

# Dalitz plot analysis of three-body charmonium decays at BaBar

Antimo Palano

*INFN and University of Bari, Italy<sup>(\*)</sup>*

On behalf of the BaBar Collaboration

## Outline

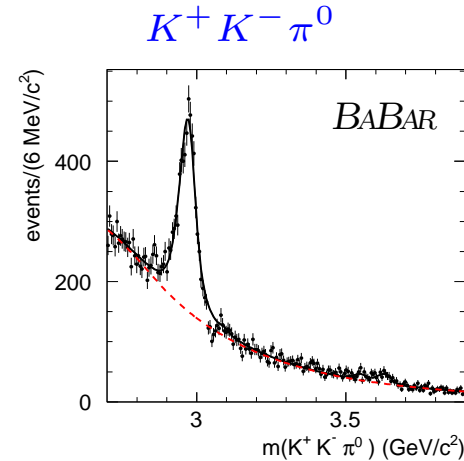
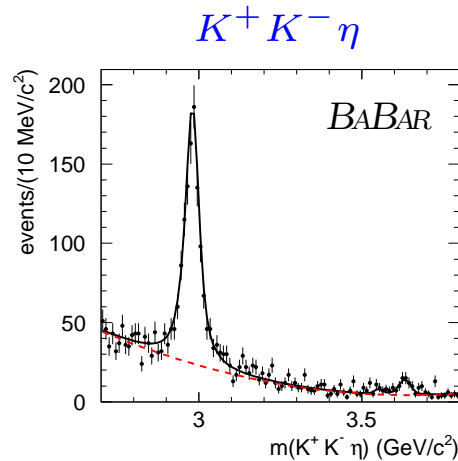
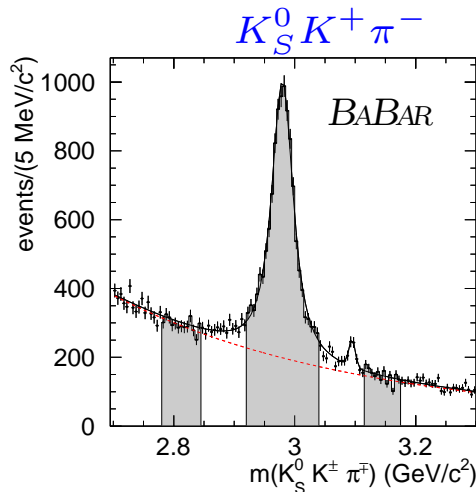
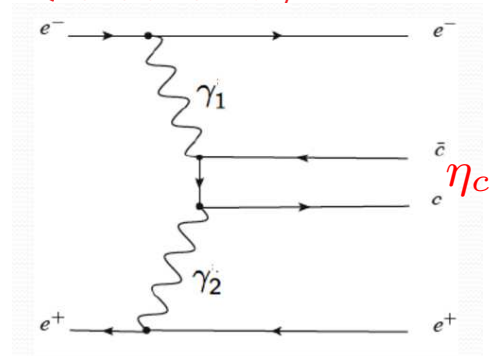
- Measurement of the  $I=1/2$   $K\pi$   $\mathcal{S}$ -wave amplitude from a Dalitz plot analysis of  $\eta_c$  decays in two-photon interactions.
- Dalitz plot analysis of three  $J/\psi$  three-body hadronic decays.

EXA 2017 - International Conference on Exotic Atoms and Related Topics,  
Wien, September 11-15, 2017

*(\*) Work supported in part by Jefferson Lab, VA, USA*

## The $K\pi$ $S$ -wave from $\eta_c$ decays

- Charmonium decays are used to obtain new information on light meson spectroscopy.
- In two-photon interactions we select events in which the  $e^+$  and  $e^-$  beam particles are scattered at small angles and remain undetected. Require  $p_T < 0.08 \text{ GeV}/c$ .
- We have studied the following final states.
  - $\eta_c \rightarrow K_S^0 K^+ \pi^-$ , 12849 evts with  $(64.3 \pm 0.4)\%$  purity.
  - $\eta_c \rightarrow K^+ K^- \pi^0$ , 6494 evts with  $(55.2 \pm 0.6)\%$  purity.
  - $\eta_c \rightarrow K^+ K^- \eta$ , 1161 evts with  $(76.1 \pm 1.3)\%$  purity.



□  $Purity = Signal / (Signal + Background)$

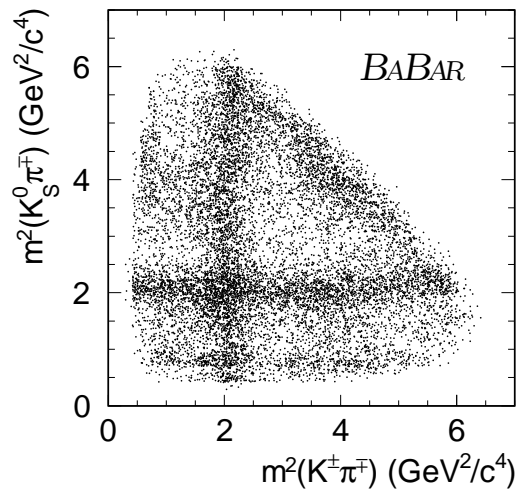
## Branching fraction and Dalitz plots.

□ We measure:

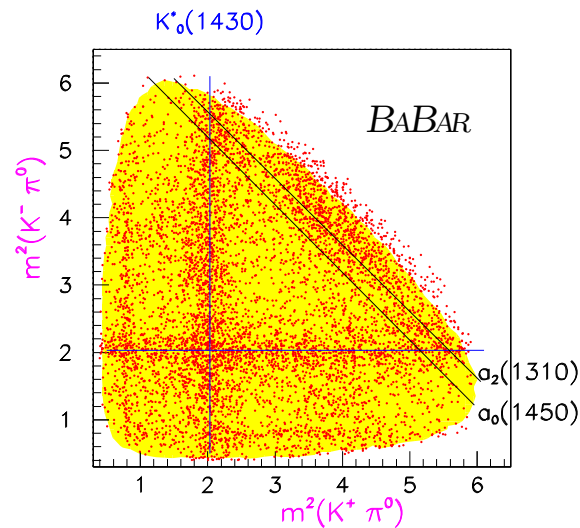
$$\mathcal{R}(\eta_c) = \frac{\mathcal{B}(\eta_c \rightarrow K^+ K^- \eta)}{\mathcal{B}(\eta_c \rightarrow K^+ K^- \pi^0)} = 0.571 \pm 0.025 \pm 0.051$$

□ Dalitz plots.

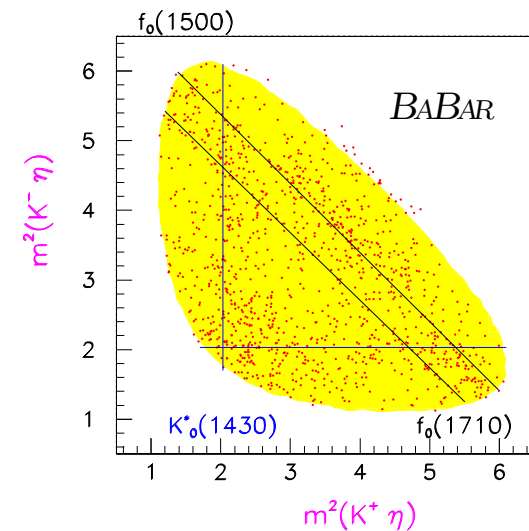
$\eta_c \rightarrow K_S^0 K^+ \pi^-$



$\eta_c \rightarrow K^+ K^- \pi^0$



$\eta_c \rightarrow K^+ K^- \eta$



□ Dominated by the presence of scalar mesons.

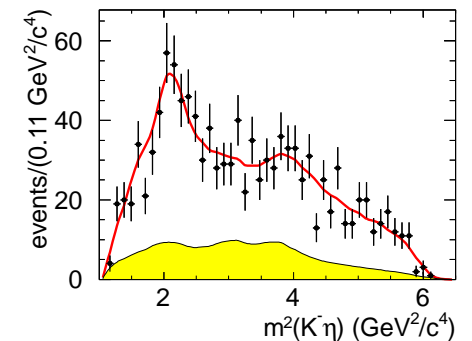
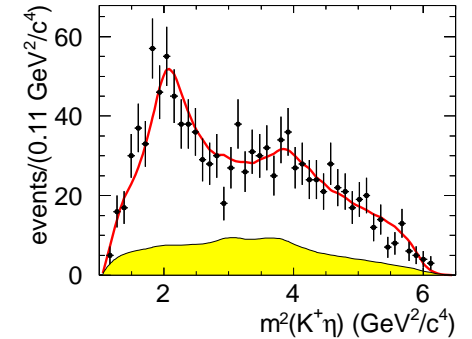
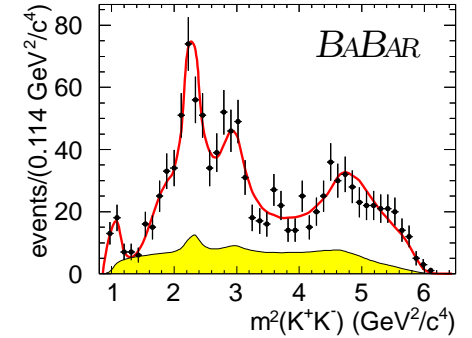
□ In particular, strong contribution from  $K_0^*(1430)$  in the three Dalitz plots.

## $\eta_c \rightarrow \eta K^+ K^-$ Dalitz plot analysis, Isobar model.

- Unbinned maximum likelihood fits.
  - Resonances described by Breit-Wigner functions.
- (D. Asner, Review of Particle Physics”, Phys. Lett. B 592, 1 (2004)).
- Results from the Dalitz analysis and fit projections.
  - Charge conjugated amplitudes symmetrized.

Final state	Fraction %	Phase (radians)
$f_0(1500)\eta$	$23.7 \pm 7.0 \pm 1.8$	0.
$f_0(1710)\eta$	$8.9 \pm 3.2 \pm 0.4$	$2.2 \pm 0.3 \pm 0.1$
$f_0(2200)\eta$	$11.2 \pm 2.8 \pm 0.5$	$2.1 \pm 0.3 \pm 0.1$
$f_0(1350)\eta$	$5.0 \pm 3.7 \pm 0.5$	$0.9 \pm 0.2 \pm 0.1$
$f_0(980)\eta$	$10.4 \pm 3.0 \pm 0.5$	$-0.3 \pm 0.3 \pm 0.1$
$f_2'(1525)\eta$	$7.3 \pm 3.8 \pm 0.4$	$1.0 \pm 0.1 \pm 0.1$
$K_0^*(1430)^+ K^-$	$16.4 \pm 4.2 \pm 1.0$	$2.3 \pm 0.2 \pm 0.1$
$K_0^*(1950)^+ K^-$	$2.1 \pm 1.3 \pm 0.2$	$-0.2 \pm 0.4 \pm 0.1$
$NR$	$15.5 \pm 6.9 \pm 1.0$	$-1.2 \pm 0.4 \pm 0.1$
Sum	$100.0 \pm 11.2 \pm 2.5$	
$\chi^2/\nu$	$87/65$	

- Largest amplitudes are  $f_0(1500)\eta$  and  $K_0^*(1430)K$ .
- First observation of  $K_0^*(1430) \rightarrow \eta K$ .



# $\eta_c \rightarrow \pi^0 K^+ K^-$ Dalitz analysis, Isobar model.

## □ Results from the Dalitz analysis and fit projections.

Final state	Fraction %			Phase (radians)
$K_0^*(1430)^+ K^-$	$33.8 \pm 1.9 \pm 0.4$			0.
$K_0^*(1950)^+ K^-$	$6.7 \pm 1.0 \pm 0.3$			$-0.67 \pm 0.07 \pm 0.03$
$K_2^*(1430)^+ K^-$	$6.8 \pm 1.4 \pm 0.3$			$-1.67 \pm 0.07 \pm 0.03$
$a_0(980)\pi^0$	$1.9 \pm 0.1 \pm 0.2$			$0.38 \pm 0.24 \pm 0.02$
$a_0(1450)\pi^0$	$10.0 \pm 2.4 \pm 0.8$			$-2.4 \pm 0.05 \pm 0.03$
$a_2(1320)\pi^0$	$2.1 \pm 0.1 \pm 0.2$			$0.77 \pm 0.20 \pm 0.04$
$NR$	$24.4 \pm 2.5 \pm 0.6$			$1.49 \pm 0.07 \pm 0.03$
Sum	$85.8 \pm 3.6 \pm 1.2$			
$\chi^2/\nu$	$212/130$			

□ Largest amplitudes are  $K_0^*(1430)K$  and  $a_0(1450)\pi^0$ .

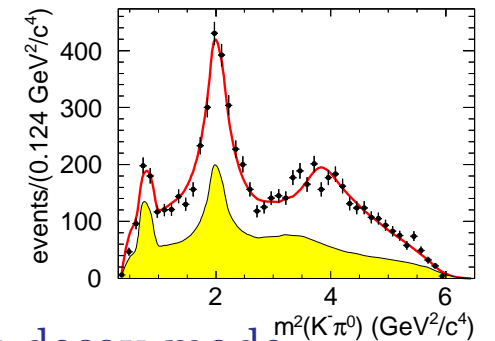
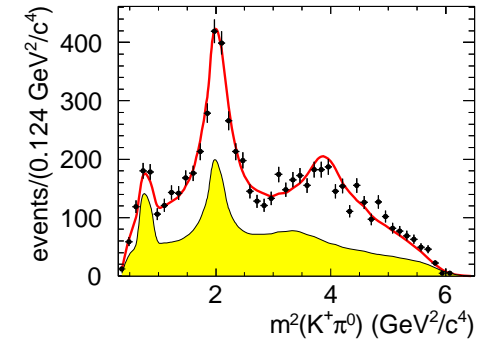
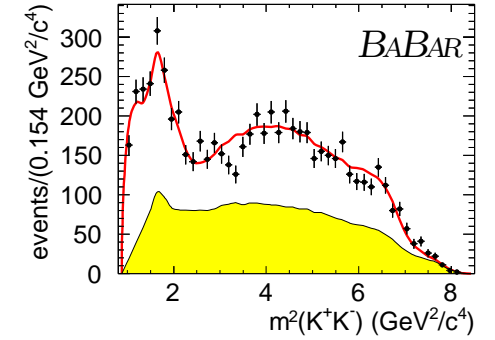
□  $K^*(890)$  contribution entirely from background.

□ No spin-one resonances expected in  $\gamma\gamma$  collisions.

□ We measure:

$$\frac{\mathcal{B}(K_0^*(1430) \rightarrow \eta K)}{\mathcal{B}(K_0^*(1430) \rightarrow \pi K)} = \mathcal{R}(\eta_c) \frac{f_{\eta K}}{f_{\pi K}} = 0.092 \pm 0.025^{+0.010}_{-0.025}$$

where  $f_{\pi K}$  denotes  $f_{\pi^0 K}$  fraction after correcting for the  $K^0\pi$  decay mode.



## Model Independent Partial Wave Analysis

(Phys. Rev. D **73**, 032004 (2006)).

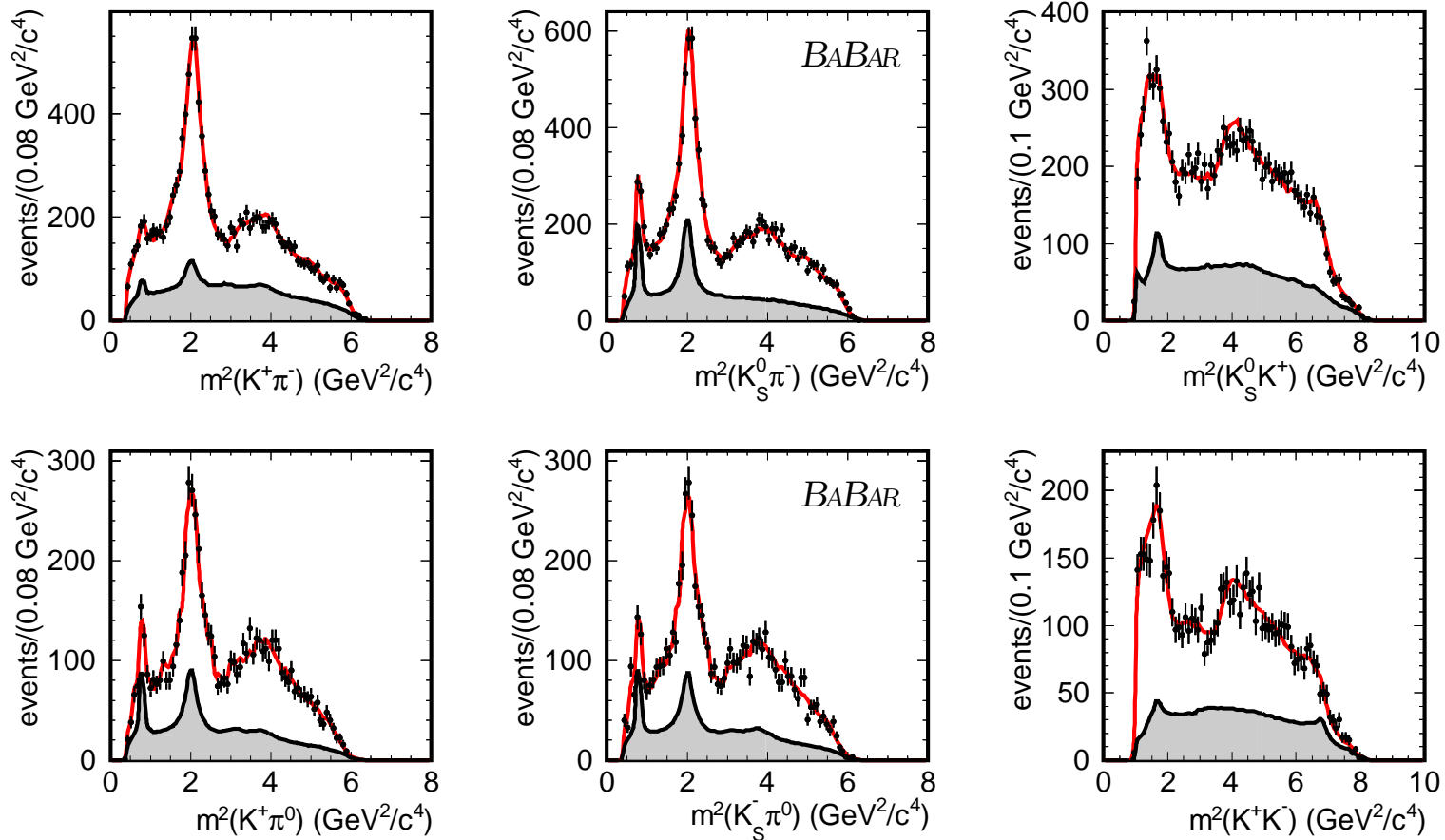
- The  $K\pi$   $\mathcal{S}$ -wave ( $A_1$ ) is taken as the reference amplitude.

$$A = A_1 + c_2 A_2 e^{i\phi_2} + c_3 A_3 e^{i\phi_3} + \dots$$

- The  $K\pi$  mass spectrum is divided into 30 equally spaced mass intervals 60 MeV wide and for each bin we add to the fit two new free parameters, the amplitude and the phase of the  $K\pi$   $\mathcal{S}$ -wave (constant inside the bin).
- Interference between the two  $K\pi$  modes is determined by Isospin conservation.
- The  $K_2^*(1420)$ ,  $a_0(980)$ ,  $a_0(1400)$ ,  $a_2(1310)$ , ... contributions are modeled as relativistic Breit-Wigner functions multiplied by the corresponding angular functions.
- Backgrounds are fitted separately and interpolated into the  $\eta_c$  signal regions.

## Dalitz plots mass projections

- Dalitz plot projections with fit results for  $\eta_c \rightarrow K_S^0 K^+ \pi^-$  (top) and  $\eta_c \rightarrow K^+ K^- \pi^0$  (bottom)



- Shaded is contribution from the interpolated background.

## Fit fractions from the MIPWA. Comparison with the Isobar Model

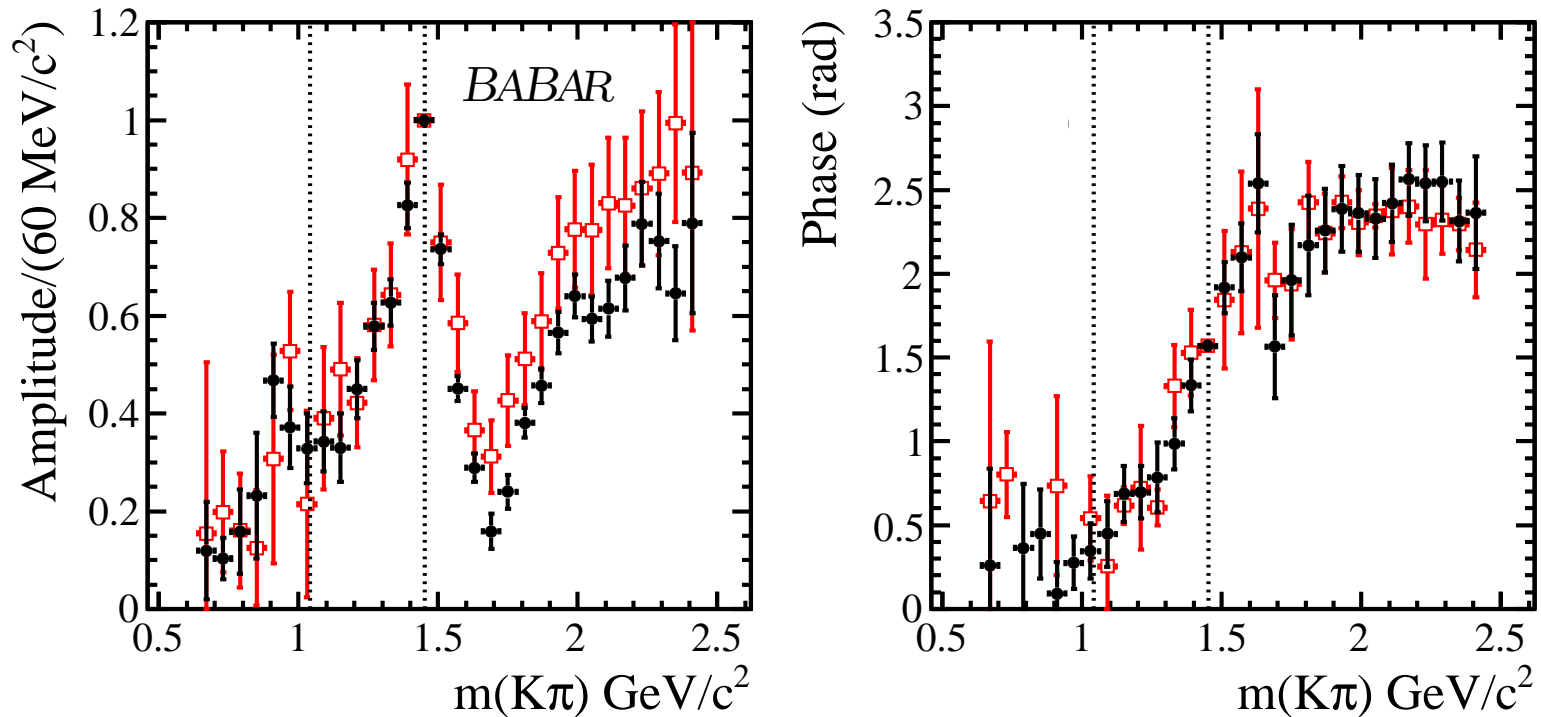
Amplitude	$\eta_c \rightarrow \mathbf{K}_S^0 \mathbf{K}^+ \pi^-$ Fraction (%)	$\eta_c \rightarrow \mathbf{K}^+ \mathbf{K}^- \pi^0$ Fraction (%)
$(K\pi \mathcal{S}\text{-wave}) K$	$107.3 \pm 2.6 \pm 17.9$	$125.5 \pm 2.4 \pm 4.2$
$a_0(1950)\pi$	$3.1 \pm 0.4 \pm 1.2$	$4.4 \pm 0.8 \pm 0.7$
$K_2^*(1430)^0 K$	$4.7 \pm 0.9 \pm 1.4$	$3.0 \pm 0.8 \pm 4.4$
$+a_0(980), a_0(1450), a_0(1950)$		
$+a_2(1320), K_2^*(1430)$		
$\chi_2/N_{cells}$	<b>301/254=1.17</b>	<b>283.2/233=1.22</b>
<b>Isobar Model</b>		
$(K_0^*(1430)K)+$	$73.6 \pm 3.7$	$63.6 \pm 5.6$
$(K_0^*(1950)K)+$		
<i>Nonresonant</i>		
$+a_0(980), a_0(1450), a_0(1950)$		
$+a_2(1320), K_2^*(1430)$		
$\chi_2/N_{cells}$	<b>467/256=1.82</b>	<b>383/233=1.63</b>

- For MIPWA, good agreement between the two  $\eta_c$  decay modes.
- $(K\pi \mathcal{S}\text{-wave})K$  amplitude dominant with small contributions from  $K_2^*(1430)^0 K$  and  $a_0(1950)\pi$ .
- Good description of the data with MIPWA.
- Worse description of the data with the Isobar Model.



## The $I=1/2$ $K\pi$ S-wave

- Fitted amplitude and phase. Average systematic uncertainty is 16%.
- Red:  $\eta_c \rightarrow K^+ K^- \pi^0$ . Black:  $\eta_c \rightarrow K_S^0 K^+ \pi^-$ .
- Clear  $K_0^*(1430)$  resonance and corresponding phase motion.
- At high mass broad  $K_0^*(1950)$  contribution.



- Dashed lines are  $K\eta$  and  $K\eta'$  thresholds.
- Good agreement between the two  $\eta_c$  decay modes.

## Comparison with the LASS and E791 experiments

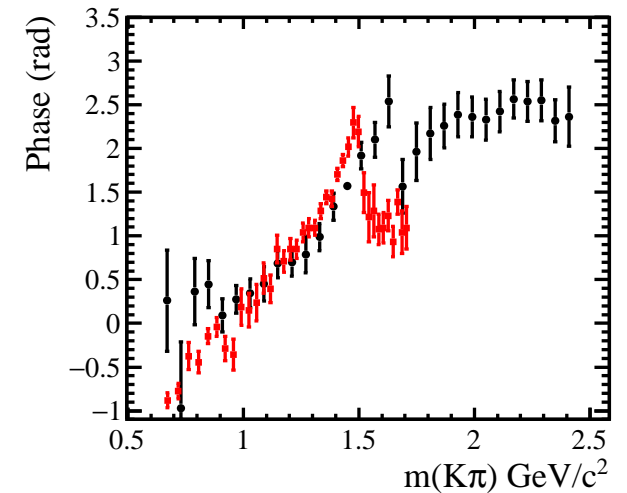
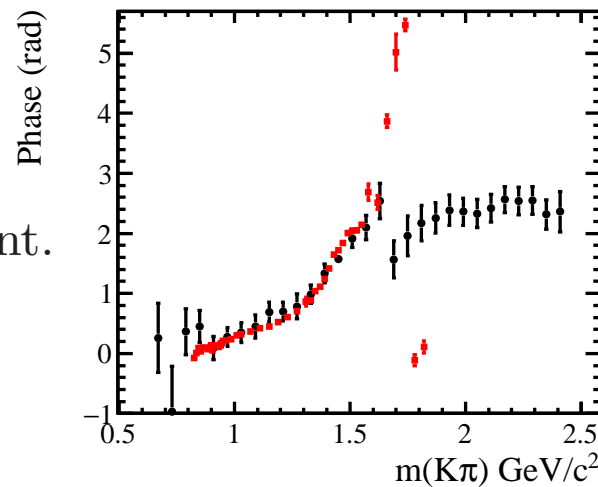
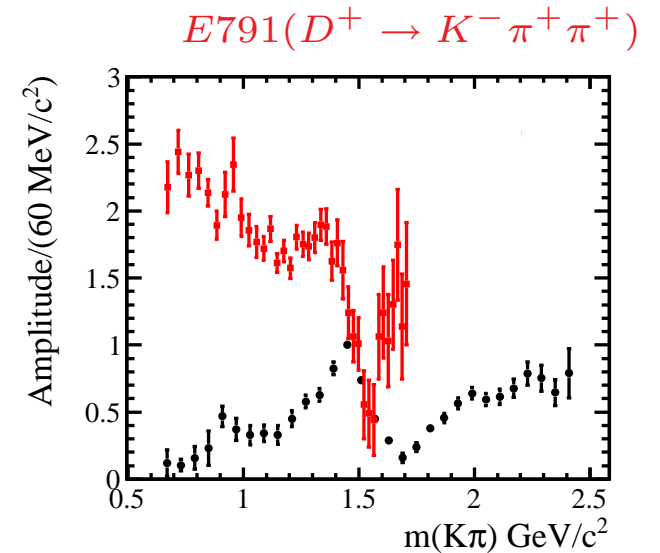
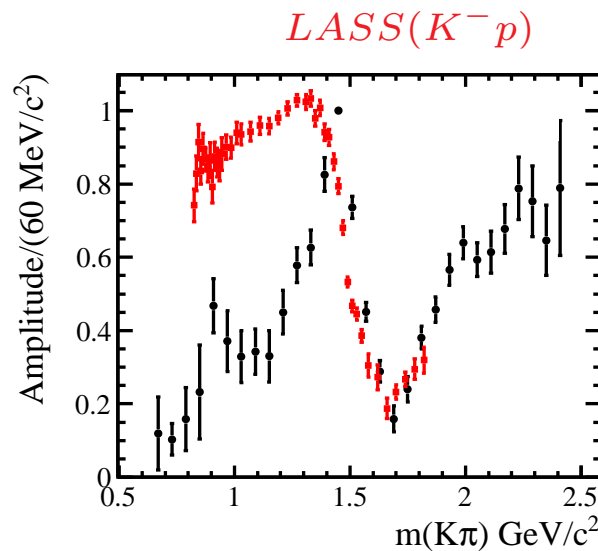
□ Black is  $\eta_c \rightarrow K_S^0 K^+ \pi^-$ .

□ Normalization is arbitrary.

□ LASS analysis has two solutions above 1.9 GeV.

□ Phases before the  $K\eta'$  threshold are similar, as expected from Watson theorem.

□ Amplitudes are very different.

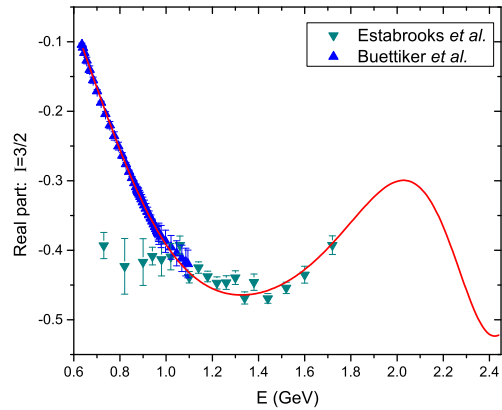


(LASS: Nucl. Phys. B **296**, 493 (1988)), (E791: Phys. Rev. D **73**, 032004 (2006)), (K.M. Watson, Phys. Rev. **88**, 1163 (1952))

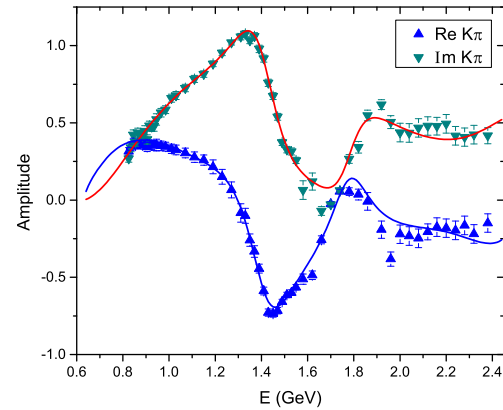
# Overall fit of LASS and $\eta_c$ data.

□ K-matrix fit. (A. Palano, M. Pennington, arXiv:1701.04881)

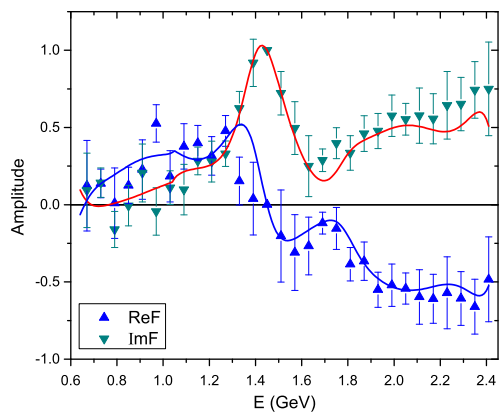
Elastic  $K^- \pi^- \rightarrow K^- \pi^-$



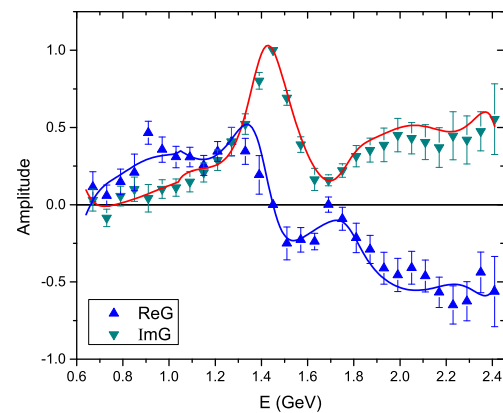
LASS :  $K^- \pi^+ \rightarrow K^- \pi^+$



BABAR  $\eta_c \rightarrow K^+ K^- \pi^0$



BABAR  $\eta_c \rightarrow K_S^0 K^+ \pi^-$



- Data fitted in terms of Real and Imaginary parts of the complex amplitudes.
- Solution A for the LASS data.
- Curves are fit results. Red: Imaginary, Blue: Real.

## Overall K-matrix fit of LASS and $\eta_c$ data.

### □ Measured pole positions.

$$\text{Pole 1 } E_{P1} = 659 - i302 \text{ MeV on Sheet II,}$$

$$\text{Pole 2 } E_{P2} = 1409 - i128 \text{ MeV on Sheet III,}$$

$$\text{Pole 3 } E_{P3} = 1768 - i107 \text{ MeV on Sheet III.}$$

□ Pole 1 is identified with the  $\kappa$ , the pole position of which was found to be at  $[(658 \pm 7) - i(278 \pm 13)]$  MeV, in the dispersive analysis of (arXiv:0310283, Eur.Phys.J. C33, 409 (2004)).

□ Pole 2 is identified with  $K_0^*(1430)$ , to be compared with  $[(1438 \pm 8 \pm 4) - i(105 \pm 20 \pm 12)]$  MeV using the Breit-Wigner form.

□ Pole 3 may be identified with the  $K_0^*(1950)$  with a pole mass closer to that of the reanalysis of the LASS by Anisovich (Phys. Lett.B413, 137 (1997)) with a pole at  $E = (1820 \pm 20) - i(125 \pm 50)$  MeV.

□ For pole 2, the  $K_0^*(1430)$ , we have a ratio of  $K\eta$  to  $K\pi$  decay of 0.05 consistent with the branching ratio of  $(0.092 \pm 0.025_{-0.025}^{+0.010})$  determined from the Dalitz plot analysis of  $\eta_c \rightarrow K^+ K^- \eta / \pi^0$  decays.

## Dalitz plot analysis of $J/\psi \rightarrow$ three body decays

□  $J/\psi$  samples are obtained from the Initial State Radiation (ISR) process. (Phys.Rev. D95 (2017), 072007)

□ Only  $J^{PC} = 1^{--}$  states can be produced.

□ We study the following reactions:

$$e^+e^- \rightarrow \gamma_{\text{ISR}} \pi^+ \pi^- \pi^0,$$

$$e^+e^- \rightarrow \gamma_{\text{ISR}} K^+ K^- \pi^0,$$

$$e^+e^- \rightarrow \gamma_{\text{ISR}} K_S^0 K^\pm \pi^\mp,$$

where  $\gamma_{\text{ISR}}$  indicates the (undetected) ISR photon.

□ We compute:

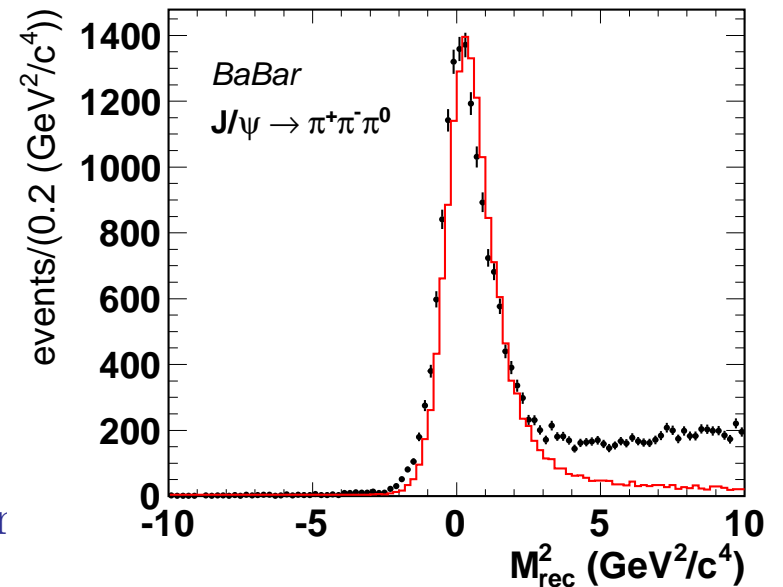
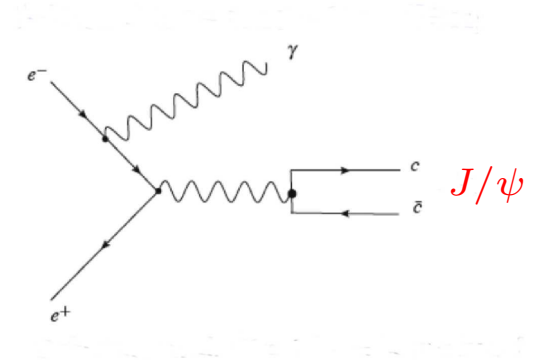
$$M_{\text{rec}}^2 \equiv (p_{e^-} + p_{e^+} - p_{h1} - p_{h2} - p_{h3})^2$$

where  $h = \pi/K/K_S^0$ .

□ This quantity should peak near zero for ISR events

□ Plot of  $M_{\text{rec}}^2$  in the  $J/\psi$  signal region.

*In red is Monte Carlo simulation.*



## J/ψ signals and branching fractions

- We select events in the ISR region by requiring  $|M_{\text{rec}}^2| < 2 \text{ GeV}^2/c^4$  ( $|M_{\text{rec}}^2| < 1.5 \text{ GeV}^2/c^4$ ).
- We fit the mass spectra using the Monte Carlo resolution functions described by a Crystal Ball+Gaussian functions and obtain the yields:

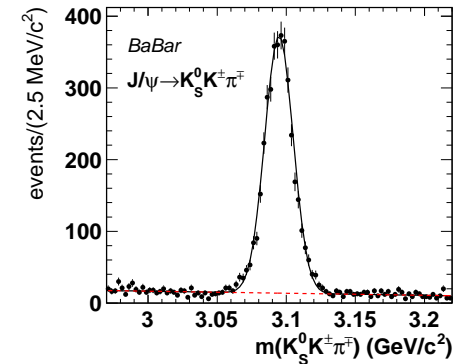
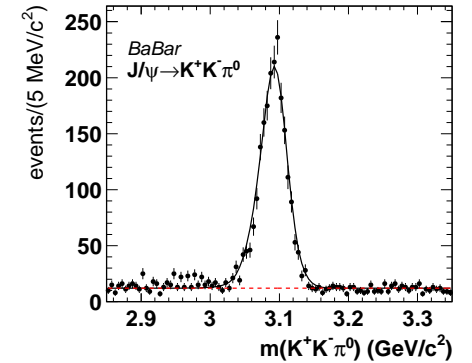
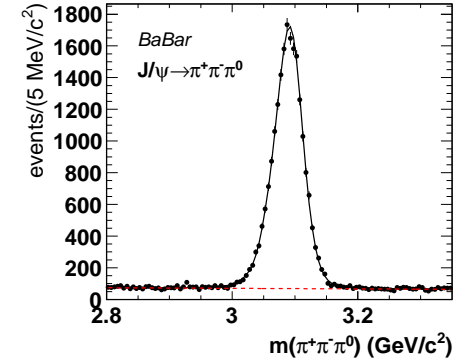
J/ψ decay mode	Signal region (GeV/c <sup>2</sup> )	Event yields	Purity %
$\pi^+ \pi^- \pi^0$	3.028-3.149	20417	$91.3 \pm 0.2$
$K^+ K^- \pi^0$	3.043-3.138	2102	$88.8 \pm 0.7$
$K_S^0 K^\pm \pi^\mp$	3.069-3.121	3907	$93.1 \pm 0.4$

- We measure the following branching fractions:

$$\mathcal{R}_1 = \frac{\mathcal{B}(J/\psi \rightarrow K^+ K^- \pi^0)}{\mathcal{B}(J/\psi \rightarrow \pi^+ \pi^- \pi^0)} = 0.120 \pm 0.003(\text{stat}) \pm 0.009(\text{sys})$$

$$\mathcal{R}_2 = \frac{\mathcal{B}(J/\psi \rightarrow K_S^0 K^\pm \pi^\mp)}{\mathcal{B}(J/\psi \rightarrow \pi^+ \pi^- \pi^0)} = 0.265 \pm 0.005(\text{stat}) \pm 0.021(\text{sys})$$

- $\mathcal{R}_1^{PDG} = 0.133 \pm 0.038$ , in agreement with our measurement.
- $\mathcal{R}_2^{PDG} = 0.123 \pm 0.033$ , which deviates by  $3.6\sigma$  from our measurement.



# $J/\psi \rightarrow \pi^+ \pi^- \pi^0$ Dalitz plot and projections

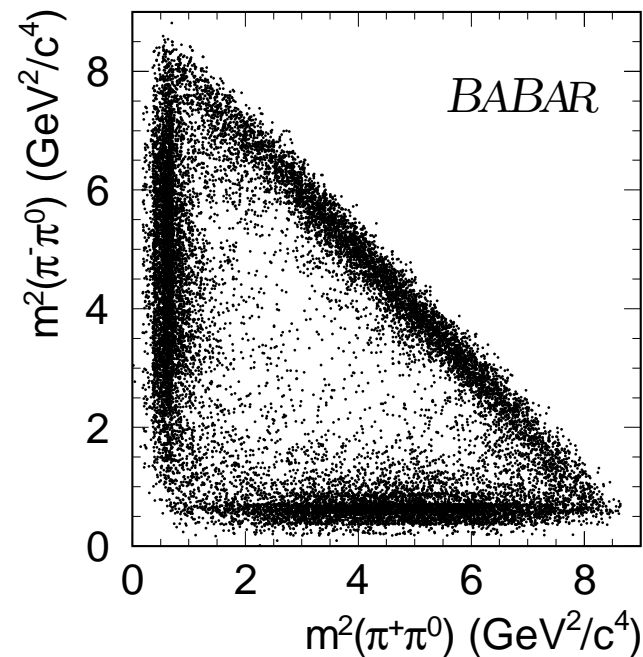
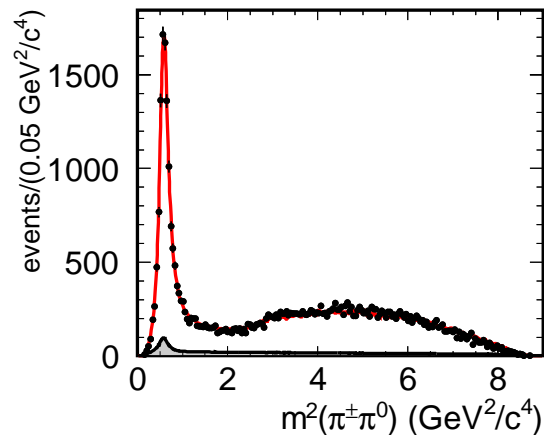
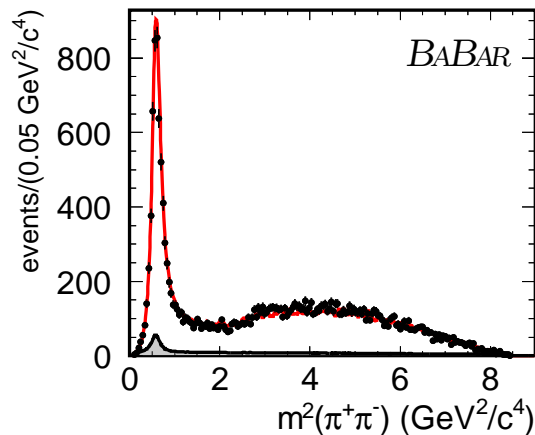
□ Dominated by three  $\rho(770)\pi$  contributions.

□ Dalitz plot analysis performed using:

- Isobar model using Zemach tensors;  
C. Zemach, Phys Rev. **133**, B1201 (1964),  
C. Dionisi et. al., Nucl. Phys. **B169**, 1 (1980).
- Veneziano model.

(A. P. Szczepaniak, M.R. Pennington, Phys. Lett. **B737**, 283 (2014)).

□ Dalitz plot projections.

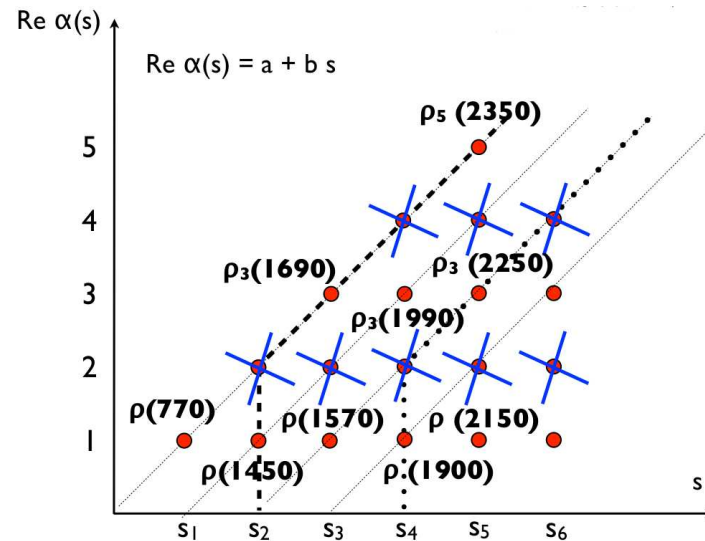


□ Shaded is the background interpolated by sidebands.

## $J/\psi \rightarrow \pi^+ \pi^- \pi^0$ Dalitz plot analysis with Veneziano model

□ The Veneziano model deals with trajectories rather than single resonances

(arXiv:1403.5782, Phys.Lett. N737, 283 (2014)).



□ The amplitudes are written as:

$$A_{X \rightarrow abc} = \sum_{n,m} c_{X \rightarrow abc}(n, m) A_{n,m}$$

with  $1 \leq m \leq n$ .

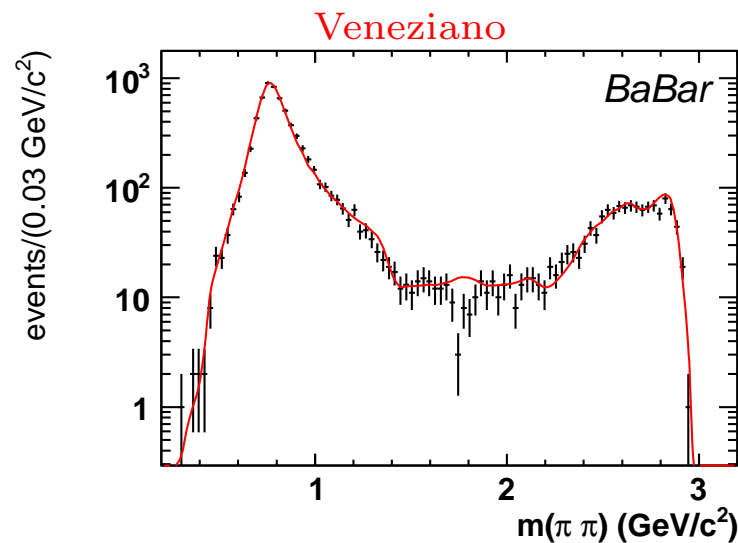
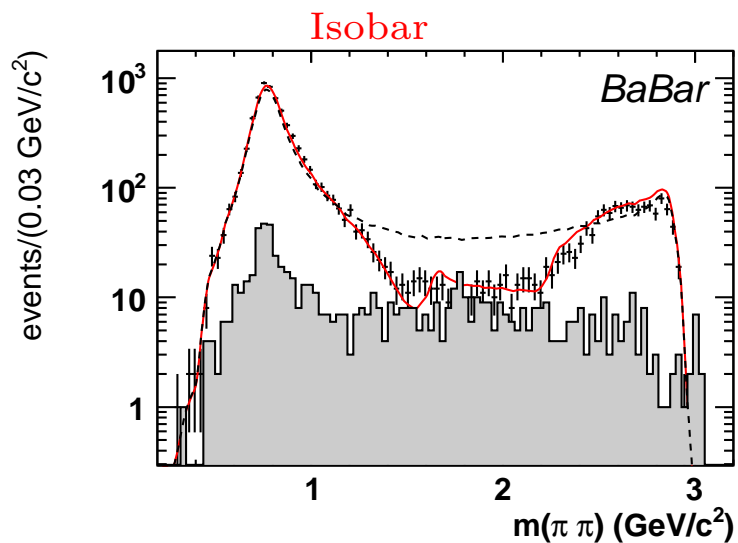
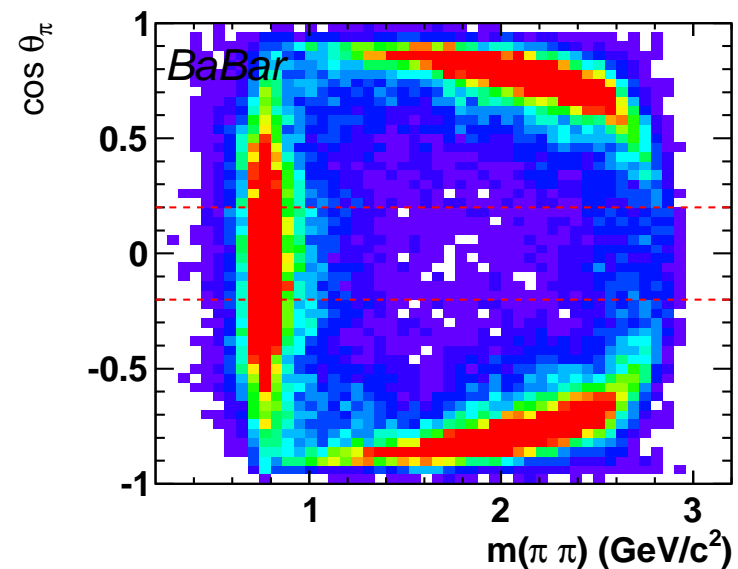
□ The complexity of the model is related to  $n$ , the number of Regge trajectories included in the fit.

□ The fit requires  $n=7$ , with 19 free parameters.



$J/\psi \rightarrow \pi^+\pi^-\pi^0$  Dalitz plot analysis.

- Combinatorial  $\pi$  helicity angle vs.  $m(\pi\pi)$ .
- $m(\pi\pi)$  mass projections for  $|\cos\theta_\pi| < 0.2$  in log scale.
- The cut removes the reflections from the other combinations.
- Gray is the background contribution.
- Dashed line: fit without  $\rho'$  contributions.



## $J/\psi \rightarrow \pi^+\pi^-\pi^0$ Dalitz plot analysis.

Final state	Amplitude	Isobar fraction (%)	Phase (radians)	Veneziano fraction (%)
$\rho(770)\pi$	1.	$114.2 \pm 1.1 \pm 2.6$	0.	$133.1 \pm 3.3$
$\rho(1450)\pi$	$0.513 \pm 0.039$	$10.9 \pm 1.7 \pm 2.7$	$-2.63 \pm 0.04 \pm 0.06$	$0.80 \pm 0.27$
$\rho(1700)\pi$	$0.067 \pm 0.007$	$0.8 \pm 0.2 \pm 0.5$	$-0.46 \pm 0.17 \pm 0.21$	$2.20 \pm 0.60$
$\rho(2150)\pi$	$0.042 \pm 0.008$	$0.04 \pm 0.01 \pm 0.20$	$1.70 \pm 0.21 \pm 0.12$	$6.00 \pm 2.50$
$\omega(783)\pi^0$	$0.013 \pm 0.002$	$0.08 \pm 0.03 \pm 0.02$	$2.78 \pm 0.20 \pm 0.31$	
$\rho_3(1690)\pi$				$0.40 \pm 0.08$
Sum		$127.8 \pm 2.0 \pm 4.3$		$142.5 \pm 2.8$
$\chi^2/\nu$		$687/519 = 1.32$		$596/508 = 1.17$

□ The two models have similar quality, but different fractions. The Veneziano model fits better the data.

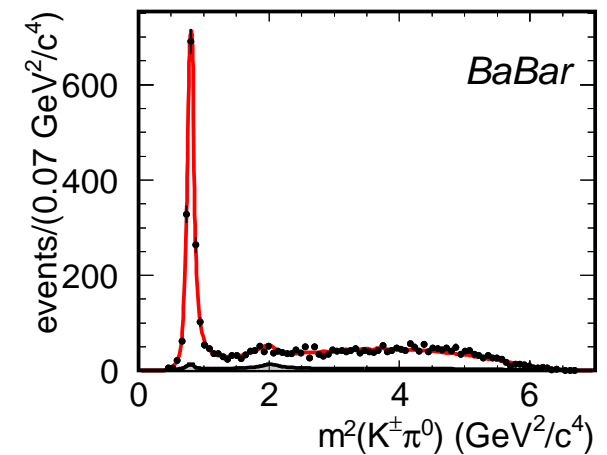
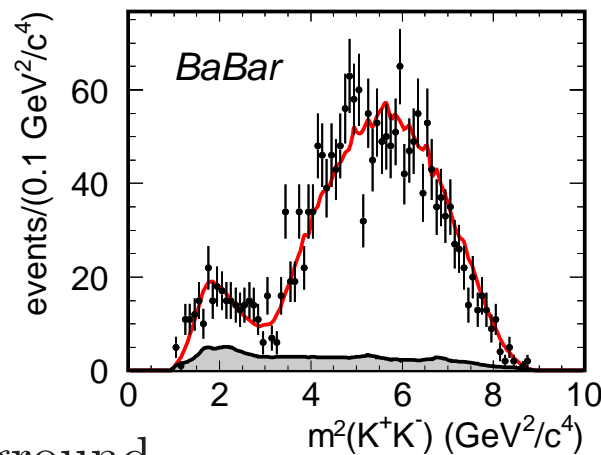
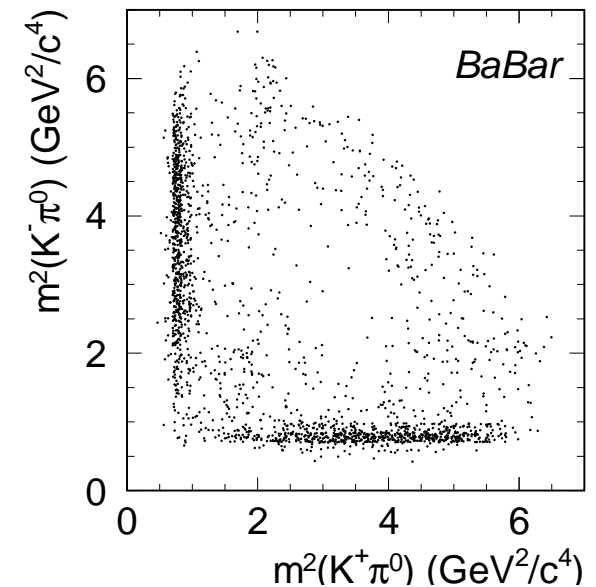
□ This may indicate that other resonances are contributing to the decay.

## $J/\psi \rightarrow K^+ K^- \pi^0$ Dalitz plot analysis

- Clear  $K^{*+}$  and  $K^{*-}$  bands.
- Broad structure in the low  $K^+ K^-$  mass region.
- We make use of the Isobar model only.

Final state	fraction (%)	phase (radians)
$K^*(892)^\pm K^\mp$	$92.4 \pm 1.5 \pm 3.4$	0.
$\rho(1450)^0 \pi^0$	$9.3 \pm 2.0 \pm 0.6$	$3.78 \pm 0.28 \pm 0.08$
$K^*(1410)^\pm K^\mp$	$2.3 \pm 1.1 \pm 0.7$	$3.29 \pm 0.26 \pm 0.39$
$K_2^*(1430)^\pm K^\mp$	$3.5 \pm 1.3 \pm 0.9$	$-2.32 \pm 0.22 \pm 0.05$
Total	$107.4 \pm 2.8$	
$\chi^2/\nu$	$132/137 = 0.96$	

- Dalitz plot projections:

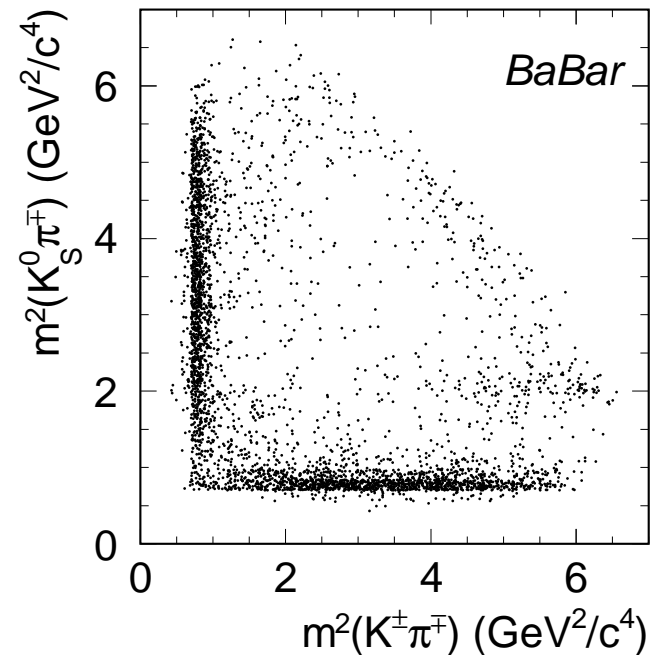


- Shaded is the background.

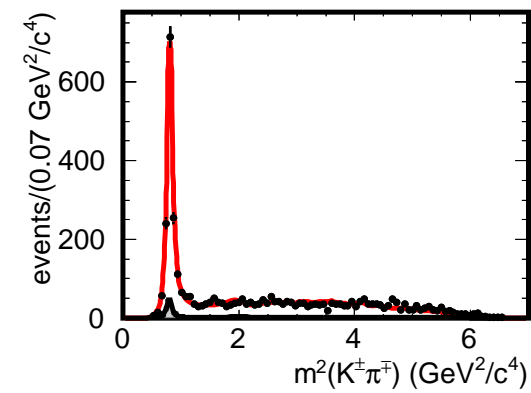
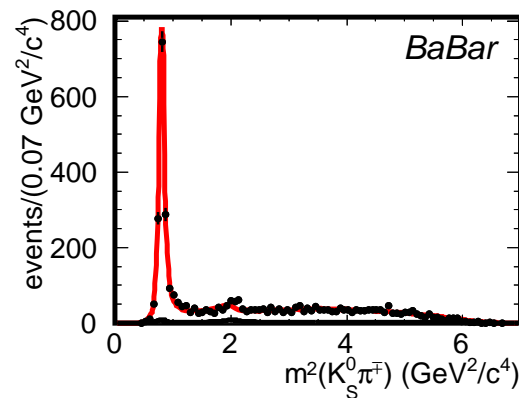
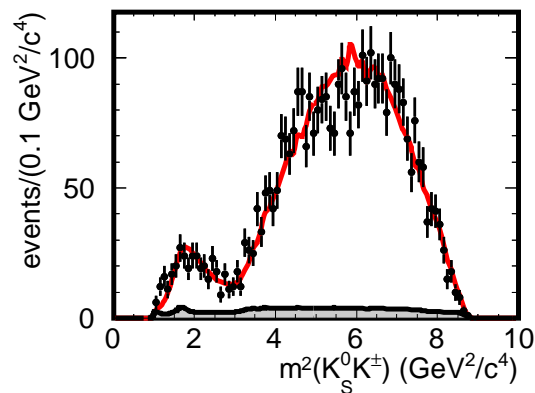
# $J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$ Dalitz plot analysis

- Clear  $K^{*+}$  and  $K^{*-}$  bands.
- Broad structure in the low  $K^+ K^-$  mass region.
- We make use of the Isobar model only.

Final state	fraction (%)	phase (radians)
$K^*(892)\bar{K}$	$90.5 \pm 0.9 \pm 3.8$	0.
$\rho(1450)^\pm \pi^\mp$	$6.3 \pm 0.8 \pm 0.6$	$-3.25 \pm 0.13 \pm 0.21$
$K_1^*(1410)\bar{K}$	$1.5 \pm 0.5 \pm 0.9$	$1.42 \pm 0.31 \pm 0.35$
$K_2^*(1430)\bar{K}$	$7.1 \pm 1.3 \pm 1.2$	$-2.54 \pm 0.12 \pm 0.12$
Total	$105.3 \pm 3.1$	
$\chi^2/\nu$	$274/217 = 1.26$	



- Dalitz plot projections:



## $\rho(1450)$ branching fraction

□ We find the parameters of the low mass  $K\bar{K}$  structure consistent with being associated to  $\rho(1450)$ .

□ From the Dalitz-plot analysis of  $J/\psi \rightarrow \pi^+\pi^-\pi^0$  we obtain:

$$\begin{aligned}\mathcal{B}_1 &= \frac{\mathcal{B}(J/\psi \rightarrow \rho(1450)^0\pi^0)\mathcal{B}(\rho(1450)^0 \rightarrow \pi^+\pi^-)}{\mathcal{B}(J/\psi \rightarrow \pi^+\pi^-\pi^0)} \\ &= (3.6 \pm 0.6(\text{stat}) \pm 0.9(\text{sys}))\%.\end{aligned}$$

□ From the Dalitz-plot analysis of  $J/\psi \rightarrow K^+K^-\pi^0$  we obtain:

$$\begin{aligned}\mathcal{B}_2 &= \frac{\mathcal{B}(J/\psi \rightarrow \rho(1450)^0\pi^0)\mathcal{B}(\rho(1450)^0 \rightarrow K^+K^-)}{\mathcal{B}(J/\psi \rightarrow K^+K^-\pi^0)} \\ &= (9.3 \pm 2.0(\text{stat}) \pm 0.6(\text{sys}))\%.\end{aligned}$$

□ We therefore obtain:

$$\begin{aligned}\frac{\mathcal{B}(\rho(1450)^0 \rightarrow K^+K^-)}{\mathcal{B}(\rho(1450)^0 \rightarrow \pi^+\pi^-)} \\ = 0.307 \pm 0.084(\text{stat}) \pm 0.082(\text{sys}).\end{aligned}$$

## Summary

- We show results on the Dalitz plot analyses of  $\eta_c \rightarrow K_S^0 K^+ \pi^-$ ,  $\eta_c \rightarrow K^+ K^- \pi^0$  and  $\eta_c \rightarrow K^+ K^- \eta$  produced in two-photon interactions.  
(Phys.Rev. D89 (2014) no.11, 112004, Phys.Rev. D93 (2016) 012005)
- We extract for the first time the  $I=1/2$   $K\pi$   $\mathcal{S}$ -wave amplitude and phase using the MIPWA method up to a mass of  $2.5 \text{ GeV}/c^2$ . We find a very different amplitude with respect to that measured by previous experiments in different processes.  
(Phys.Rev. D93 (2016) 012005)
- A K-matrix formalism is able to obtain a good description of the  $I=1/2$   $K\pi$   $\mathcal{S}$ -wave.  
(A. Palano, M. Pennington, arXiv:1701.04881)
- We show results on Dalitz plot analyses of  $J/\psi \rightarrow \pi^+ \pi^- \pi^0$ ,  $J/\psi \rightarrow K^+ K^- \pi^0$  and  $J/\psi \rightarrow K_S^0 K^\pm \pi^\mp$  produced in Initial State Radiation events using the isobar and Veneziano models.  
(Phys.Rev. D95 (2017), 072007)