

# Heavy hadron interactions from Lattice QCD

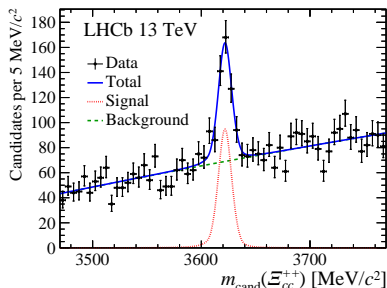
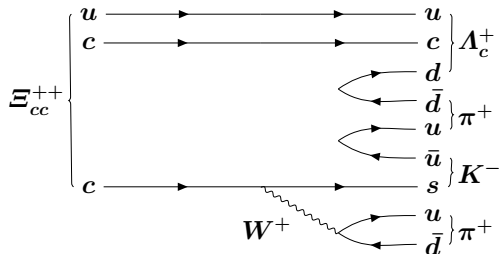
Daniel Mohler

Wien, Sep 14th, 2017



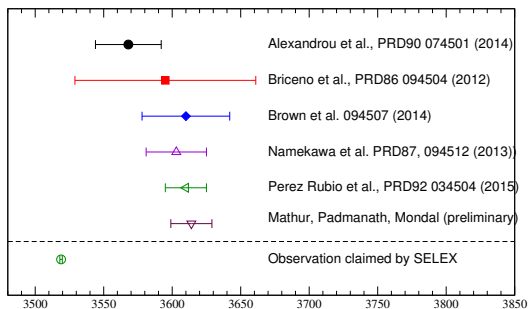
# Observation of a doubly-charmed $\Xi_{cc}^{++}$ by LHCb

Roel Aaij et al., LHCb collaboration arXiv:1707.01621



- $\Xi_{cc}^{++}$  with mass  $3621.40 \pm 0.72 \pm 0.27 \pm 0.14$  seen in both 13 TeV and 8 TeV data
- Previous claim of  $\Xi_{cc}$  by SELEX with mass  $\approx 3519$  MeV not seen by BaBar, Belle, LHCb
- What about Lattice QCD Predictions?

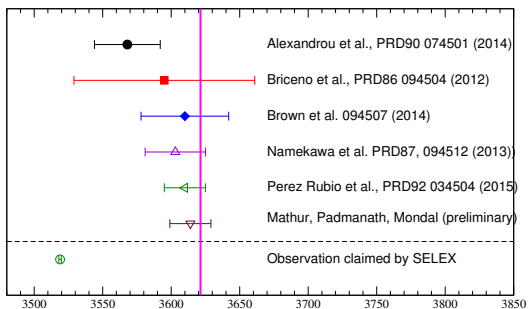
# $\Xi_{cc}$ – Recent Lattice QCD predictions



- Full symbols: Good control of systematic uncertainty  
Empty symbols: Continuum extrapolation missing
- All simulations neglect isospin splittings  $\Xi_{cc}^{++} - \Xi_{cc}^{+}$
- Publications also contain a number of further predictions and successful postdictions
- History of earlier calculations, most notably

Mathur, Lewis, Woloshyn, PRD 64 094509 (2001); PRD 66 014502

# $\Xi_{cc}$ – Recent Lattice QCD predictions

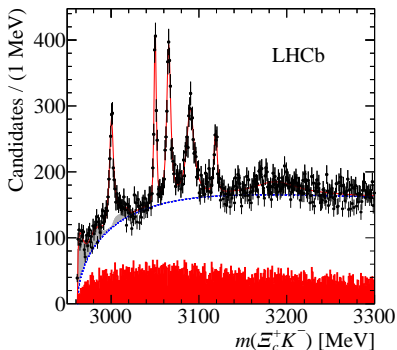


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# Observation of 5 narrow $\Omega_c$ states

Roel Aaij et al., LHCb collaboration PRL 118 182001 (2017)

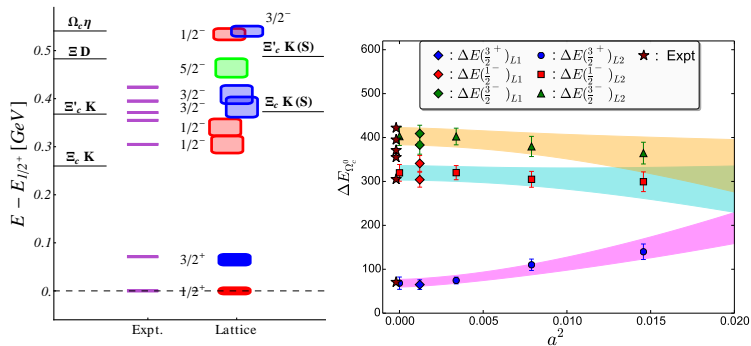


Resonance	Mass [MeV]	$\Gamma$ [MeV]
$\Omega_c(3000)^0$	$3000.4 \pm 0.2 \pm 0.1^{+0.3}_{-0.5}$	$4.5 \pm 0.6 \pm 0.3$
$\Omega_c(3050)^0$	$3050.2 \pm 0.1 \pm 0.1^{+0.3}_{-0.5}$	$0.8 \pm 0.2 \pm 0.1$
$\Omega_c(3066)^0$	$3065.6 \pm 0.1 \pm 0.3^{+0.3}_{-0.5}$	$3.5 \pm 0.4 \pm 0.2$
$\Omega_c(3090)^0$	$3090.2 \pm 0.3 \pm 0.5^{+0.3}_{-0.5}$	$8.7 \pm 1.0 \pm 0.8$
$\Omega_c(3119)^0$	$3119.1 \pm 0.3 \pm 0.9^{+0.3}_{-0.5}$	$1.1 \pm 0.8 \pm 0.4$

- Observation of 5 new  $\Omega_c$  resonances
- $J^P$  not identified
- All states are narrow  $\rightarrow$  can compare to Lattice QCD simulations treating them as stable

# What can Lattice QCD say about their spin-parity?

Padmanath, Mathur, PRL 119 042001 (2017)



- Pattern of lattice states agrees well with experiment
- second study with smaller basis can not resolve two states with same  $J^P$ ; checks systematics
- Scattering thresholds somewhat unphysical

# Outline

- 1 Motivation – Charmed baryons
- 2 Recent progress in Lattice QCD
- 3  $D$ ,  $D_s$ , and  $B_s$
- 4 Charmonium(-like) states
  - Charmonium-like states:  $Z_c$
  - Charmonium-like states:  $X(3915)$
  - Charmonium-like states:  $X(3872)$
- 5 Exotic doubly-heavy states

# Lattice Quantum Chromodynamics: What do we calculate?

Regularization of QCD by a 4-d Euclidean space-time lattice. (Kenneth Wilson 1974)

Provides a calculational method for QCD



Euclidean correlator of two Hilbert-space operators  $\hat{O}_1$  and  $\hat{O}_2$ .

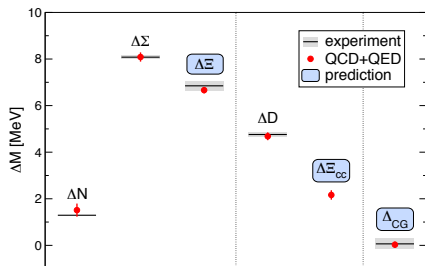
$$\begin{aligned}\langle \hat{O}_2(t)\hat{O}_1(0) \rangle &= \sum_n e^{-t\Delta E_n} \langle 0|\hat{O}_2|n\rangle \langle n|\hat{O}_1|0\rangle \\ &= \frac{1}{Z} \int \mathcal{D}[\psi, \bar{\psi}, U] e^{-S_E} O_2[\psi, \bar{\psi}, U] O_1[\psi, \bar{\psi}, U]\end{aligned}$$

- Path integral over the Euclidean action  $S_{E,QCD}[\psi, \bar{\psi}, U]$ ; (a sum over quantum fluctuations)
- Can be evaluated with *Markov Chain Monte Carlo* (using methods well established in statistical physics)



# Recent progress in Lattice QCD

- Dynamical simulations with 2+1(+1) flavors of sea quarks
- Simulations at physical pion (light-quark) masses
- Isospin splitting and QCD+QED simulations
- Improved heavy quark actions for charm
- Efficient methods for all-to-all propagation (disconnected diagrams)



BMW Collaboration, Borsanyi et al. Science 347 1452 (2015)

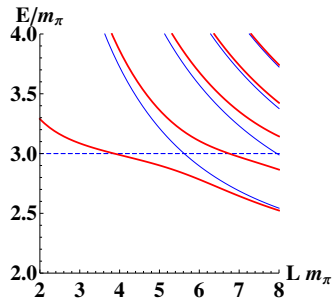
# Progress from an old idea: Lüscher's finite-volume method

M. Lüscher Commun. Math. Phys. 105 (1986) 153;  
Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

*Basic observation:* Finite volume, multi-particle energies are shifted with regard to the free energy levels due to the interaction

$$E = E(p_1) + E(p_2) + \Delta E$$

- Energy shifts encode scattering amplitude(s)
- Original method: Elastic scattering in the rest-frame in multiple spatial volumes  $L^3$
- Coupled 2-hadron channels well understood
- $2 \leftrightarrow 1$  and  $2 \leftrightarrow 2$  transitions well understood (example  $\pi\pi \rightarrow \pi\gamma^*$ )
- significant progress for 3-particle scattering



Reviews by R. A. Briceño arXiv:1411.6944 and M. Hansen arXiv:1511.04737

# Fully systematic calculation vs. exploratory study

Important lattice systematics from

- Taking the *continuum limit*:  $a(g, m) \rightarrow 0$
- Taking the *infinite volume limit*:  $L \rightarrow \infty$
- Calculation at (or extrapolation to) the physical pion mass

I cover many *exploratory* results

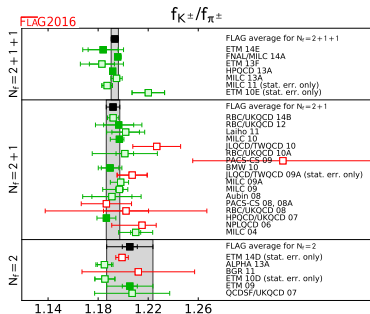
- Should be compared only qualitatively to experiment
- Provide an outlook on future Lattice QCD results

Example for fully systematic results:

- Flavor physics results listed in the FLAG review

<http://flag.unibe.ch/>

→ See talk by U. Wenger



# Exotic $D_s$ and $B_s$ candidates

Established s and p-wave  $D_s$  and  $B_s$  hadrons:

$D_s$  ( $J^P = 0^-$ ) and  $D_s^*$  ( $1^-$ )  
 $D_{s0}^*$ (2317) ( $0^+$ ),  $D_{s1}$ (2460) ( $1^+$ ),  
 $D_{s1}$ (2536) ( $1^+$ ),  $D_{s2}^*$ (2573) ( $2^+$ )

$B_s$  ( $J^P = 0^-$ ) and  $B_s^*$  ( $1^-$ )  
 $B_{s1}$ (5830) ( $1^+$ ),  $B_{s2}^*$ (5840) ( $2^+$ )

- Corresponding  $D_0^*$ (2400) and  $D_1$ (2430) are broad resonances
- Peculiarity:  $M_{c\bar{s}} \approx M_{c\bar{d}} \rightarrow$  exotic structure? (tetraquark, molecule)
- $B_s$  cousins of the  $D_{s0}^*$ (2317) and  $D_{s1}$ (2460) not (yet) seen in experiment
- The LHCb experiment at CERN should be able to see these

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$$\begin{array}{ll} D_s (J^P = 0^-) \text{ and } D_s^* (1^-) & B_s (J^P = 0^-) \text{ and } B_s^* (1^-) \\ D_{s0}^* (2317) (0^+), D_{s1} (2460) (1^+), & ? \\ D_{s1} (2536) (1^+), D_{s2}^* (2573) (2^+) & B_{s1} (5830) (1^+), B_{s2}^* (5840) (2^+) \end{array}$$

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# $D_{s0}^*(2317)$ : D-meson – Kaon s-wave scattering

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Nucl. Phys. B 354 (1991) 531; Nucl. Phys. B 364 (1991) 237.

## Charm-light hadrons

$D_{s0}^*(2317)^\pm$

$I(J^P) = 0(0^+)$   
 $J, P$  need confirmation.

$J^P$  is natural, low mass consistent with  $0^+$ .

Mass  $m = 2317.7 \pm 0.6$  MeV

$m_{D_{s0}^*(2317)^\pm} - m_{D^\pm} = 349.4 \pm 0.6$  MeV

Full width  $\Gamma < 3.8$  MeV, CL = 95%

$$p \cot \delta_0(p) = \frac{2}{\sqrt{\pi}L} Z_{00} \left( 1; \left( \frac{L}{2\pi} p \right)^2 \right) \\ \approx \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

Mohler et al. PRL 111 222001 (2013)

Lang, DM et al. PRD 90 034510 (2014)

## Results for ensembles (1) and (2)

$$a_0 = -0.756 \pm 0.025 \text{ fm} \quad (1)$$

$$r_0 = -0.056 \pm 0.031 \text{ fm}$$

$$a_0 = -1.33 \pm 0.20 \text{ fm} \quad (2)$$

$$r_0 = 0.27 \pm 0.17 \text{ fm}$$

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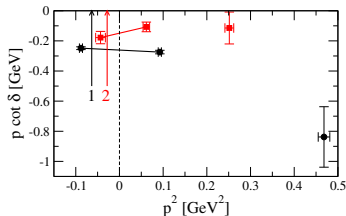
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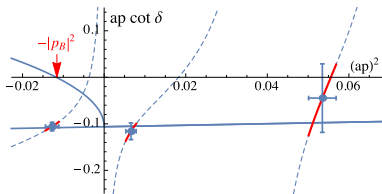
$$a_0 = -1.33 \pm 0.20 \text{ fm} \quad (2)$$

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# $B_{s0}^*$ and $B_{s1}$ : Results

Lang, Mohler, Prelovsek, Woloshyn PLB 750 17 (2015)

$B_{s0}^*$

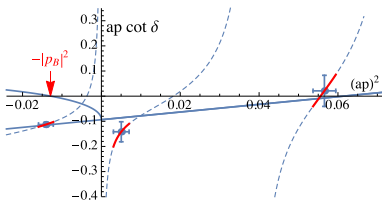


$$a_0^{BK} = -0.85(10) \text{ fm}$$

$$r_0^{BK} = 0.03(15) \text{ fm}$$

$$M_{B_{s0}^*} = 5.711(13) \text{ GeV}$$

$B_{s1}$



$$a_0^{B^*K} = -0.97(16) \text{ fm}$$

$$r_0^{B^*K} = 0.28(15) \text{ fm}$$

$$M_{B_{s1}} = 5.750(17) \text{ GeV}$$

- Energy from the difference to the  $B^{(*)}K$  threshold

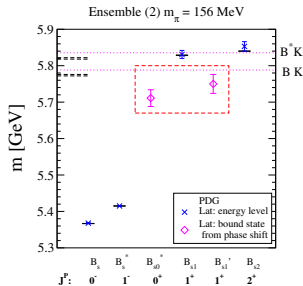
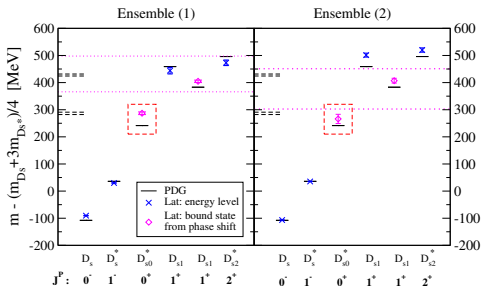


# $D_s$ and $B_s$ : Spectrum results

Mohler et al. PRL 111 222001 (2013)

Lang, Mohler et al. PRD 90 034510 (2014)

Lang, Mohler, Prelovsek, Woloshyn PLB 750 17 (2015)

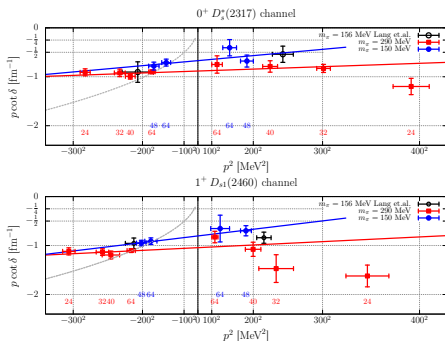


- Discretization uncertainties sizeable for charm
- Many improvements possible for the  $D_s$  states

- Full uncertainty estimate only for magenta  $B_s$  states
- Prediction of exotic states from Lattice QCD!

# Positive parity $D_s$ : More comprehensive results from RQCD

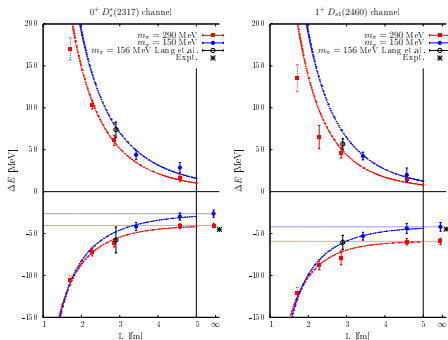
Bali, Collins, Cox, Schäfer, arXiv:1706.01247



- Study with different volumes at pion masses of 150, 290 MeV
- Remaining discretization effects non-negligible
- Results confirm basic behavior seen in previous simulation

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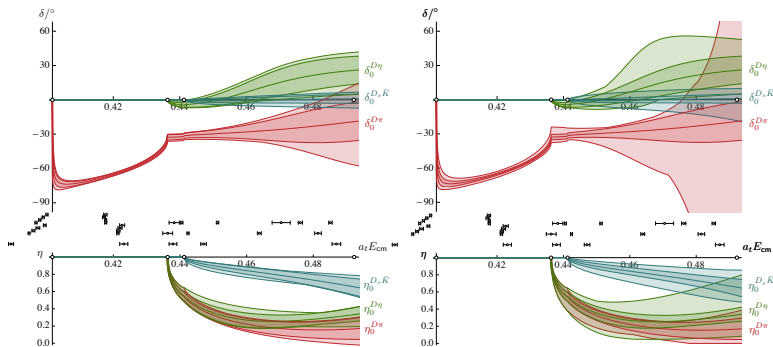
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# Coupled-channel study of $D\pi$ , $D\eta$ , $D_s K$ scattering

Moir et al., JHEP 1610 011 (2016)



- Lattice data from multiple volumes at  $m_\pi = 391$  MeV
- Shallow bound state seen in coupled channel s-wave
- Narrow spin-2 D-wave resonance seen as well
- For older single-channel results see

DM, Prelovsek, Woloshyn PRD 87 034501 (2013)

# Search for a $Z_c^+$ state from Lattice QCD

Prelovsek, Lang, Leskovec, DM, Phys.Rev. D91 014504 (2015)

- Search for a  $Z_c^+$  in the  $I^G J^{PC} = 1^+ 1^{+-}$  channel
- Aim at simulating all meson-meson states below  $\approx 4.3\text{GeV}$
- Caveat: Neglects 3-particle states
- Include tetraquark interpolators of type  $3_c \times \bar{3}_c$
- Count energy levels and identify them according to their overlaps
- Hope: See an extra level, as would be expected for a (narrow) resonance

More rigorous approach (a la Lüscher) quite challenging

- Coupled channel system with many channels
- Small shifts in finite volume and (largish) discretization effects
- Thresholds should be close to physical
- Suitable ensembles are (probably) not available at the moment.

# A look at the spectrum of scattering states

- Expect level close to non-interacting scattering states

$J/\Psi\pi$

$\eta_c\rho$

$J\Psi(1)\pi(-1)$

$DD^*$

$\Psi_{2S}\pi$

$D^*D^*$

$\Psi_{3770}\pi$

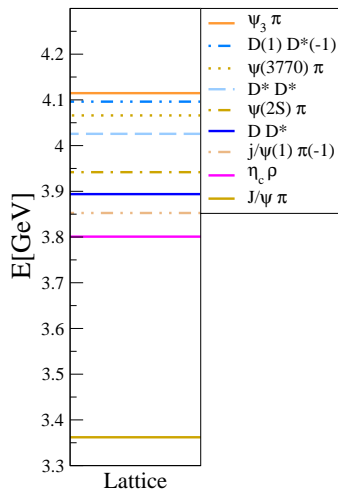
$D(1)D^*(-1)$

$\Psi_3\pi$

$J\Psi(2)\pi(-2)$

$D^*(1)D^*(-1)$

$D(2)D^*(-2)$



# Search for $Z_c^+$ with $I^G J^{PC} = 1^+ 1^{+-}$

X(3900)

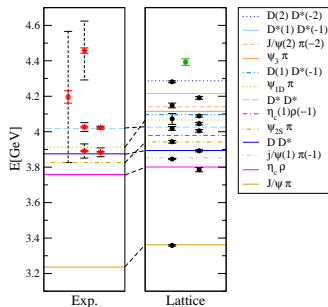
$$I^G(J^{PC}) = 1^+(1^{+-})$$

Mass  $m = 3886.6 \pm 2.4$  MeV ( $S = 1.6$ )

Full width  $\Gamma = 28.1 \pm 2.6$  MeV

Prelovsek, Lang, Leskovec, DM,

Phys.Rev. D91 014504 (2015)

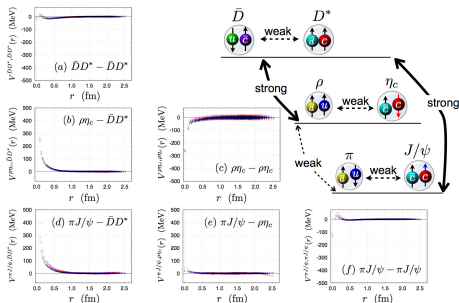


- Simple level counting approach
- We find 13 two meson states as expected
- We find no extra energy level that could point to a  $Z_c$  candidate

# $Z_c(3900)$ with the HALQCD method I

Ikeda et al. PRL 117 242001 (2016); Ikeda arXiv:1706.07300

- Coupled-channel scattering  $J/\Psi\pi, \eta_c\rho, \bar{D}D^*, I^G(J^{PC}) = 1^+1^{+-}$
- Uses 2+1 flavor gauge configurations with  $a = 0.907(13)$  and  $m_\pi = 410, 570, 700$
- HALQCD method Ishii et al. PLB 712, 437 (2012)
  - Calculate a potential as a function of distance  $r$
  - Solve Schrödinger equation with given  $V(r)$  and determine scattering phase shifts





# $Z_c(3900)$ with the HALQCD method II

$X(3900)$

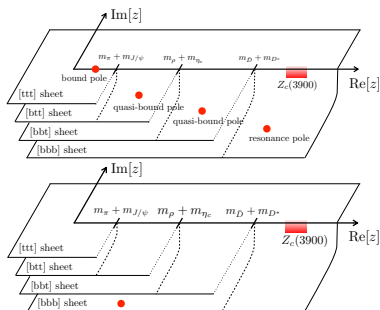
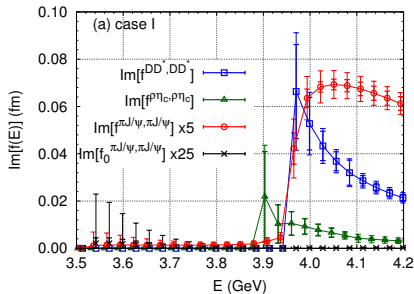
$$I^G(J^{PC}) = 1^{+}(1^{+-})$$

Ikeda *et al.* PRL 117 242001 (2016);

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Full width  $\Gamma = 28.1 \pm 2.6$  MeV

Ikeda arXiv:1706.07300



- Pole found far below  $DD^*$  threshold for all quark masses
- Authors conclude  $Z_c(3900)$  not a usual resonance but a threshold cusp
- Structure comes from strong  $\pi J/\Psi - \bar{D}D^*$  coupling
- Analysis at close-to-physical pion mass planned

# $\chi'_{c0}$ and $X/Y(3915)$

$X(3915)$   
was  $\chi_{c0}(3915)$

$$J^{G(J^{PC})} = 0^{+(0 \text{ or } 2^{++})}$$

Mass  $m = 3918.4 \pm 1.9$  MeV

Full width  $\Gamma = 20 \pm 5$  MeV ( $S = 1.1$ )

PDG interpreted  $X(3915)$  as a **regular charmonium** ( $\chi'_{c0}$ )

- Some of the reasons to doubt this assignment:

Guo, Meissner Phys. Rev. **D86**, 091501 (2012)

Olsen, PRD 91 057501 (2015)

- No evidence for fall-apart mode  $X(3915) \rightarrow \bar{D}D$
- Spin splitting  $m_{\chi_{c2}(2P)} - m_{\chi_{c0}(2P)}$  too small
- Large OZI suppressed  $X(3915) \rightarrow \omega J/\psi$
- Width should be significantly larger than  $\Gamma_{\chi_{c2}(2P)}$
- Zhou *et al.* (PRL 115 2, 022001 (2015)) argue that what is dubbed  $X(3915)$  is the spin 2 state already known and suggests that a broader state is hiding in the experiment data.
- Observation of an alternative  $\chi_{c0}(2P)$  by Belle:

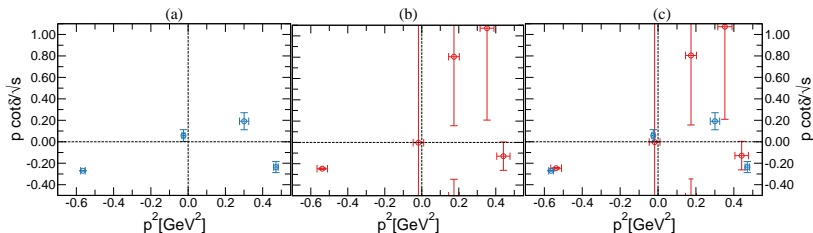
Chilikin *et al.* PRD 95 112003 (2017)

$$M = 3862^{+26+40}_{-32-13}$$

$$\Gamma = 201^{+154+88}_{-067-82}$$

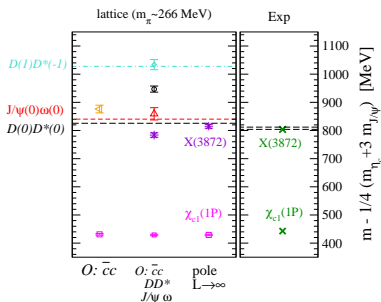
# $\chi'_{c0}$ : Exploratory lattice calculation

Lang, Leskovec, DM, Prelovsek, JHEP 1509 089 (2015)

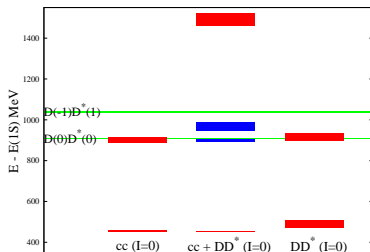


- Assumes only  $\bar{D}D$  is relevant
- Lattice data suggests a fairly narrow resonance with  $3.9\text{GeV} < M < 4.0\text{GeV}$  and  $\Gamma < 100\text{MeV}$
- Future experiment and lattice QCD results needed to clarify the situation

# An $X(3872)$ candidate from Lattice QCD



Prelovsek, Leskovec, PRL 111  
192001 (2013)

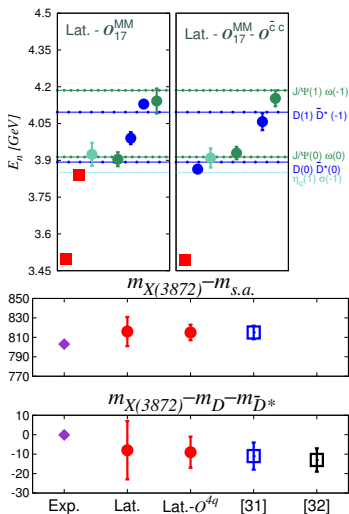


Lee, DeTar, DM, Na,  
arXiv:1411.1389

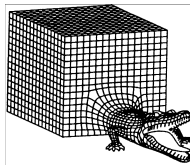
- Neglects charm annihilation and  $J/\psi\omega$
- Seen only when  $\bar{q}q$  and  $\bar{D}^*D$  are used
- The two simulations have vastly different systematics (yet results are similar)

# An $X(3872)$ candidate from Lattice QCD II

Padmanath, Lang, Prelovsek, PRD 92 034501 (2015)



- Without  $\bar{q}q$  interpolators signal vanishes
- Simulations still unphysical in many ways
- Discretization and finite volume effects sizable!



- Makes interpretation as pure molecule or pure tetraquark unlikely

# Recent simulations of charm or beauty tetraquarks

- Searches for charmed tetraquarks
  - Doubly charmed and charmed-strange tetraquarks by HALQCD  
Ikeda et al. PLB 729 85–90 (2014)
  - Search for doubly charmed tetraquarks on CLS lattices (preliminary)  
Guerrieri et al. arXiv:1411.2247
- HHLL systems with bottom quarks
  - Tetraquark bound states in heavy-light heavy-light systems  
Brown and Orginos PRD 86 114506 (2012)
  - Lattice QCD results for a bottom-bottom tetraquark  
Bicudo and Wagner PRD 87 114511 (2013)
  - Search for  $ud\bar{b}\bar{b}$   $ss\bar{b}\bar{b}$  and  $cc\bar{b}\bar{b}$  tetraquarks  
Bicudo et al., PRD 92 014507 (2015)
  - ***BB* interactions with static bottom quarks**  
Bicudo, Cichy, Peters, Wagner, PRD 93 034501 (2016);  
Bicudo, Scheunert, Wagner, PRD 95 034502 (2017)
  - **Deeply bound doubly-heavy tetraquarks with NRQCD b-quarks**  
Francis, Hudspith, Lewis, Maltman, PRL 118 142001 (2017)  
Junnarkar, Mathur, Padmanath, presented at Lattice 2017

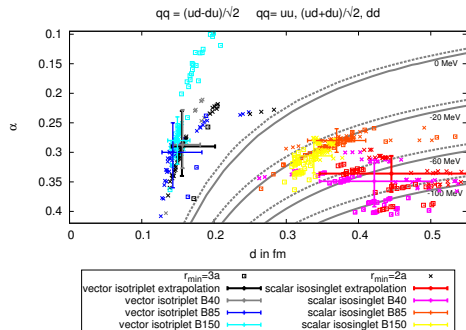
# BB interactions with static bottom quarks

Bicudo, Cichy, Peters, Wagner, PRD 93 034501 (2016)

Bicudo, Scheunert, Wagner, PRD 95 034502 (2017)

- Potentials of two static antiquarks in the presence of two light quarks
- Search for bound states (rather than resonances)
- Lattices with  $a = 0.079$  fm and  $m_\pi \approx 650, 480, 340$
- Fit function used for the lattice QCD potentials

$$V(r) = -\frac{\alpha}{r} \exp\left(-\left(\frac{r}{d}\right)^p\right) + V_0$$



Resulting binding energy:

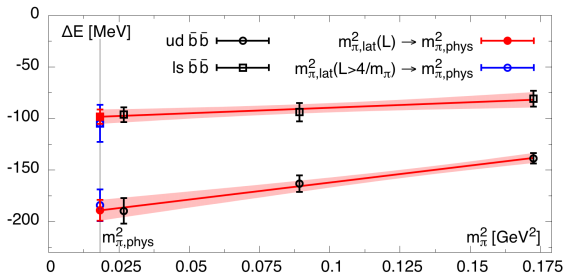
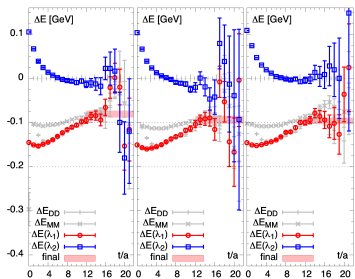
$$E_B = 90_{-36}^{+43} \text{ MeV}$$

Including heavy  $\bar{b}$  spins:

$$E_B = 59_{-38}^{+30} \text{ MeV}$$

# Doubly bottom $J^P = 1^+$ tetraquarks with NRQCD b-quarks

Francis, Hudspith, Lewis, Maltman, PRL 118 142001 (2017)

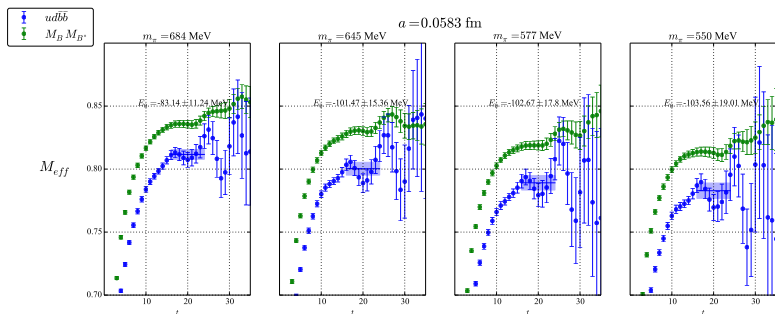


- Study at a single lattice spacing and three pion masses  
 $164\text{MeV} \leq M_\pi \leq 415\text{MeV}$
- Authors obtain bound 4-quark states for both  $ud\bar{b}\bar{b}$  and  $ls\bar{b}\bar{b}$
- Potential issues
  - Binding energies extracted from ratios can be misleading
  - For a bound state, excited state naively expected above threshold
  - Finite volume effects alter the binding energy



# $ud\bar{b}\bar{b} J^P = 1^+$ tetraquarks on HISQ lattices

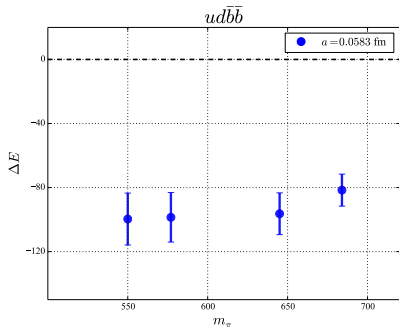
Junnarkar, Padmanath, Mathur, reported at Lattice 2017



- Results from  $48^3 \times 144$  HISQ ensemble with  $M_{\pi, sea} \approx 310 \text{ MeV}$
- Four valence pion masses in range  $550 \text{ MeV} \leq M_\pi \leq 684 \text{ MeV}$
- Same quantum numbers than Francis *et al.*  
(but notable difference in binding energy)
- Preliminary results confirm existence of a bound state

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Thank you!

Thanks to Parikshit Junnarkar and Nilmani Mathur for sending me unpublished updates

# The X(5568)

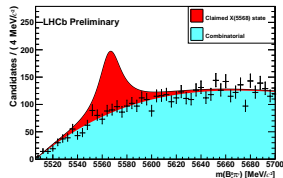
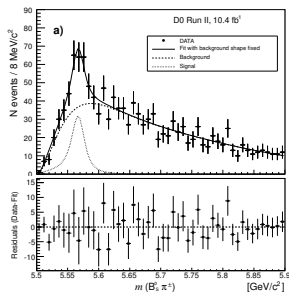
Abazov *et al.* PRL 117, 022003 (2016).

- The D0 collaboration is reporting evidence for a peak in the  $B_s\pi^+$  invariant mass not far above threshold
- D0 attributes this to resonance X(5568)

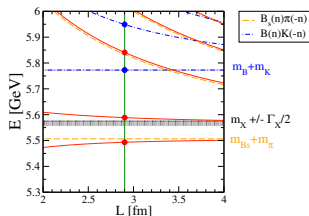
$$m_X = 5567.8 \pm 2.9^{+0.9}_{-1.9} \text{ MeV}$$

$$\Gamma_X = 21.9 \pm 6.4^{+5.0}_{-2.5} \text{ MeV}$$

- Decay to  $B_s\pi^+$  implies exotic flavor structure  $\bar{b}s\bar{d}u$
- Most model studies accommodating a X(5568) propose  $J^P = 0^+$
- LHCb did not find any peak in the  $B_s\pi^+$  invariant mass (with increased statistics)



# Expected signatures of the X(5568)

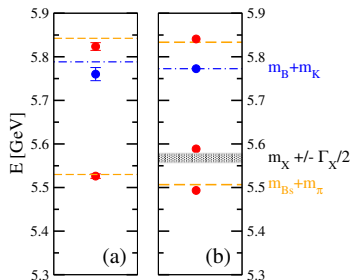


Analytic predictions for energies of eigenstates for an elastic resonance in  $B_s\pi$  (with  $J^P = 0^+$ ) as a function of the lattice size  $L$ .

- Orange (blue) dashed lines show the  $B_s\pi$  ( $BK$ ) eigenstates when  $B_s$  and  $\pi$  ( $B$  and  $K$ ) do not interact
- Red lines show the expectation for lattice energy levels in elastic  $B_s\pi$  scattering (decoupled from  $BK$ ) for a resonance with a mass and width of the X(5568).
- In the scenario of a deeply bound  $BK$  state, the simulation would result in an eigenstate with  $E \approx m_X$  up to exponentially small corrections in  $L$ .

# Lattice results and conclusions

Lang, DM, Prelovsek, PRD 94 074509 (2016)

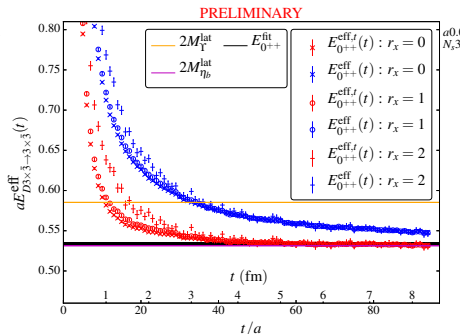
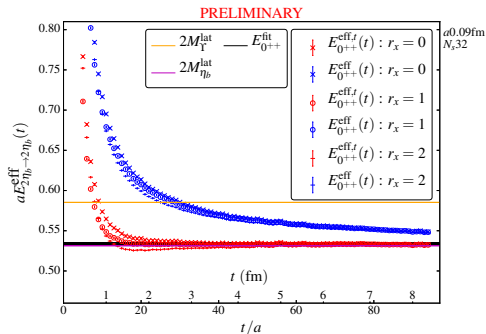


- Exploratory calculation with a single ensemble at  $M_\pi = 156$  MeV
- Left pane: eigenenergies of the  $\bar{b}s\bar{d}u$  system with  $J^P = 0^+$
- Right pane: Analytic prediction based on the  $X(5568)$
- Results stable under variations of the fit methodology
- Lattice simulation at close-to-physical quark masses does not yield a second low-lying energy level (expected for the case of the  $X(5568)$ )
- Results do not support the existence of  $X(5568)$  with  $J^P = 0^+$ .

# Search for a deeply bound $bb\bar{b}\bar{b}$ state

C. Hughes, C. Davies, E. Eichten, presented at Lattice 2017

- A number of recent potential model predictions of a very deeply bound  $bb\bar{b}\bar{b}$  state
- Lattice study using NRQCD b quarks, 3 different lattice spacings
- Upshot: No deeply bound states but deficiencies in models understood



# Search for charmed tetraquarks by HALQCD

Ikeda et al. PLB 729 85–90 (2014)

- Search for bound states or resonances in  $DD$ ,  $\bar{K}D$ ,  $DD^*$  and  $\bar{K}D^*$  interactions with flavor structure  $cc\bar{u}\bar{d}$  and  $cs\bar{u}\bar{d}$
- These contain no quark line diagrams with quark annihilation
- Uses 2+1 flavor gauge configurations with  $a = 0.907(13)$  and  $m_\pi = 410, 570, 700$
- HALQCD method

Ishii et al. PLB 712, 437 (2012)

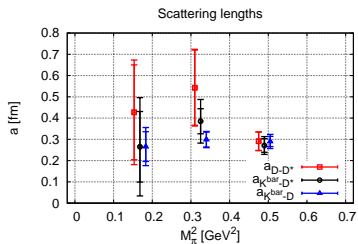
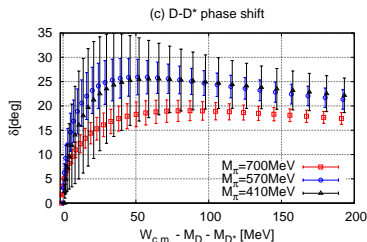
- Calculate a potential as a function of distance  $r$
- Solve Schrödinger equation with given  $V(r)$  and determine scattering phase shifts
- Uses variant of the Fermilab method (relativistic heavy quark action)



# Tetraquarks with the HALQCD method: Results

Ikeda et al. PLB 729 85–90 (2014)

- Repulsive interaction in all  $I = 1$  channels considered
- Attractive interaction in all  $I = 0$  channels considered



- No bound states or resonances at simulated  $m_\pi$
- Attraction becomes more prominent at light pion masses
- Authors have some indication that  $BB^*$  with  $IJ^P = 01^+$  is bound

# Lattice simulation

- We use lattices with 2+1 flavors of Wilson-Clover quarks by PACS-CS

$N_L^3 \times N_T$	$N_f$	$a[\text{fm}]$	$L[\text{fm}]$	#configs	$m_\pi[\text{MeV}]$	$m_K[\text{MeV}]$
$32^3 \times 64$	2+1	0.0907(13)	2.90	196	156(7)(2)	504(1)(7)

- For a description of the heavy-quark methodology see

Lang, DM, Prelovsek, Woloshyn PLB 750 17 (2015)

- Interpolator basis

$$O_{1,2}^{B_s(0)\pi(0)} = [\bar{b}\Gamma_{1,2}s](\mathbf{p}=0) [\bar{d}\Gamma_{1,2}u](\mathbf{p}=0)$$
$$O_{1,2}^{B_s(1)\pi(-1)} = \sum_{\mathbf{p}=\pm\mathbf{e}_{x,y,z} 2\pi/L} [\bar{b}\Gamma_{1,2}s](\mathbf{p}) [\bar{d}\Gamma_{1,2}u](\mathbf{-p})$$
$$O_{1,2}^{B(0)K(0)} = [\bar{b}\Gamma_{1,2}u](\mathbf{p}=0) [\bar{d}\Gamma_{1,2}s](\mathbf{p}=0)$$