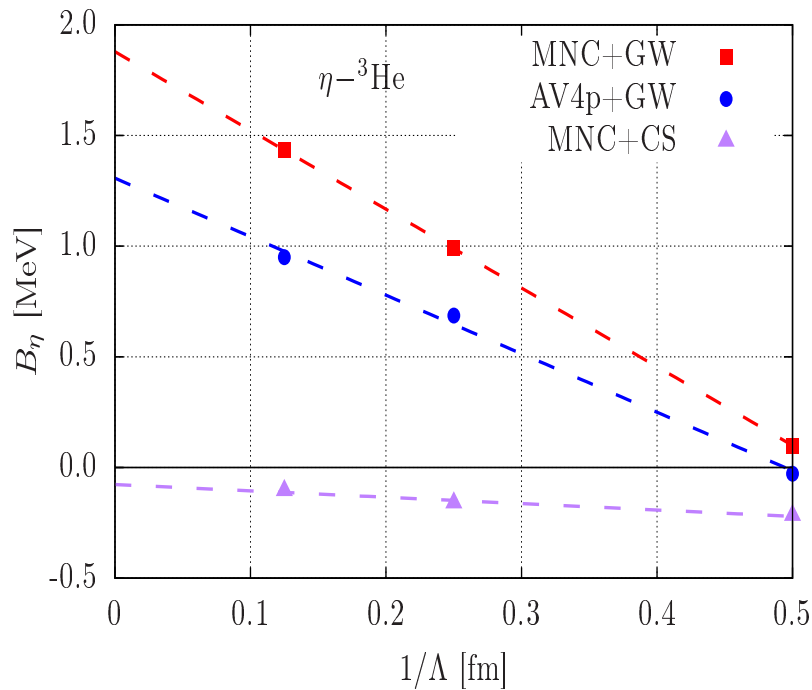


## Self consistency plot

$\eta^4\text{He}$  bound-state energy  $E$ ,  $\langle\delta\sqrt{s}\rangle$  &  $\langle H_N = H_A \rangle$ , for AV4'  $v_{NN}$  & GW  $v_{\eta N}(E)$  with scale  $\Lambda=4 \text{ fm}^{-1}$ .

- **Stochastic Variational Method** calculations with correlated Gaussian trial wavefunctions, resulting in:
- $\eta$ d is definitely unbound in both GW and CS (2015).
- $\eta^3\text{He}$  is nearly or just bound in GW & unbound in CS.
- $\eta^4\text{He}$  is bound in GW and just or nearly bound in CS.

# Scale dependence; semi-realistic NN

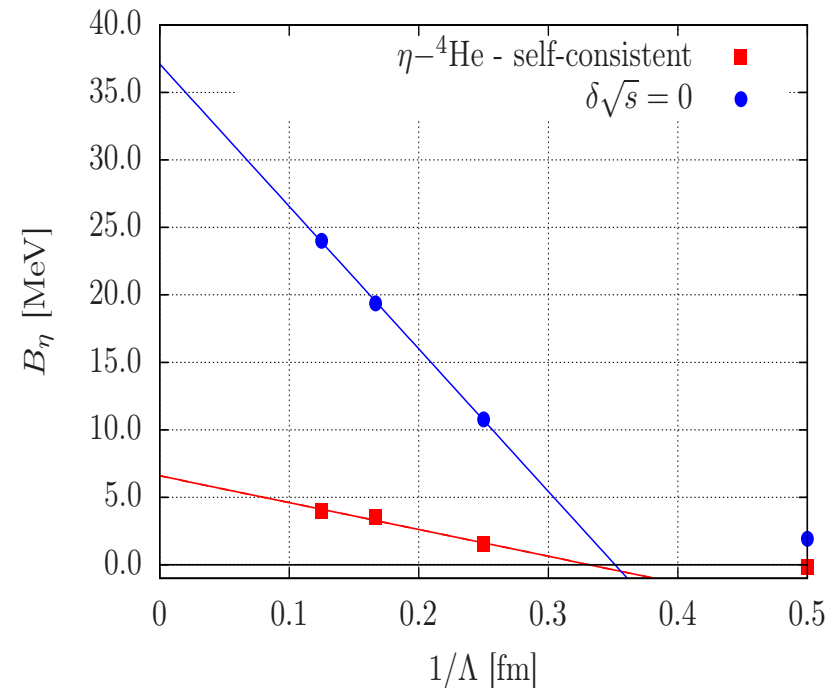
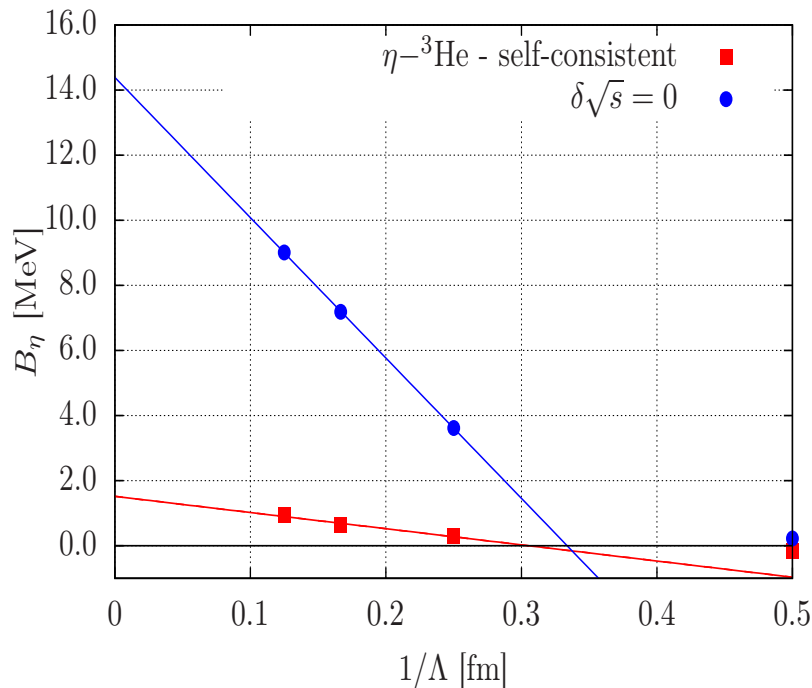


$B_\eta$  as a function of  $1/\Lambda$

- These bindings will decrease by  $\leq 0.3$  MeV when  $\text{Im } v$  is added. **GW just binds  $\eta^3\text{He}$ , & definitely binds  $\eta^4\text{He}$ .**
- AV4p (Argonne) more realistic than MNC (Minnesota).
- CS does not bind  $\eta^3\text{He}$  & is unlikely to bind  $\eta^4\text{He}$ .



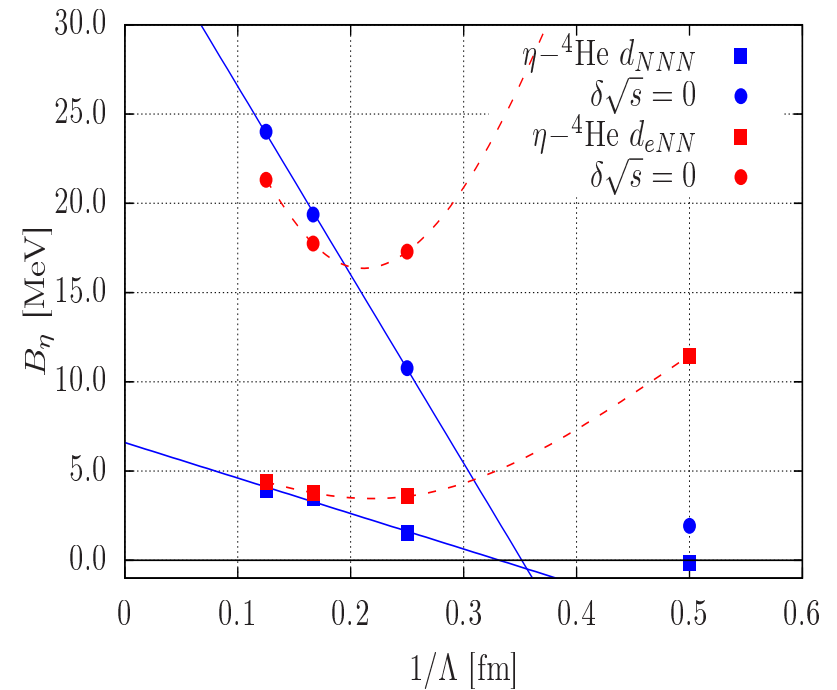
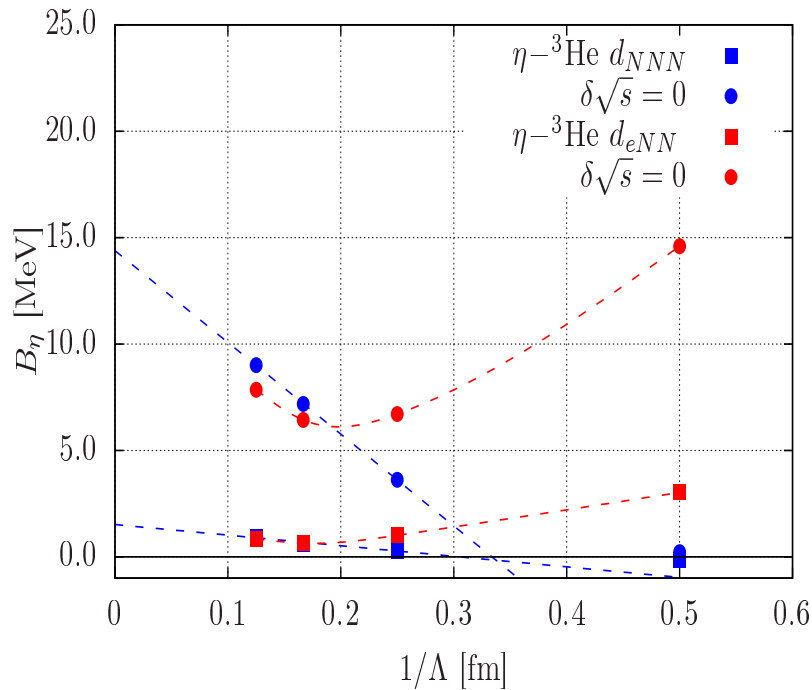
# Scale dependence; pionless EFT at LO



$B_\eta(\Lambda)$  using  $v_{\eta N}^{\text{GW}}(\mathbf{E})$ : **with** & **w/o** self consistency

- Nuclear dynamics generated from two NN & one NNN contact terms (CT). NNN CT averts  ${}^3\text{He}$  collapse.
- Add one  $\eta\text{N}$  & one  $\eta\text{NN}$  CT; given no  $\eta\text{NN}$  datum, use  $\text{CT}(\eta\text{NN}) = \text{CT}(\text{NNN})$  to start with.

# Pionless EFT at LO; $\eta$ NN CT



Dependence of  $B_\eta(\Lambda)$  on choice of  $\eta$ NN CT  
from Erratum to PLB 771 (2017) 297

- $\eta$ NN=NNN CTs vs. fitting to **assumed**  $B_\eta(\eta$ NN)=0.
- Appreciable model dependence for  $\Lambda \leq m_\rho \approx 4$  fm $^{-1}$ .  
**Need data beyond  $\eta$ N modeling.**

# Summary

- Subthreshold behavior of  $f_{\eta N}$  is crucial in studies of  $\eta$ -nuclear bound states to decide whether (i) such states exist, (ii) can they be resolved (i.e. widths), and (iii) which nuclear targets and reactions to try.
- Binding  $\eta$   $^3\text{He}$  requires a minimum value of  $\text{Re } a_{\eta N}$  close to 1 fm, yielding then a few MeV  $B_{\eta}(\eta$   $^4\text{He})$ . Binding  $\eta$   $^4\text{He}$  requires a lower value of  $\text{Re } a_{\eta N}$ , roughly exceeding 0.7 fm. Calculated widths of near-threshold states are a few MeV.

Thanks to my collaborators N. Barnea, B. Bazak, A. Cieplý, E. Friedman, J. Mareš