

*EXA 2017 @ Wien, 2017, Sep.*

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$\bar{K}N$ - $\pi\Sigma$  coupled-channel potential  
derived from Chiral  $SU(3)$   
dynamics

K. Miyahara (Kyoto univ.)  
T. Hyodo (YITP)  
W. Weise (ECT\* and TUM)

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- ❖ Motivation
- ❖ Formulation
- ❖  $\bar{K}N$  single-channel potential
  - Potential with SIDDHARTA constraint
  - Application to  $\Lambda(1405)$  and  $\bar{K}$ -nuclei
- ❖  $\bar{K}N$ - $\pi\Sigma$  coupled-channel potential
- ❖ Summary

# Motivation

## ❖ $\bar{K}N$ interaction

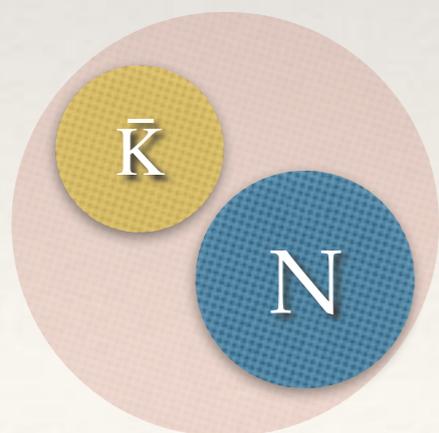
- strongly attractive
- without repulsive core



Interesting states with  $\bar{K}$  and  $N$ 's are expected.

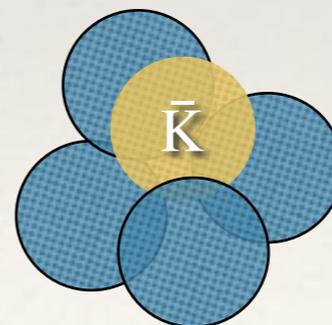
Akaishi, Yamazaki, Phys. Rev. C 65 (2002) 044005  
Hyodo, Jido, Prog. Part. Nucl. Phys. 67 (2012) 55

### “molecule picture” of $\Lambda(1405)$



- exotic state ?
- double pole ?

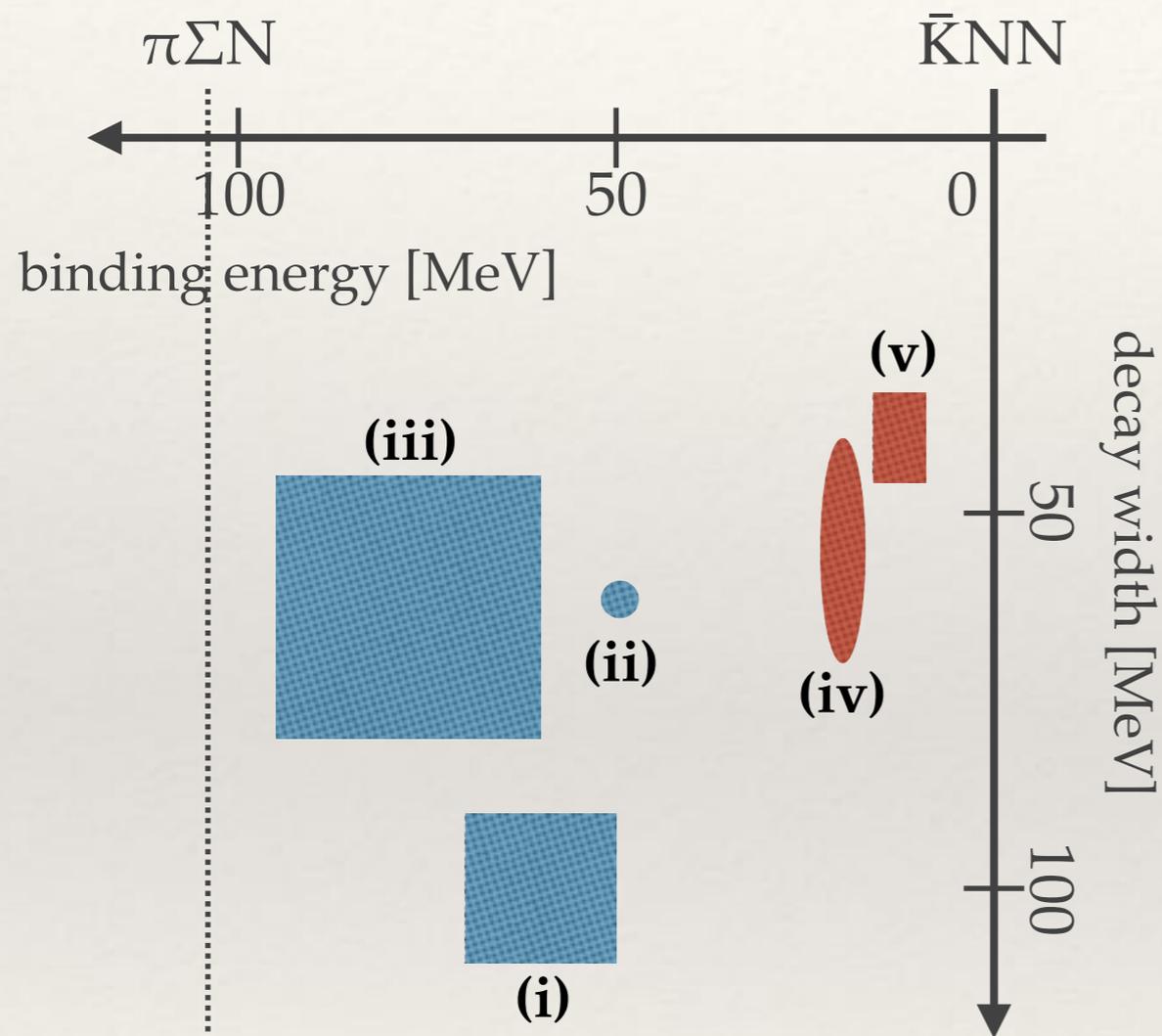
### $\bar{K}$ nuclei



- deeply bound ?
- compact state ?  
(high density)

# Motivation

## ❖ $\bar{K}NN$ state ( $I=1/2, J^P=0^-$ )



### ▶ 3-body calculation

– variational



– Faddeev



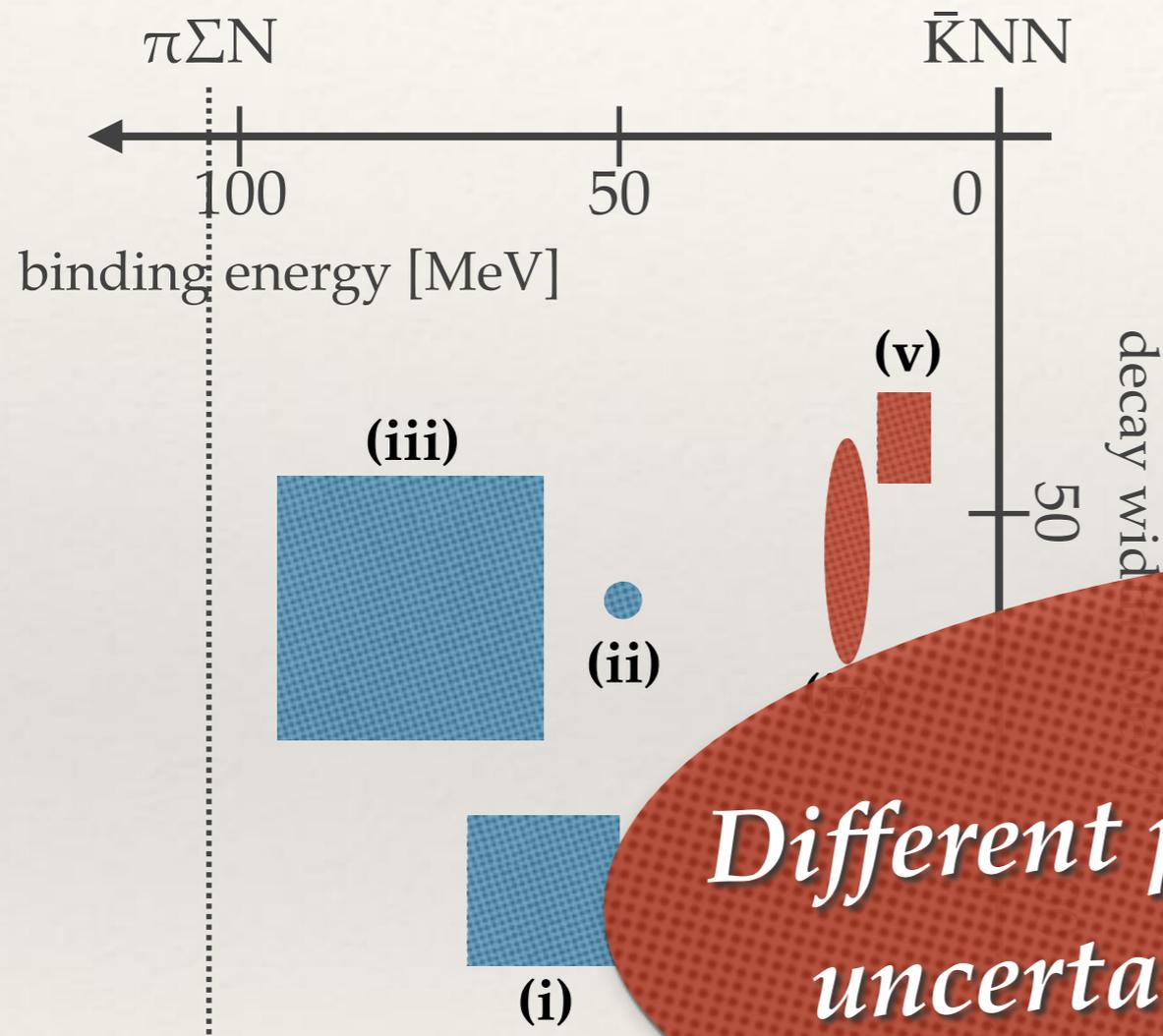
### theory

- (i) Shevchenko, Gal, Mares, PRC 76 (2007) 044004
- (ii) Yamazaki, Akaishi, PRC 76 (2007) 045201
- (iii) Ikeda, Sato, PRC 76 (2007) 035203
- (iv) Dote, Hyodo, Weise, PRC 79 (2009) 014003
- (v) Ikeda, Kamano, Sato, PTP 124 (2010) 533

**Conclusive result has not been achieved.**

# Motivation

## ❖ $\bar{K}NN$ state ( $I=1/2, J^P=0^-$ )



### ▶ 3-body calculation

– variational



– Faddeev



### theory

(i) Shevchenko, Gal, Mares, PRC 76 (2007) 044004

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(iii) Ikeda, Sato, PRC 76 (2007) 035203

(iv) ... PRC 79 (2009) 014003

(v) ... (2010) 533

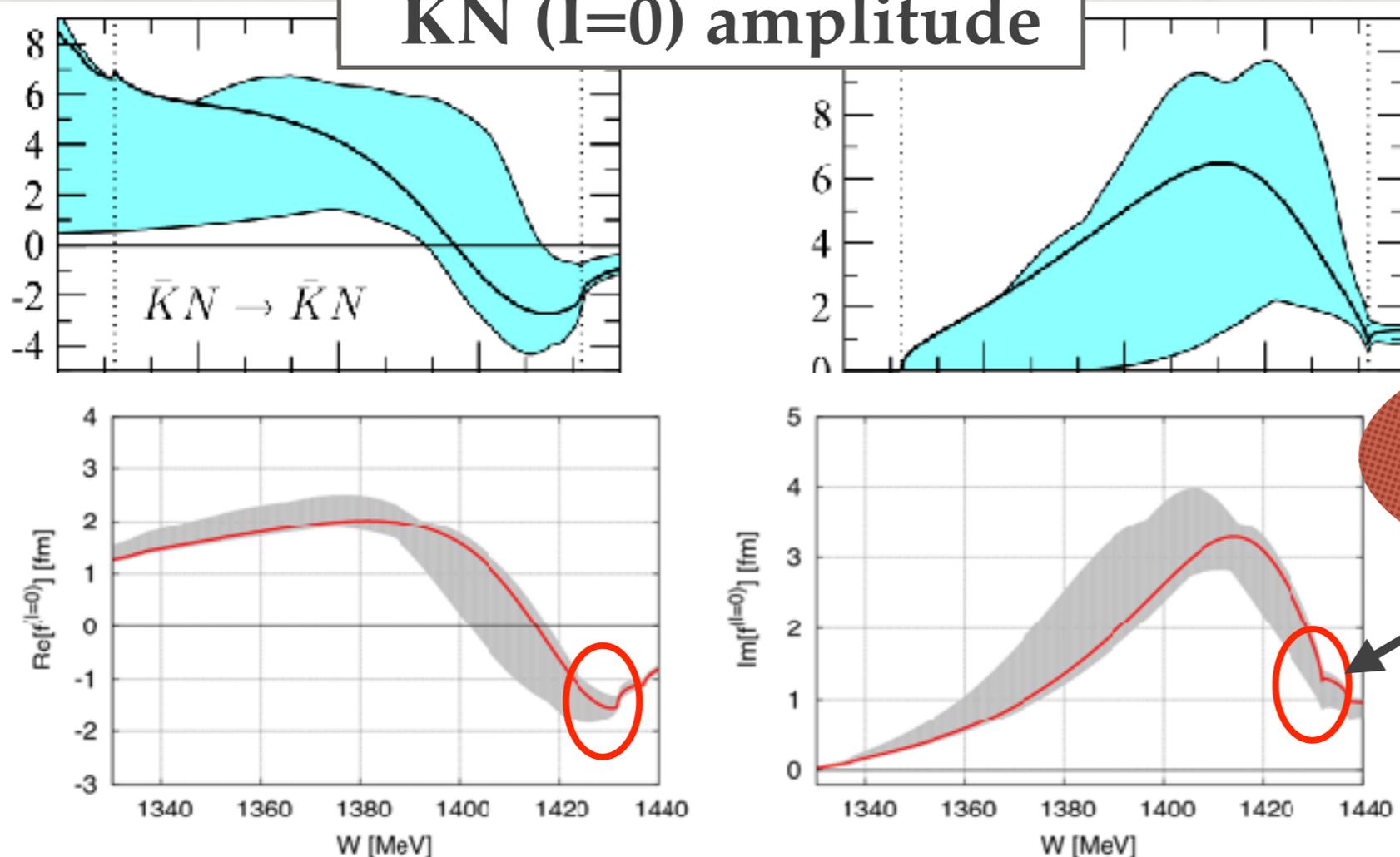
*Different predictions are caused by uncertainty of  $\bar{K}N$  interaction.*

**Conclusive result has not been achieved.**

# Motivation

❖ New constraint from precise exp.

$\bar{K}N$  ( $I=0$ ) amplitude



- Borasoy et al. PRC 74 (2006) 055201
- R.Nissler, Ph.D thesis (2007)

Bazzi et al. PLB 704, 113 (2011)

**SIDDHARTA**

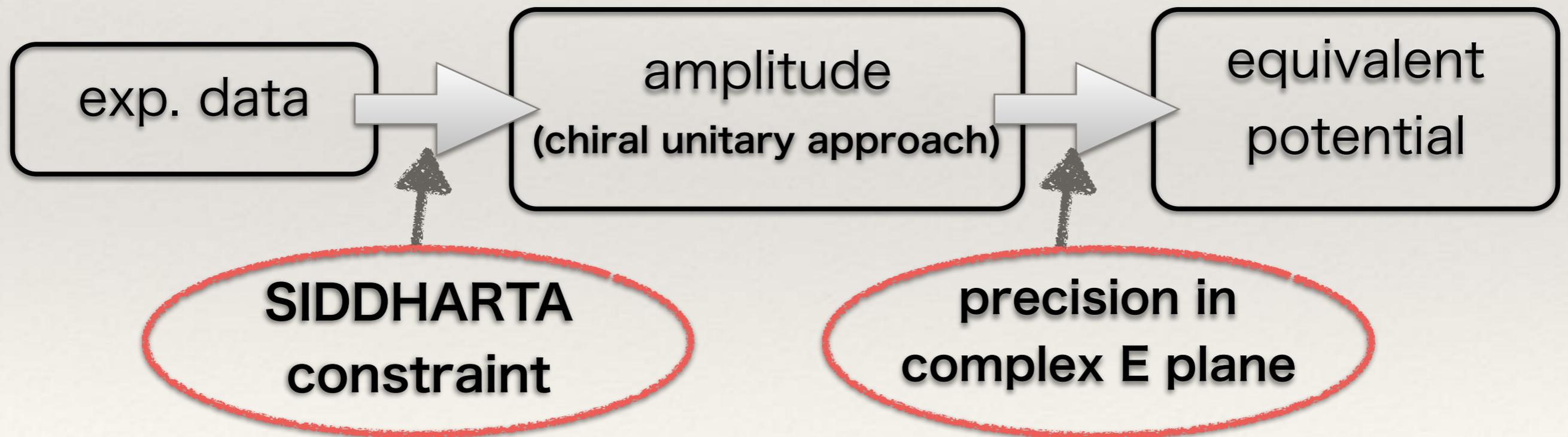
- Ikeda, Hyodo, Weise, NPA 881 (2012) 98
- Kamiya, et al., NPA 954 (2016) 41

SIDDHARTA exp. has enabled  
quantitative discussion.

# Motivation

Construct  $r$ -dep. realistic potential  
with SIDDHARTA constraint.

- 
- $\Lambda(1405)$  analysis
  - few-body calculation

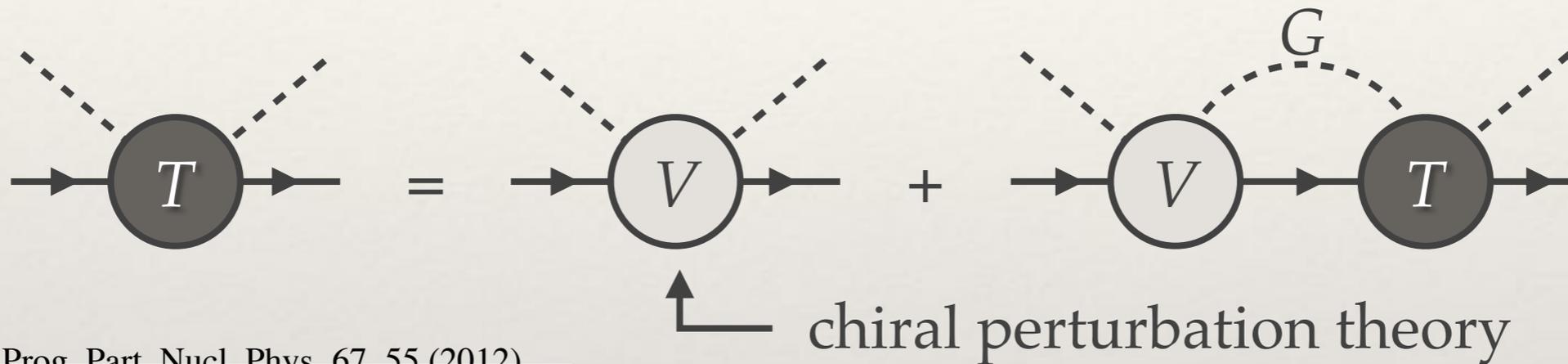


# Formulation

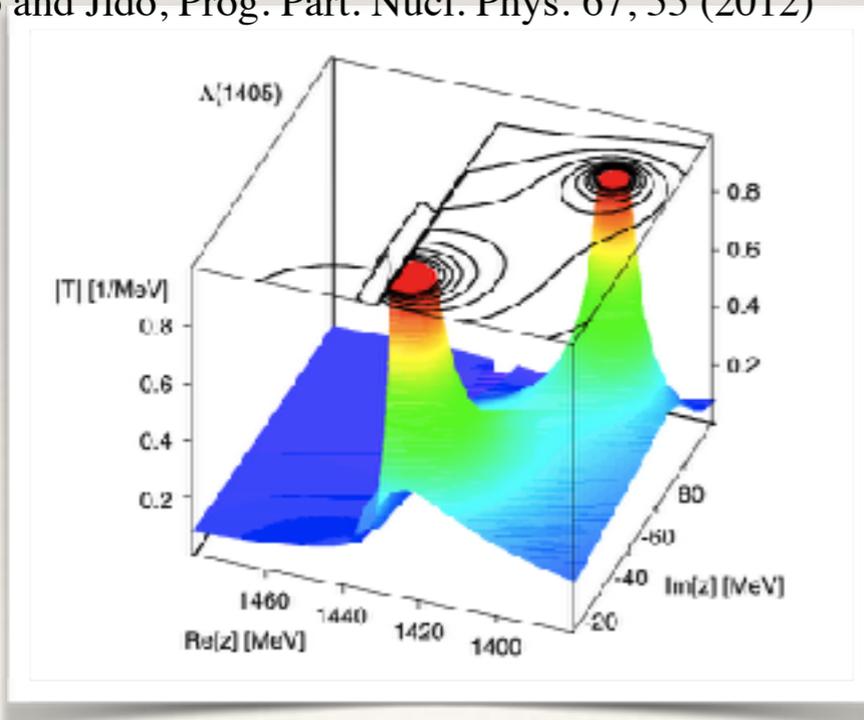
cf.) Hyodo, Weise,  
PRC77 (2008) 035204



## ❖ chiral unitary approach



Hyodo and Jido, Prog. Part. Nucl. Phys. 67, 55 (2012)



## channel coupling

attraction in  $\bar{K}N$  and  $\pi\Sigma$  channels

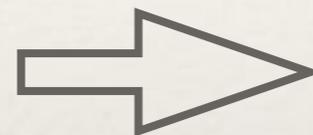
→ double pole structure  
of  $\Lambda(1405)$

# Formulation

cf.) Hyodo, Weise,  
PRC77 (2008) 035204



$$-\frac{1}{2\mu} \frac{d^2 u(r)}{dr^2} + \underline{V^{\text{equiv}}(r, E)} u(r) = E u(r)$$



$$\underline{F^{\text{equiv}} = F^{\text{Ch}}}$$

$$\underline{V^{\text{equiv}}(r, E)} = \underline{g(r)} N(E) \left[ V^{\text{eff}}(E) + \Delta V \right]$$

assume Gaussian  $r$ -dependence

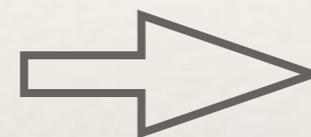
$$g(r) = \frac{1}{\pi^{3/2} b^3} e^{-r^2/b^2} \quad ( b : \text{range parameter} )$$

# Formulation

cf.) Hyodo, Weise,  
PRC77 (2008) 035204



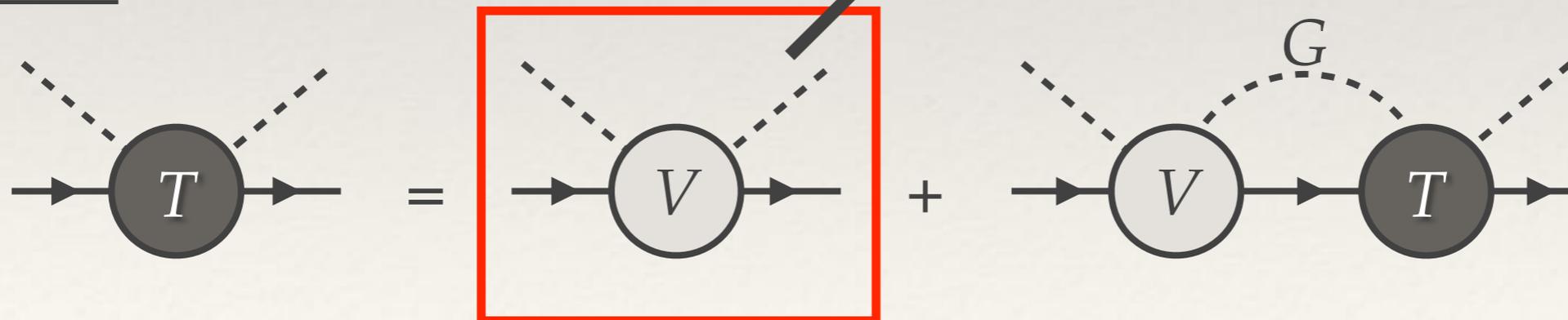
$$-\frac{1}{2\mu} \frac{d^2 u(r)}{dr^2} + \underline{V^{\text{equiv}}(r, E)} u(r) = E u(r)$$



$$\boxed{F^{\text{equiv}} = F^{\text{Ch}}}$$

$$V^{\text{equiv}}(r, E) = g(r) N(E) \left[ \underline{V^{\text{eff}}(E)} + \Delta V \right]$$

Ch-U.

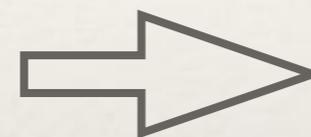


# Formulation

cf.) Hyodo, Weise,  
PRC77 (2008) 035204



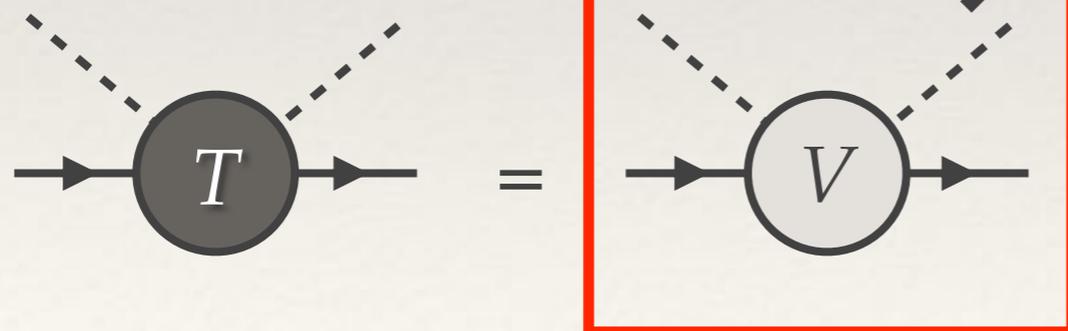
$$-\frac{1}{2\mu} \frac{d^2 u(r)}{dr^2} + \underline{V^{\text{equiv}}(r, E)} u(r) = E u(r)$$



$$F^{\text{equiv}} = F^{\text{Ch}}$$

$$V^{\text{equiv}}(r, E) = g(r) N(E) \left[ \underline{V^{\text{eff}}(E)} + \Delta V \right]$$

Ch-U.



channel coupling

Ch-U. :  $\pi\Sigma, \bar{K}N, \eta\Lambda, K\Xi$

Schrödinger :  $(\pi\Sigma), \bar{K}N$

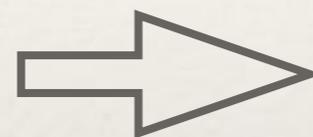
**→ Feshbach projection**

# Formulation

cf.) Hyodo, Weise,  
PRC77 (2008) 035204



$$-\frac{1}{2\mu} \frac{d^2 u(r)}{dr^2} + \underline{V^{\text{equiv}}(r, E)} u(r) = E u(r)$$



$$\underline{F^{\text{equiv}} = F^{\text{Ch}}}$$

$$V^{\text{equiv}}(r, E) = g(r) N(E) \left[ V^{\text{eff}}(E) + \underline{\Delta V} \right]$$

**correction for model difference**

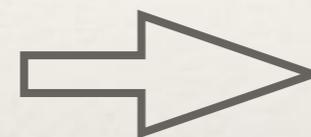
**Ch-U. ↔ Schrödinger eq.**

# Formulation

cf.) Hyodo, Weise,  
PRC77 (2008) 035204

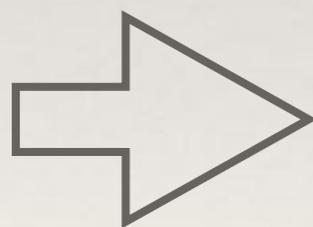


$$-\frac{1}{2\mu} \frac{d^2 u(r)}{dr^2} + \underline{V^{\text{equiv}}(r, E)} u(r) = E u(r)$$



$$F^{\text{equiv}} = F^{\text{Ch}}$$

$$V^{\text{equiv}}(r, E) = g(r) N(E) \left[ V^{\text{eff}}(E) + \Delta V \right]$$



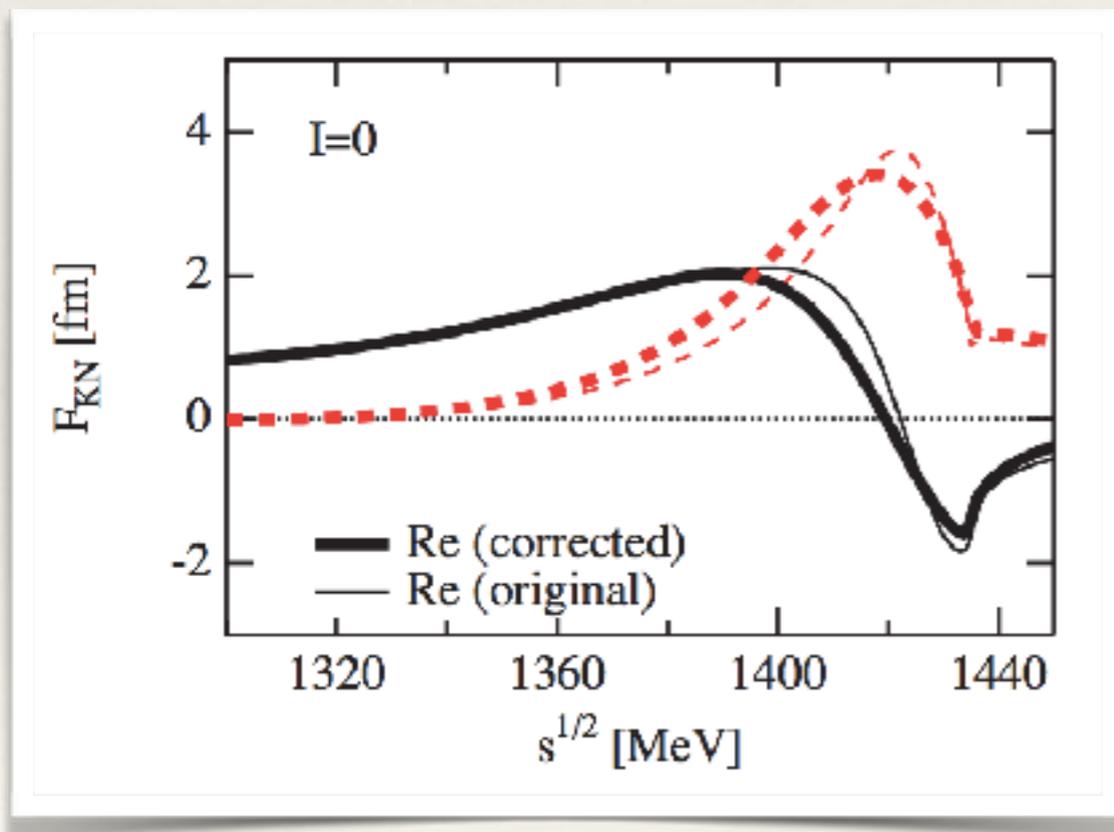
fit by polynomial

$$V^{\text{equiv}} = g(r) N(E) \left[ \sum_i c_i E^i \right]$$

for convenience and analytical continuation

# single-channel $\bar{K}N$ potential

- ❖ Previous work : Hyodo, Weise, PRC77 (2008) 035204
  - $\Delta V$  : real
  - fit range : 1300-1400 MeV
  - polynomial type : 3rd order in  $E$

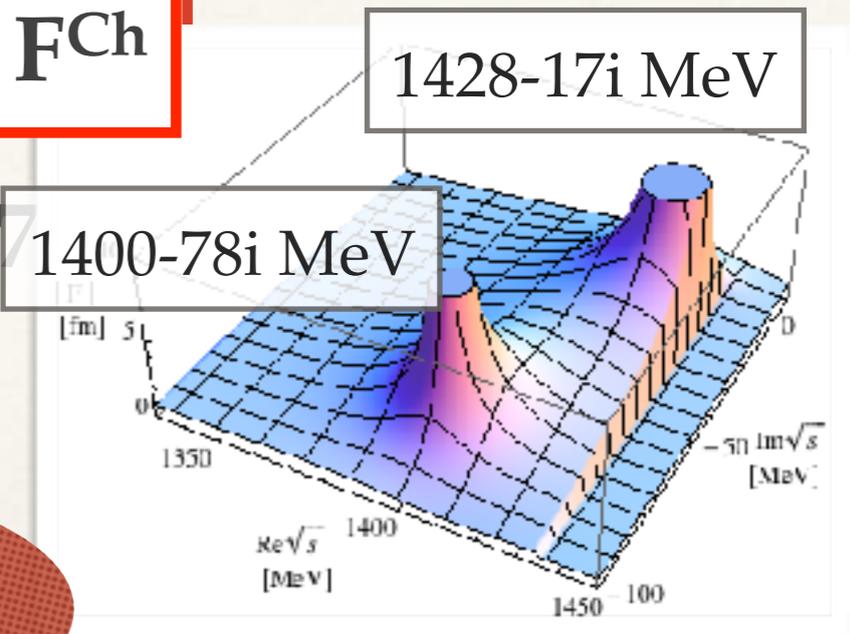


$F^{\text{Ch}}$  was almost reproduced on real  $E$  axis.

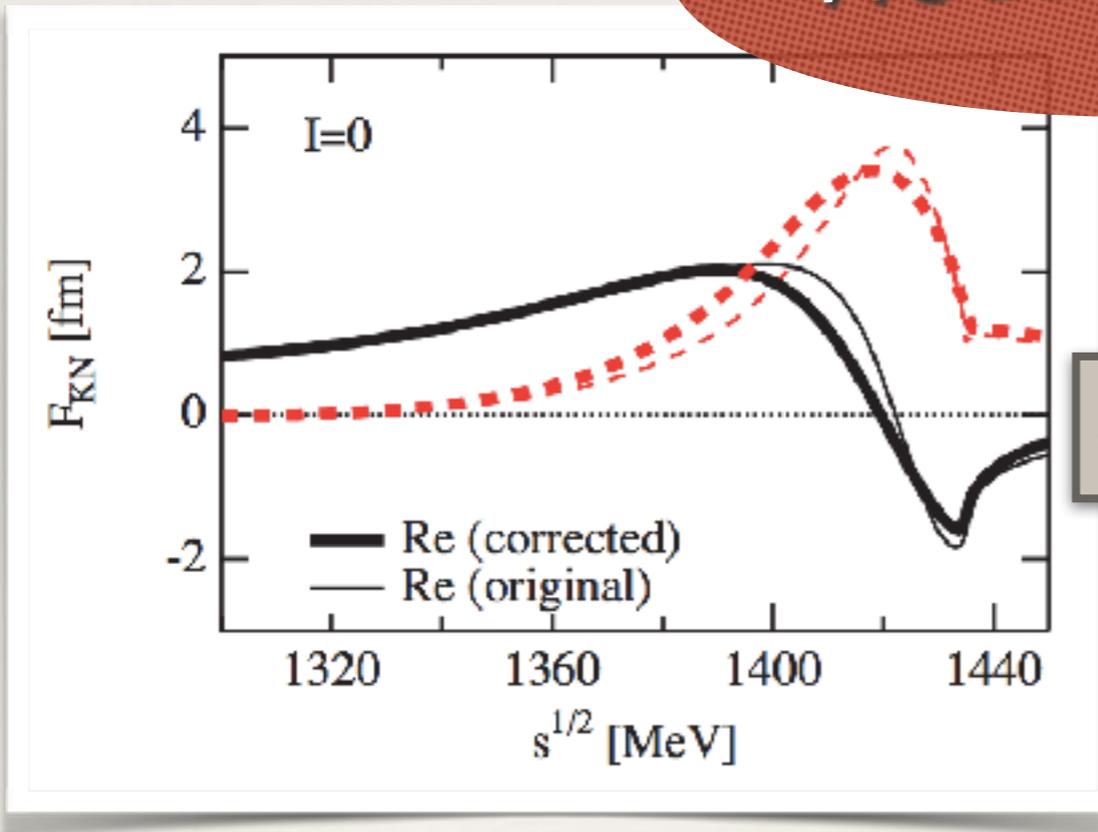
# single-channel $\bar{K}N$ potentials **FCh**

❖ Previous work : Hyodo, Weise, PRC77

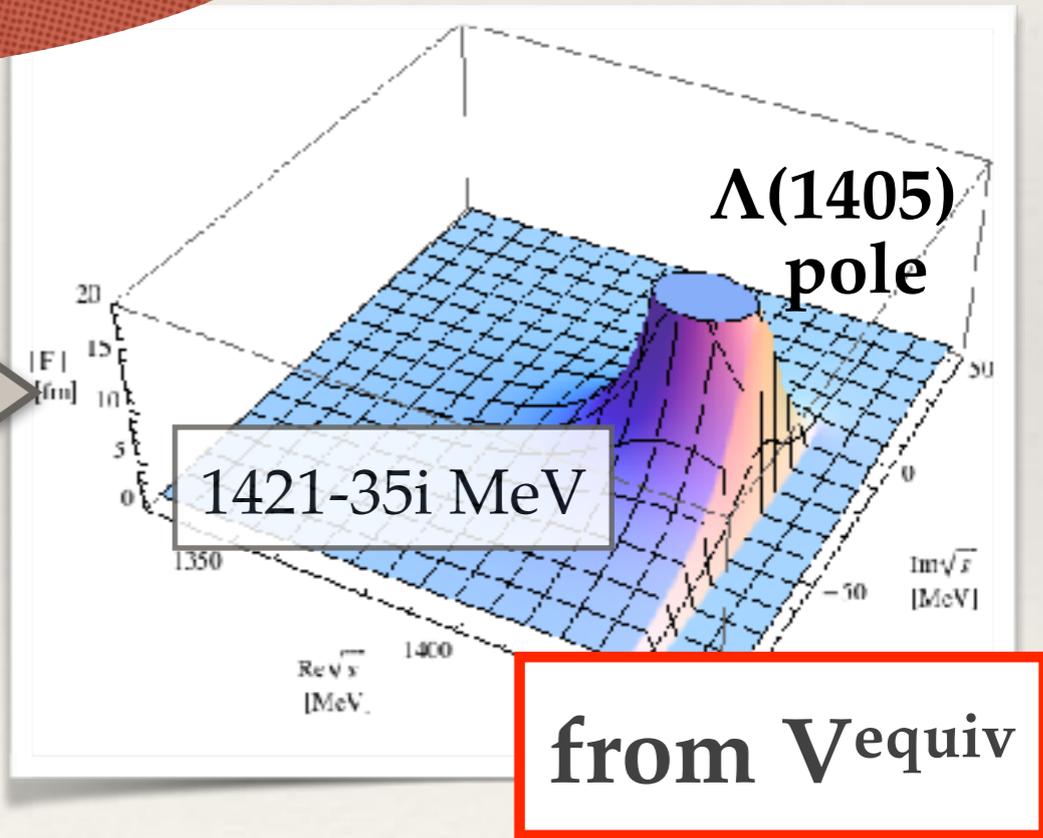
- $\Delta V$  : real
- fit range : 1300-1400 MeV
- polynomial



**We reanalyzed ...**



comp  $E$



**from  $V_{equiv}$**

$V_{equiv}$  does not reproduce pole structure of FCh.

# single-channel $\bar{K}N$ potential

## ❖ Our work : Miyahara, Hyodo, PRC93 (2016) 015201

- $\Delta V$  : ~~real~~  $\longrightarrow$  complex
- fit range : ~~1300-1400~~ MeV  $\longrightarrow$  1332-1450
- polynomial type : 3rd order in  $E$

$$V^{\text{equiv}}(r, E) = g(r)N(E) \left[ \underline{V^{\text{eff}}(E)} + \Delta V \right]$$

$$V_{ij}^{\text{Ch}} \in \mathbb{R} \quad \Rightarrow \quad V_{\bar{K}N, \bar{K}N}^{\text{eff}} \in \mathbb{C}$$

Feshbach projection

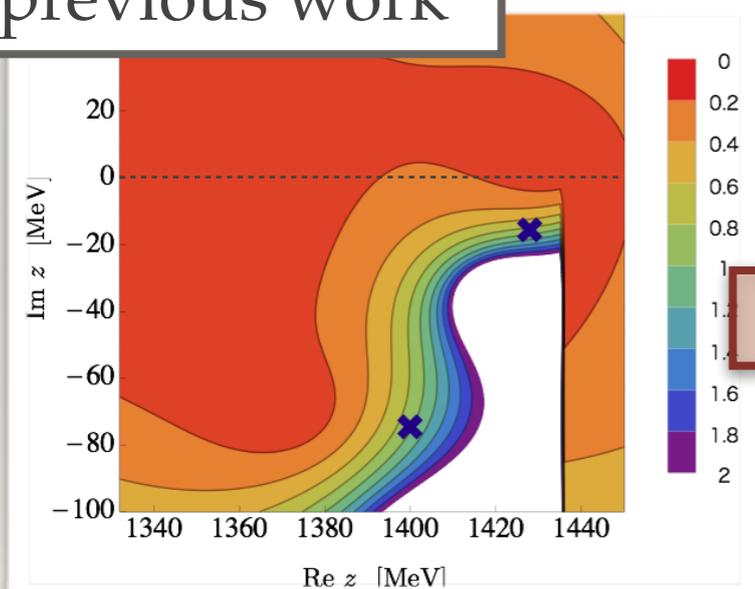
eliminate  $\pi\Sigma$  decay channel

# single-channel $\bar{K}N$ potential

❖ Our work : Miyahara, Hyodo, PRC93 (2016) 015201

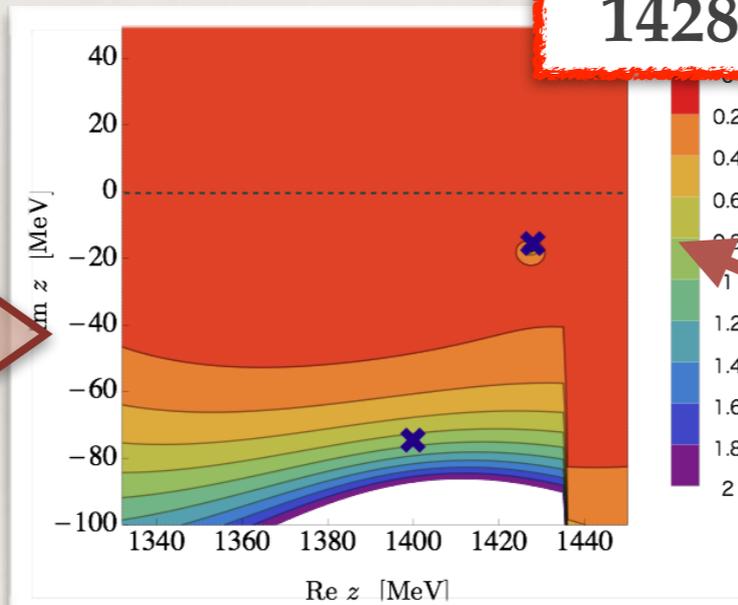
- $\Delta V$  : ~~real~~  $\longrightarrow$  complex
- fit range : ~~1300-1400~~ MeV  $\longrightarrow$  1332-1450
- polynomial type : 3rd order in  $E$

previous work



pole : 1421-35i MeV

original pole (Ch-U.) :  
1428-17i MeV, 1400-76i MeV



1427-17i MeV

“deviation” in complex  $E$

$$\Delta F = \left| \frac{F^{\text{Ch}} - F^{\text{equiv}}}{F^{\text{Ch}}} \right|$$

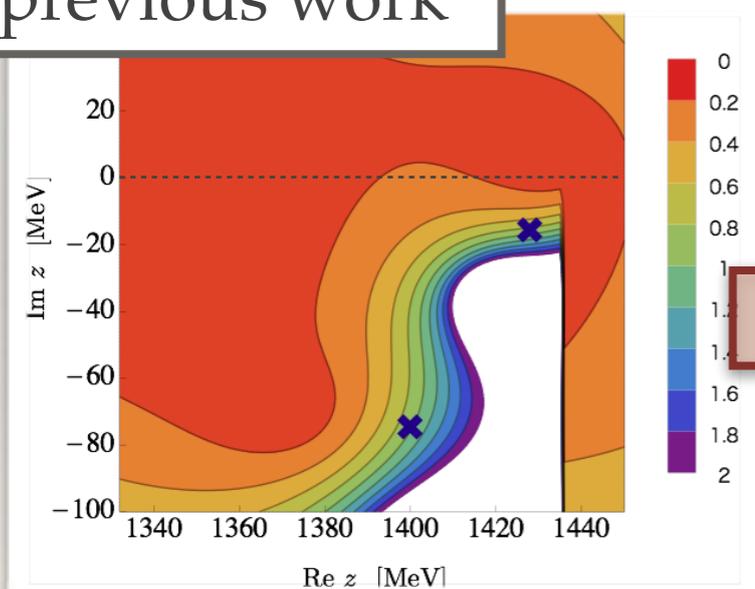
Precision on real  $E$  and higher pole are improved.

# single-channel $\bar{K}N$ potential

❖ Our work : Miyahara, Hyodo, PRC93 (2016) 015201

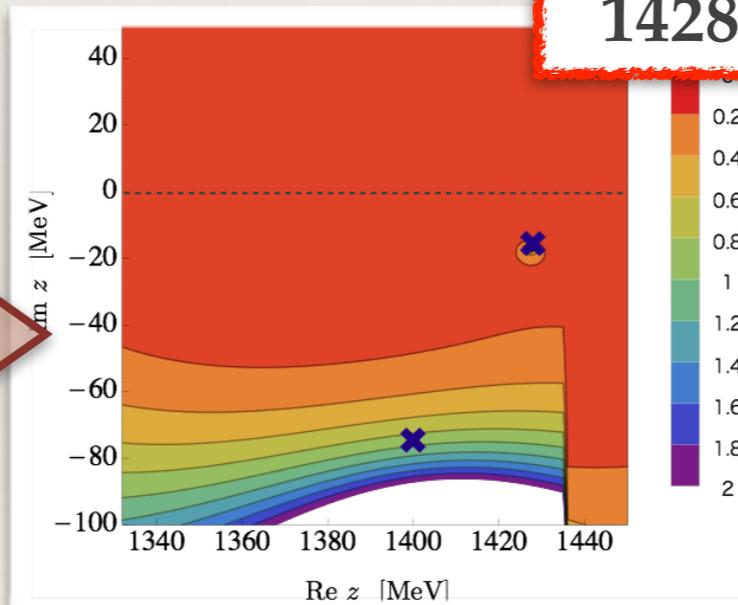
- $\Delta V$  : ~~real~~  $\longrightarrow$  complex
- fit range : ~~1300-1400~~ MeV  $\longrightarrow$  1332-1450
- polynomial type : 3rd order in  $E$

previous work



pole : 1421-35i MeV

original pole (Ch-U.) :  
1428-17i MeV, 1400-76i MeV



1427-17i MeV

no lower pole

Precision on real  $E$  and higher pole are improved.

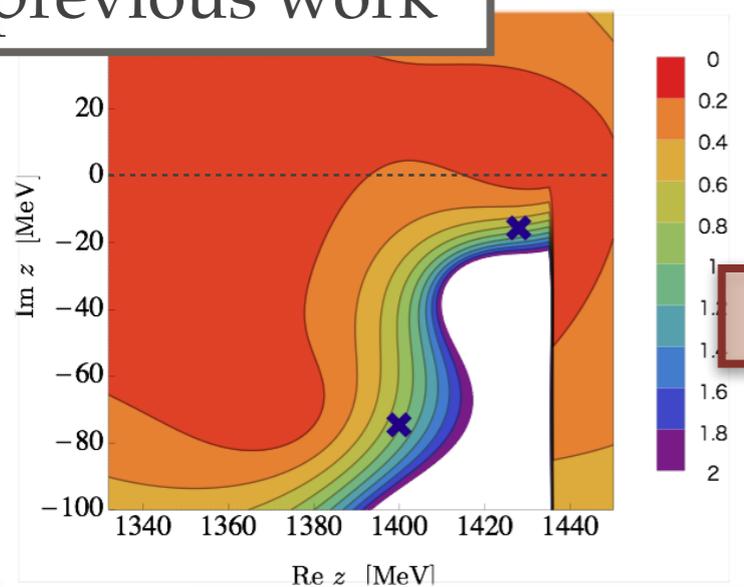
# single-channel $\bar{K}N$ potential

❖ Our work : Miyahara, Hyodo, PRC93 (2016) 015201

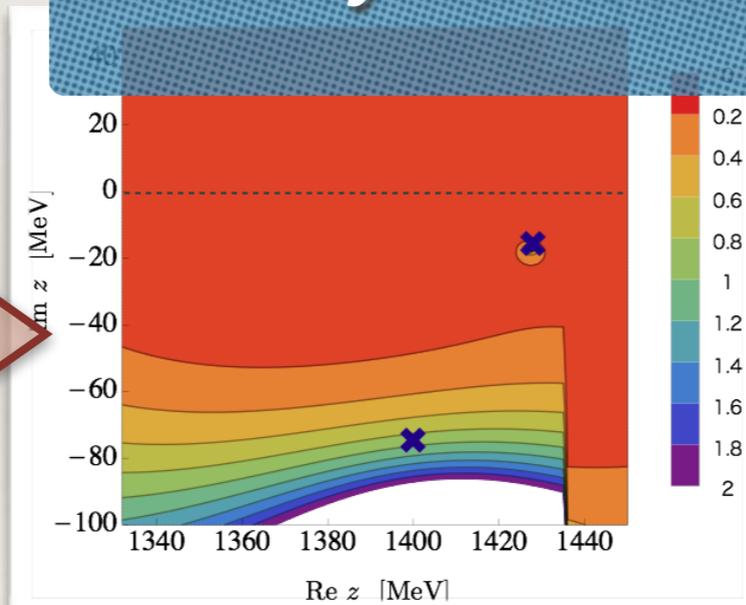
- $\Delta V$  : ~~real~~  $\longrightarrow$  complex
- fit range : ~~1300-1400~~ MeV  $\longrightarrow$  ~~1332-1450~~  $\longrightarrow$  1332-1520
- polynomial type : 3rd order in  $E$

Analytic continuation is unique

previous work



pole : 1421-35i MeV



1427-17i MeV

no lower pole

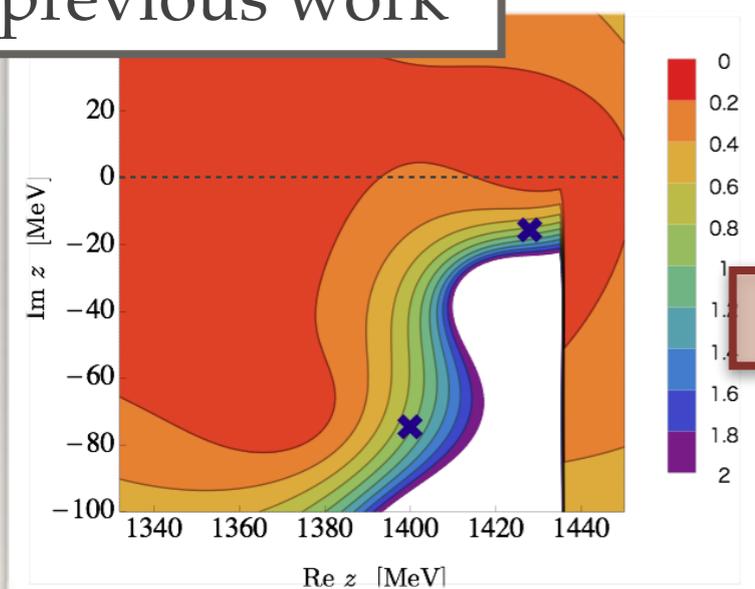
Precision on real  $E$  and higher pole are improved.

# single-channel $\bar{K}N$ potential

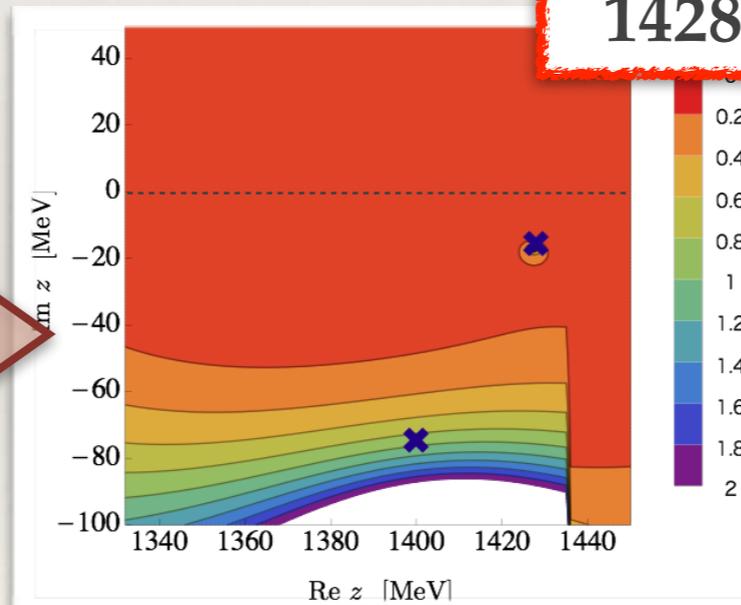
❖ Our work : Miyahara, Hyodo, PRC93 (2016) 015201

- $\Delta V$  : ~~real~~  $\longrightarrow$  complex
- fit range : ~~1300-1400 MeV~~  $\longrightarrow$  ~~1332-1450~~  $\longrightarrow$  1332-1520
- polynomial type : ~~3rd order in E~~  $\longrightarrow$  10th order

previous work

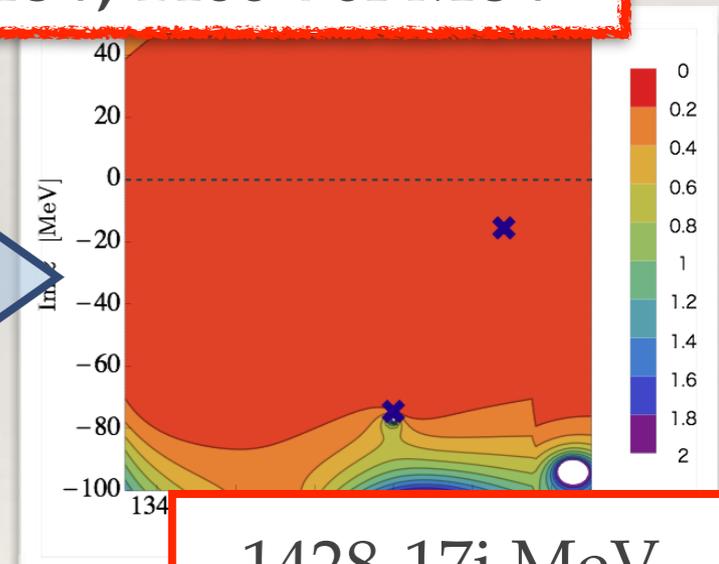


pole : 1421-35i MeV



1427-17i MeV

original pole (Ch-U.) :  
1428-17i MeV, 1400-76i MeV



1428-17i MeV  
1400-77i MeV

Two poles are well reproduced.

# single-channel $\bar{K}N$ potential

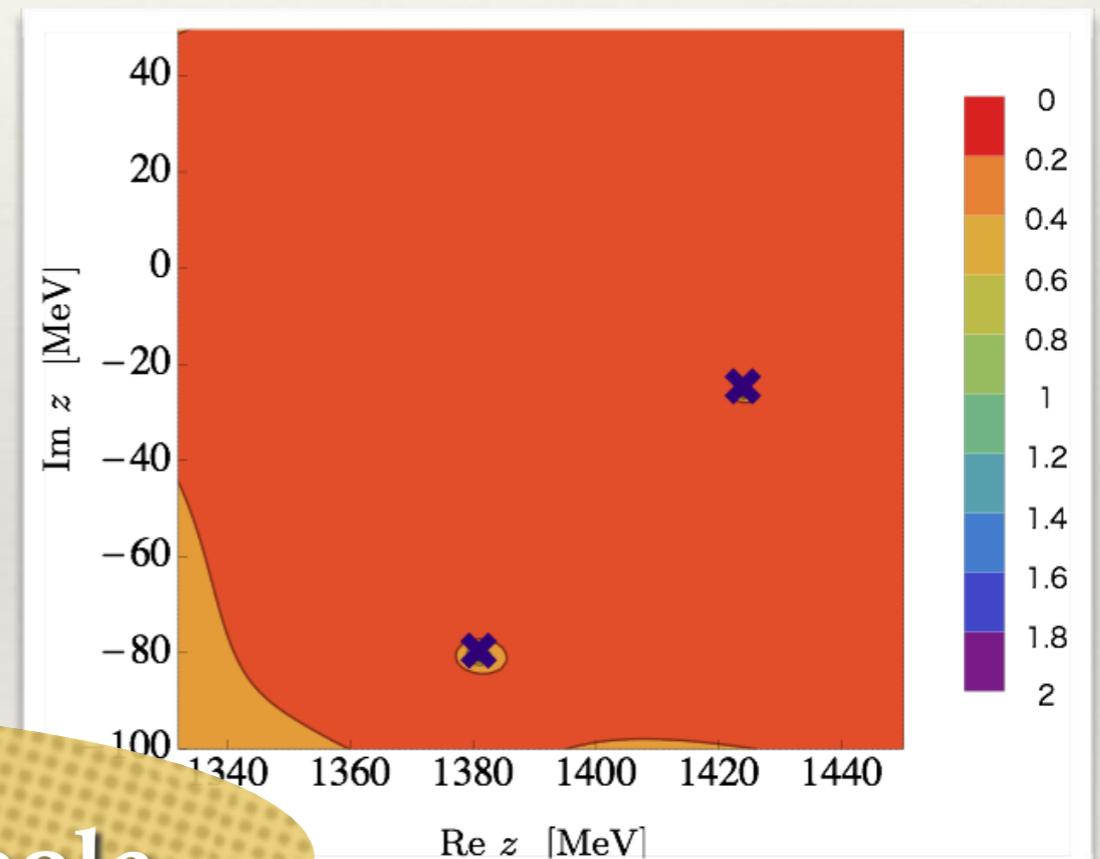
❖ Our work : Miyahara, Hyodo with SIDDHARTA constraint

Ch-U. with SIDDHARTA : Ikeda, Hyodo, Weise, NPA881 (2012) 98

- $\Delta V$  : complex
- fit range : 1332-1657 MeV
- polynomial type : 10th order

pole :  
**1424-26i MeV**  
**1381-81i MeV**

same as original pole



Reliable  $\bar{K}N$  potential is obtained.

# single-channel $\bar{K}N$ potential

## ❖ Application

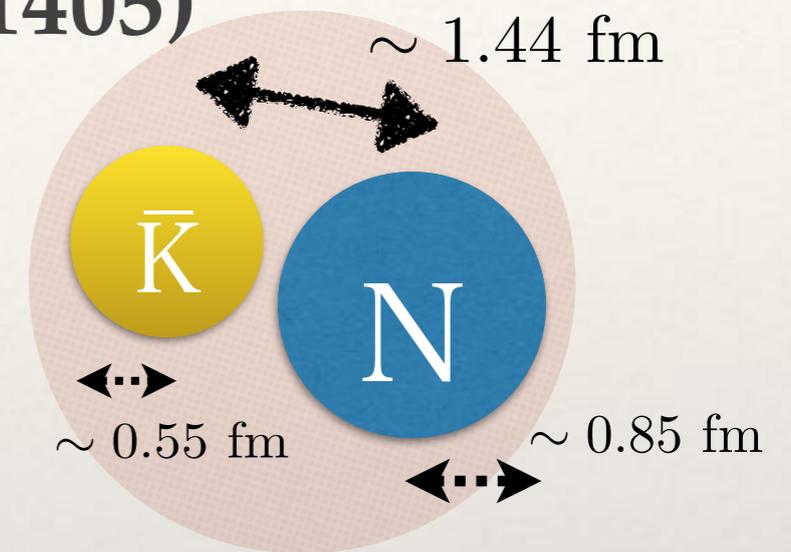
### 1. spatial size of $\Lambda(1405)$

Miyahara, Hyodo, PRC93 (2016) 015201

$$\sqrt{\langle r^2 \rangle} \sim 1.44 \text{ fm}$$

→ “molecular picture”

$\Lambda(1405)$



### 2. $\bar{K}$ -nuclei

Ohnishi et al., PRC95 (2017) 065202

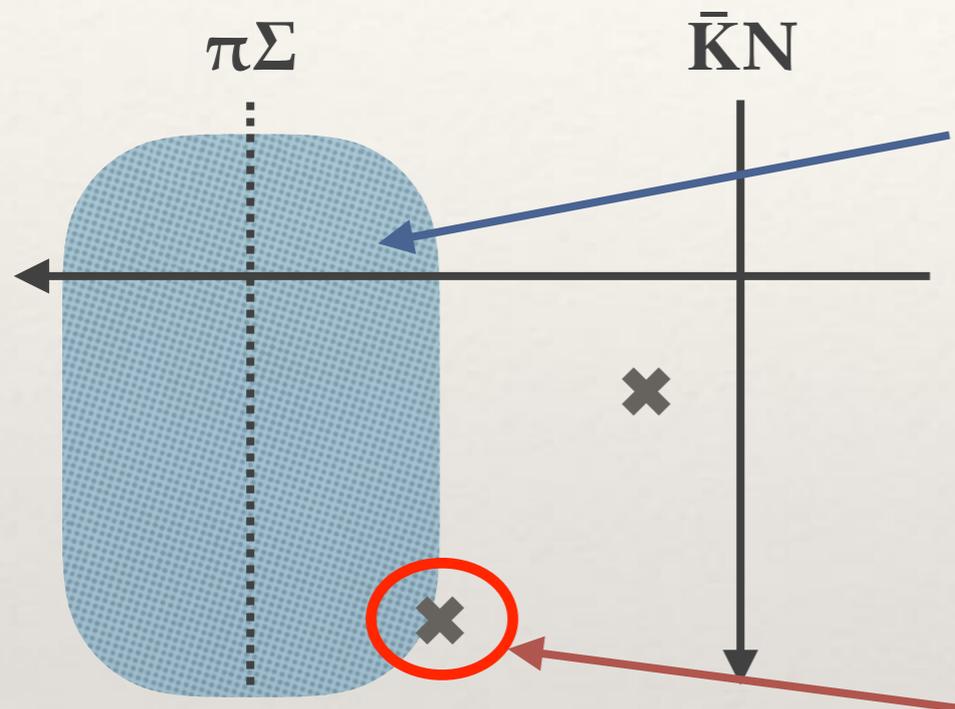
- variational method →  $\bar{K}NN, \dots, \bar{K}NNNNNNN$

$$\bar{K}NN : (B, \Gamma) = (26.1-27.9, 30.9-59.3) \text{ MeV}$$

cf.) with AY potential : (48.7, 61.9) MeV

# coupled-channel $\bar{K}N$ - $\pi\Sigma$ potential

## ❖ explicit treatment of $\pi\Sigma$



- reliability in lower  $E$  region  
cf.) B.E. of some  $\bar{K}$ -nuclei  
 $\sim 70$  MeV

Ohnishi et al., PRC95 (2017) 065202

- dynamically generated by  $\pi\Sigma$   
This may affect the  $\bar{K}$ -nuclei result.

We construct  $\bar{K}N$ - $\pi\Sigma$  potential.

# coupled-channel $\bar{K}N$ - $\pi\Sigma$ potential

## ❖ explicit treatment of $\pi\Sigma$

- $\Delta V$ : real
- fit range : 1300-1500 MeV
- polynomial type : 3rd order in  $E$

$$V^{\text{equiv}}(r, E) = g(r)N(E) \left[ \underline{V^{\text{eff}}(E)} + \Delta V \right]$$

$$V_{ij}^{\text{Ch}} \in \mathbb{R} \quad \Rightarrow \quad V_{ij}^{\text{eff}} \in \mathbb{R}$$

Feshbach projection

$\pi\Sigma, \bar{K}N, \eta\Lambda, \bar{K}\bar{E}$

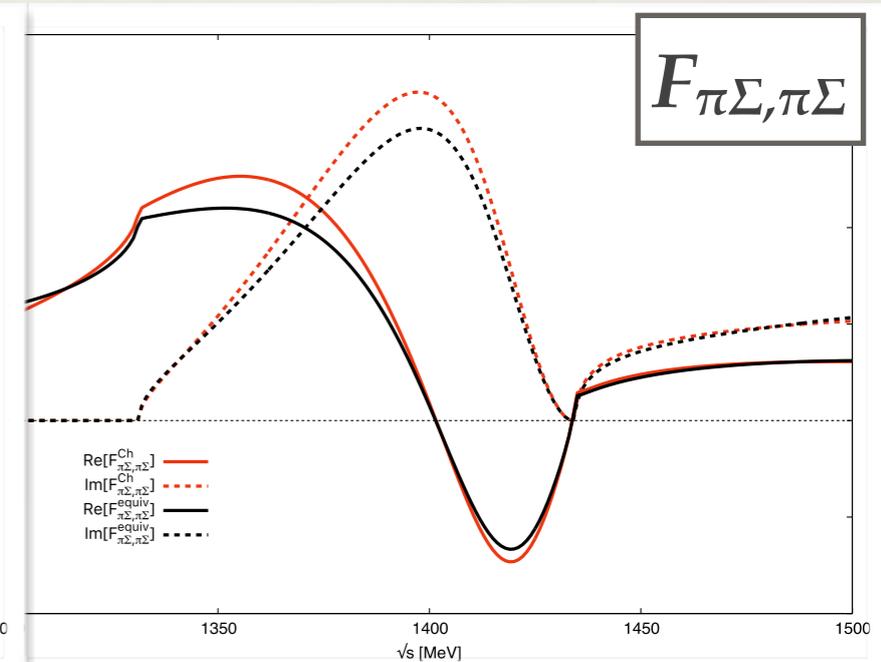
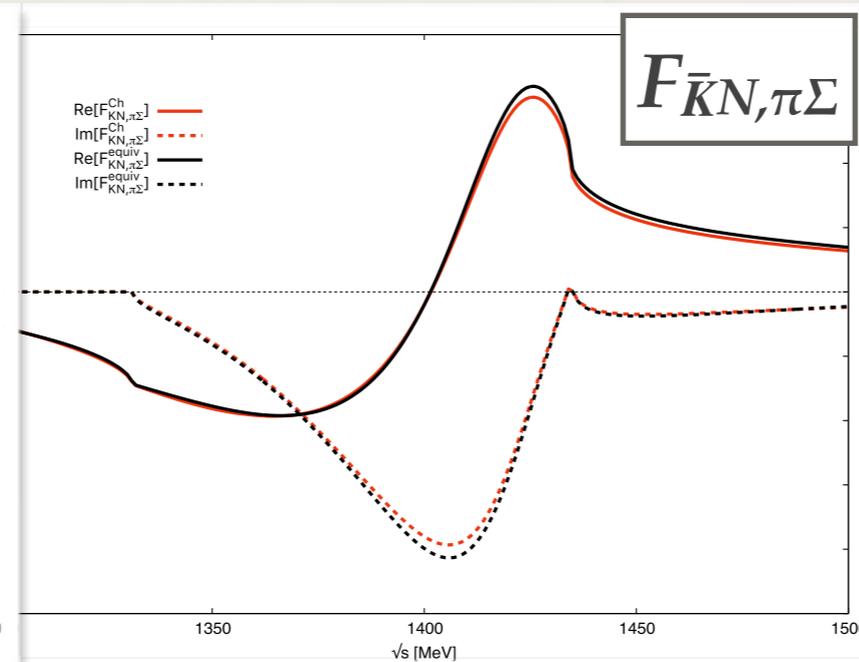
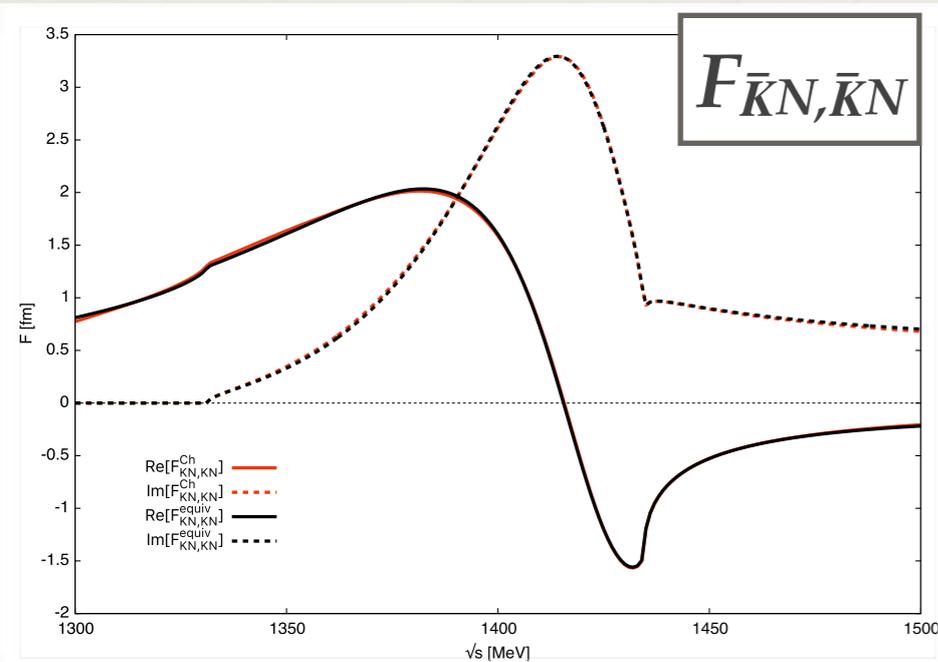
# coupled-channel $\bar{K}N$ - $\pi\Sigma$ potential

## ❖ explicit treatment of $\pi\Sigma$

- $\Delta V$  : real
- fit range : 1300-1500 MeV
- polynomial type : 3rd order in  $E$

original pole (Ch-U.) :  
1424-26i MeV, 1381-81i MeV

pole from equiv. potential :  
1424-28i MeV, 1401-85i MeV



$V^{equiv}$  almost reproduces  $F^{Ch}$  including poles.

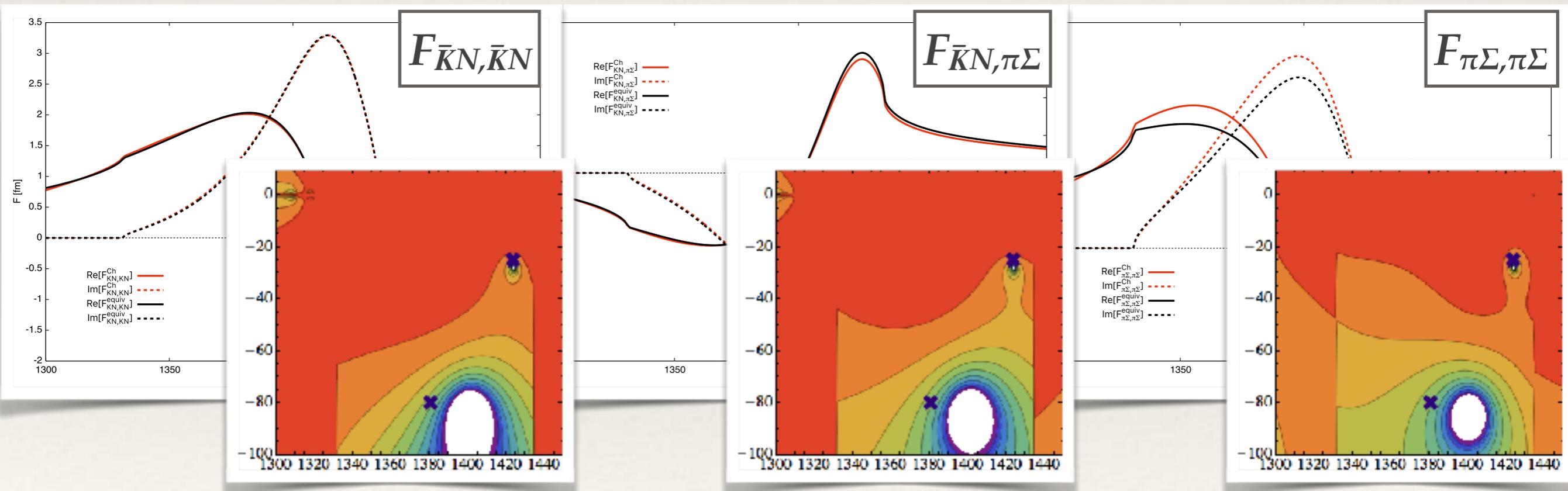
# coupled-channel $\bar{K}N$ - $\pi\Sigma$ potential

## ❖ explicit treatment of $\pi\Sigma$

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# Summary

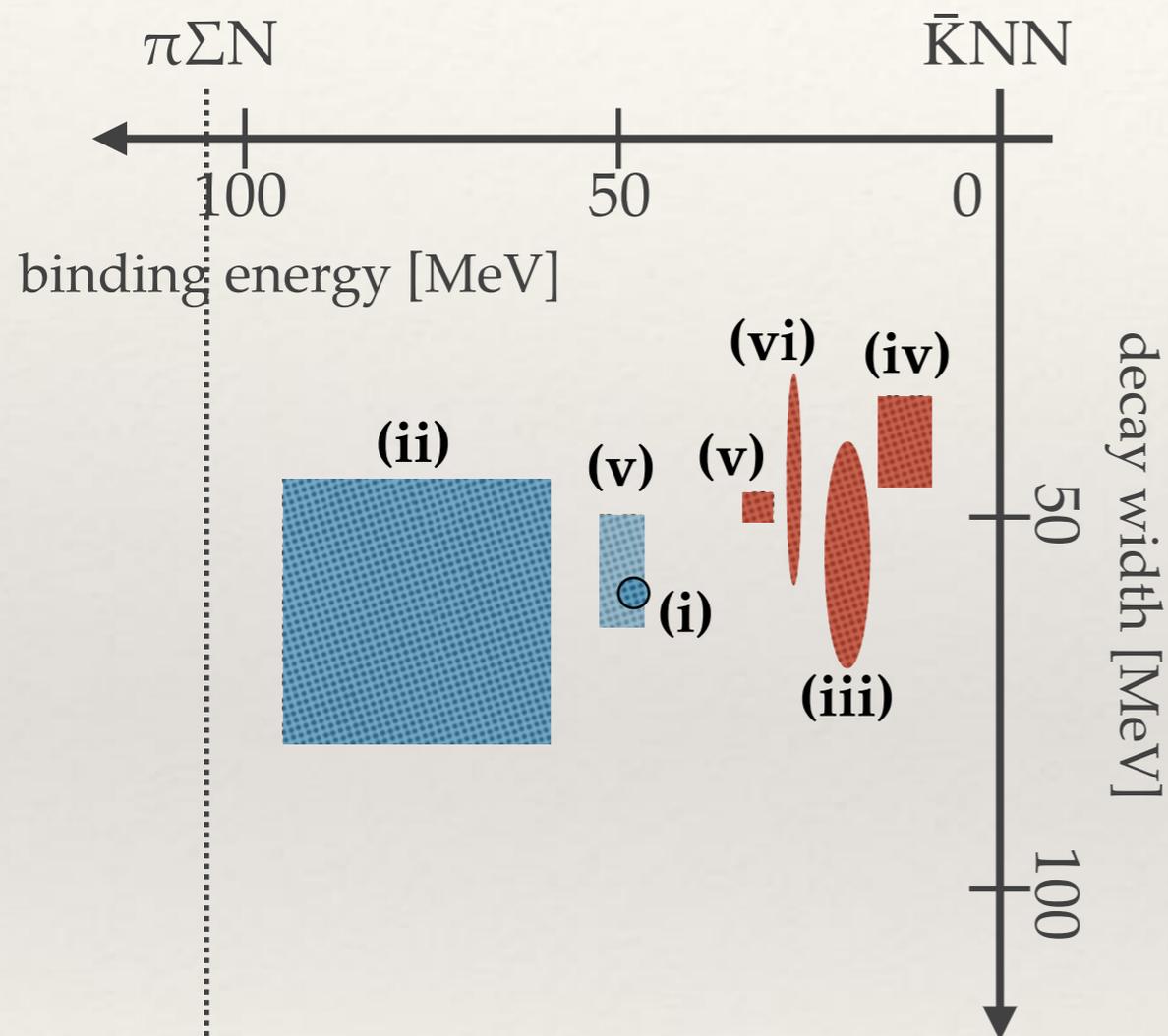
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- ❖ We have improved the potential construction procedure paying attention to  $F$  in complex  $E$ .
- ❖ Realistic  $\bar{K}N$  potential has been obtained with SIDDHARTA constraint.
  - Application  $\Lambda(1405) : \sqrt{\langle r^2 \rangle} = 1.44 \text{ fm}$   
Miyahara, Hyodo, PRC93 (2016) 015201
  - $\bar{K}NN : (B, \Gamma) = (26.1-27.9, 30.9-59.3) \text{ MeV}$   
Ohnishi et al., PRC95 (2017) 065202
- ❖ Similarly, we have constructed reliable  $\bar{K}N$ - $\pi\Sigma$  coupled-channel potential.  
Miyahara, Hyodo, Weise, in preparation

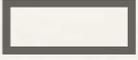
Backup slides

# Motivation

## ❖ $\bar{K}NN$ state ( $I=1/2, J^P=0^-$ )



### ▶ 3-body calculation

- variational 
- Faddeev 

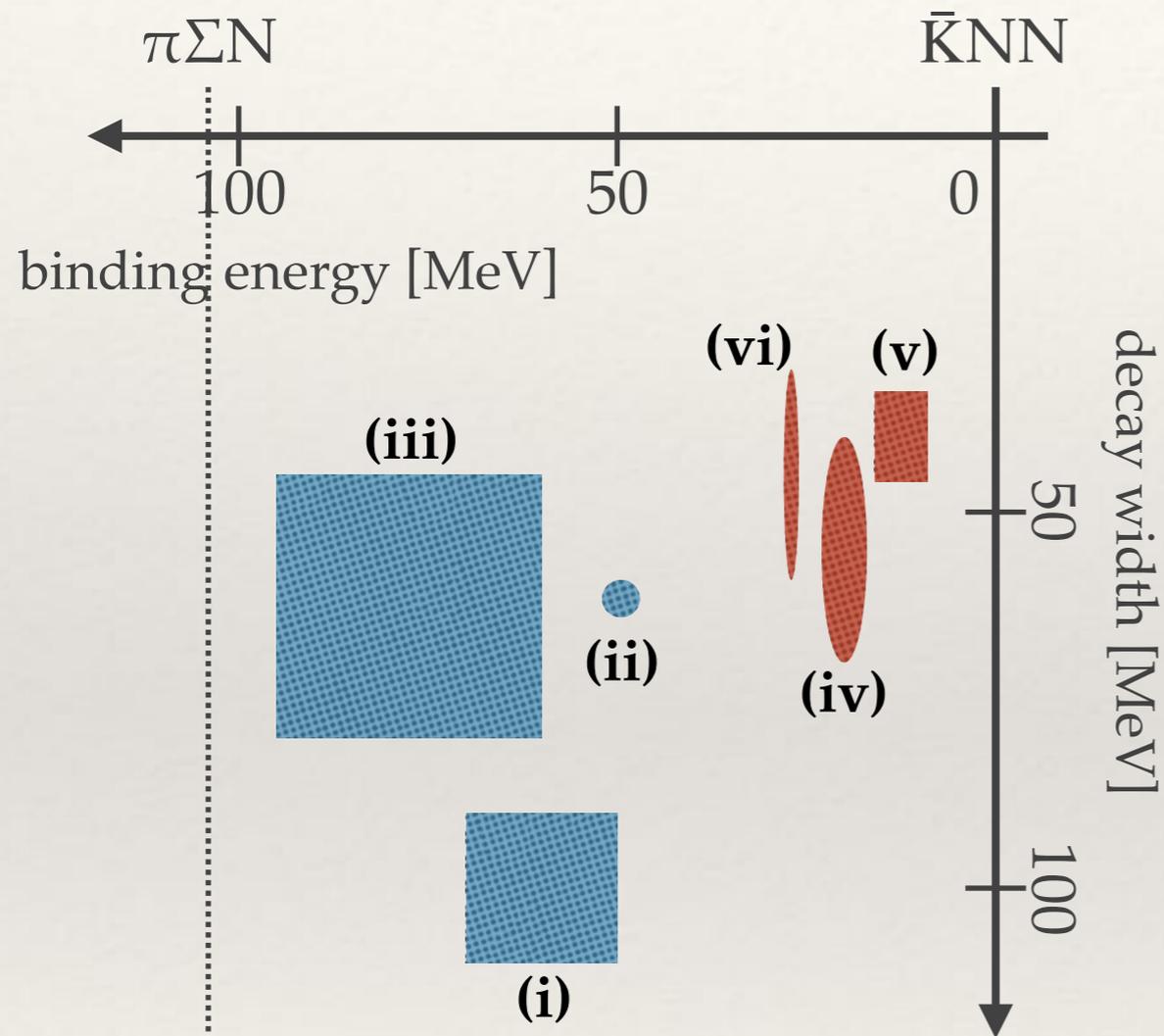
### theory

- (i) Yamazaki, Akaishi, PRC 76 (2007) 045201
- (ii) Ikeda, Sato, PRC 76 (2007) 035203
- (iii) Dote, Hyodo, Weise, PRC 79 (2009) 014003  
Barnea, Gal, Liverts, PLB 712 (2012) 132
- (iv) Ikeda, Kamano, Sato, PTP 124 (2010) 533
- (v) Revai, Shevchenko, PRC 90 (2014) 034004
- (vi) Ohnishi et al. PRC 95 (2017) 065202

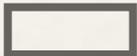
**Conclusive result has not been achieved.**

# Motivation

## ❖ $\bar{K}NN$ state ( $I=1/2, J^P=0^-$ )



### ▶ 3-body calculation

- variational 
- Faddeev 

### ▶ $\bar{K}N$ interaction

- E-indep. ( $\leftrightarrow$  single pole) 
- E-dep. ( $\leftrightarrow$  double pole) 

- (i) Shevchenko, Gal, Mares, PRC 76 (2007) 044004
- (ii) Yamazaki, Akaishi, PRC 76 (2007) 045201
- (iii) Ikeda, Sato, PRC 76 (2007) 035203
- (iv) Dote, Hyodo, Weise, PRC 79 (2009) 014003
- (v) Ikeda, Kamano, Sato, PTP 124 (2010) 533
- (vi) Ohnishi et al. PRC 95 (2017) 065202

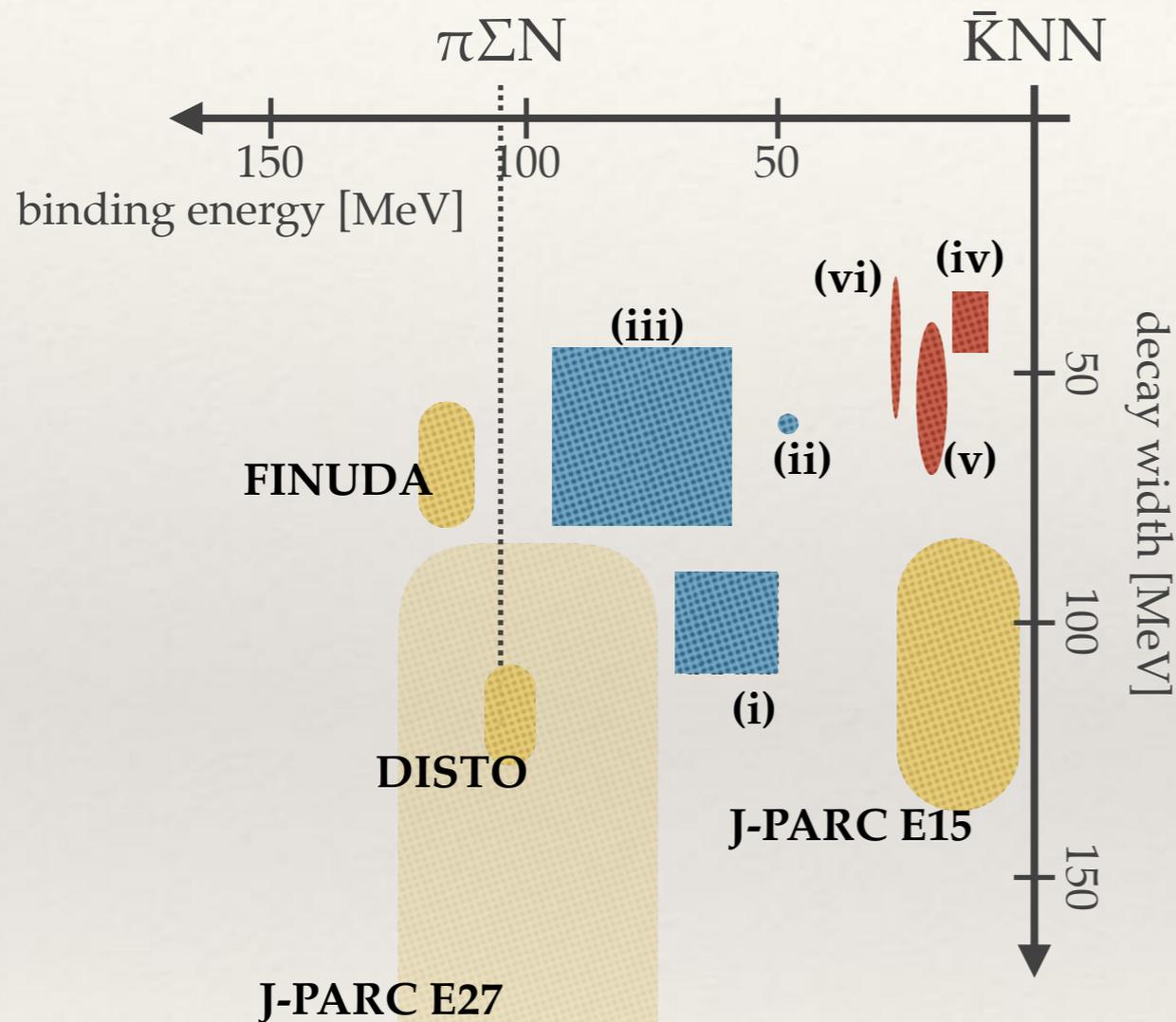
## exp. situation

	Group	Reaction	( $B_{\bar{K}NN}$ , $\Gamma_{\bar{K}NN}$ ) [MeV]
2005年	FINUDA	$K^-A \rightarrow A'(\Lambda p)$	( 115, 67 ) Angello et al., PRL 94 (2005) 212303
2010年	DISTO	$pp \rightarrow K^+(\Lambda p)$	( 103, 118 ) Yamazaki et al., PRL 104 (2010) 132502
2014年	LEPS	$\gamma d \rightarrow K^+\pi^-X$	no peak Tokiyasu et al., PLB728 (2014) 616
2015年	J-PARC E27	$\pi^+d \rightarrow K^+X$	( 95, 162 ) Ichikawa et al., PTEP 2015, 021D01
2015年	HADES	$pp \rightarrow K^+(\Lambda p)$	no peak Agakishiev et al., PLB742 (2015) 242
2015年	J-PARC E15	$K^- {}^3\text{He} \rightarrow nX$	no peak Hashimoto et al., PTEP 2015, 061D01

**We do not have conclusive results.**

# Motivation

## ❖ $\bar{K}NN$ state ( $I=1/2, J^P=0^-$ )



### ▶ 3-body calculation

- variational
- Faddeev



### theory

- (i) Shevchenko, Gal, Mares, PRC 76 (2007) 044004
- (ii) Yamazaki, Akaishi, PRC 76 (2007) 045201
- (iii) Ikeda, Sato, PRC 76 (2007) 035203
- (iv) Dote, Hyodo, Weise, PRC 79 (2009) 014003
- (v) Ikeda, Kamano, Sato, PTP 124 (2010) 533
- (vi) Ohnishi et al. PRC 95 (2017) 065202

### exp.

- [FINUDA] Angello et al., PRL 94 (2005) 212303
- [DISTO] Yamazaki et al., PRL 104 (2010) 132502
- [J-PARC E27] Ichikawa et al., PTEP 2015, 021D01
- [J-PARC E15] Sada et al., PTEP 2016, 051D01

**Conclusive result has not been achieved.**

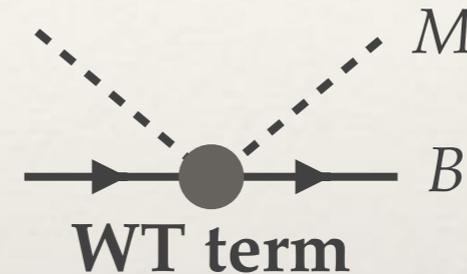
# Chiral unitary approach

## ❖ Ch-EFT

- nonlinear sigma model based on SU(3) chiral sym.
- expand in terms of small energy, momentum, and mass

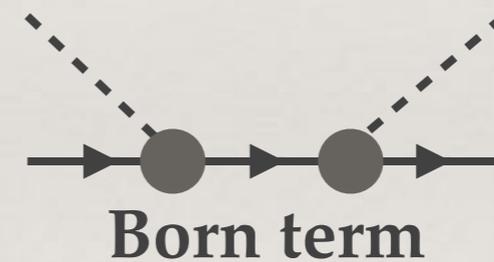
on-shell  
s wave

$$\mathcal{L}_{\text{WT}}^{(1)} = \frac{1}{4f^2} \text{Tr} (\bar{B} i \gamma^\mu [\Phi \partial_\mu \Phi - (\partial_\mu \Phi) \Phi], B)$$



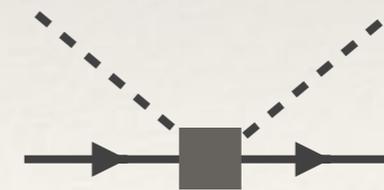
$\mathcal{O}(p)$

$$\mathcal{L}_{\text{Yukawa}}^{(1)} = -\frac{1}{\sqrt{2}f} \text{Tr} (D(\bar{B} \gamma^\mu \gamma_5 \{ \partial_\mu \Phi, B \}) + F(\bar{B} \gamma^\mu \gamma_5 [\partial_\mu \Phi, B]))$$



$\mathcal{O}(p^2)$

$\mathcal{L}^{(2)}$  : including other low energy constants  
**free parameter**



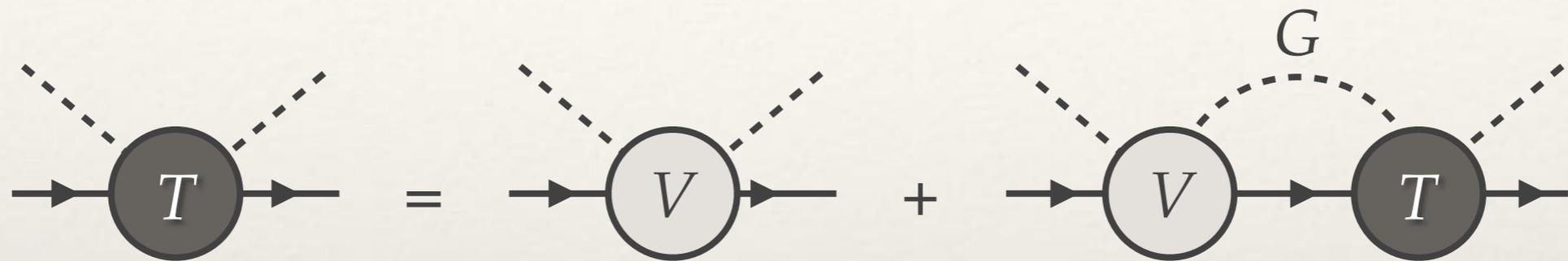
$\mathcal{O}(p^2)$

$\left( \begin{array}{l} B : \text{baryon octet} \\ \Phi : \text{meson octet} \end{array} \right.$

# Chiral unitary approach

## ❖ chiral unitary approach

- non-perturbative treatment (bound state, resonance state)



$$T = V + VGT = V + VGV + VGVGV + \dots$$

$$= (V^{-1} - G)^{-1} : \text{pole} \longleftrightarrow \text{resonance state}$$

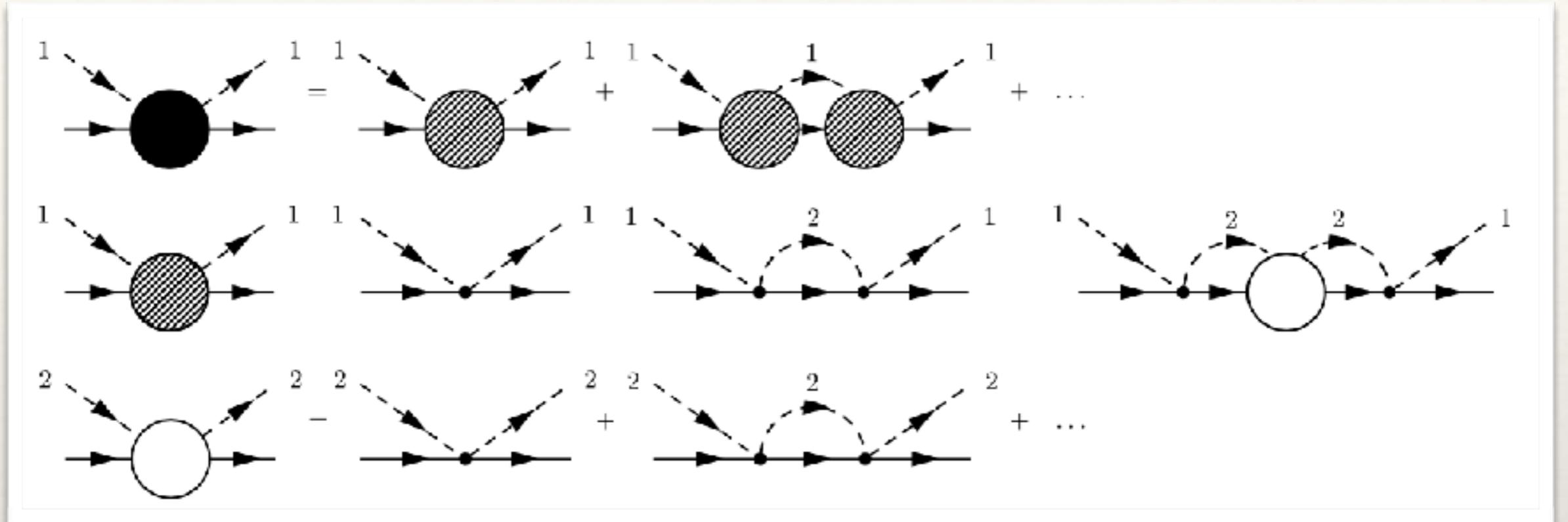
$$G(W) = i \int \frac{d^4q}{(2\pi)^4} \frac{2M}{(P - q)^2 - M^2 + i\epsilon} \frac{1}{q^2 - m^2 + i\epsilon}$$

dimensional regularization  subtraction constant  
**free parameter**

fit scattering data

 well describe resonance

# Feshbach projection

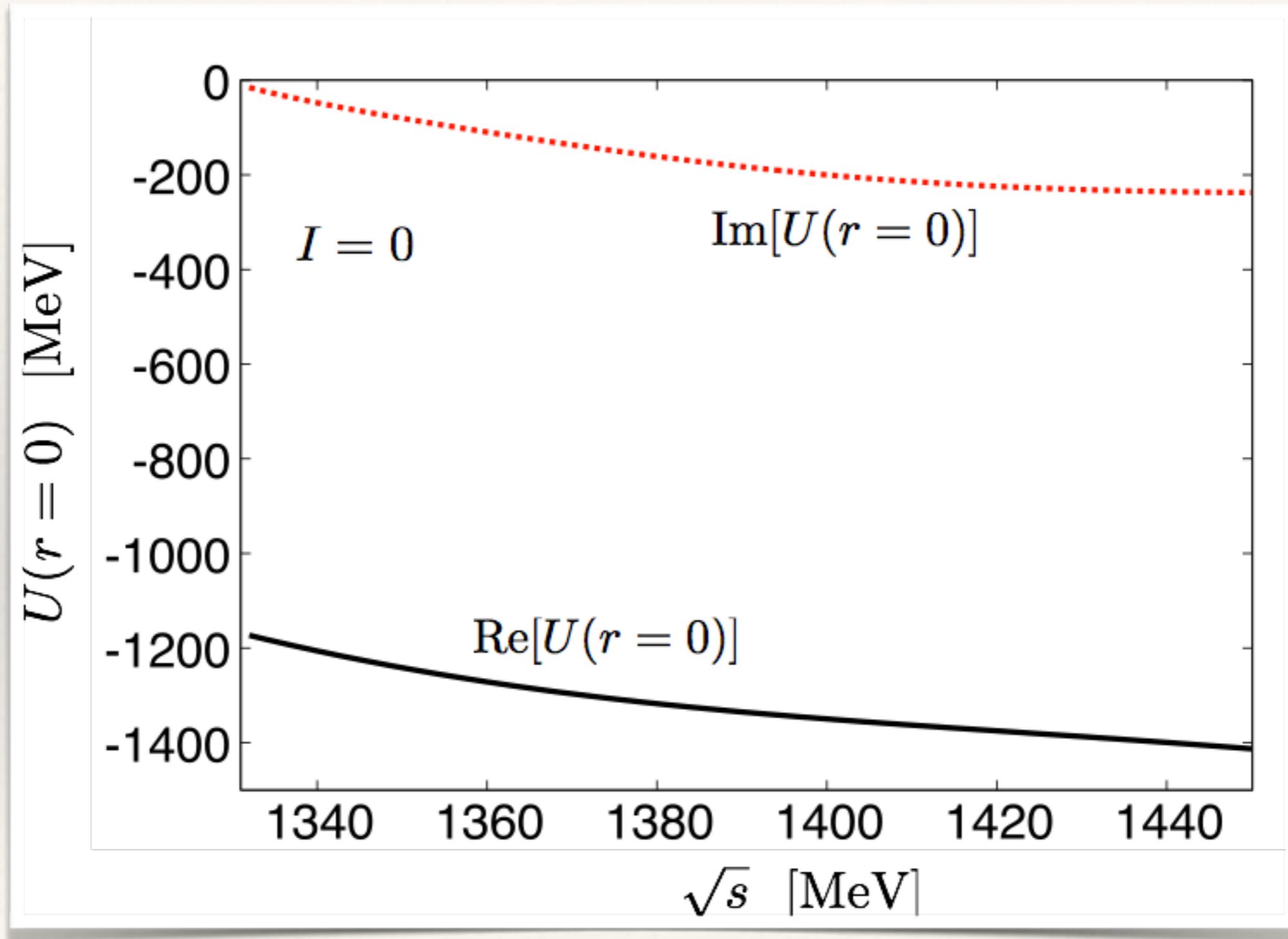


$$T_{\bar{K}N, \bar{K}N} = V_{\bar{K}, \bar{K}N}^{\text{eff}} + V_{\bar{K}, \bar{K}N}^{\text{eff}} G_{\bar{K}N} T_{\bar{K}, \bar{K}N}$$

$$V_{\bar{K}N}^{\text{eff}} = \sum_{m \neq \bar{K}N} V_{\bar{K}N, m} G_m V_{m, \bar{K}N} + \sum_{m, l \neq \bar{K}N} V_{\bar{K}N, m} G_m \tilde{T}_{m, l} G_l V_{l, \bar{K}N}$$

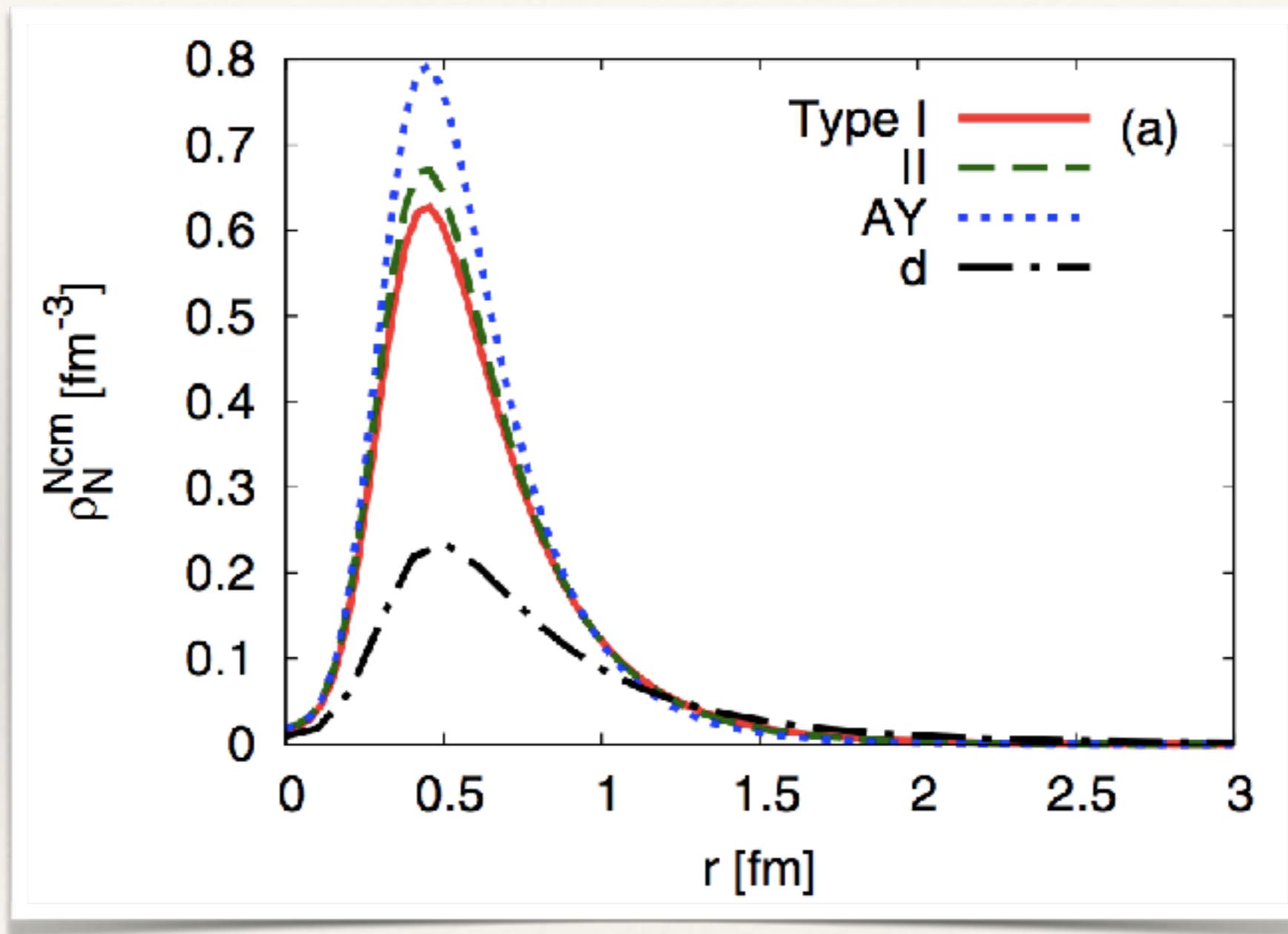
$$\tilde{T}_{m, l} = V_{m, l} + \sum_{k \neq \bar{K}N} V_{m, k} G_k \tilde{T}_{k, l}$$

# Kyoto $\bar{K}N$ potential



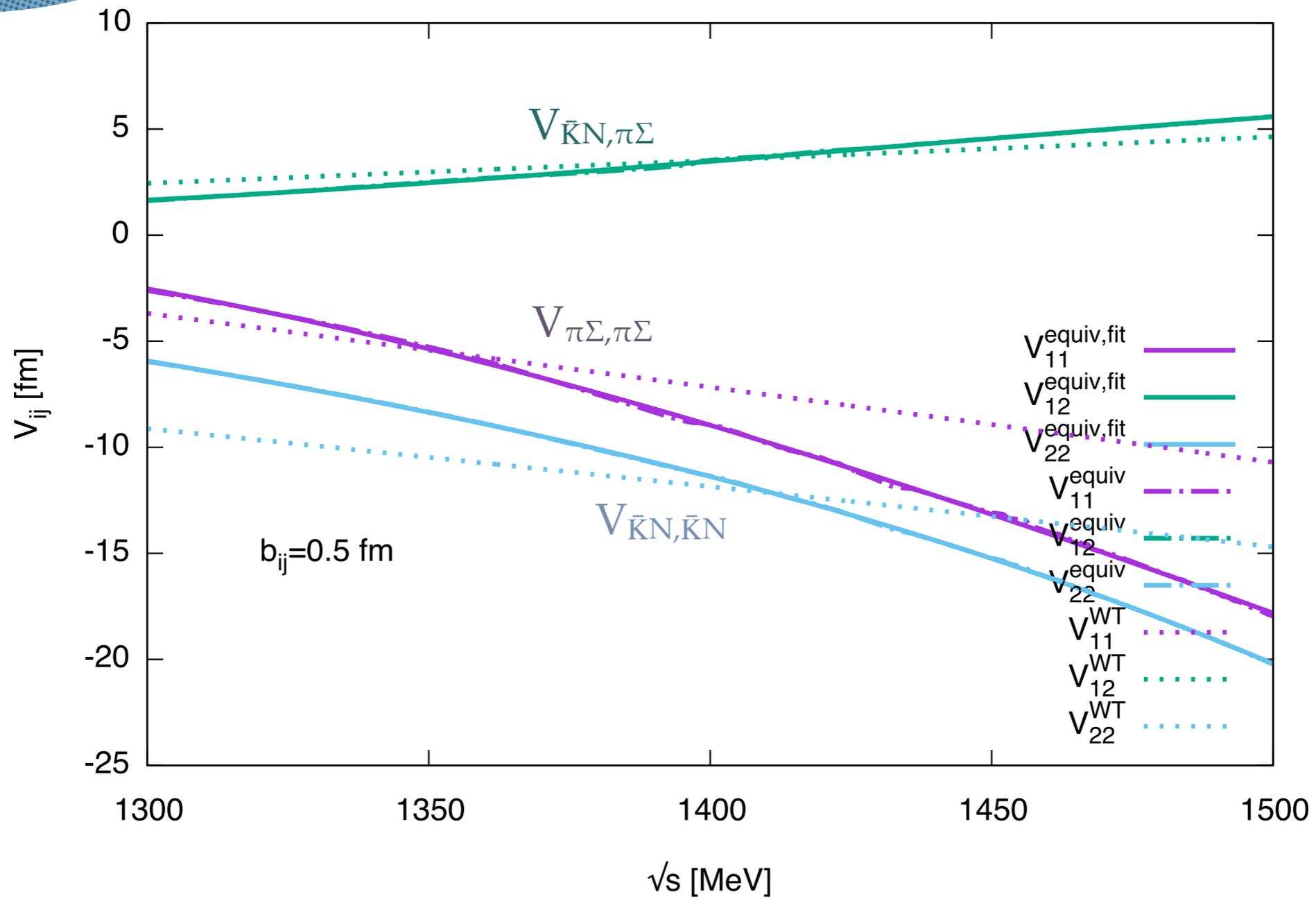
# nucleon distribution in $\bar{K}NN$

Ohnishi et al., PRC95 (2017) 065202



# $\bar{K}N-\pi\Sigma$ potential

HNJH model



# coupled-channel $\bar{K}N$ - $\pi\Sigma$ potential

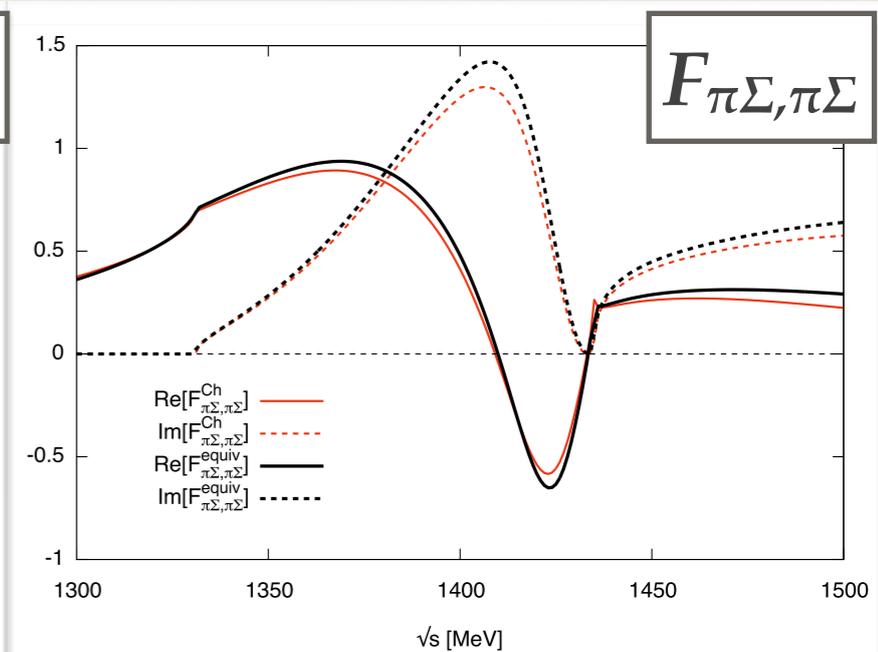
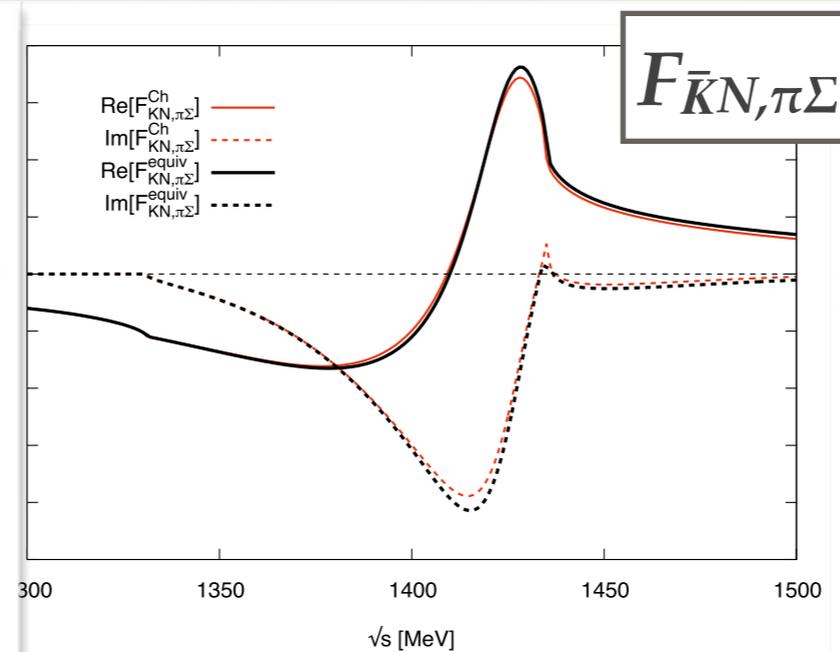
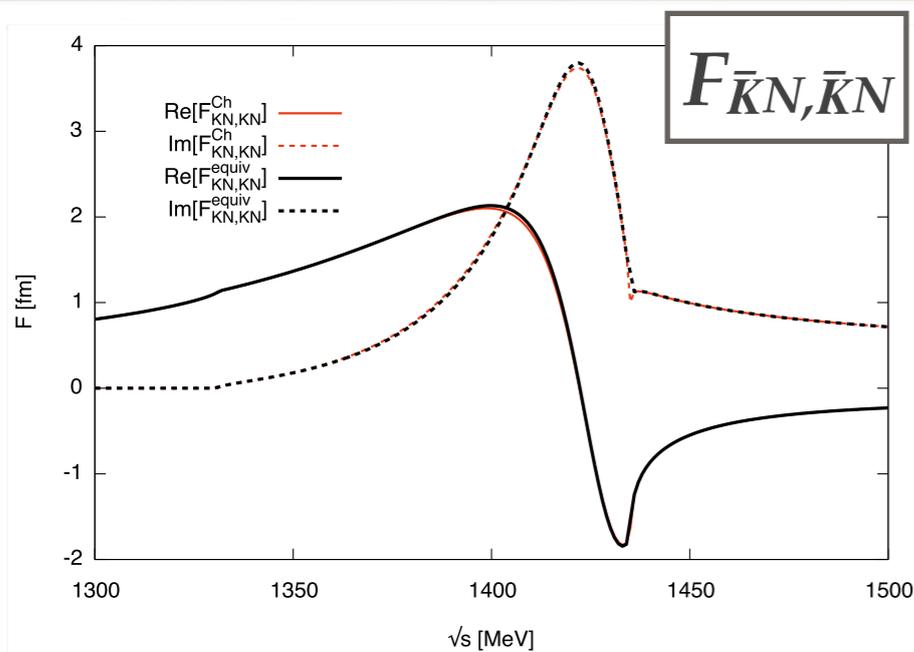
## HNJH model

### equivalent treatment of $\pi\Sigma$

- $\Delta V$  : real
- fit range : 1300-1500 MeV
- polynomial type : 3rd order in  $E$

original pole (Ch-U.) :  
1428-17i MeV, 1400-76i MeV

pole from equiv. potential :  
1427-17i MeV, 1397-83i MeV



$V^{\text{equiv}}$  almost reproduces  $F^{\text{Ch}}$  including poles.