Dynamically generated hadronic states in the $\bar{K}N$ and $\eta N$ coupled-channels interactions

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Outline:

1. $\bar{K}N$ and $\eta N$ interactions
2. Model predictions
3. Dynamically generated poles
4. Summary

A. Cieplý: Dynamically generated hadronic states in the $\bar{K}N$ and $\eta N$ coupled-channels interactions

$\bar{K}N$ and $\eta N$ interactions

very basic comparison of the systems

<table>
<thead>
<tr>
<th></th>
<th>strangeness</th>
<th>$E_{th}$ (MeV)</th>
<th>resonance</th>
<th>$a_{MN}$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{K}N$</td>
<td>$S = -1$</td>
<td>1434</td>
<td>$\Lambda(1405)$</td>
<td>$-0.15 + 0.62i$</td>
</tr>
<tr>
<td>$\eta N$</td>
<td>$S = 0$</td>
<td>1486</td>
<td>$N^*(1535)$</td>
<td>$0.67 + 0.20i$</td>
</tr>
</tbody>
</table>

Involved channels: $\pi\Lambda$, $\pi\Sigma$, $\bar{K}N$, $\eta\Lambda$, $\eta\Sigma$, $K\Xi$ ($S = -1$)
$\pi N$, $\eta N$, $K\Lambda$, $K\Sigma$ ($S = 0$)

strongly interacting multichannel systems with an s-wave resonance near threshold
modern theoretical treatment based on an effective chiral Lagrangian
perturbation series do not converge in the vicinity of resonances!

Solution: construct effective potentials, then use Lippmann-Schwinger (or Bethe-Salpeter) equation to sum the major part of the perturbation series

$$T = V + VG T$$

low energies around threshold - only s-wave considered in most approaches
Chirally motivated approaches

effective potentials are constructed that match the chiral meson-baryon amplitudes up to NLO order


Schematic picture:

$\mathcal{O}(q^1)$ contact  direct s-term  crossed u-term  $\mathcal{O}(q^2)$ contact

Leading order - Weinberg-Tomozawa interaction

$$V_{ij}^{(WT)} \approx -\frac{C_{ij}^{(WT)}(\sqrt{s})}{4f_if_j}(E_i + E_j)$$

Parameters:

- $f_{\pi}, f_K, f_\eta$ - meson decay constants
- $D \sim 3/4, F \sim 1/2$ - axial vector couplings, $g_A = F + D$
- $b_0, b_D, b_F$, four $d$'s - second order couplings
- $M_0$ - baryon octet mass
\( \bar{K}N \) models

- **Kyoto-Munich (KM)**  

- **Murcia (M)**  

- **Bonn (B)**  

- **Prague (P)**  

Model parameters (couplings, inverse interaction ranges or subtraction constants) fixed in fits to low energy meson-nucleon data.

**Kaonic hydrogen characteristics:**  
1s level **energy shift** and **absorption width** due to strong interaction

\[
E(1s) = E_{em}(1s) + \Delta E_N(1s) - \frac{i}{2} \Gamma(1s)
\]

**SIDDHARTA (2011):**  
\( \Delta E_N(1s) = 283 \pm 36(\text{stat.}) \pm 6(\text{syst.}) \) eV  
\( \Gamma(1s) = 541 \pm 89(\text{stat.}) \pm 22(\text{syst.}) \) eV
Model predictions - $\bar{K}N$ results

kaonic hydrogen 1s level characteristics and $\Lambda(1405)$ poles

- the models are in close agreement reproducing the SIDDHARTA data
- two poles are generated in the $\Lambda(1405)$ sector
- all models tend to agree on the position of the $\bar{K}N$ related pole
- the data are not very sensitive to the position of the $\pi\Sigma$ related pole
Model predictions - $\bar{K}N$ results

$K^- p$ and $K^- n$ elastic amplitudes

$B_2$ (dotted, purple), $B_4$ (dot-dashed, red), $M_I$ (dashed, blue), $M_{II}$ (long-dashed, green), $P_{NLO}$ (dot-long-dashed, violet), $KM_{NLO}$ (continuous, black).
the theoretical ambiguities below the $\bar{K}N$ threshold are much larger than indicated by uncertainty bounds derived from variations of the $K^- p$ scattering length for a specific model and a given $\chi^2$ local minima !!!

the $l = 1$ sector is not restricted by the fitted experimental data leading to varied predictions for the $K^- n$ scattering amplitude

New data needed !!!

- new precise data for the isovector $K^- p \rightarrow \pi^0 \Lambda$ and isoscalar $K^- p \rightarrow \pi^0 \Sigma^0$ reactions at as low energies as possible are highly desired

- $\pi \Sigma$ mass spectra at subthreshold energies should help provided we understand the process dynamics

- kaonic deuterium measurement (AMADEUS, Frascati) will also add to the picture
The model predictions for the $\pi^- p \rightarrow \eta n$ total cross section are plotted together with the experimental data. The shape of the $N^*(1535)$ resonance is reproduced quite well.
\( \eta N \) amplitude (various models)

<table>
<thead>
<tr>
<th>line</th>
<th>( a_{\eta N} ) [fm]</th>
<th>model</th>
</tr>
</thead>
<tbody>
<tr>
<td>dotted</td>
<td>0.46+i0.24</td>
<td>N. Kaiser, P.B. Siegel, W. Weise, PLB 362 (1995) 23</td>
</tr>
<tr>
<td>short-dashed</td>
<td>0.26+i0.25</td>
<td>T. Inoue, E. Oset, NPA 710 (2002) 354</td>
</tr>
<tr>
<td>dot-dashed</td>
<td>0.96+i0.26</td>
<td>A.M. Green, S. Wycech, PRC 71 (2005) 014001</td>
</tr>
<tr>
<td>long-dashed</td>
<td>0.38+i0.20</td>
<td>M. Mai, P.C. Bruns, U.-G. Meißner, PRD 86 (2012) 094033</td>
</tr>
<tr>
<td>continuous</td>
<td>0.67+i0.20</td>
<td>A.C., J. Smejkal, NPA 919 (2013) 46</td>
</tr>
</tbody>
</table>
Dynamically generated resonances/poles

Where do the poles come from?
The amplitude has poles for complex energies \( z \) (equal to \( \sqrt{s} \) on the real axis) if a determinant of the inverse matrix is equal to zero,

\[
\det|f^{-1}(z)| = \det|v^{-1}(z) - G(z)| = 0
\]

The origin of the poles can be traced to the zero coupling limit: \( C_{ij} = 0 \) for \( i \neq j \) (interchannel couplings switched off)

for \( C_{i,j\neq i} = 0 \) the condition for a pole of the amplitude becomes

\[
\prod_n \left[ \frac{1}{v_{nn}(z)} - G_n(z) \right] = 0
\]

Since \( v_{nn} \propto C_{nn} \), only states with nonzero diagonal couplings \( C_{i,j=i} \) can generate the poles!

Prague approach: There will be a pole in channel \( n \) at a Riemann sheet \([+/-]\) (phys./unphys.) if the following condition is satisfied for any complex energy \( z \):

\[
\frac{4\pi f_n^2}{C_{nn}(z)} \frac{z}{M_n} + \frac{(\alpha_n + ik_n)^2}{2\alpha_n} \left[ g_n(k_n) \right]^2 = 0; \quad \alpha_n > \alpha_n(\text{min}) = \frac{16\pi}{\tilde{C}_{nn}} \frac{f_n^2}{\omega_n}
\]
Dynamically generated resonances/poles

ZCL pole equation for the on-shell Kyoto-Munich and Murcia approaches:

\[
\frac{1}{V_{nn}(z)} + \frac{1}{(4\pi)^2} \left[ a_n(\mu) + 2 \log \frac{M_n}{\mu} + G_0(z, m, M) \right] = 0
\]

Solutions can be found as crossings of the $1/V$ plot and the meson-baryon loop function $G$. Example: $KM_{WT}$ model, $K\Xi$ channel, unphysical RS

**Conclusion:** $a_n < a_n(\text{max})$ to generate a ZCL pole
Dynamically generated resonances/poles

What channels have nonzero diagonal couplings?

For simplicity, we first look only at the leading order WT term couplings

\[
\tilde{C}_{ij}^{I=0} = \begin{pmatrix}
4 & -\sqrt{3}/2 & 0 & \sqrt{3}/2 \\
3 & 3\sqrt{1/2} & 0 & \\
0 & -3\sqrt{1/2} & 3
\end{pmatrix}
\]

\[
\tilde{C}_{ij}^{I=1} = \begin{pmatrix}
0 & 0 & -\sqrt{3}/2 & 0 & -\sqrt{3}/2 \\
2 & -1 & 0 & 1 \\
1 & -\sqrt{3}/2 & 0 & \\
0 & -\sqrt{3}/2 & 1
\end{pmatrix}
\]

For both isospins, \( I = 0 \) and \( I = 1 \) the poles can be in the \( \pi\Sigma \), \( \bar{K}N \) and \( K\Xi \) channels.

We can have as many as three isoscalar poles and three isovector poles.
Dynamically generated resonances/poles

Sample results for the $P_{WT}$ model:

<table>
<thead>
<tr>
<th>sector</th>
<th>channel</th>
<th>ZCL state</th>
<th>resonance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I = 0$</td>
<td>$\bar{K}N$</td>
<td>bound</td>
<td>$\Lambda(1405)$</td>
</tr>
<tr>
<td>$I = 1$</td>
<td>$\bar{K}N$</td>
<td>virtual</td>
<td>$K^- n$ amplitude</td>
</tr>
<tr>
<td></td>
<td>$\pi \Sigma$</td>
<td>resonance</td>
<td>$\Sigma(1750)$</td>
</tr>
<tr>
<td></td>
<td>$K \Xi$</td>
<td>virtual</td>
<td>$\Sigma(1750)$</td>
</tr>
</tbody>
</table>

In general, the exact situation (existence of a ZCL pole in a given channel) depends on the model parameters (inverse ranges or subtraction constants, NLO contributions that generate sufficiently large couplings $C_{nn}$).
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Dynamically generated resonances/poles

Pole movements upon scaling the nondiagonal interchannel couplings

$C_{i,j \neq i}$ replaced by $x \cdot C_{i,j \neq i}$

$P_{NLO}$ model, left panel: isoscalar states, right panel: isovector states

The pole positions in the physical limit are emphasized with large empty circles. The triangles at the top of the real axis indicate the channel thresholds.
Dynamically generated resonances/poles

Our findings

<table>
<thead>
<tr>
<th>resonance</th>
<th>models / ZCL channels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{NLO}$</td>
</tr>
<tr>
<td>$\Lambda_1(1405)$</td>
<td>$\pi\Sigma$</td>
</tr>
<tr>
<td>$\Lambda_2(1405)$</td>
<td>$\bar{K}N$</td>
</tr>
<tr>
<td>$\Lambda(1670)$</td>
<td>$K\Xi$</td>
</tr>
<tr>
<td>$\bar{K}N(I = 1)$</td>
<td>$\bar{K}N$</td>
</tr>
<tr>
<td>$\Sigma(1750)$</td>
<td>$K\Xi$</td>
</tr>
</tbody>
</table>

$M_{II}$ and $B_2$ models generate the $\Lambda_2(1405)$ pole from the $\eta\Lambda$ ZCL bound state.

earlier reports on the isovector $\bar{K}N$ related pole:

A.C., E. Friedman, A. Gal, D. Gazda, J. Mareš - PRC84 (2011) 045206
A.C., J. Smejkal - Few Body Syst. 54 (2013) 1183
origin of the $N^*(1535)$ and $N^*(1650)$ poles

Movement of the poles $z_1$ [assigned to $N^*(1535)$] and $z_2$ [assigned to $N^*(1650)$] upon gradually switching off the inter-channel couplings. The positions of the poles in a physical limit are encircled and marked by the labels that also denote the Riemann sheets the poles are located on.
Summary

- Chirally motivated coupled channels models provide a realistic description of the $\bar{K}N$ and $\eta N$ interactions at energies close to threshold.

- The predictions for the elastic $\bar{K}N$ and $\eta N$ amplitudes vary significantly from one model to another, especially below the thresholds.

- Pole movements on the complex energy manifold give us additional insights on the dynamically generated meson-baryon resonances. The origin of the poles can be traced to the nonzero diagonal inter-channel couplings.

- Two poles of the $\Lambda(1405)$ generated dynamically. The models can (in principle) account for the $\Lambda(1670)$ and the $\Sigma(1750)$ resonances as well. Some models predict the existence of an isovector pole close to the $\bar{K}N$ threshold.

- In our model, the resonances $N^*(1535)$ and $N^*(1650)$ originate from the same $K\Sigma$ virtual state but their poles develop at different Riemann sheets.

- A similar study of vector meson interactions with baryons is in progress.

Thanks to my collaborators !!!
M. Mai (Bonn), U.-G. Meißner (Bonn), J. Smejkal (Prague)