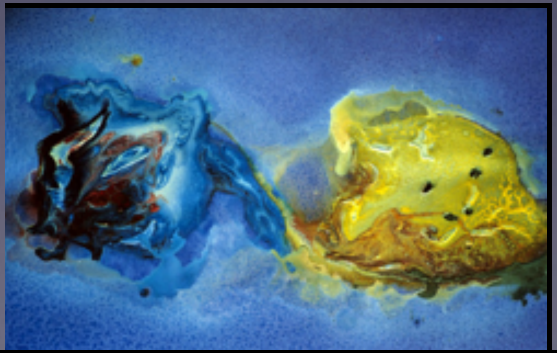
Quarkonium: Theoretical Aspects

Nora Brambilla



Physik Department TUM
T30f

- Quarkonium is a nonrelativistic (NR) multiscale systems—> golden probe of strong interactions

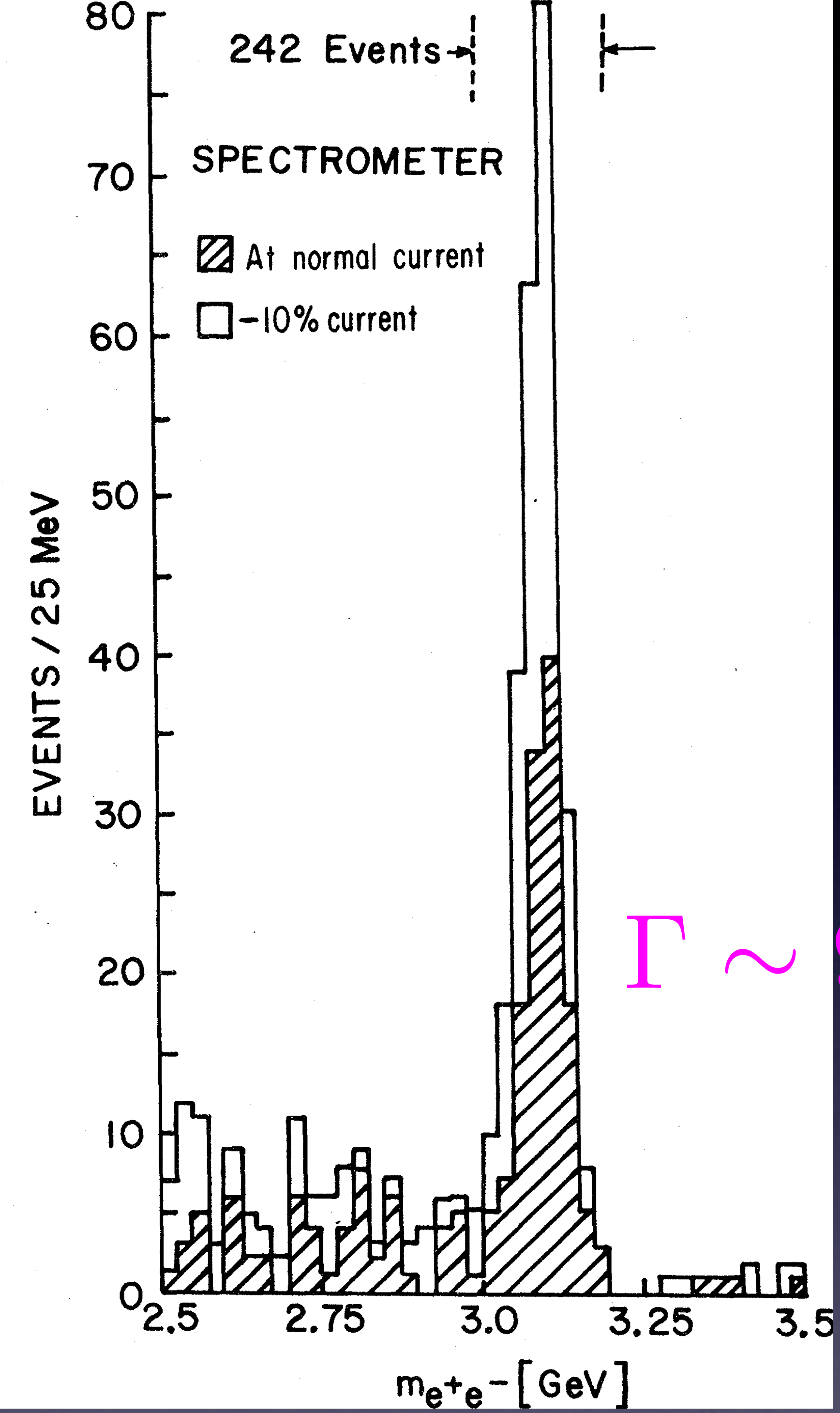
- State of the art theory tools: Effective Field Theories (EFTs) and lattice

- Same techniques can be used for studies of electromagnetic (NR) bound states: atoms and molecules

- Exotic quarkonium: EFT of Born-Oppenheimer and Van der Waals

- Quarkonium (=bound state of a heavy quark and a heavy antiquark) has been instrumental for the establishing of QCD, the theory of strong interaction, and the Standard Model of ParticlePhysics

The November revolution in 1974: the J/ψ discovery



$\Gamma \sim 90\,\mathrm{KeV}$

Aubert et al. BNL 74

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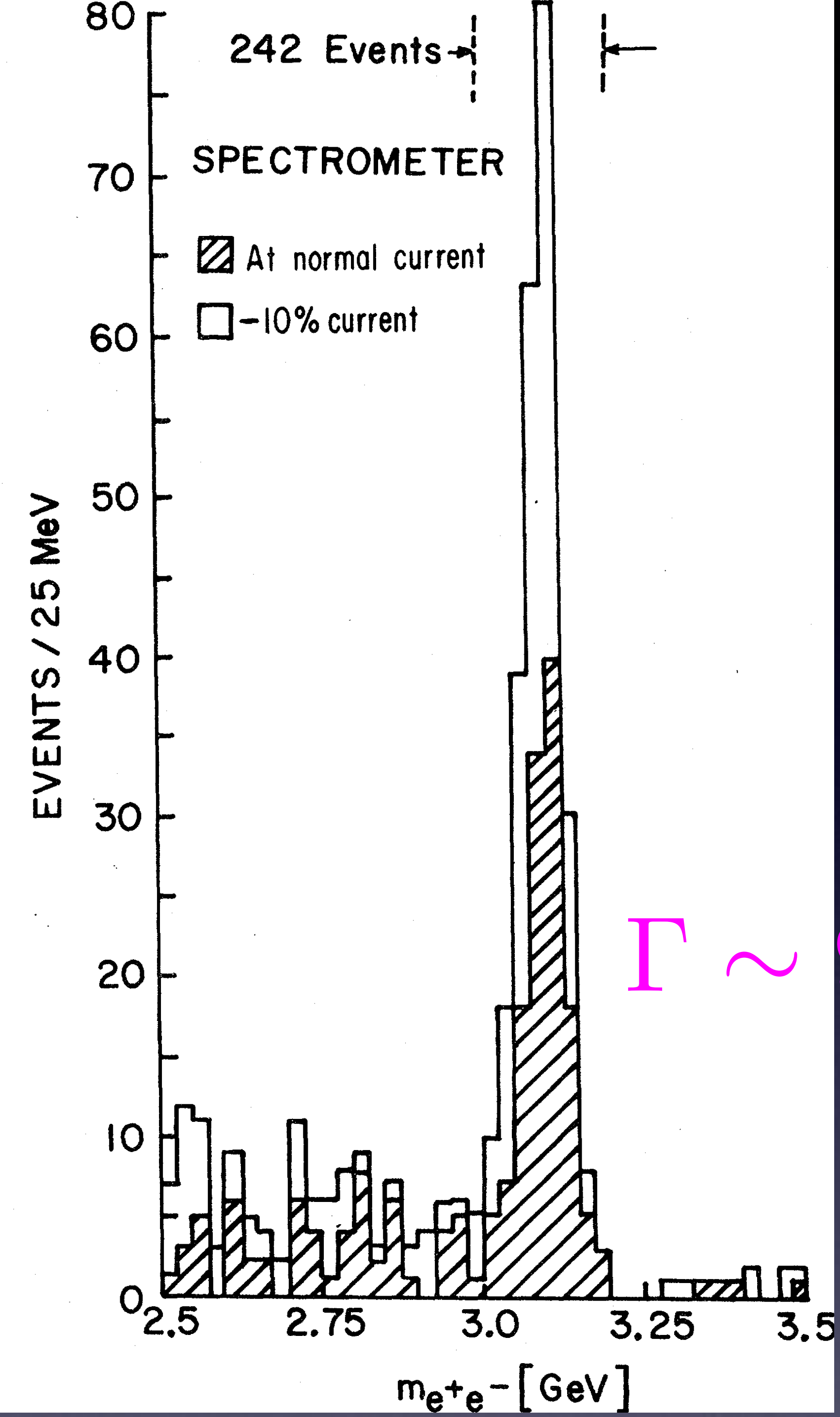

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Samuel Ting: "It is like to stumble on a village where people live 70000 years"

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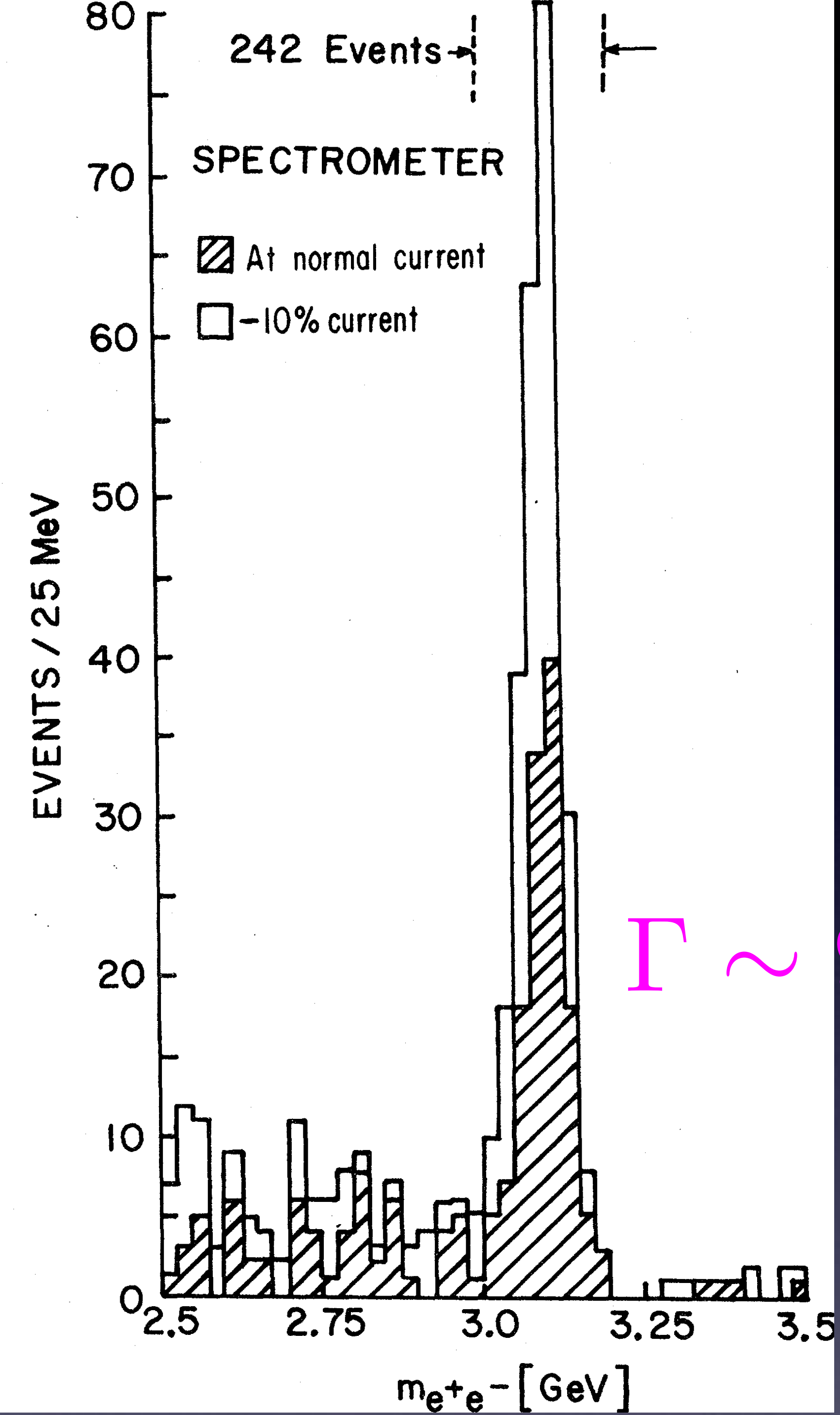

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Samuel Ting: "It is like to stumble on a village where people live 70000 years"

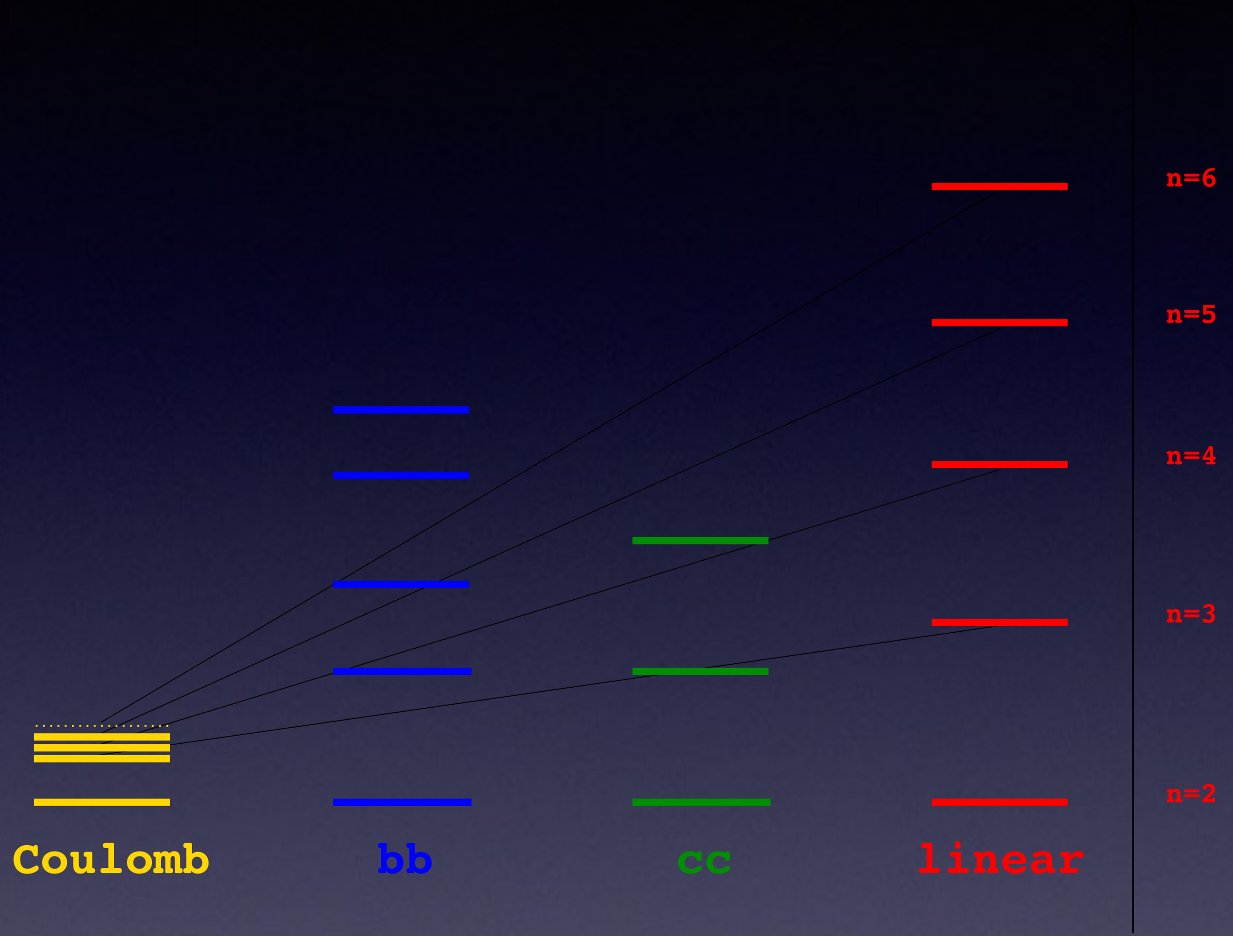
it has been the confirmation of the charm quark prediction and of QCD (strong int theory) foundations

narrow width and asymptotic freedom

annihilation at large scale controlled by small α_s

first discovery of a quark of large mass moving "slowly"

The November revolution in the '70s: more quarkonia



bbar and ccbar energy levels in comparison to Coulomb and linear potential energy levels

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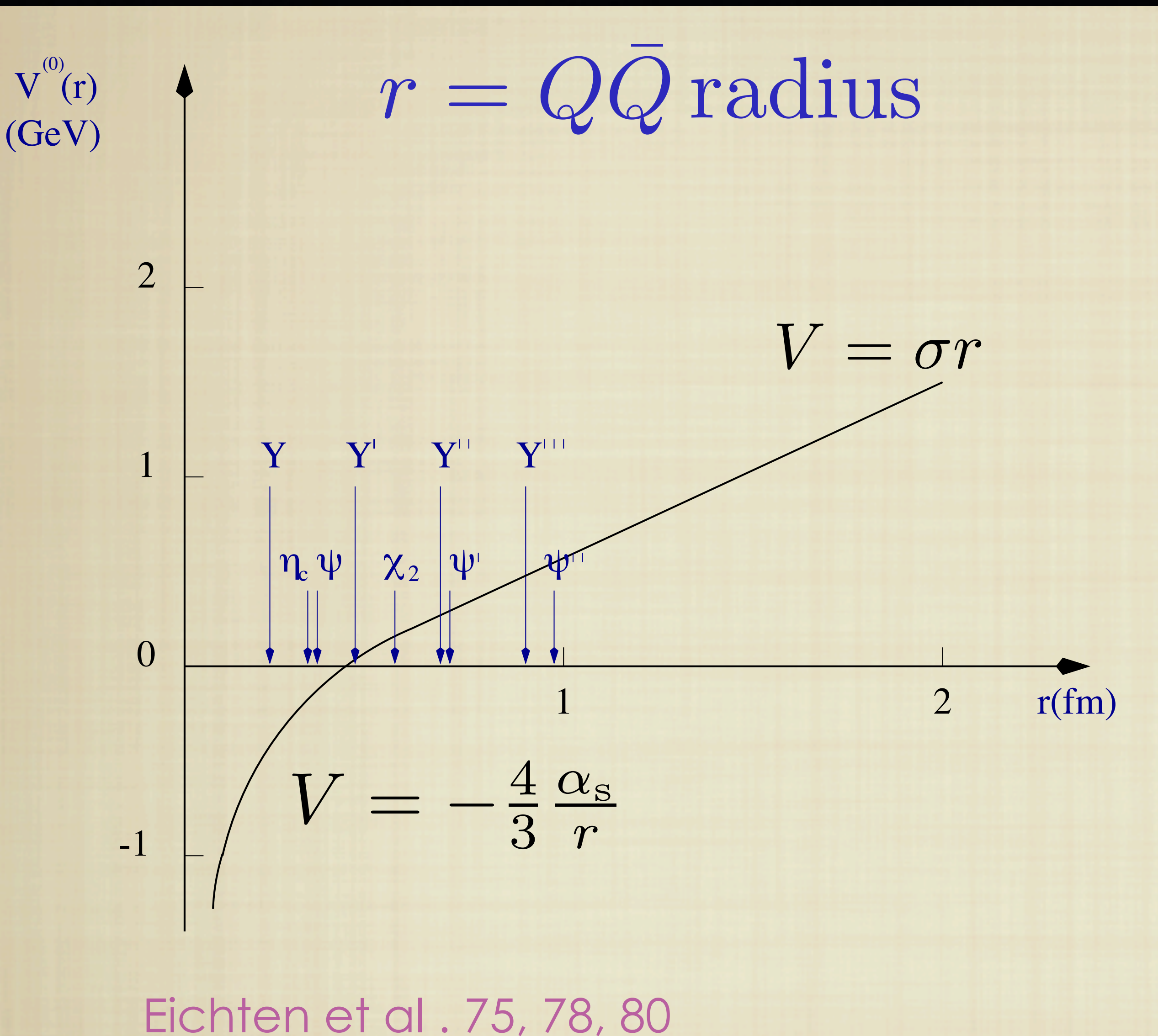

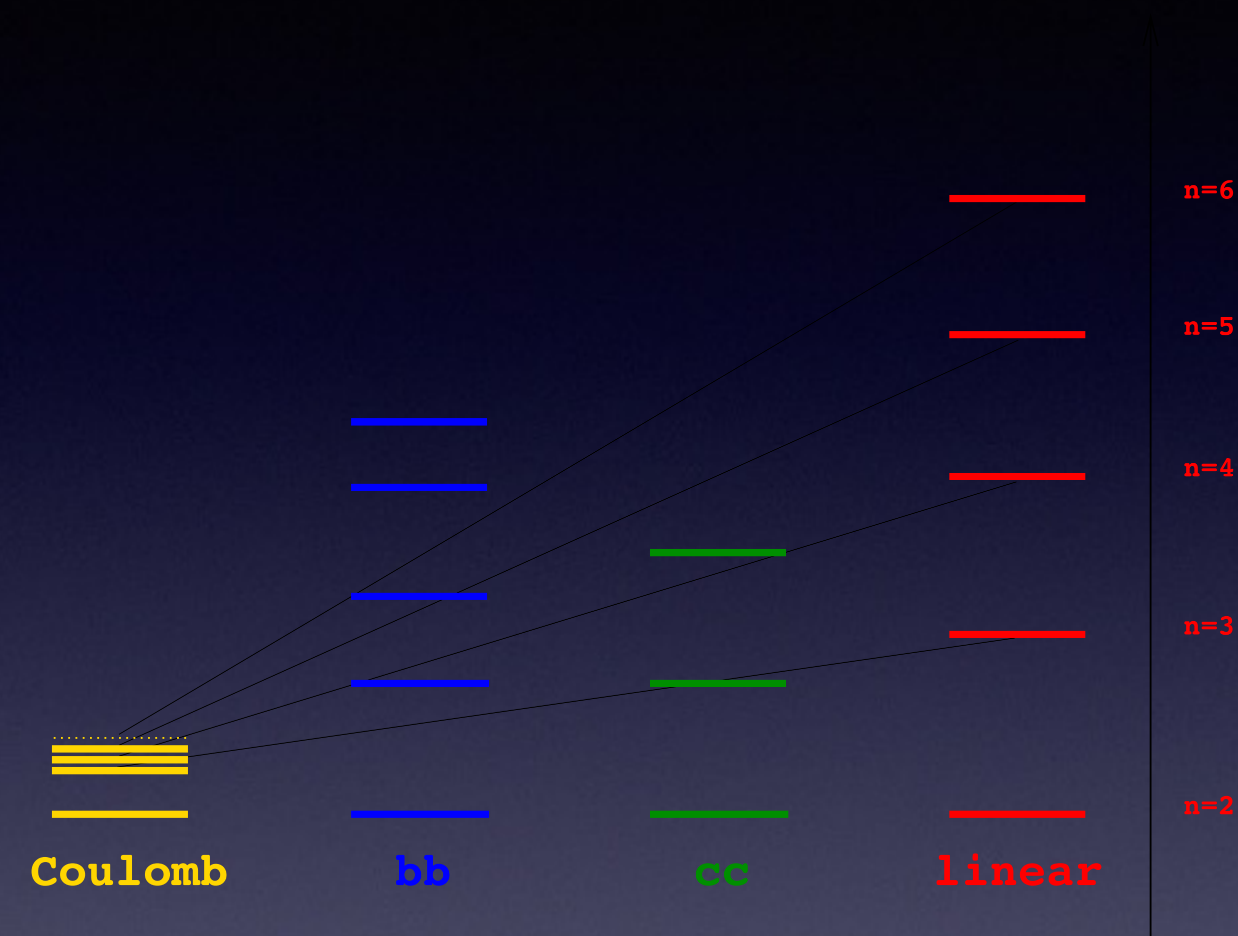

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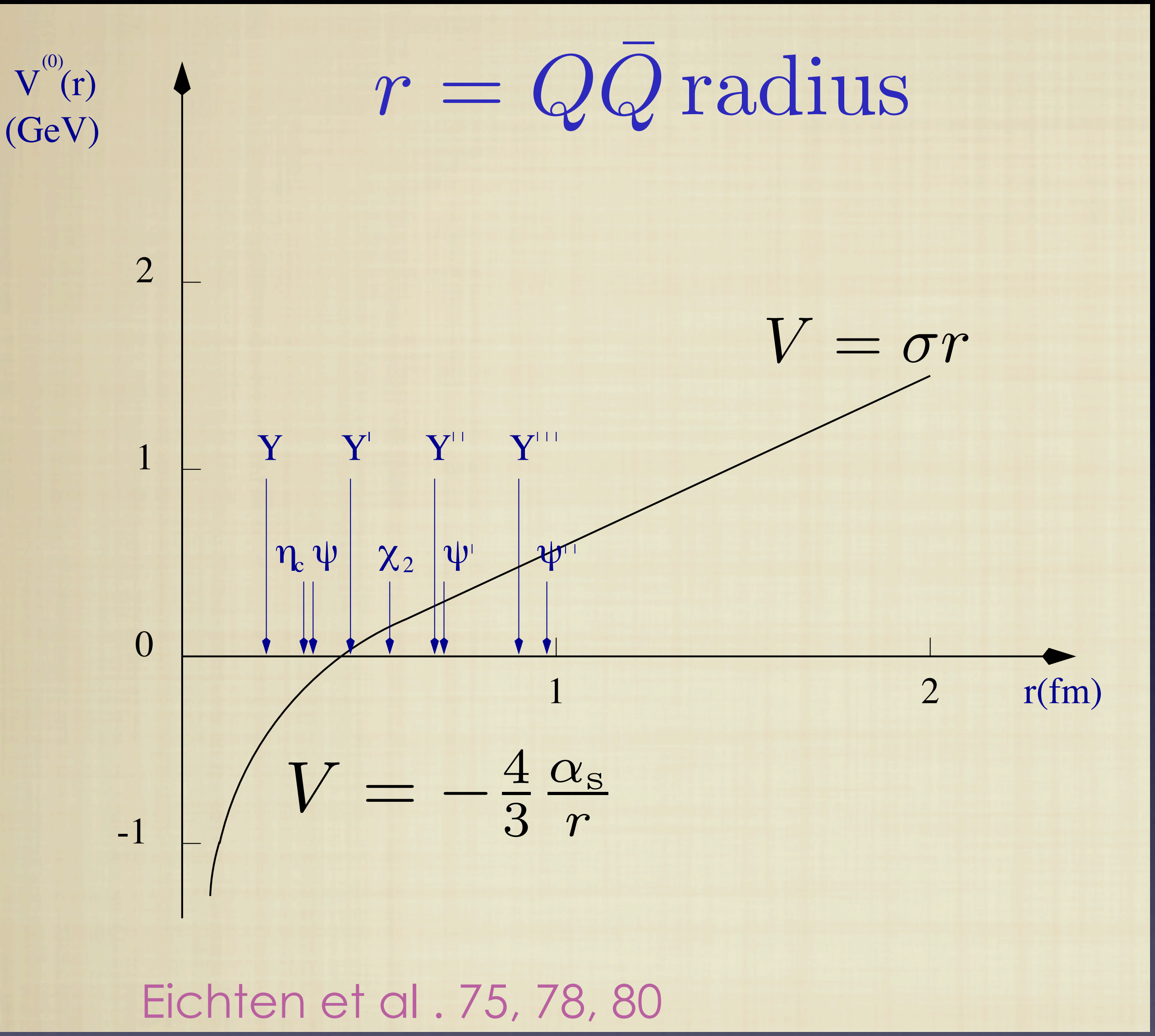
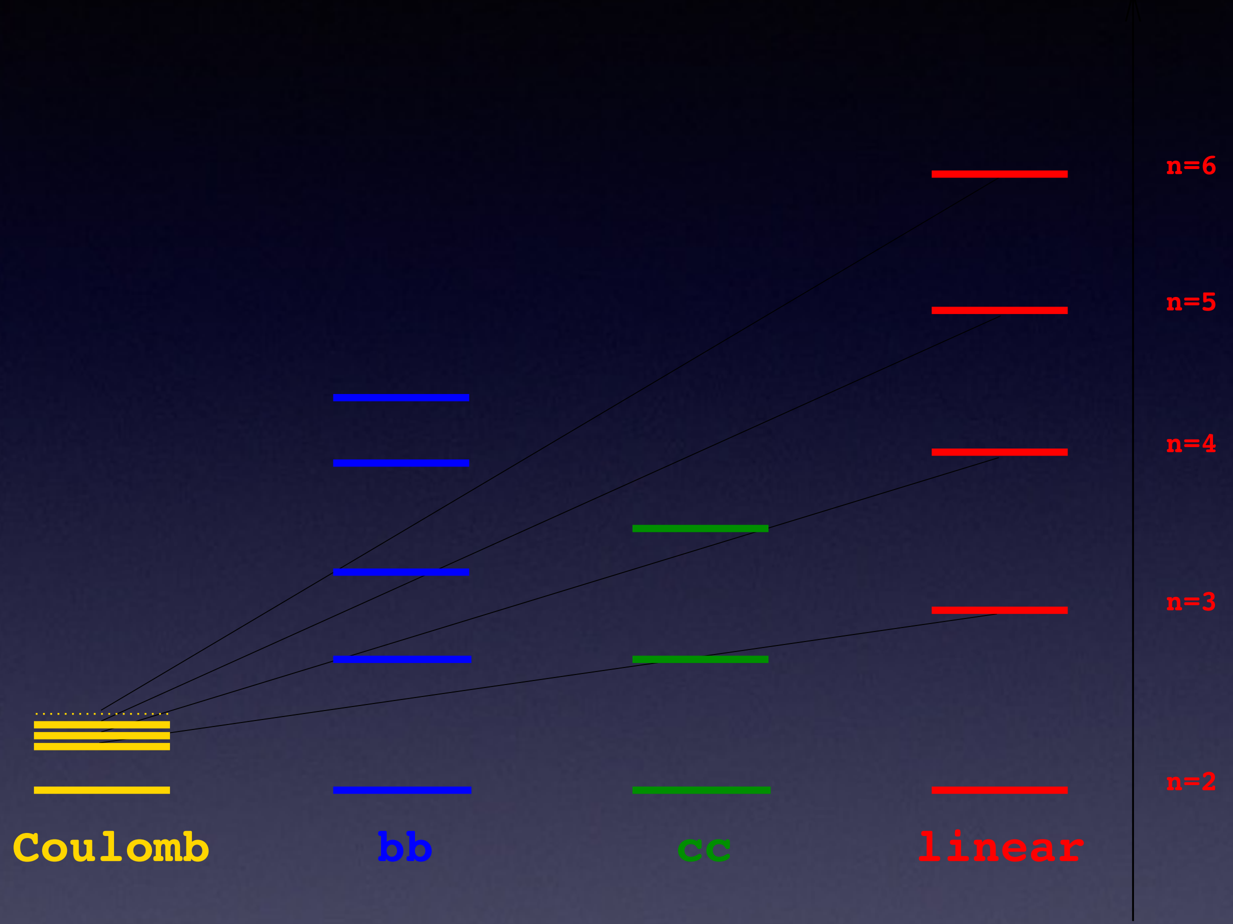




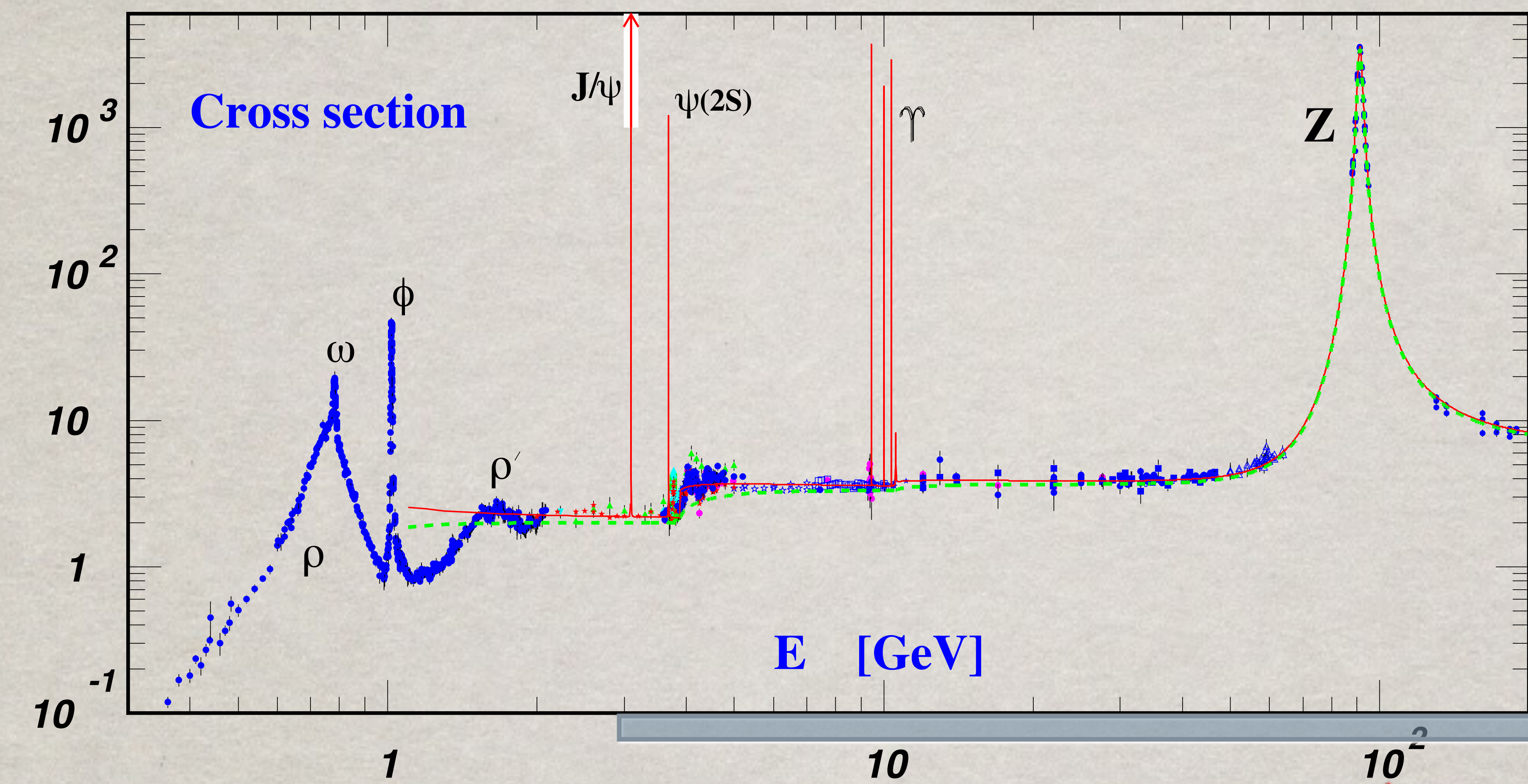
bbar and ccbar energy levels in comparison to Coulomb and linear potential energy levels

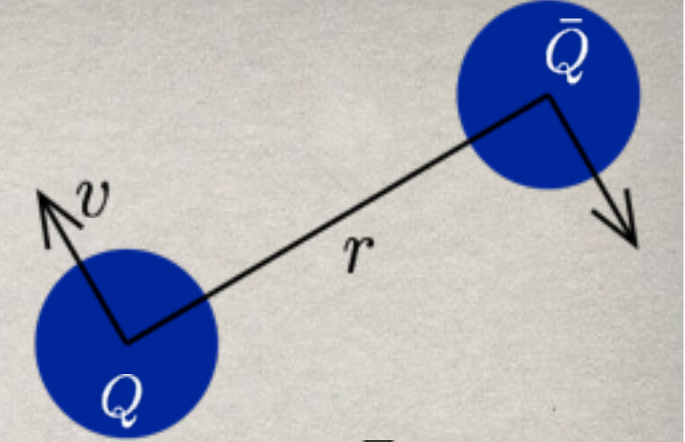
Variety of potential models used

Confinement and asymptotic freedom--> main properties of QCD

Quarkonium is
a golden system to study strong interactions

Heavy quarks offer a privileged access





with $Q, \bar{Q} = c, b, t$

$m_c \sim 1.5$GeV

$m_b \sim 5$GeV

$m_t \sim 170$GeV

A large scale $m_Q \gg \Lambda_{QCD}$ $\alpha_s(m_Q) \ll 1$

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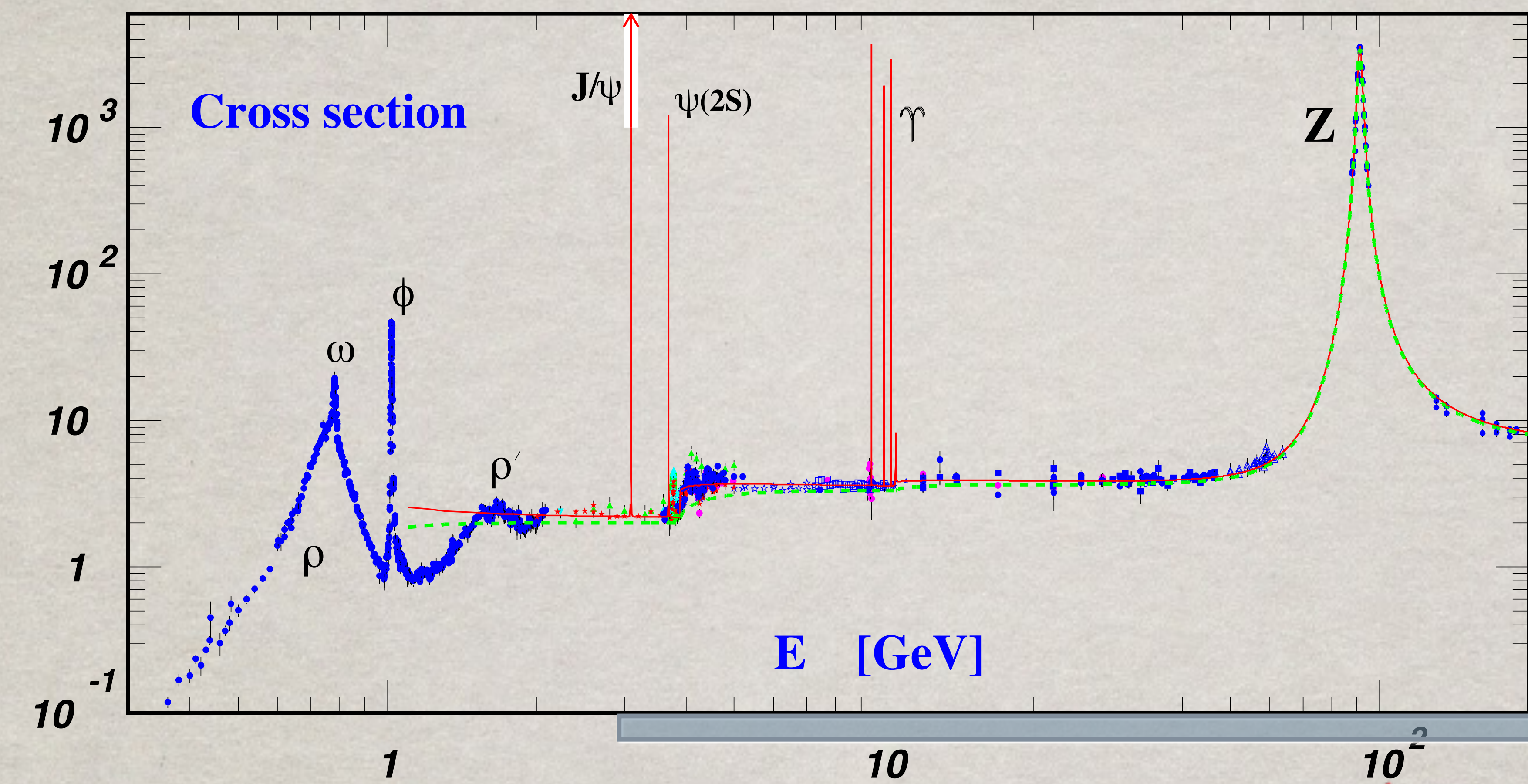

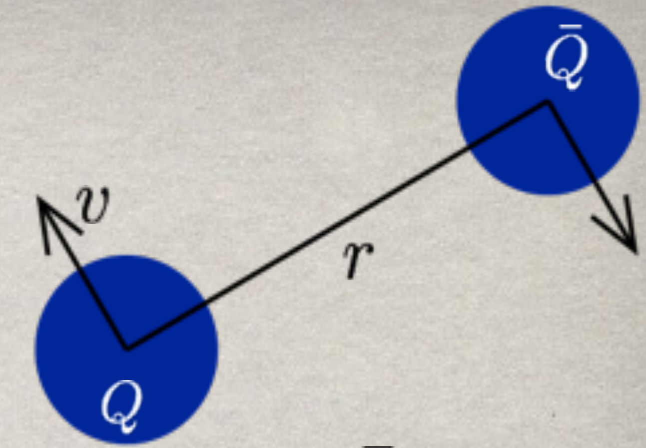

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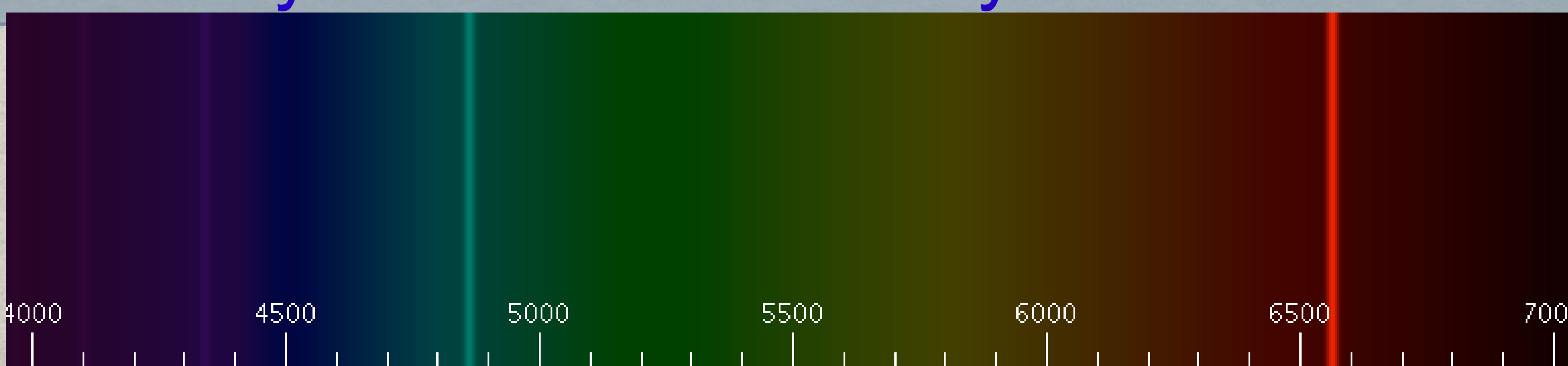
Heavy quarkonia are nonrelativistic bound systems: multiscale systems



ELECTROMAGNETIC BOUND STATES: ATOMS, MOLECULES,

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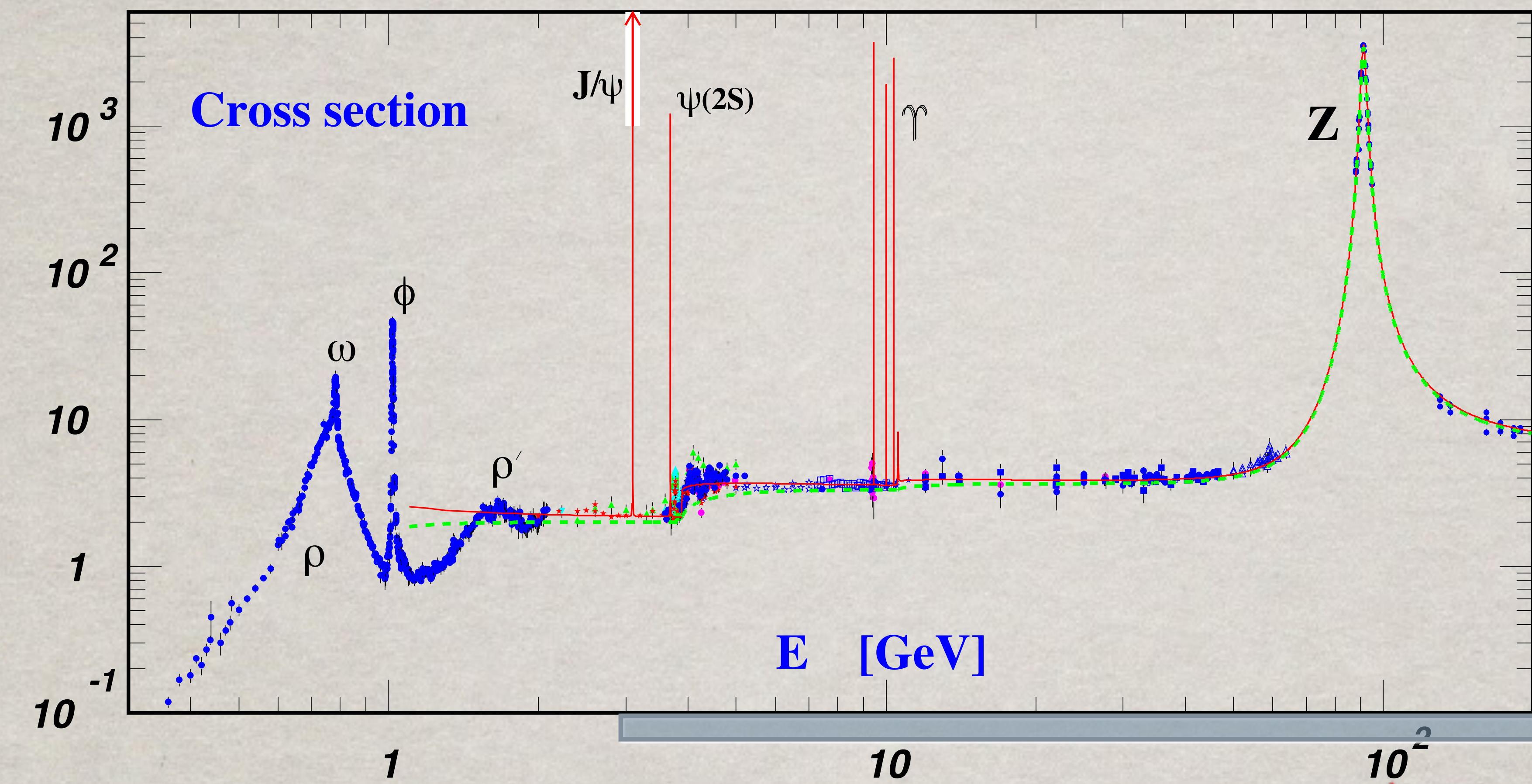

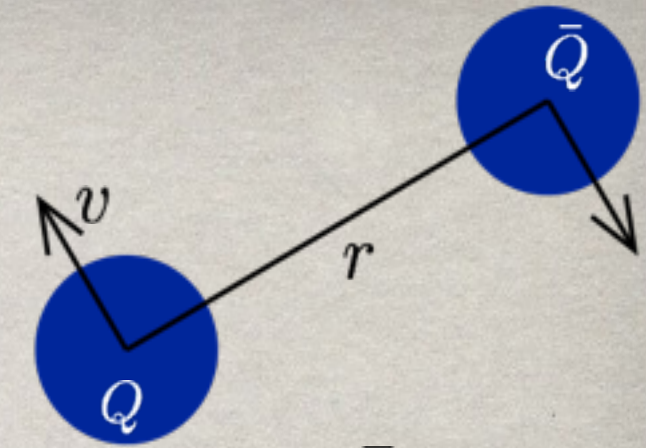

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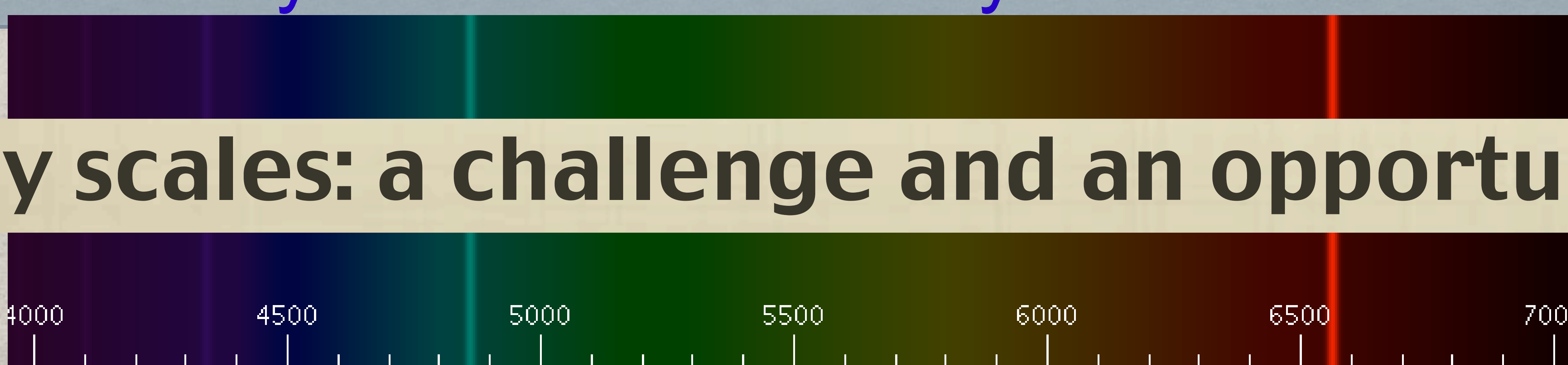
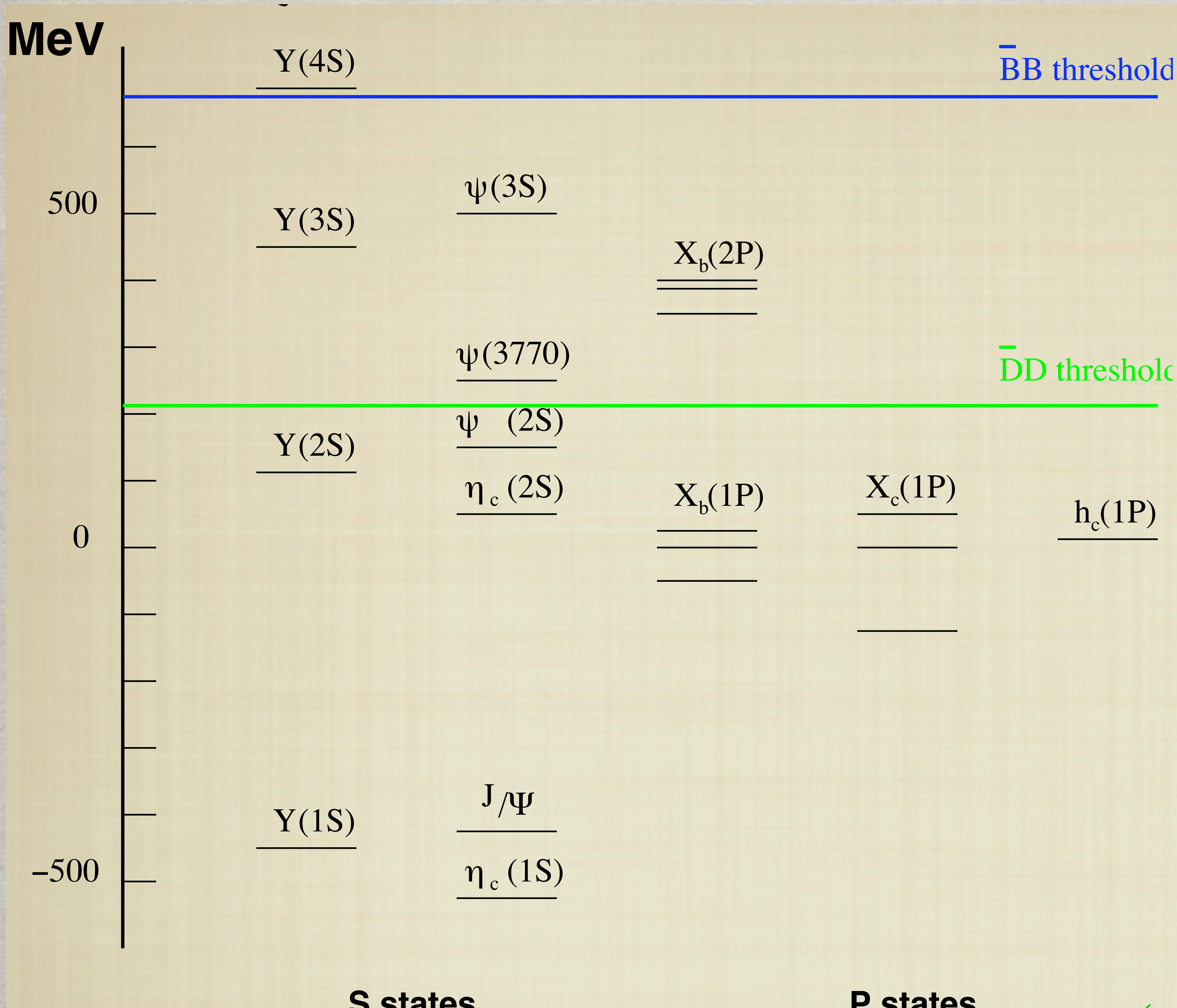


many scales: a challenge and an opportunity

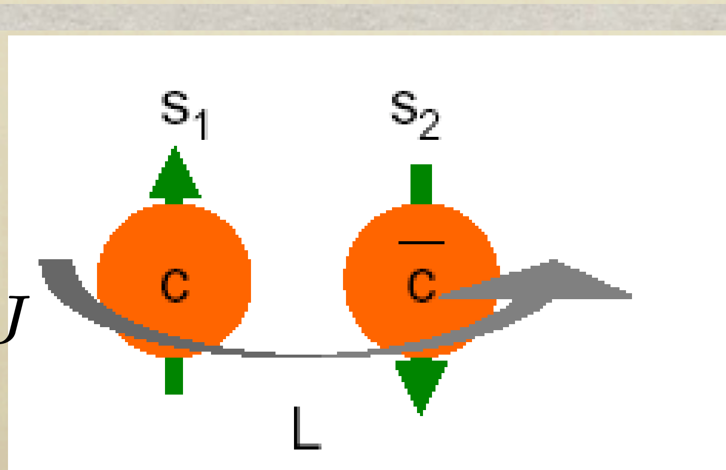
ELECTROMAGNETIC BOUND STATES: ATOMS, MOLECULES,

Quarkonium scales



Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

$^{2S+1}L_J$

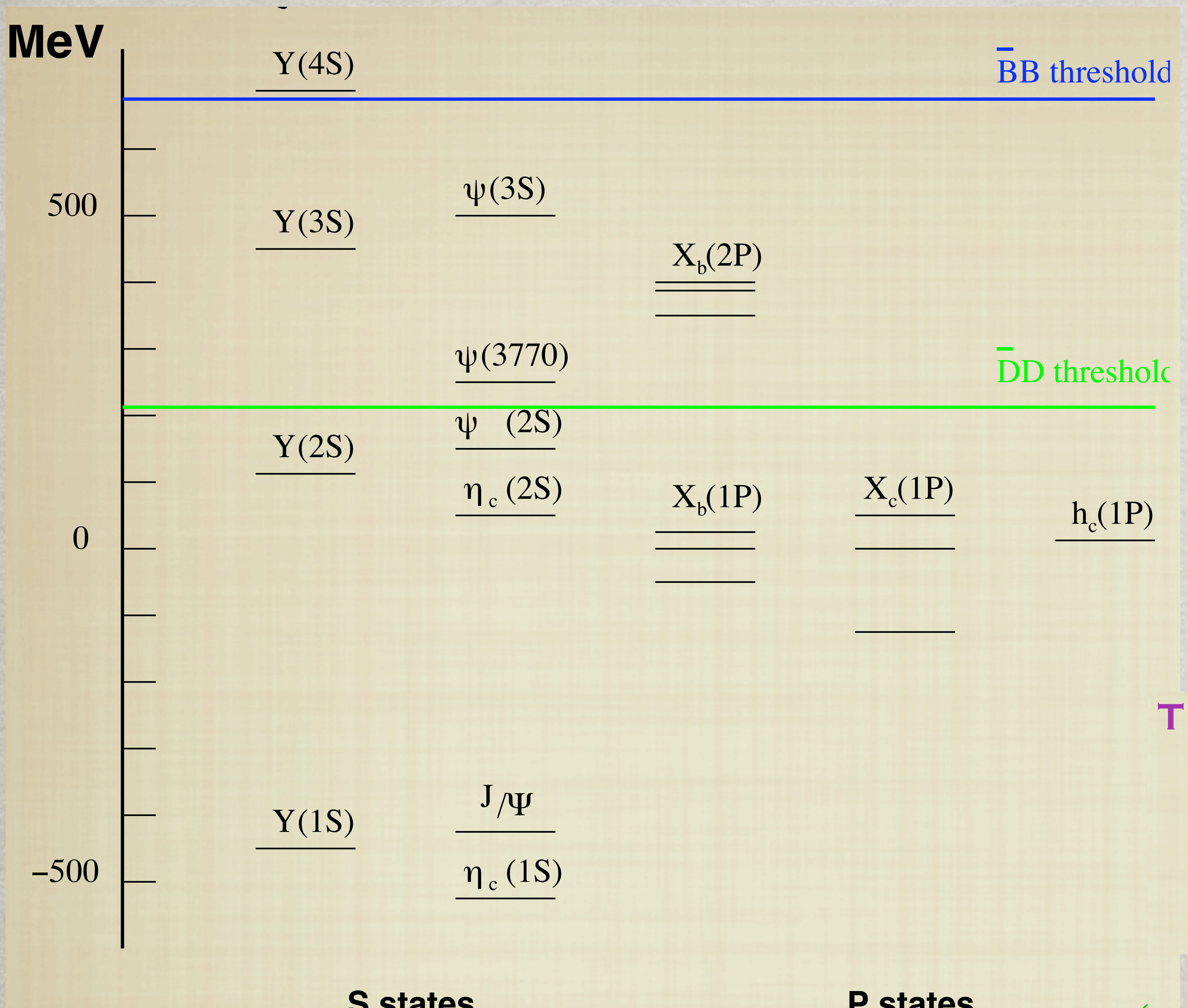


The mass scale is perturbative

$$m_Q \gg \Lambda_{\rm QCD}$$

$$m_b \simeq 5\,{\rm GeV}; m_c \simeq 1.5\,{\rm GeV}$$

Quarkonium scales



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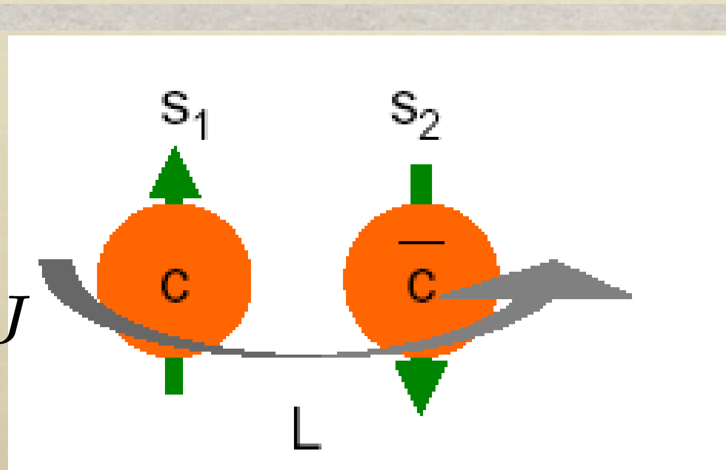

The system is nonrelativistic(NR)

$$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$$

$$v_b^2 \sim 0.1, v_c^2 \sim 0.3$$

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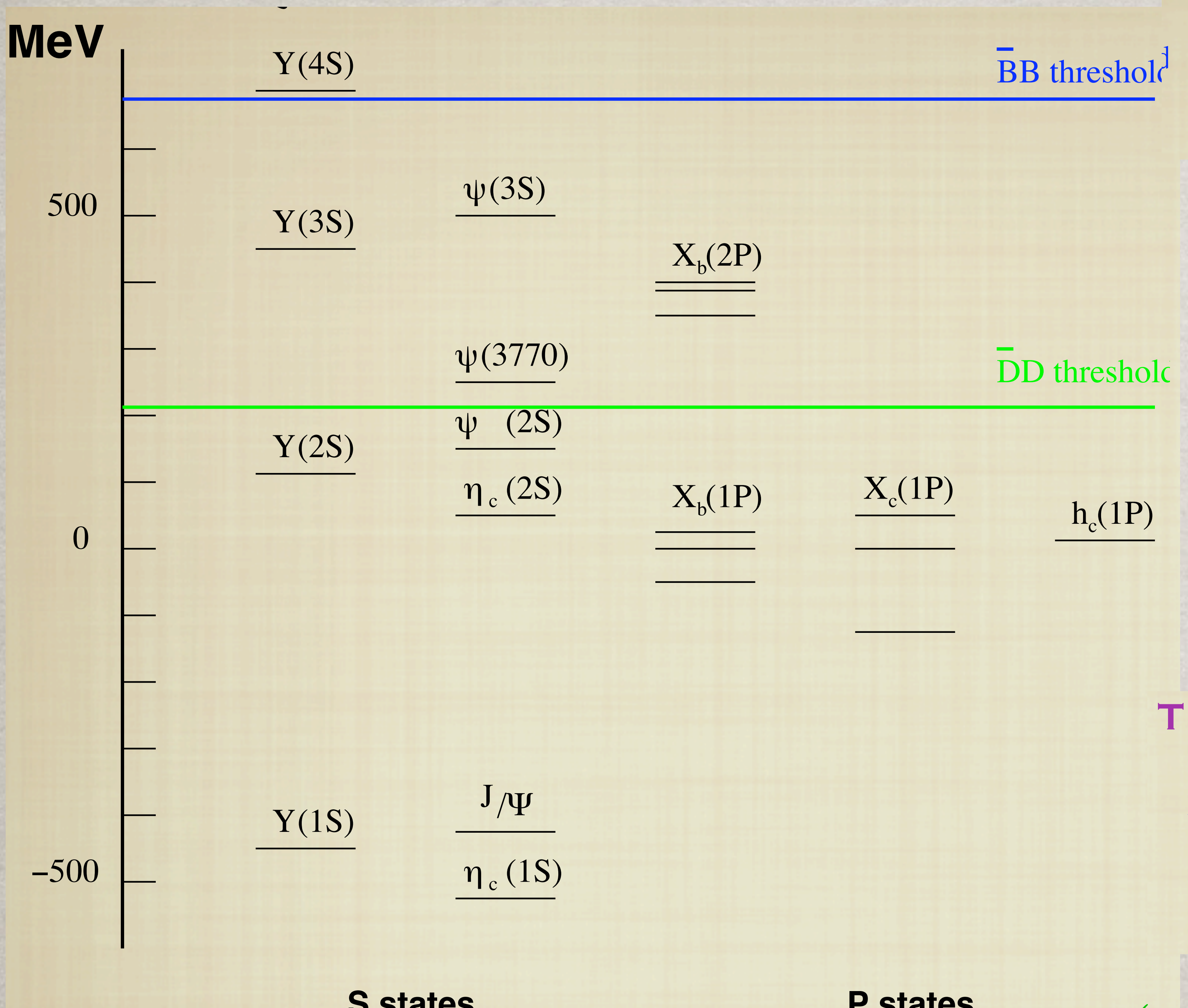

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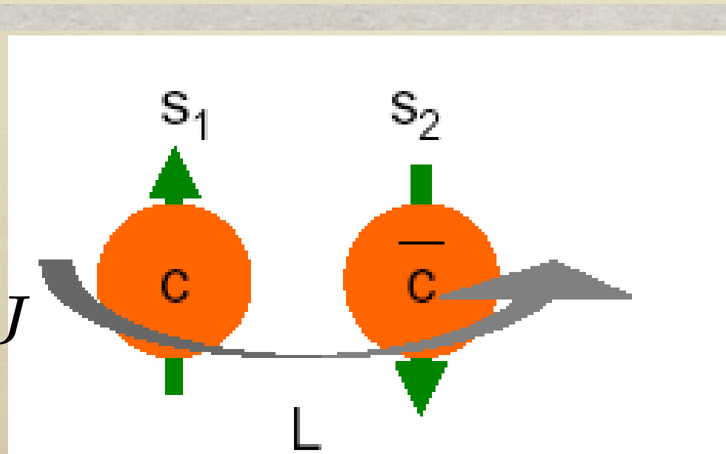

NR BOUND STATES HAVE AT LEAST 3 SCALES

$$m \gg mv \gg mv^2 \qquad v \ll 1$$

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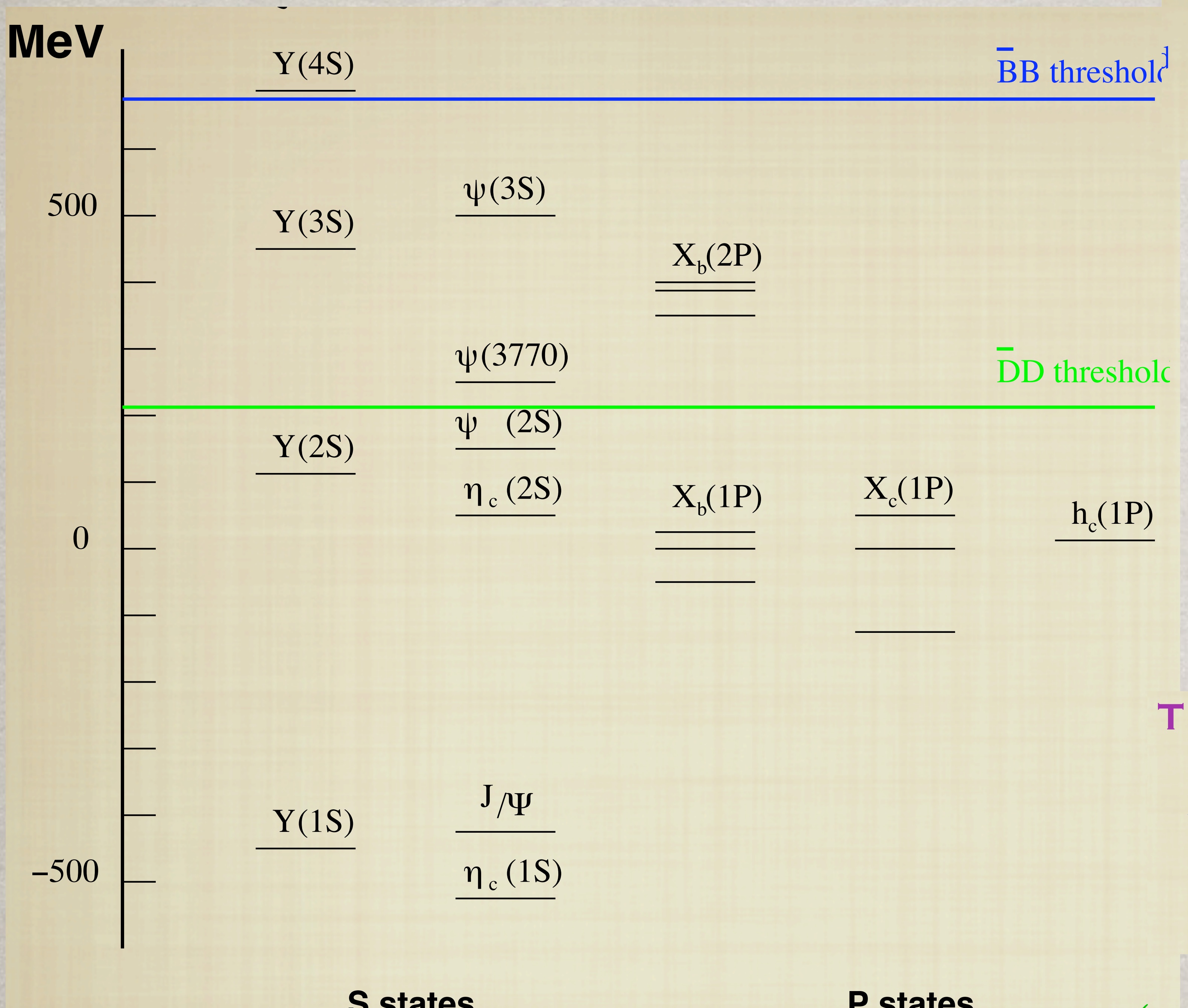

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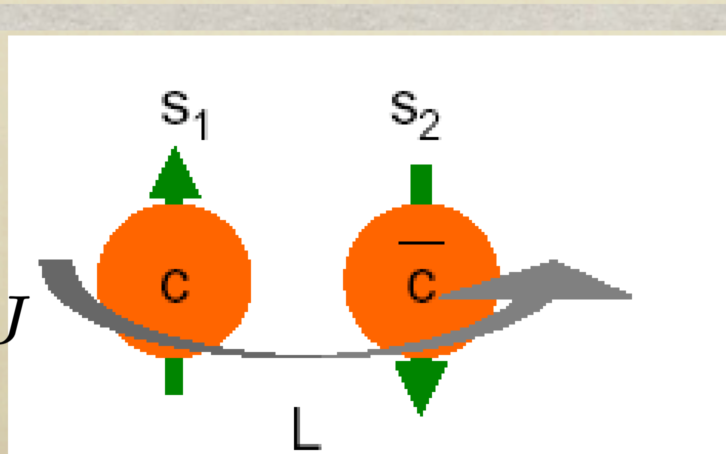

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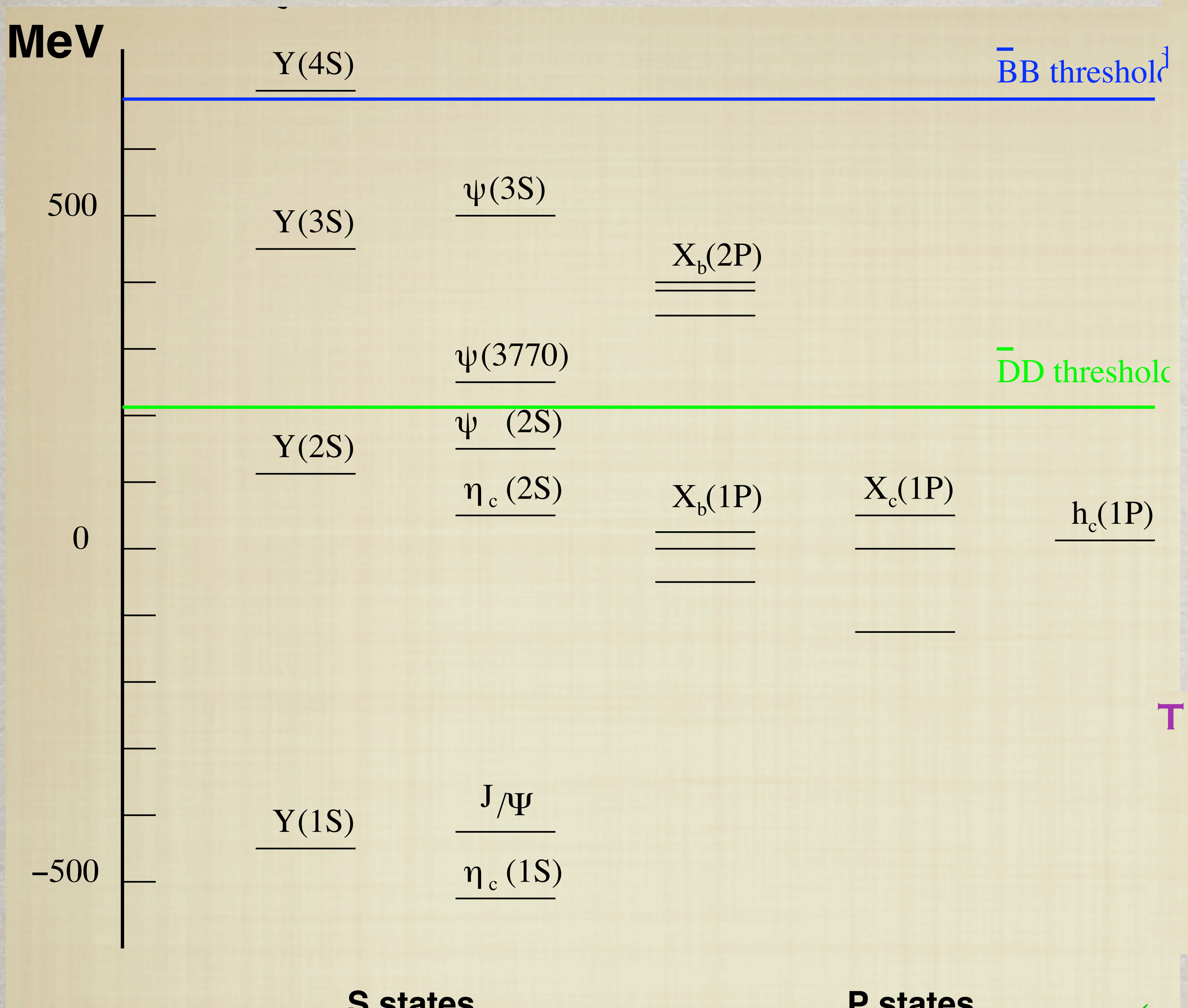

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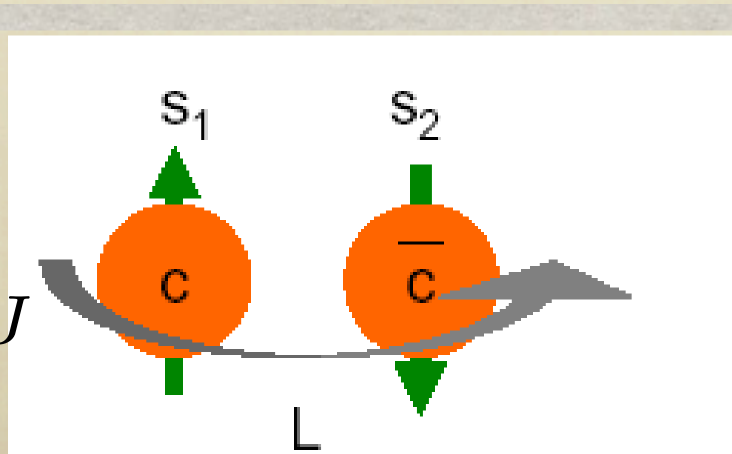

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Quarkonium as a confinement and deconfinement probe

The rich structure of separated energy scales makes QQbar an ideal probe

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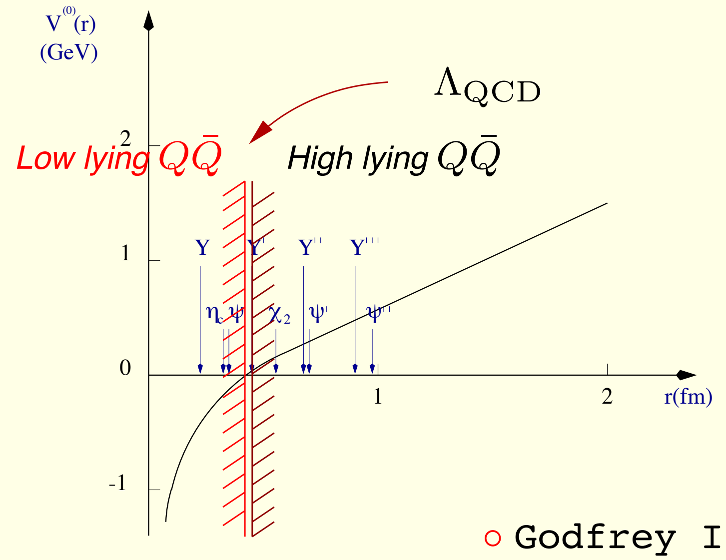
At zero temperature

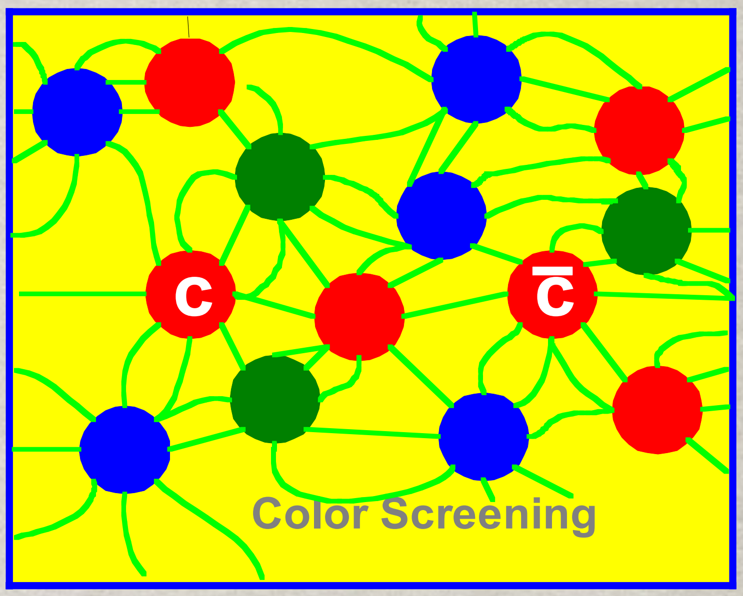
The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.



○ Godfrey Isgur PRD 32(85)189

quarkonia probe the perturbative (high energy) and non perturbative region (low energy) as well as the transition region in dependence of their radius r

At finite T



Debye charge screening

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$$r \sim \frac{1}{m_D} \longrightarrow$$ Bound state dissolves

quarkonia dissociate at different temperature in dependence of their radius: they are a Quark Gluon Plasma thermometer

Quarkonium as an exploration tool of physics of Standard Model and beyond

- Quarkonium can serve for the precise extraction of Standard Model parameters: heavy quark masses and strong coupling constant α_s

- Quarkonium in its exotic manifestations probes the nonstandard characteristics of a nonabelian gauge theory: hybrids, multi quark configurations

- The large m makes Quarkonium an ideal probe of new light particles

BaBar light-Higgs & dark-photon searches

Mode	Mass range (GeV)	BF upper limit (90% CL)
$\Upsilon(2S,3S) \to \gamma A^0, A^0 \to \mu^+\mu^-$	$0.21 < m_A < 9.3$	$(0.3-8.3)\times 10^{-6}$
$\Upsilon(3S) \to \gamma A^0, A^0 \to \tau^+\tau^-$	$4.0 < m_A < 10.1$	$(1.5-16)\times 10^{-5}$
$\Upsilon(2S,3S) \to \gamma A^0, A^0 \to$ hadrons	$0.3 < m_A < 7.0$	$(0.1-8)\times 10^{-5}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to \chi\bar{\chi}$	$m_\chi < 4.5\,\text{GeV}$	$(0.5-24)\times 10^{-5}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to$ invisible	$m_A < 9.2\,\text{GeV}$	$(1.9-37)\times 10^{-6}$
$\Upsilon(3S) \to \gamma A^0, A^0 \to$ invisible	$m_A < 9.2\,\text{GeV}$	$(0.7-31)\times 10^{-6}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to g\bar{g}$	$m_A < 9.0\,\text{GeV}$	$10^{-6}-10^{-2}$
$\Upsilon(1S) \to \gamma A^0, A^0 \to s\bar{s}$	$m_A < 9.0\,\text{GeV}$	$10^{-5}-10^{-3}$

invisible decays of Y(1S) at Belle

Quarkonium today is
a golden system to study strong interactions

many experimental data and opportunities

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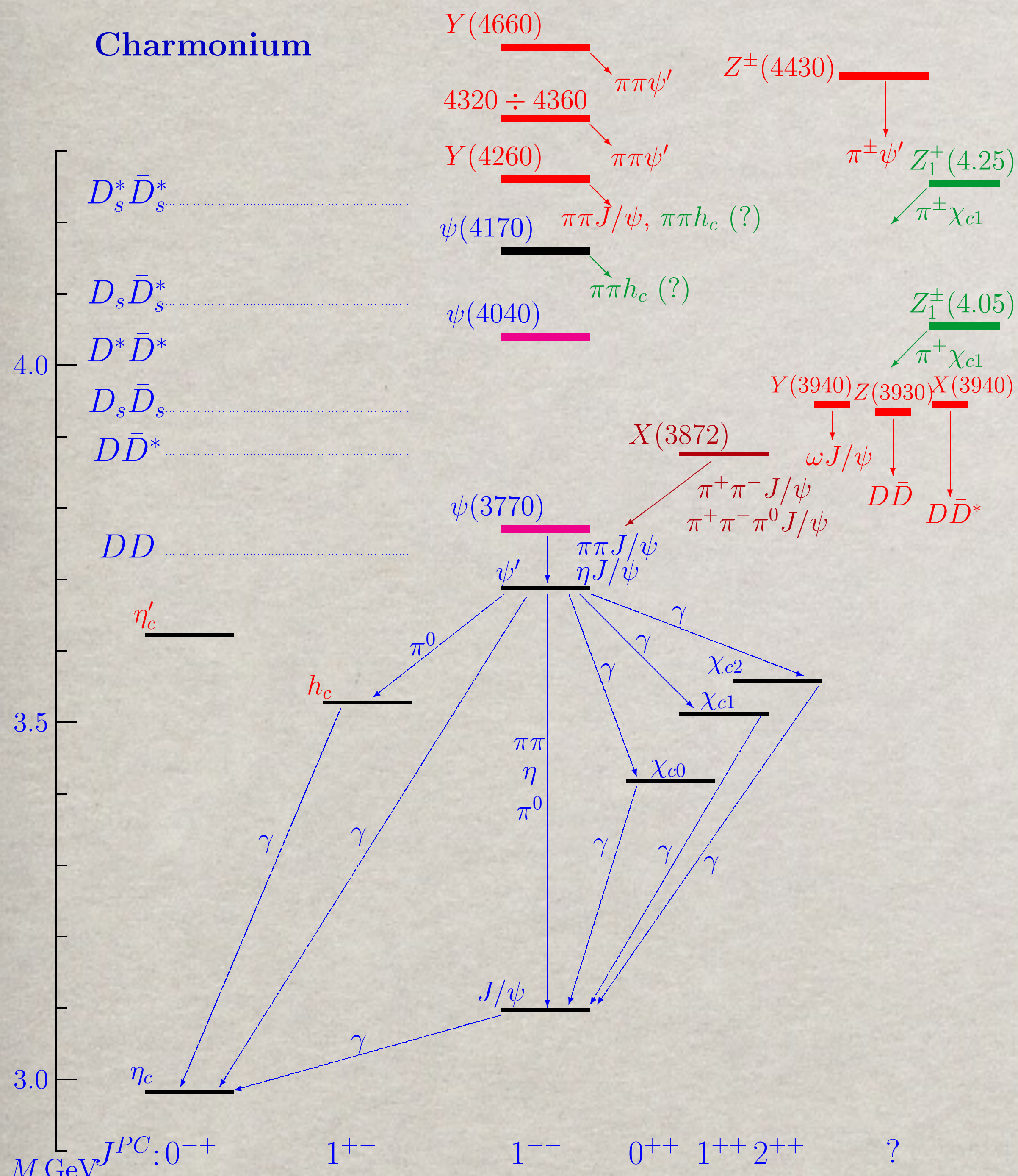
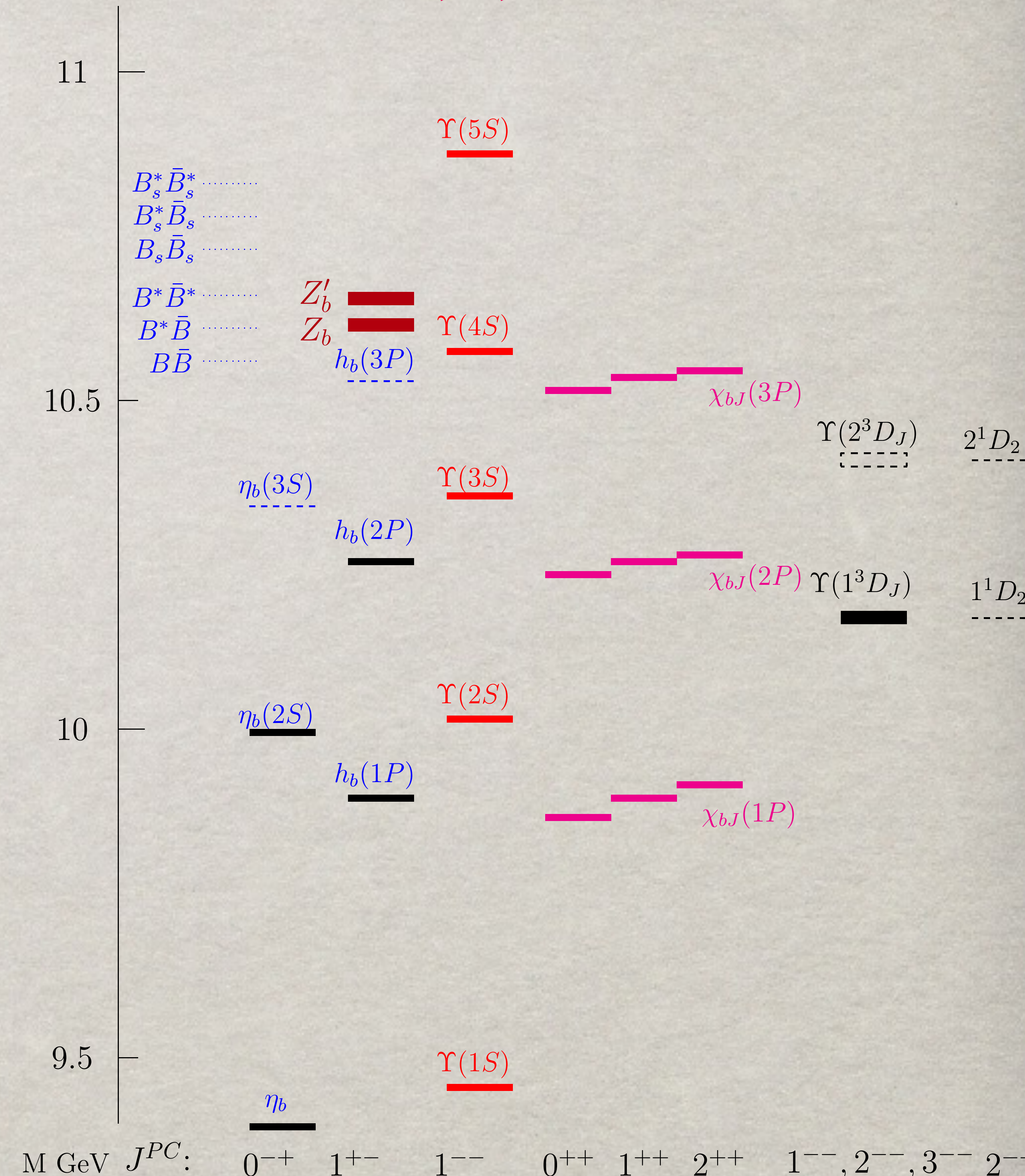
new theoretical tools:
Effective Field Theories (EFTs) of QCD
and progress in lattice QCD

Experimental New Revolution: X, Y, Z states





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THE COLLIDER DETECTOR AT FERMILAB

QCD theory of Quarkonium: a very hard problem

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Close to the bound state $\alpha_s \sim v$

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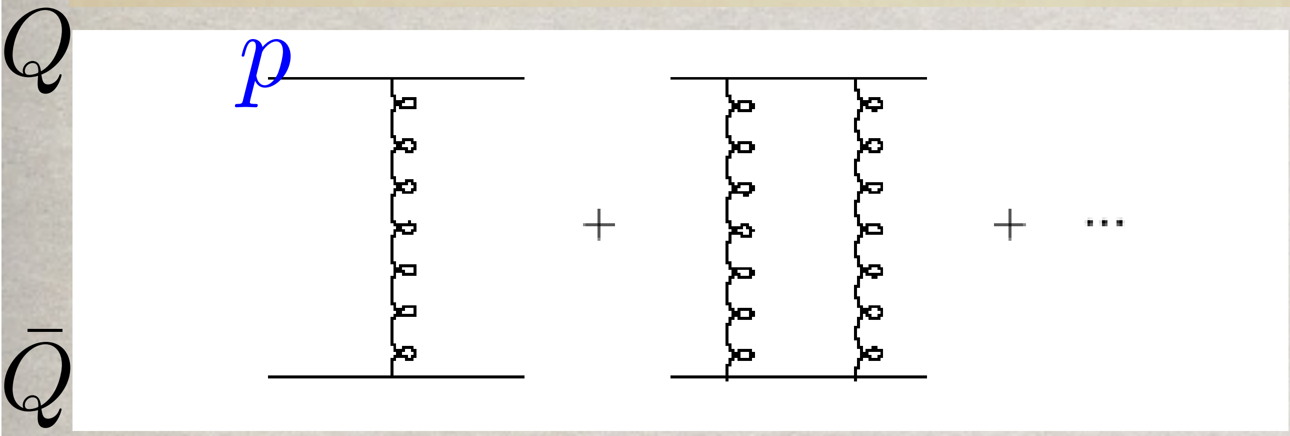

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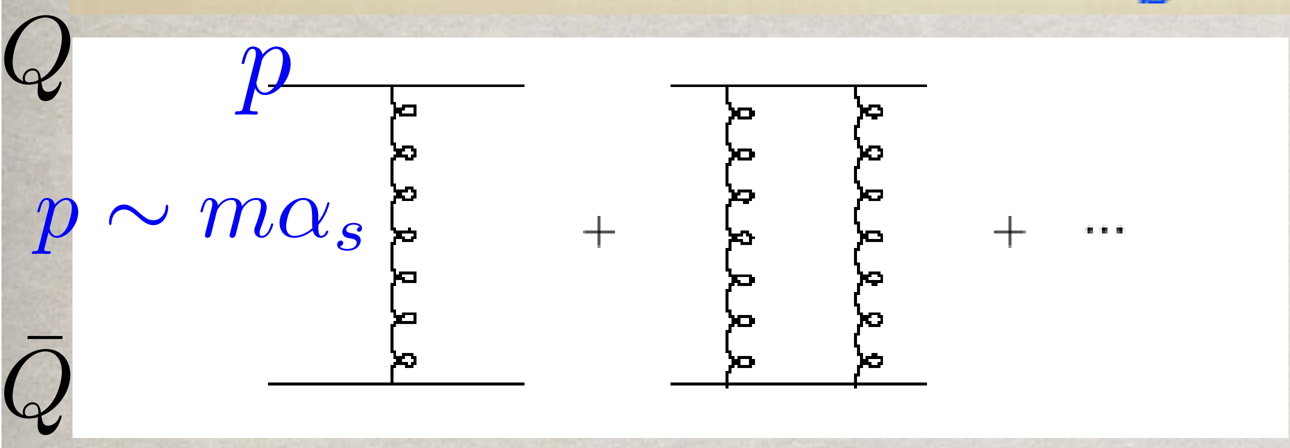

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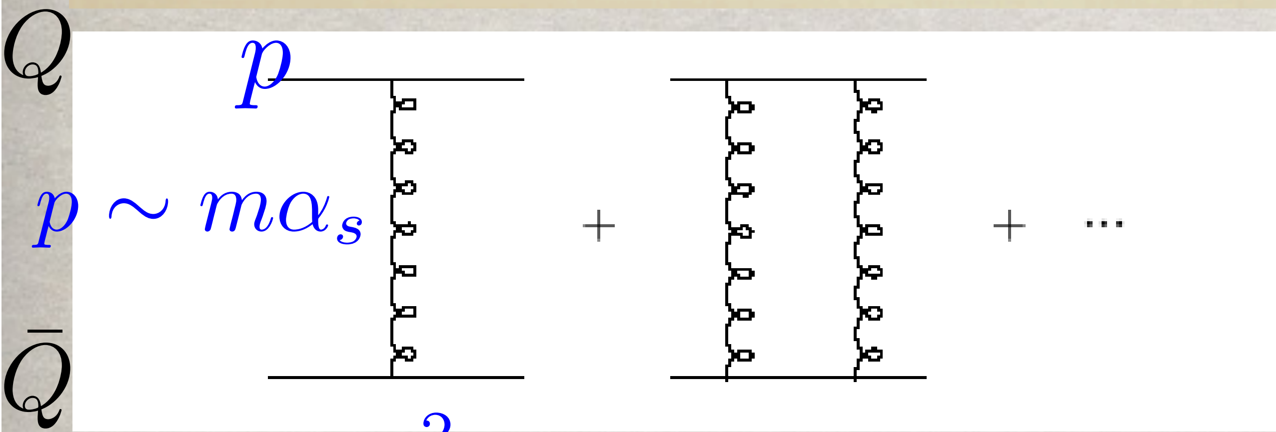

$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

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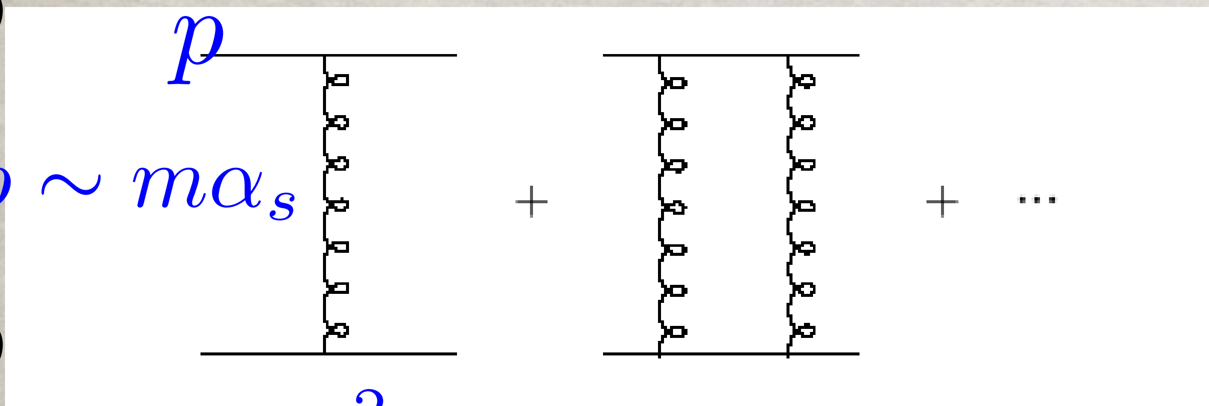
Q p

$p \sim m\alpha_s$



$\bar{Q}$

$$\frac{g^2}{p^2}\left(1 + \frac{m\alpha_s}{p}\right)$$

$$\sim \frac{1}{E - (\frac{p^2}{m} + V)}$$

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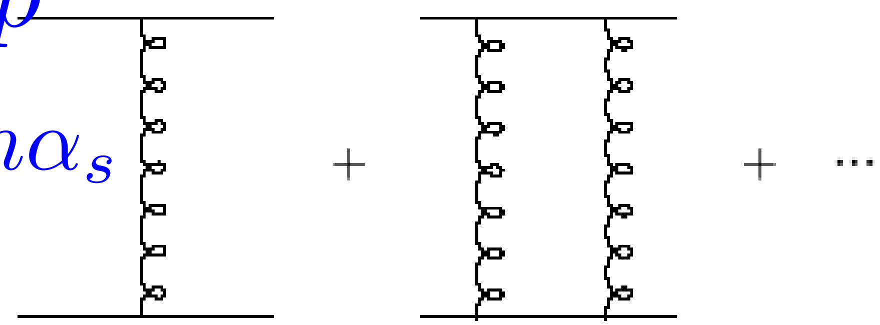

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- From $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$.

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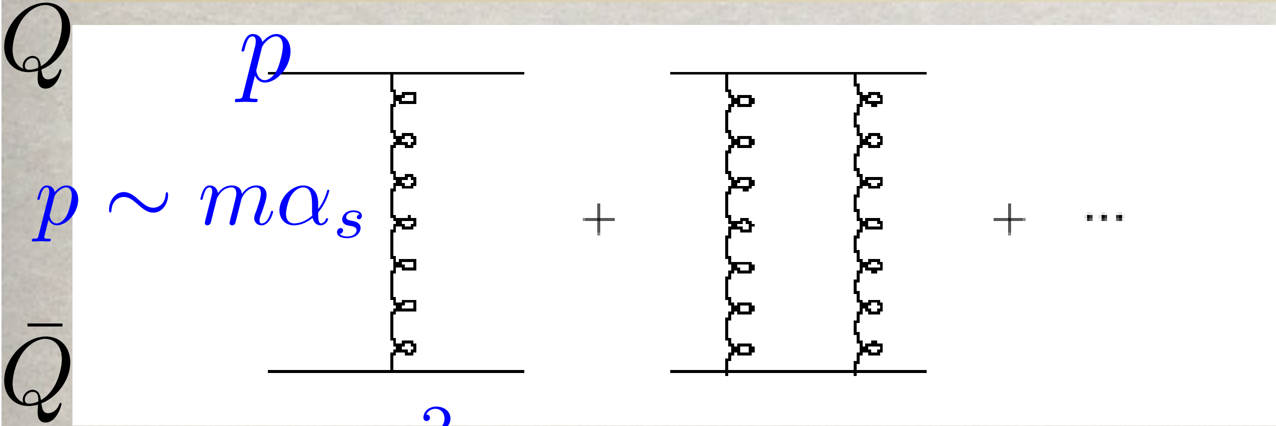

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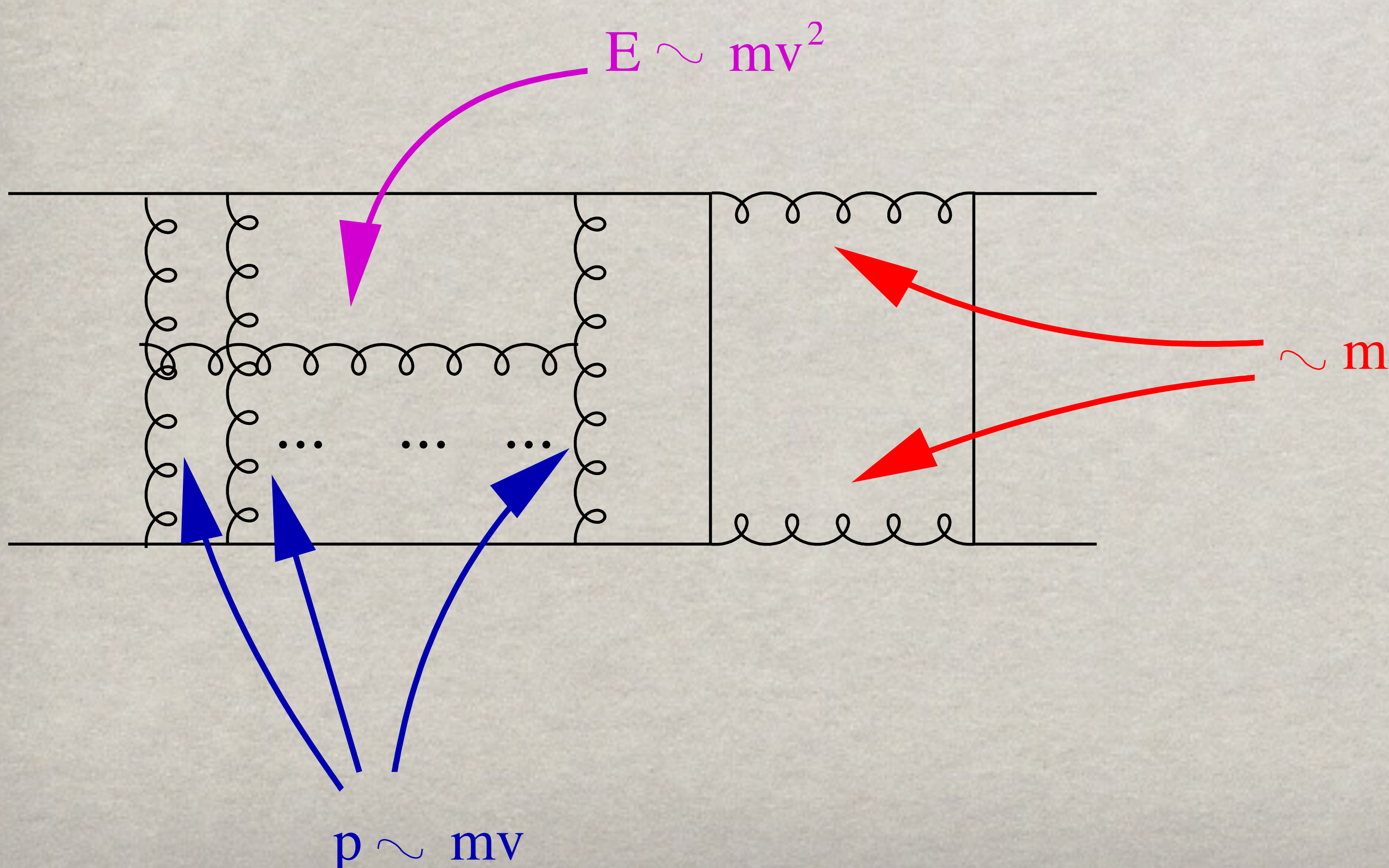

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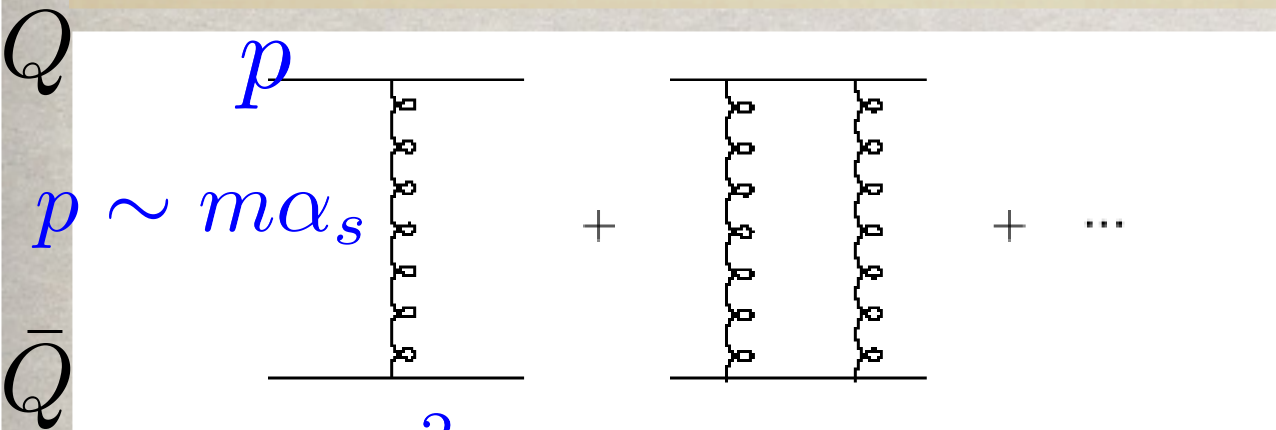

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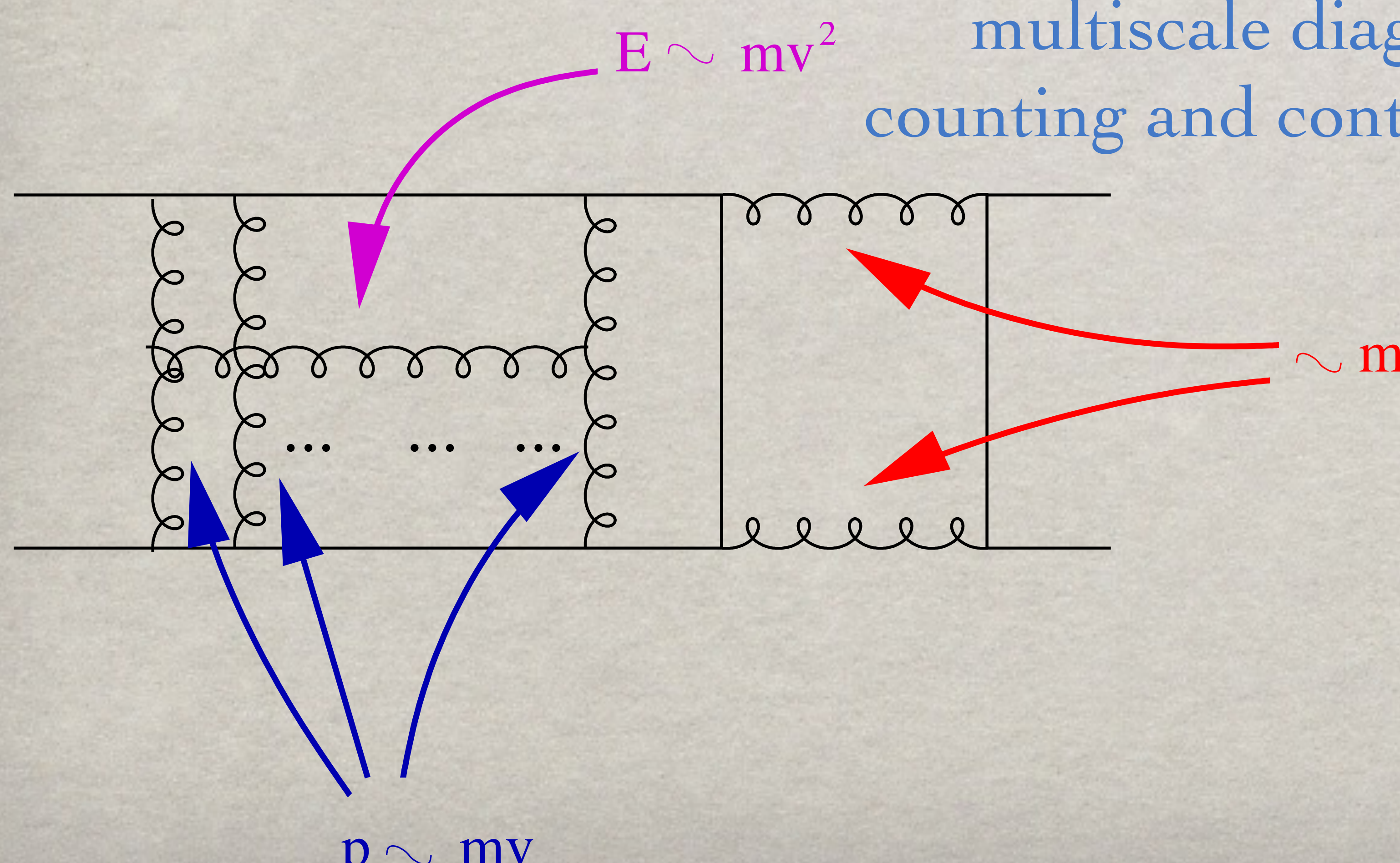
multiscale diagrams have a complicate power counting and contribute to all orders in the coupling



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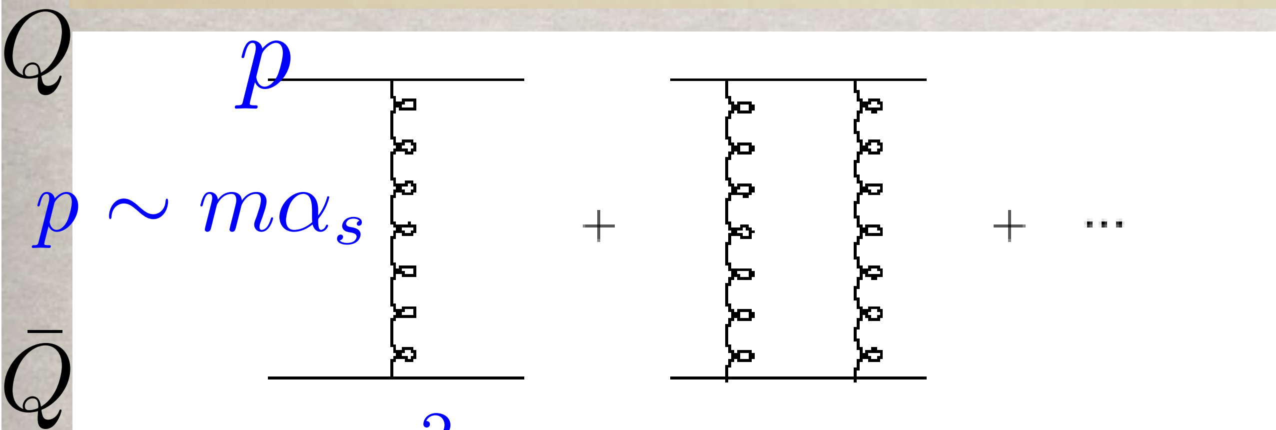

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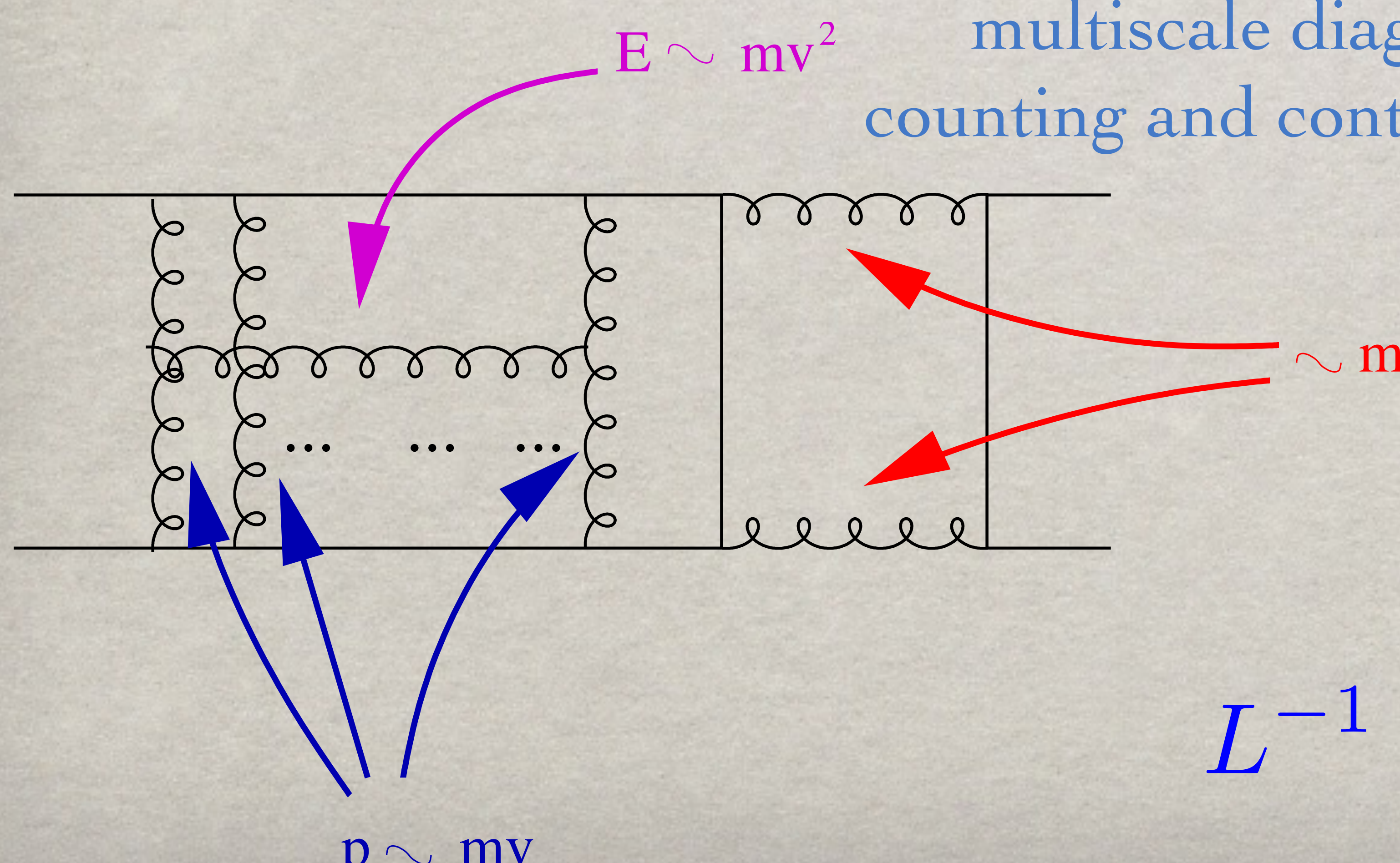

Difficult also for the lattice!

$$L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}$$

Effective field theories factories scales at the Lagrangian level

Disentangling the bound-state scales at the Lagrangian level has advantages.

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(I) It facilitates higher-order perturbative calculations.

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E.g. it took twenty-five years to go from the calculation of the $m\alpha^5$ correction in the hyperfine splitting of the positronium ground state to the $m\alpha^6 \ln \alpha$ term!

- Karplus Klein PR 87(52)848, Caswell Lepage PRA (20)(79)36
  Bodwin Yennie PR 43(78)267

Relevant for
- atomic physics: Hydrogen atom (e.g. proton radius), positronium (e.g. width, hfs), ...
- $t\bar{t}$ threshold production, ...
- ...

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Disentangling the bound-state scales at the Lagrangian level has advantages.

(I) It facilitates higher-order perturbative calculations.

E.g. it took twenty-five years to go from the calculation of the $m\alpha^5$ correction in the hyperfine splitting of the positronium ground state to the $m\alpha^6 \ln \alpha$ term!

- Karplus Klein PR 87(52)848, Caswell Lepage PRA (20)(79)36 Bodwin Yennie PR 43(78)267

Relevant for
- atomic physics: Hydrogen atom (e.g. proton radius), positronium (e.g. width, hfs), ...
- $t\bar{t}$ threshold production, ...
- ...

(II) In QCD, it factorizes automatically high-energy (perturbative) contributions from low-energy (non-perturbative, thermal, ...) ones.

Relevant for
- pionium and precision chiral dynamics, ...
- nucleon-nucleon systems, ...
- quarkonia and new quarkonium states
- confinement and lattice calculations, ...
- quarkonium in heavy ion collisions: factorization of thermal contributions.

(III) İt makes apparent the zero order problem which is a Schoedinger eq.

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More conceptually: it provides a field theoretical foundation of the Schrödinger equation:

$$\mathcal{L}_{\mathrm{EFT}} = \phi^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V \right) \phi + \Delta\mathcal{L}$$

The Lagrangian $\mathcal{L}_{\mathrm{EFT}}$, which factorizes the dynamics of the two-particle field ϕ, from the low-energy dynamics encoded in $\Delta\mathcal{L}$ defines an effective field theory.

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The Lagrangian $\mathcal{L}_{\mathrm{EFT}}$, which factorizes the dynamics of the two-particle field ϕ, from the low-energy dynamics encoded in $\Delta\mathcal{L}$ defines an effective field theory.

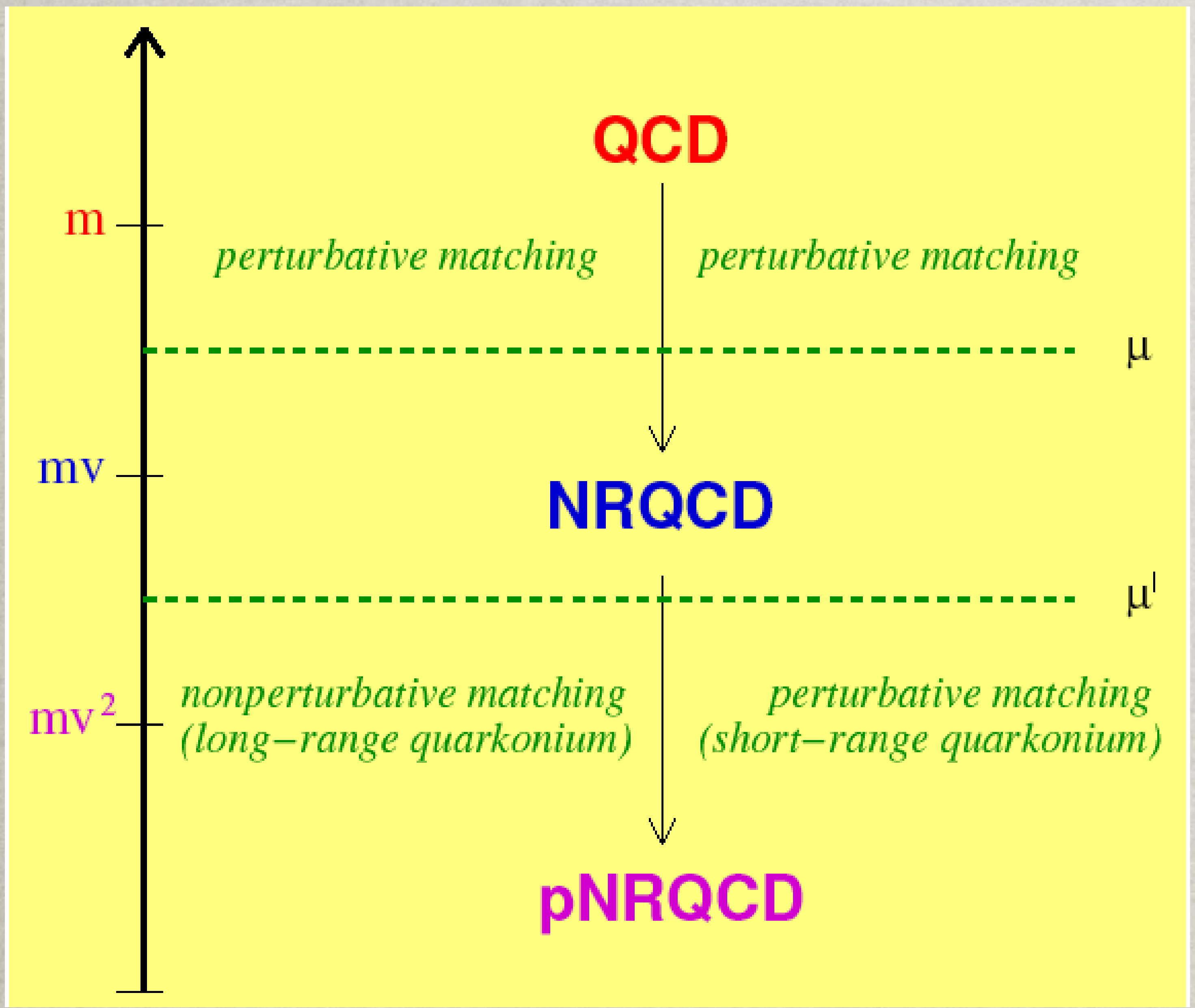
—the potentials come directly from QFT—> everything finite in perturbation theory

—the non-potentials corrections come directly from QFT

—Poincare' invariance is intact at QM level—> exact relations among potentials

Quarkonium with NR EFT

Color degrees of freedom
3X3=1+8
singlet and octet QQbar



Hard

Soft
(relative
momentum)

Ultrasoft
(binding energy)

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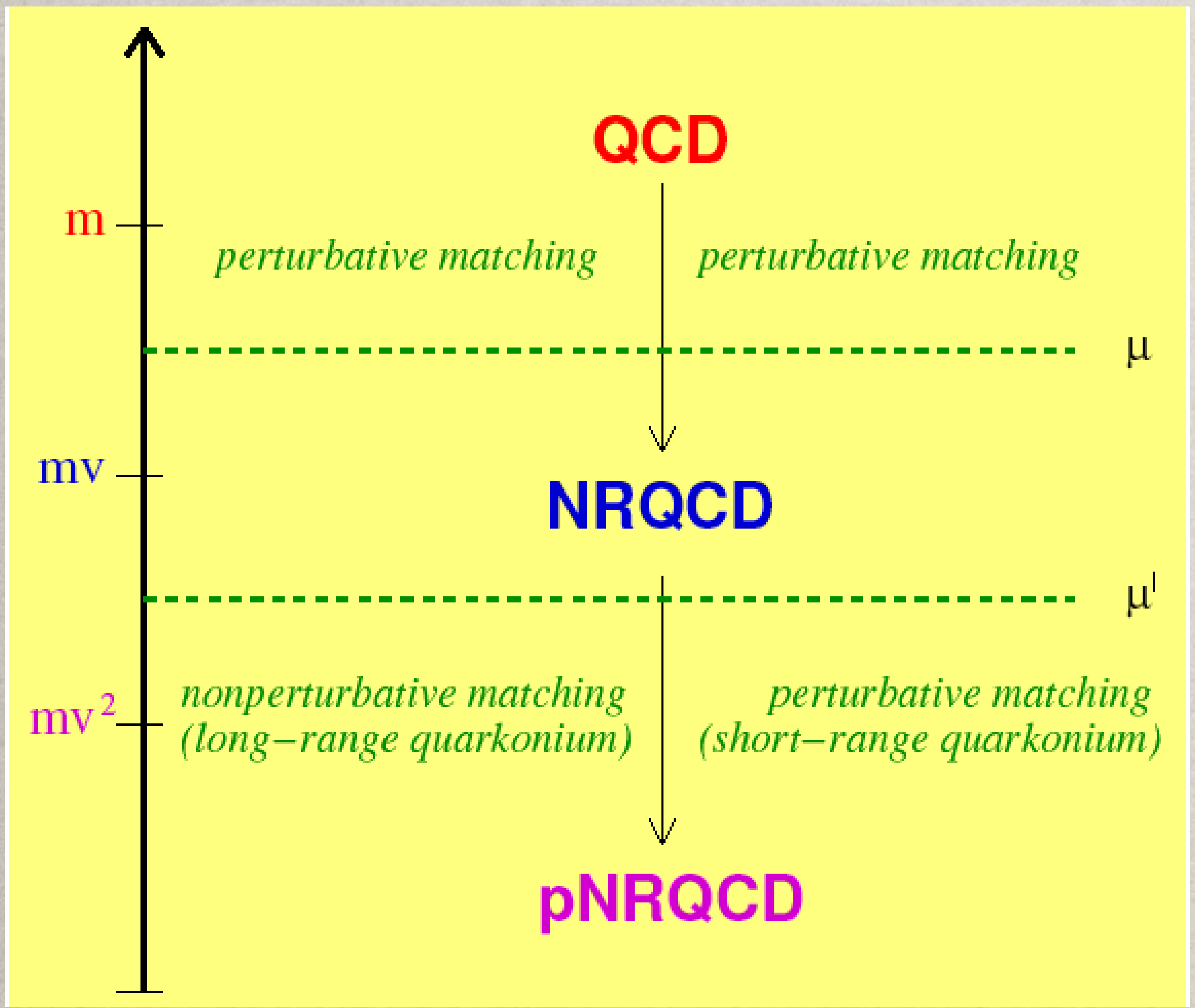

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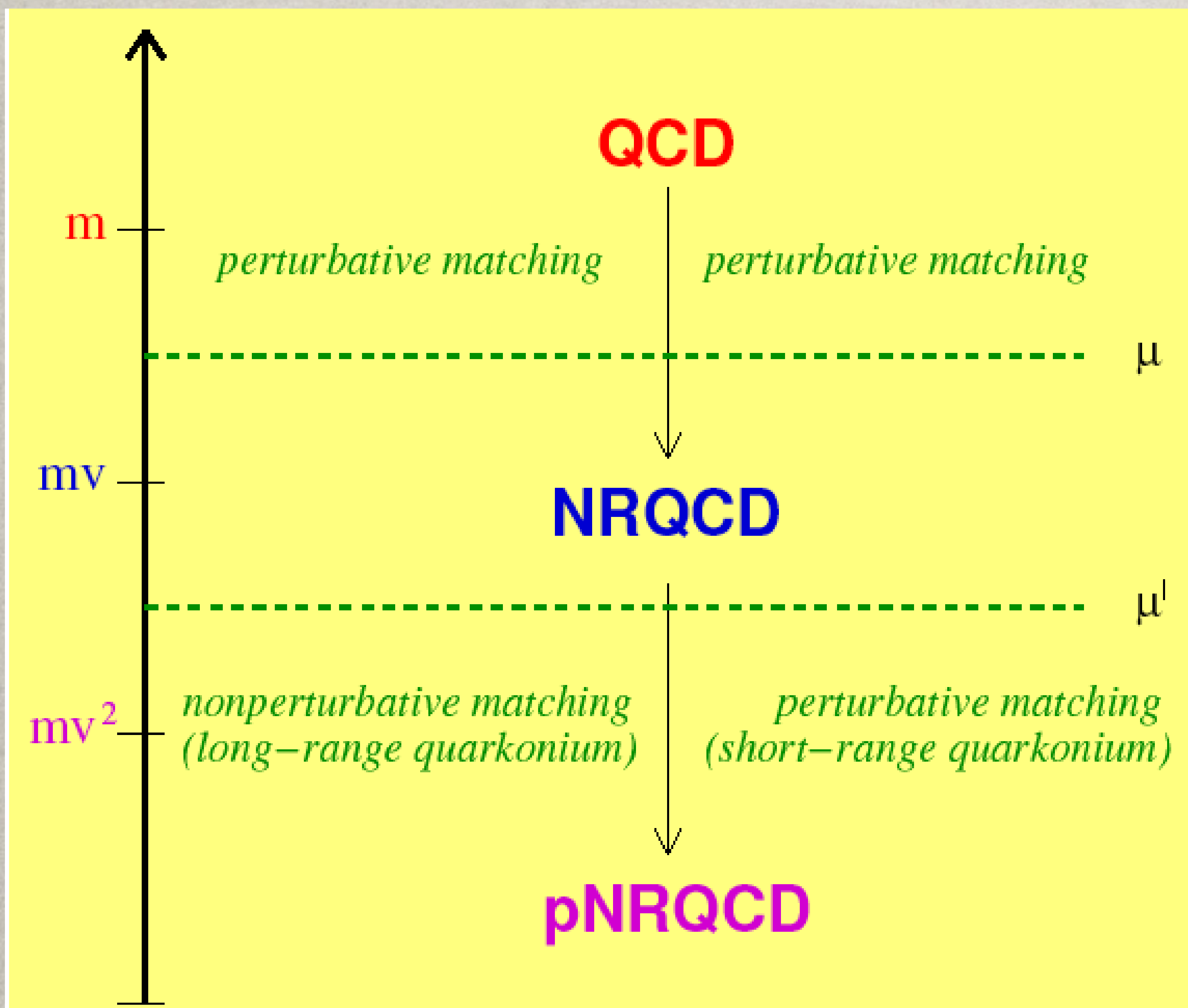

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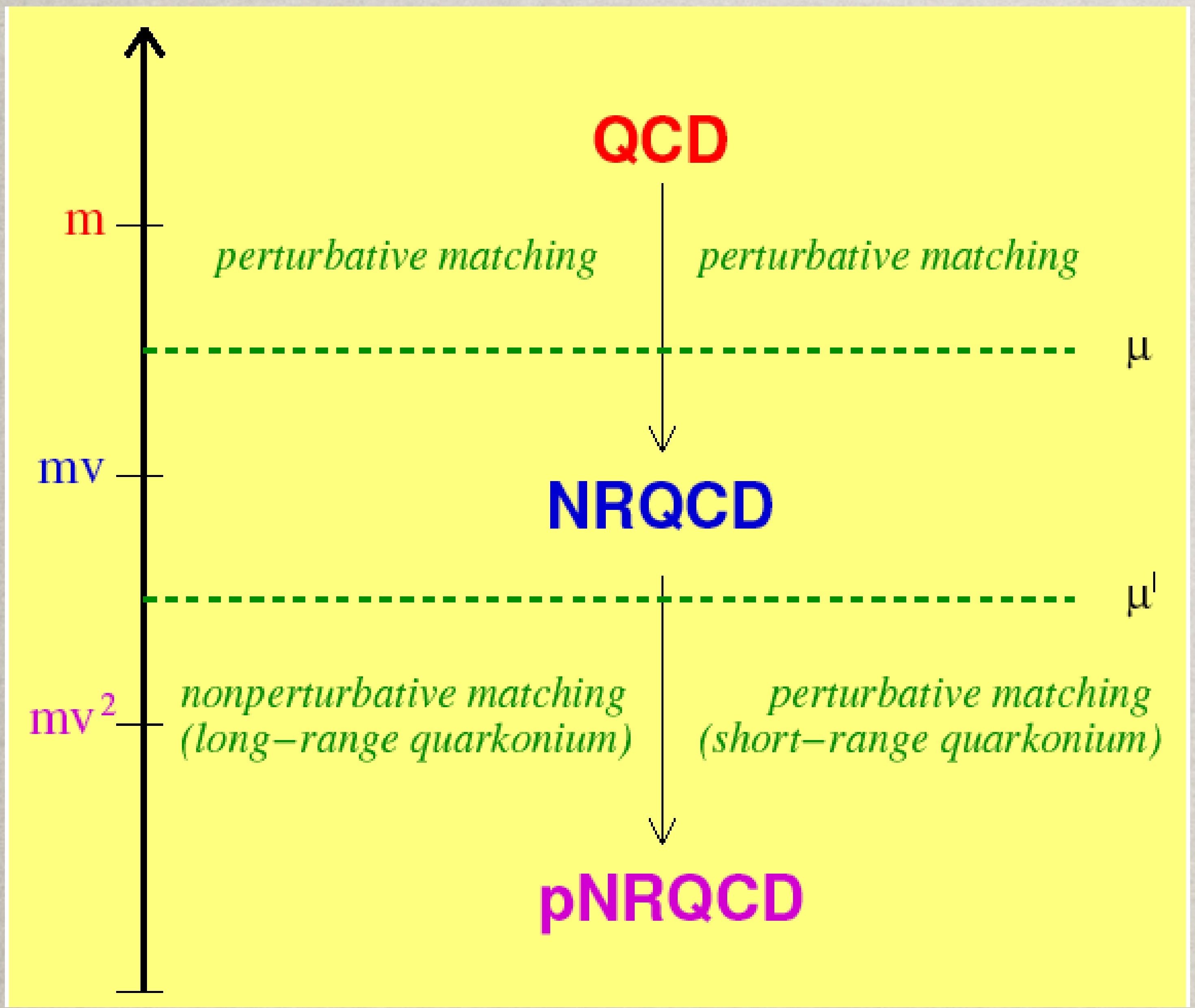

$$\frac{E_\lambda}{E_\Lambda} = \frac{mv}{m}$$

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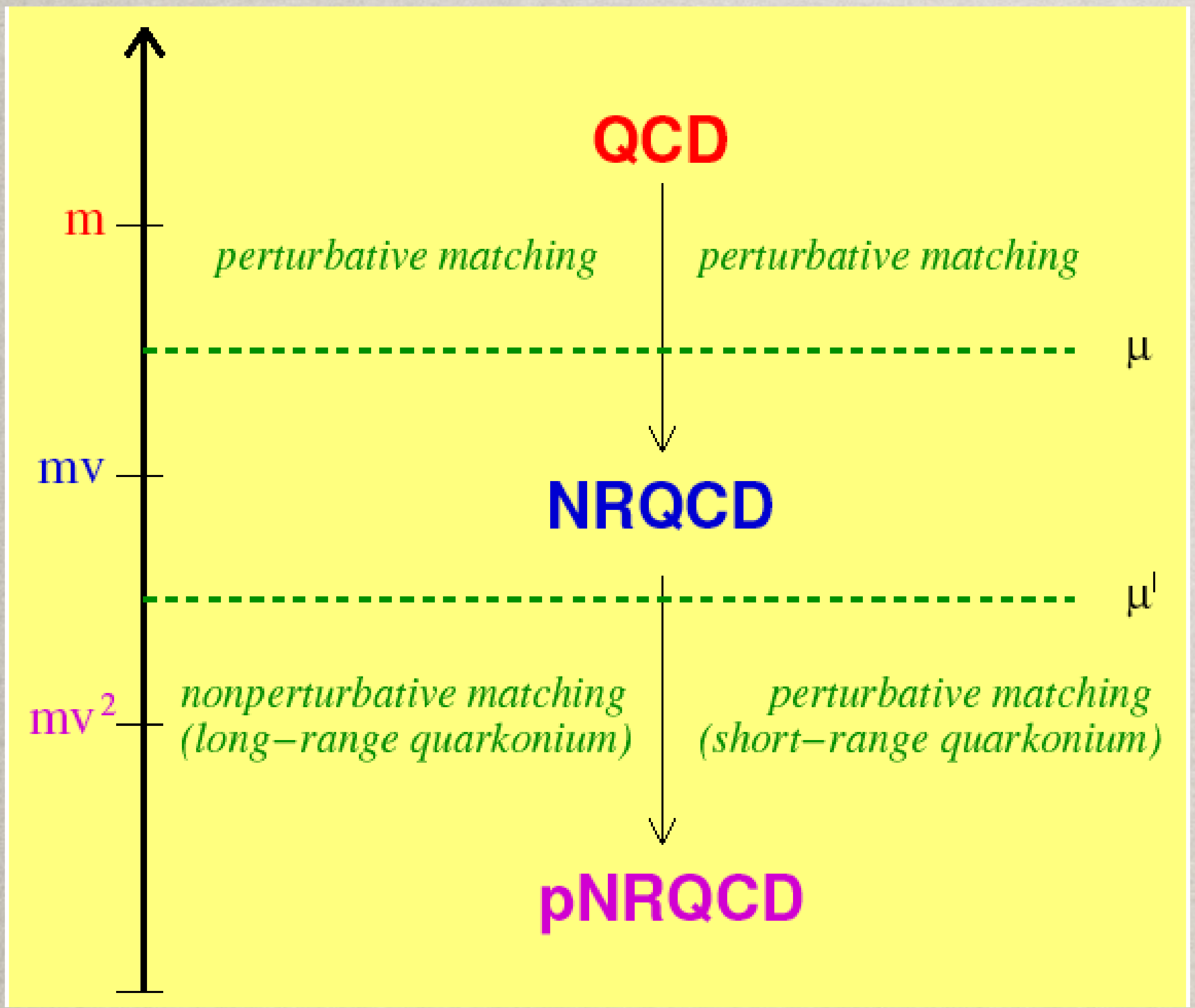

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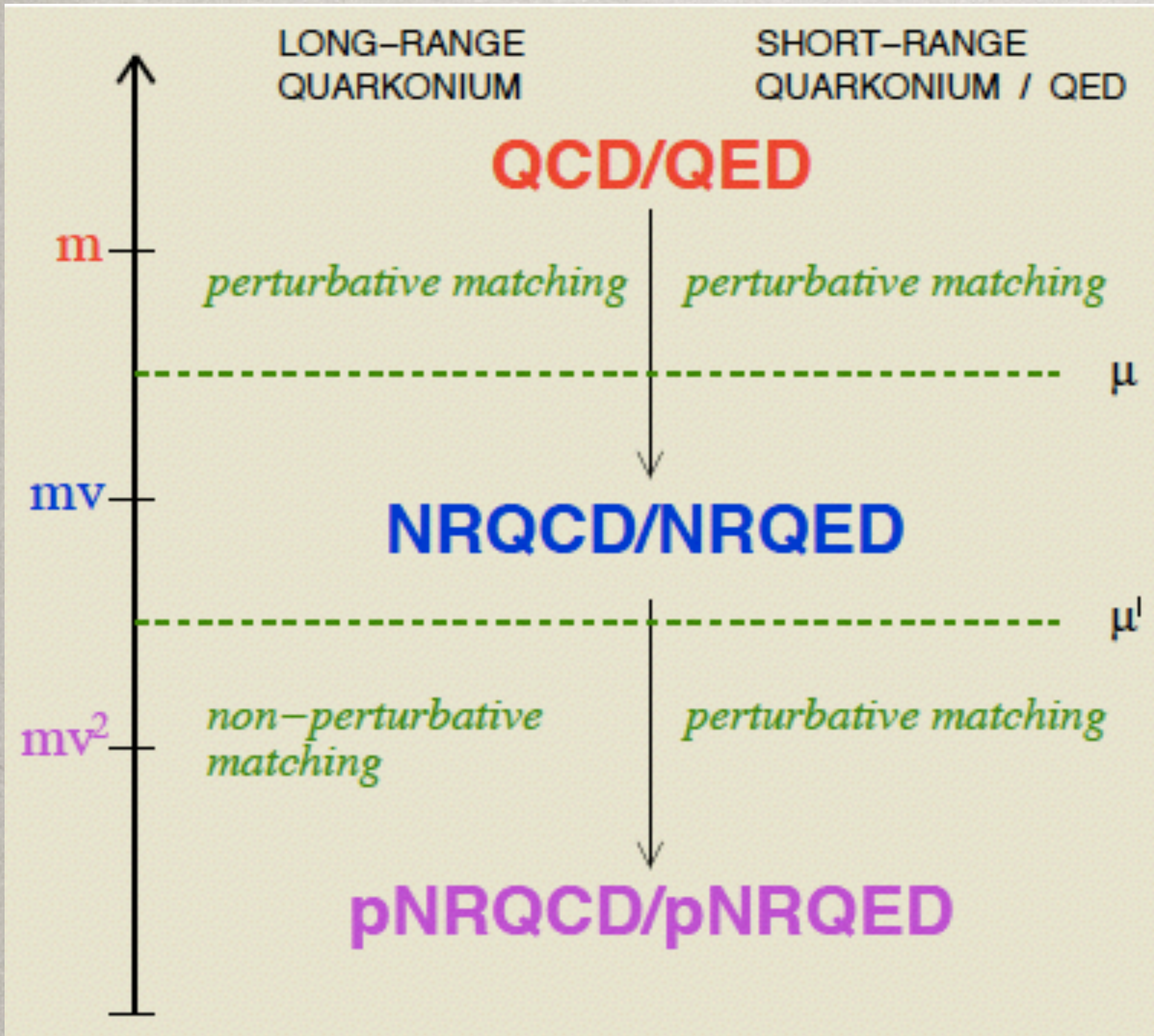

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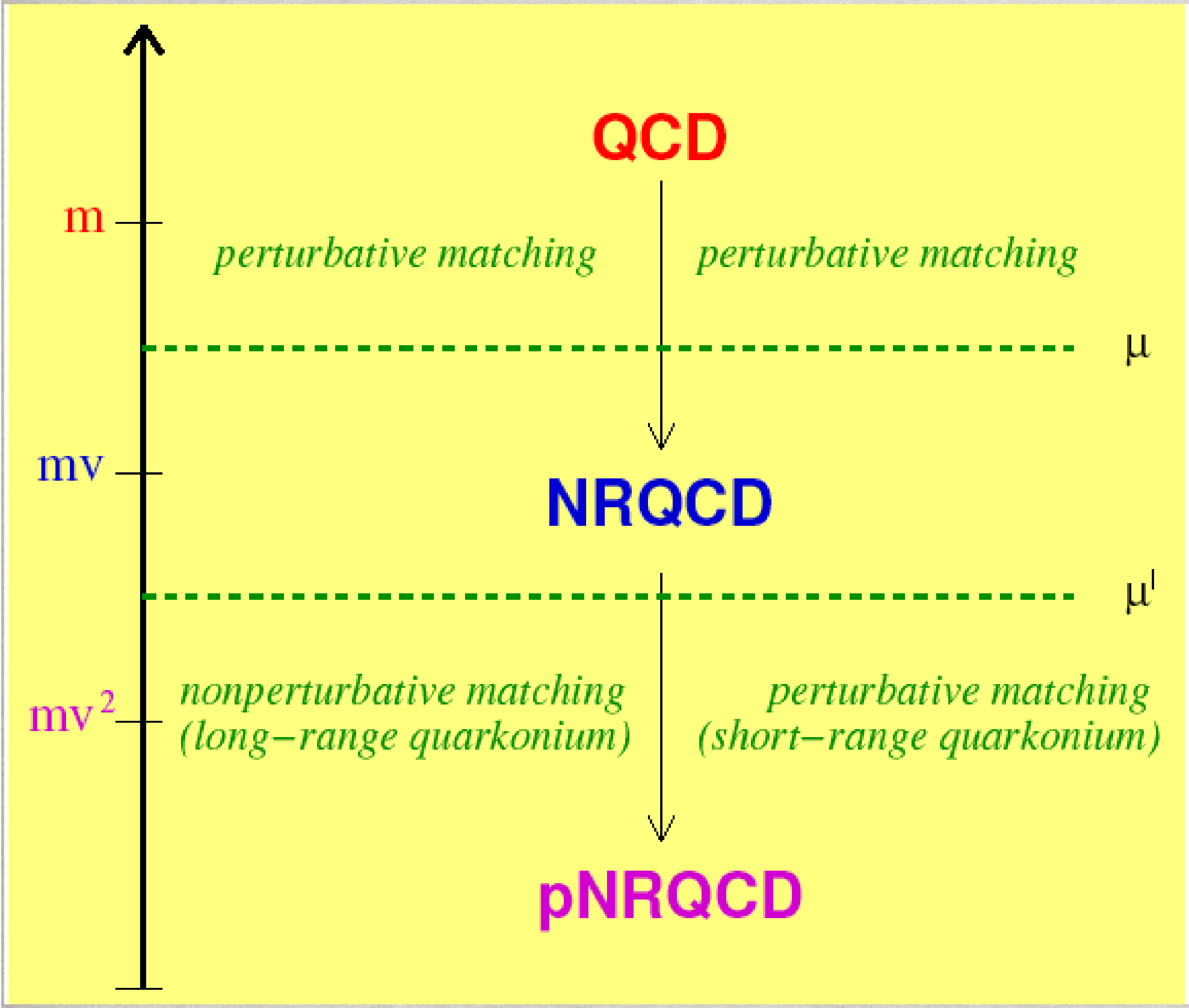
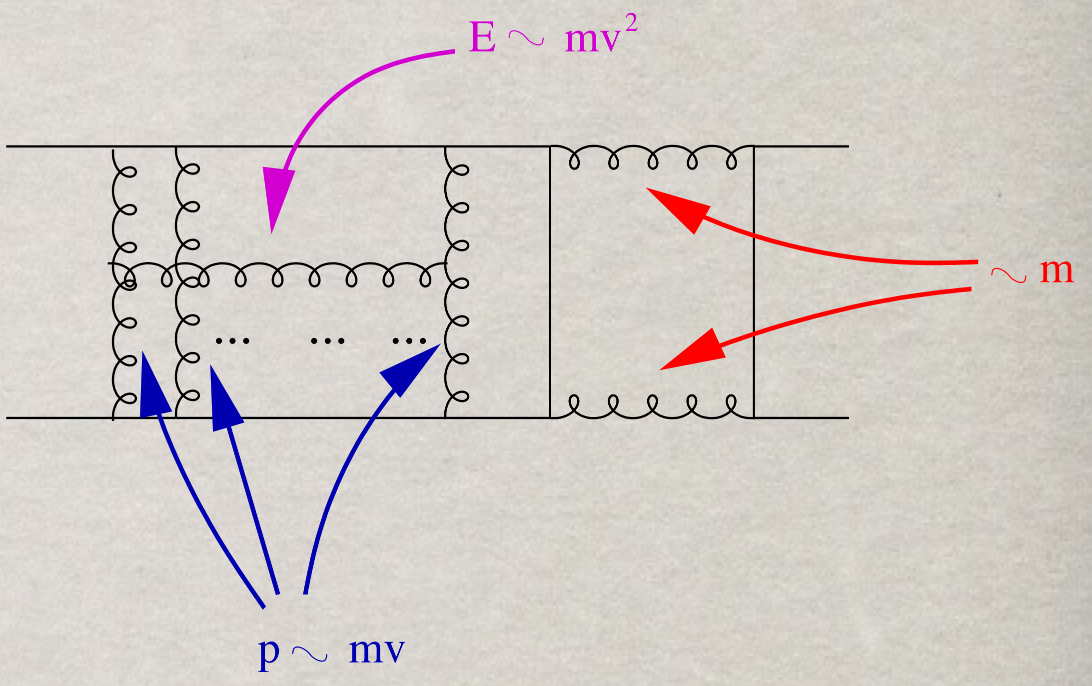
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)





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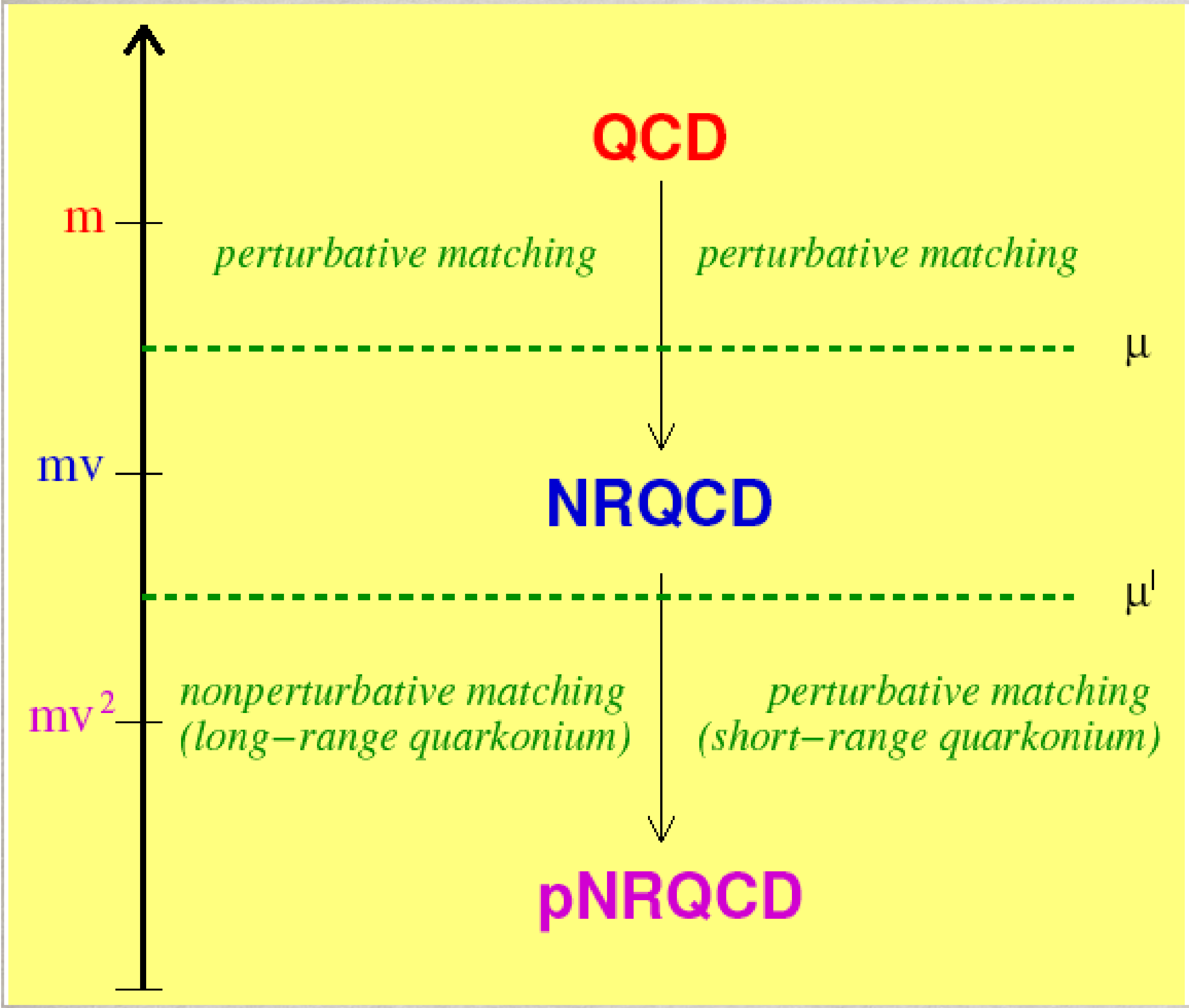

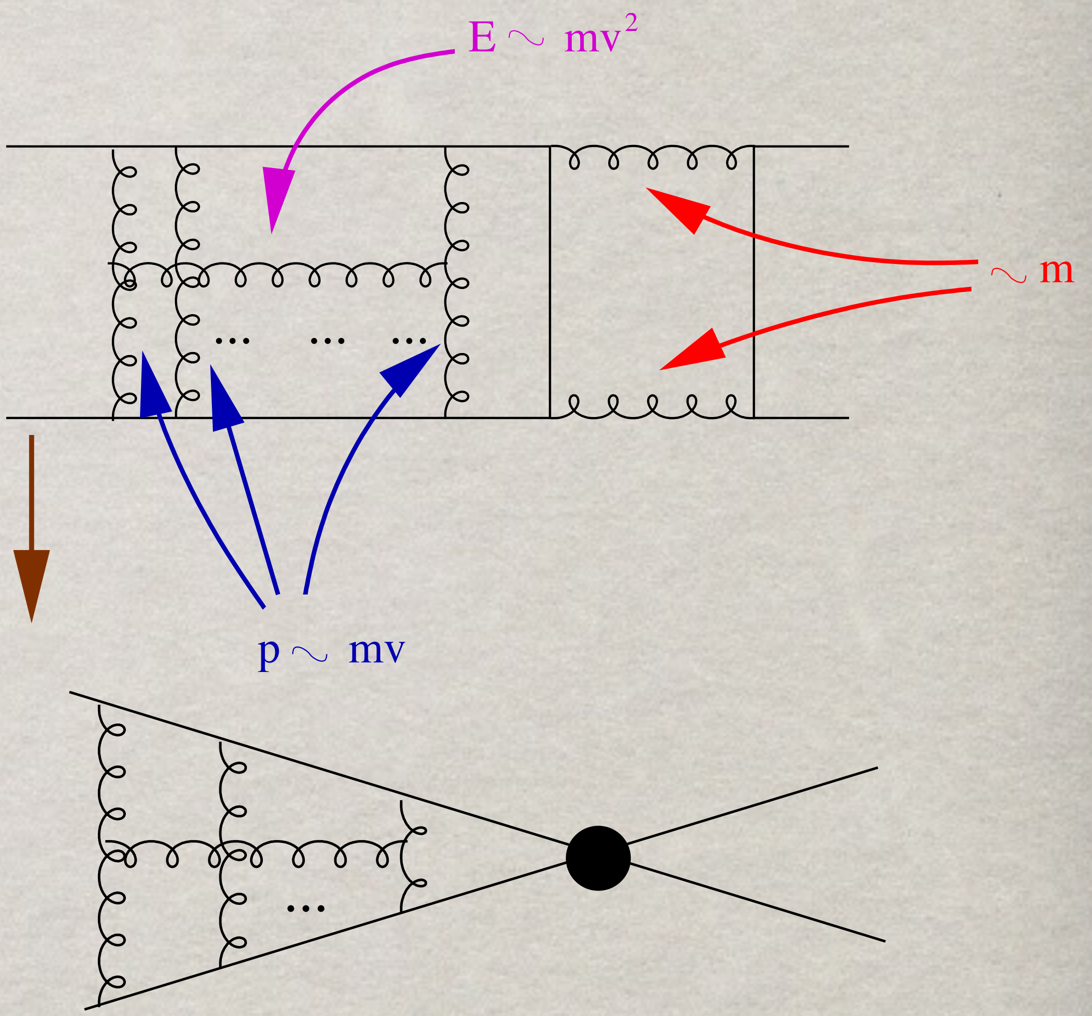

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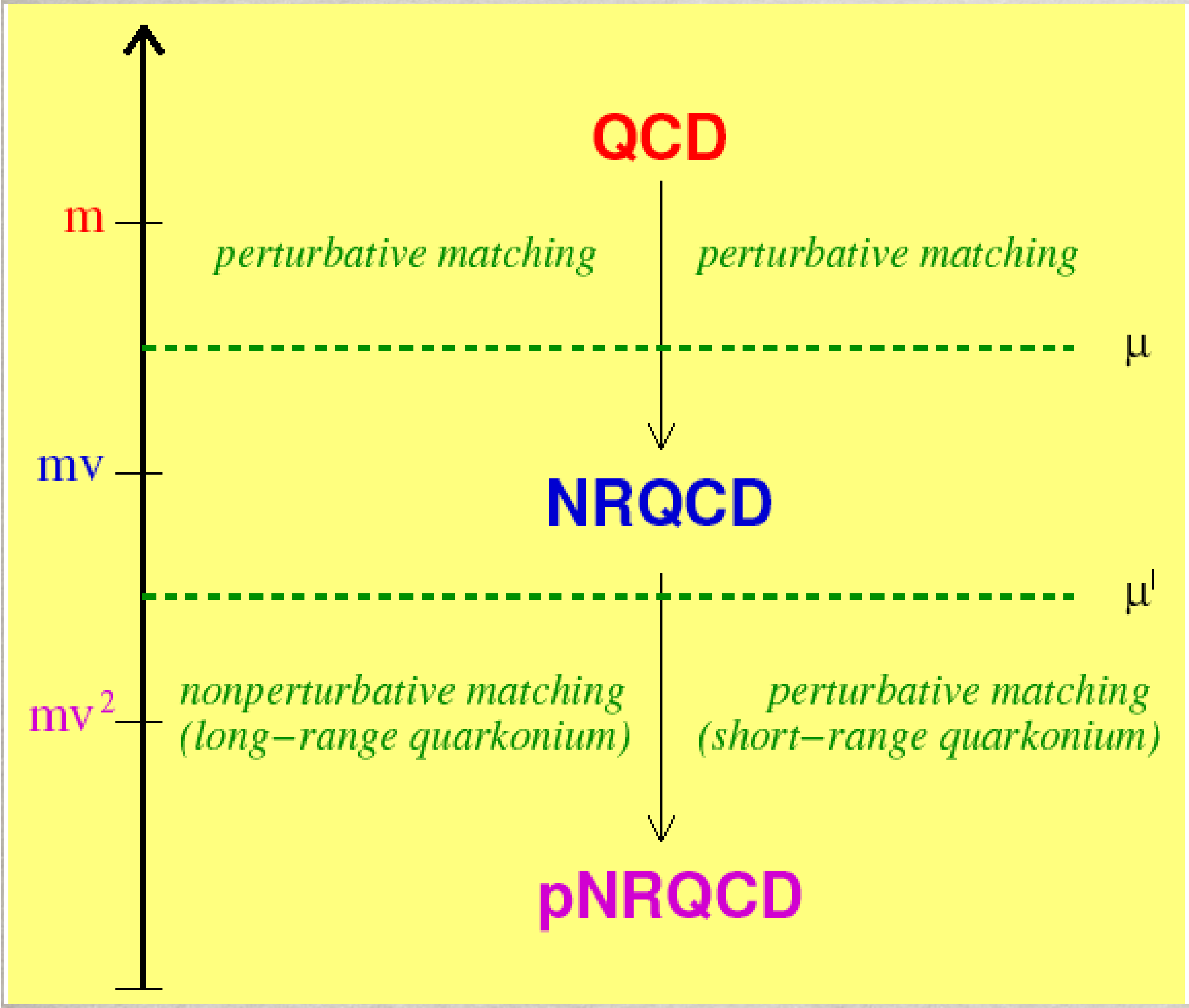
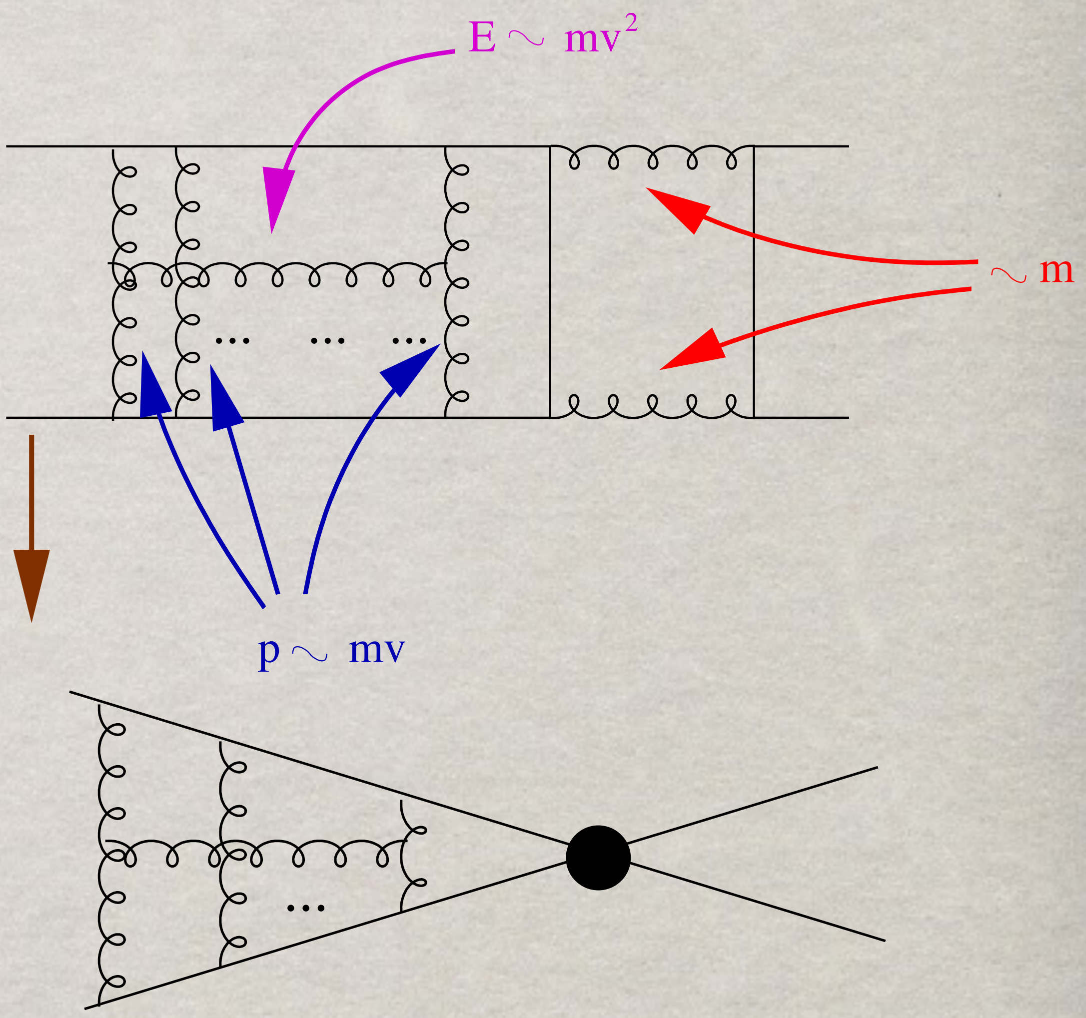
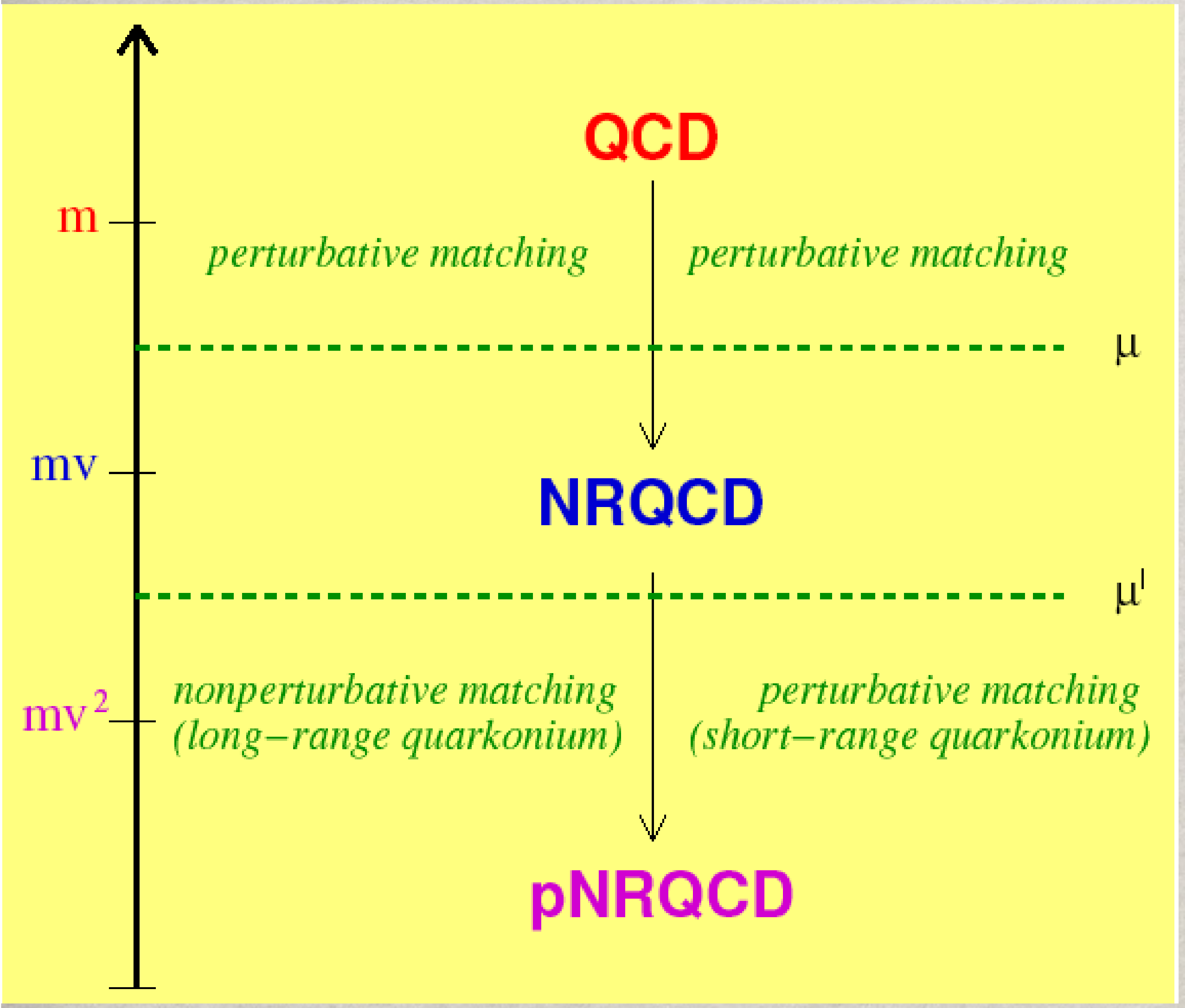
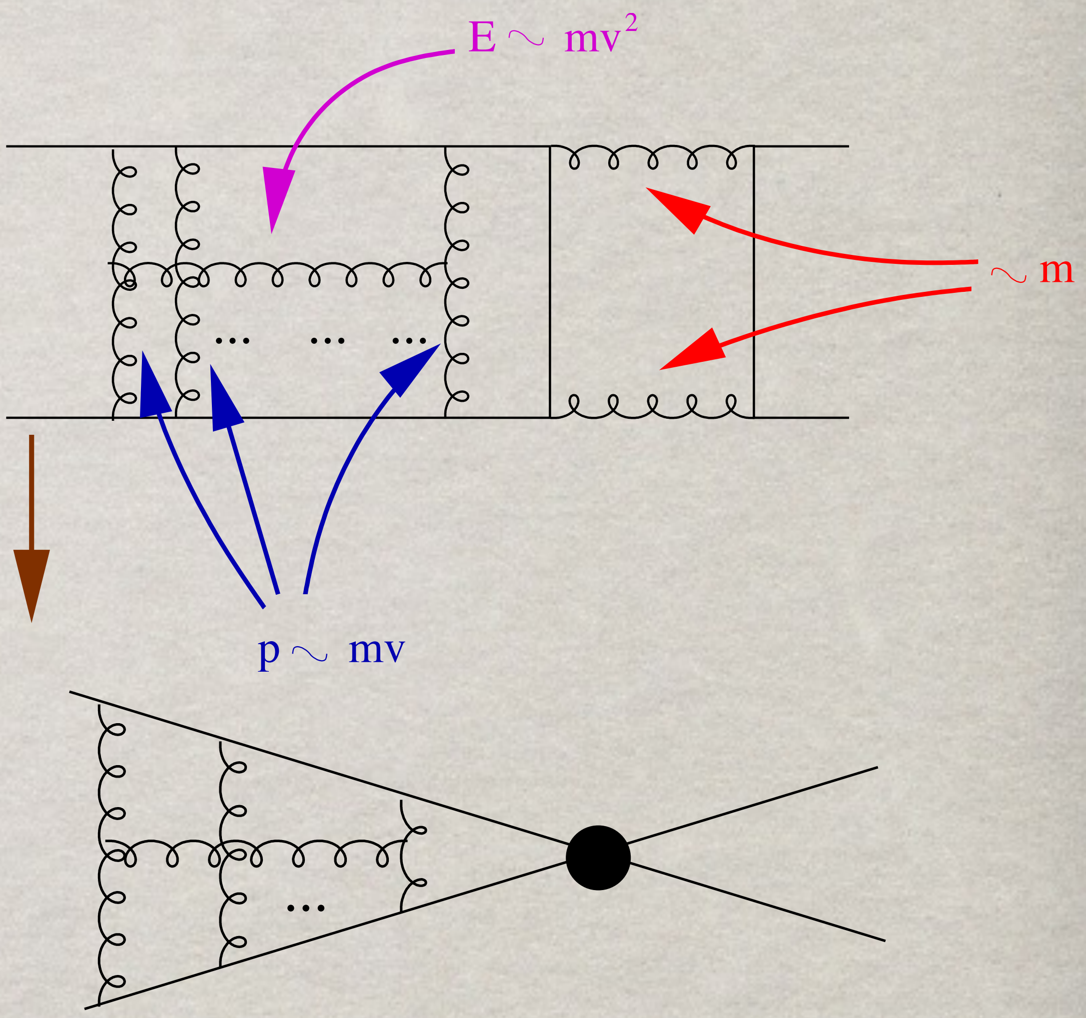




$$\mathcal{L}_{\rm NRQCD} = \sum_n c(\alpha_{\rm s}(m/\mu)) \times \frac{O_n(\mu,\lambda)}{m^n}$$

Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)





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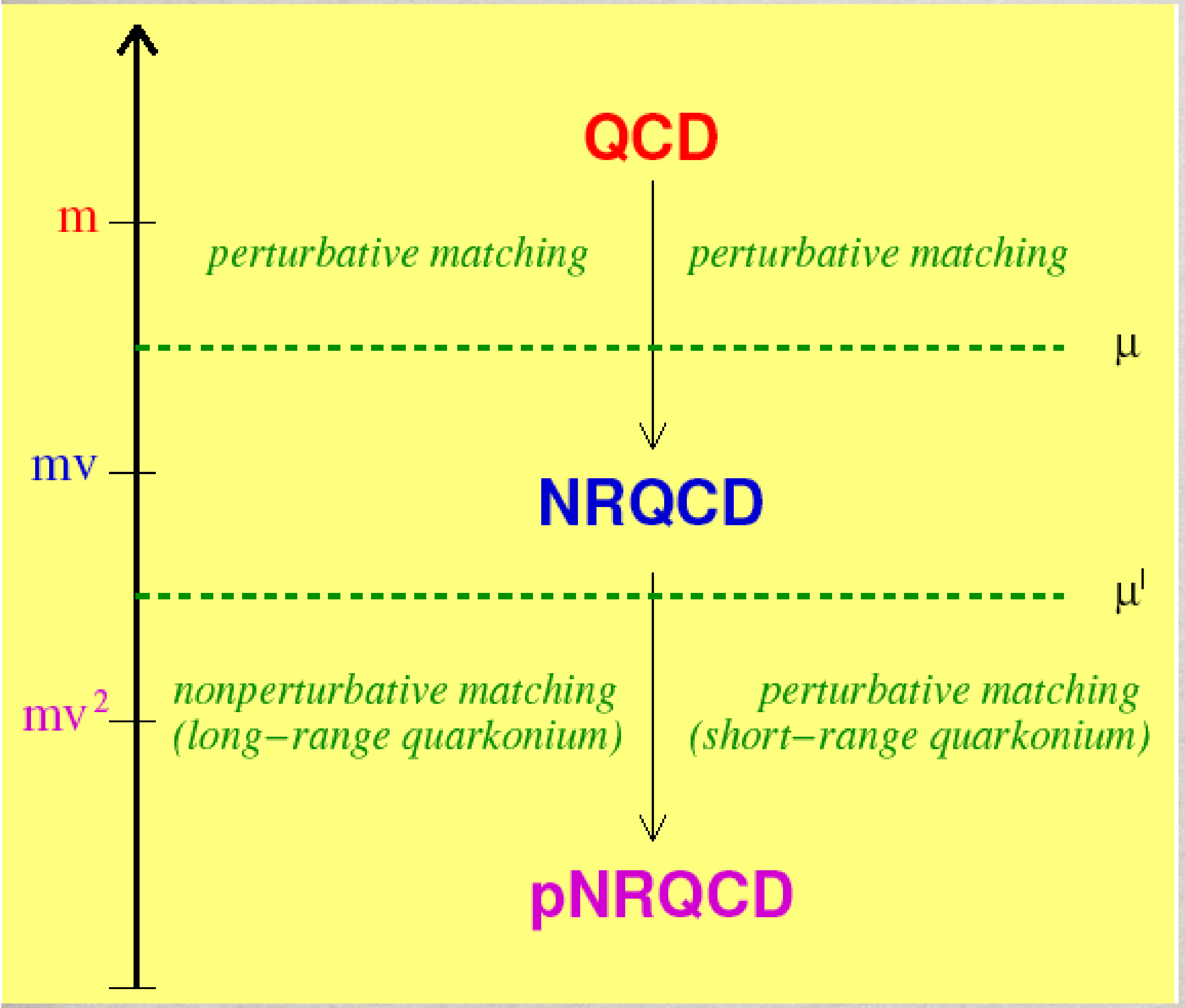

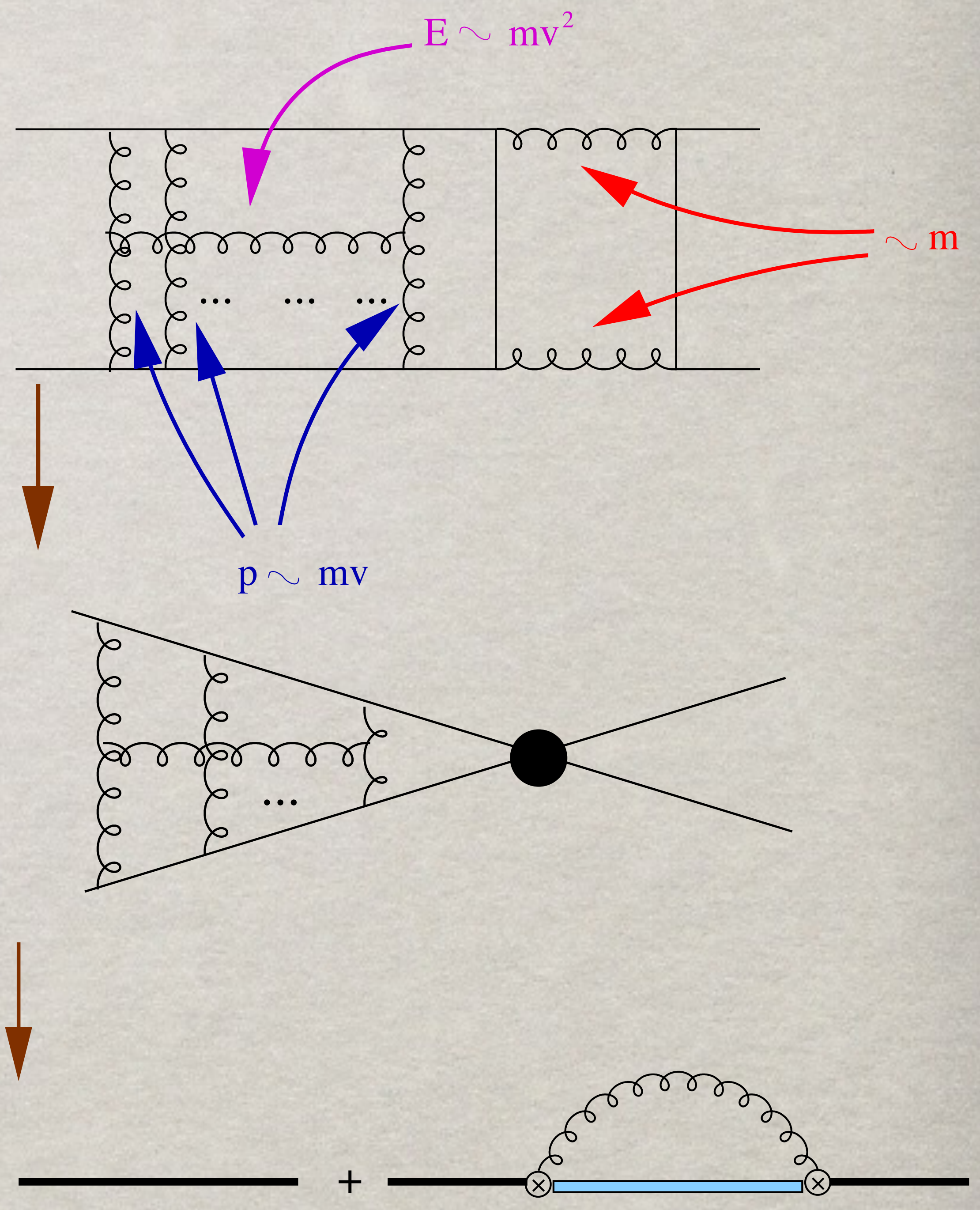

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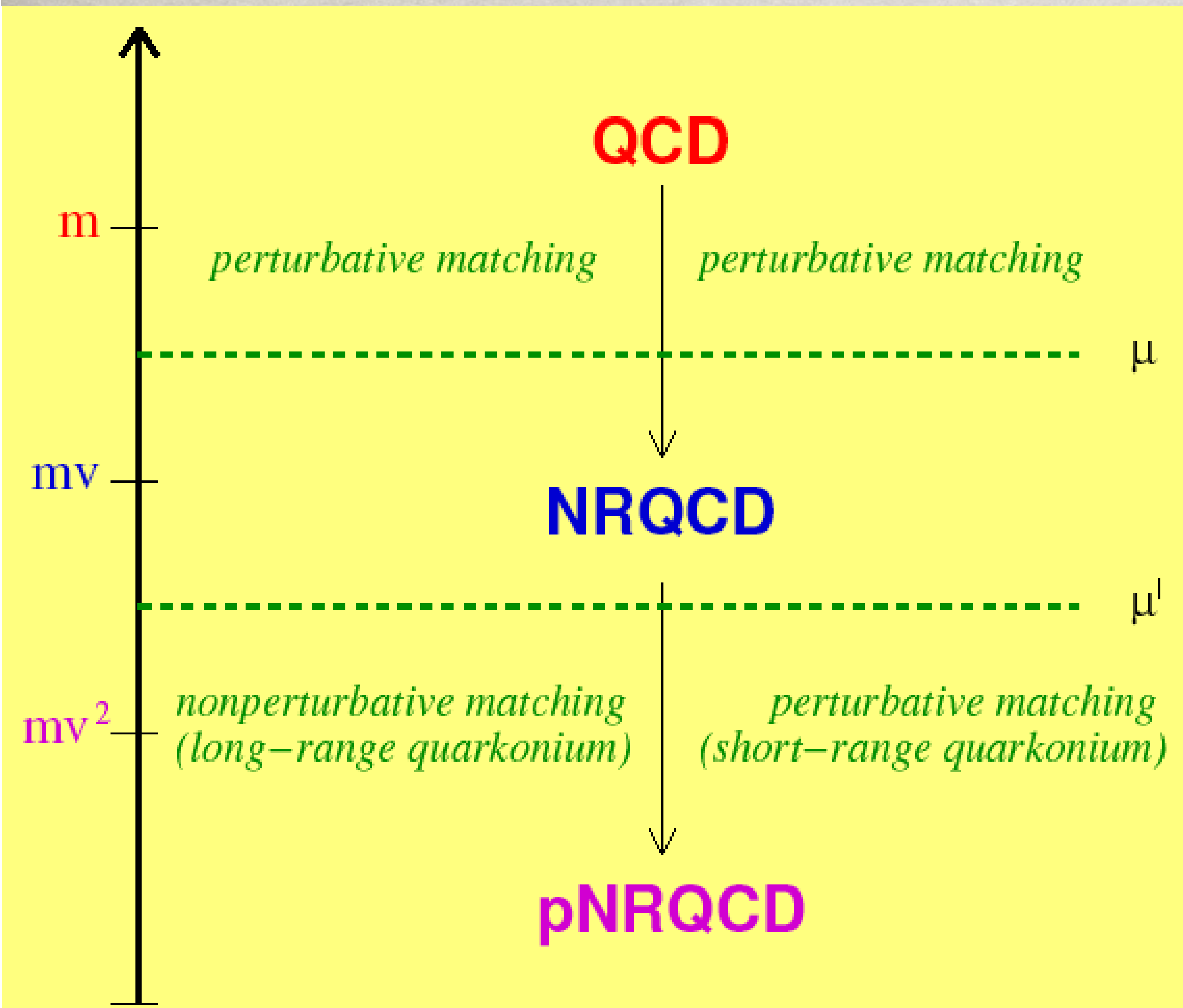

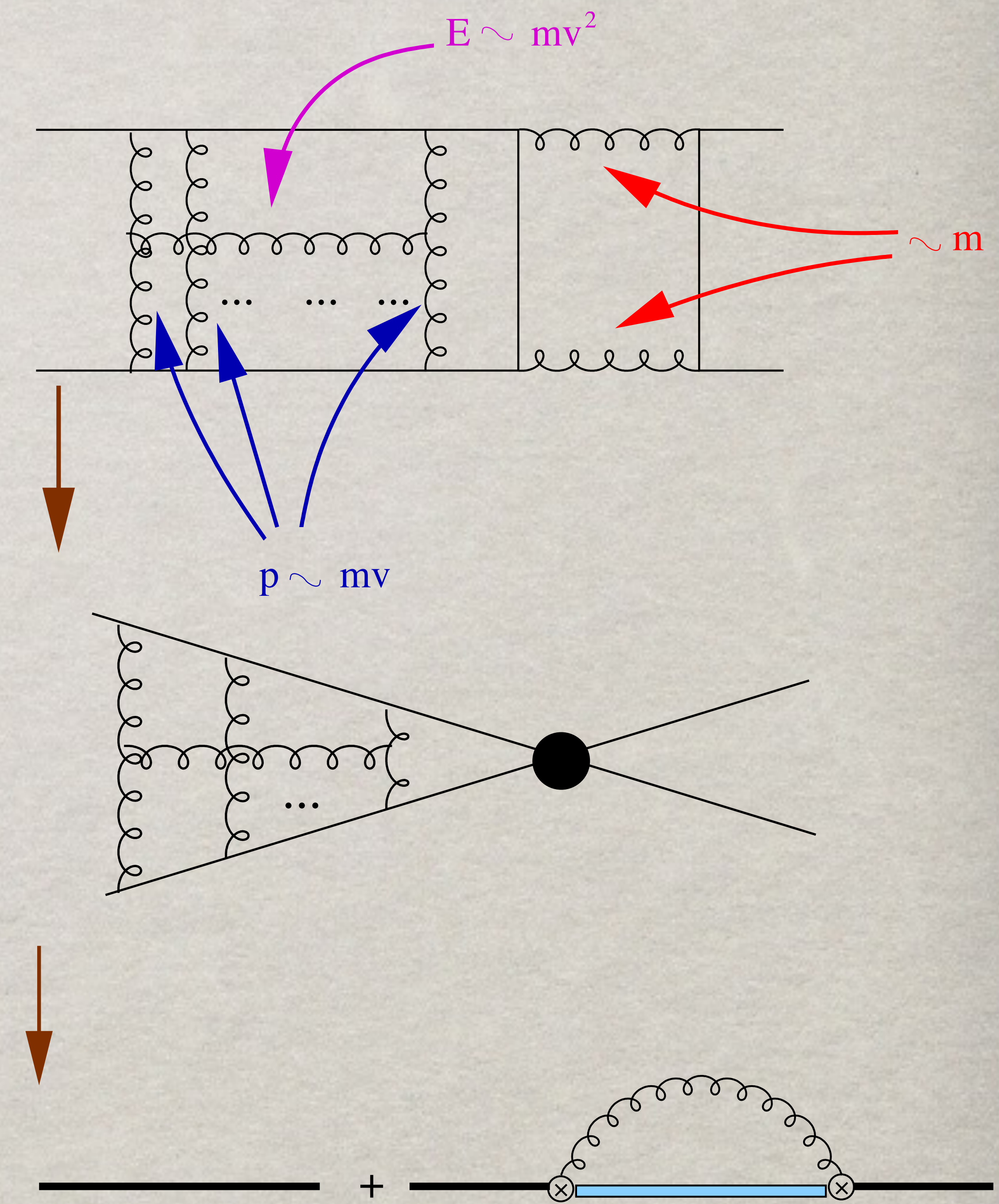

$$\mathcal{L}_{\rm pNRQCD} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_{\rm s}(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda)\, r^n$$

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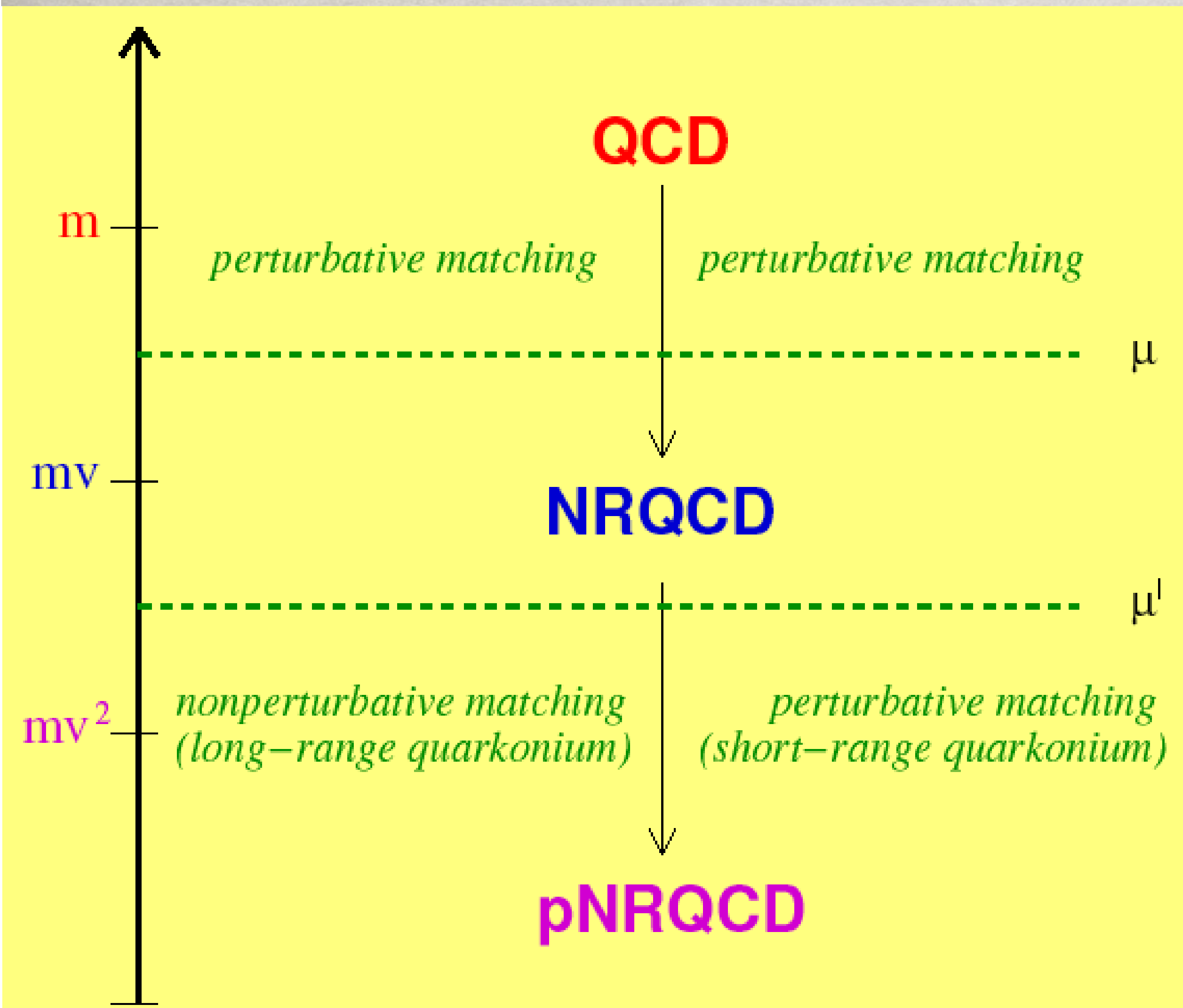

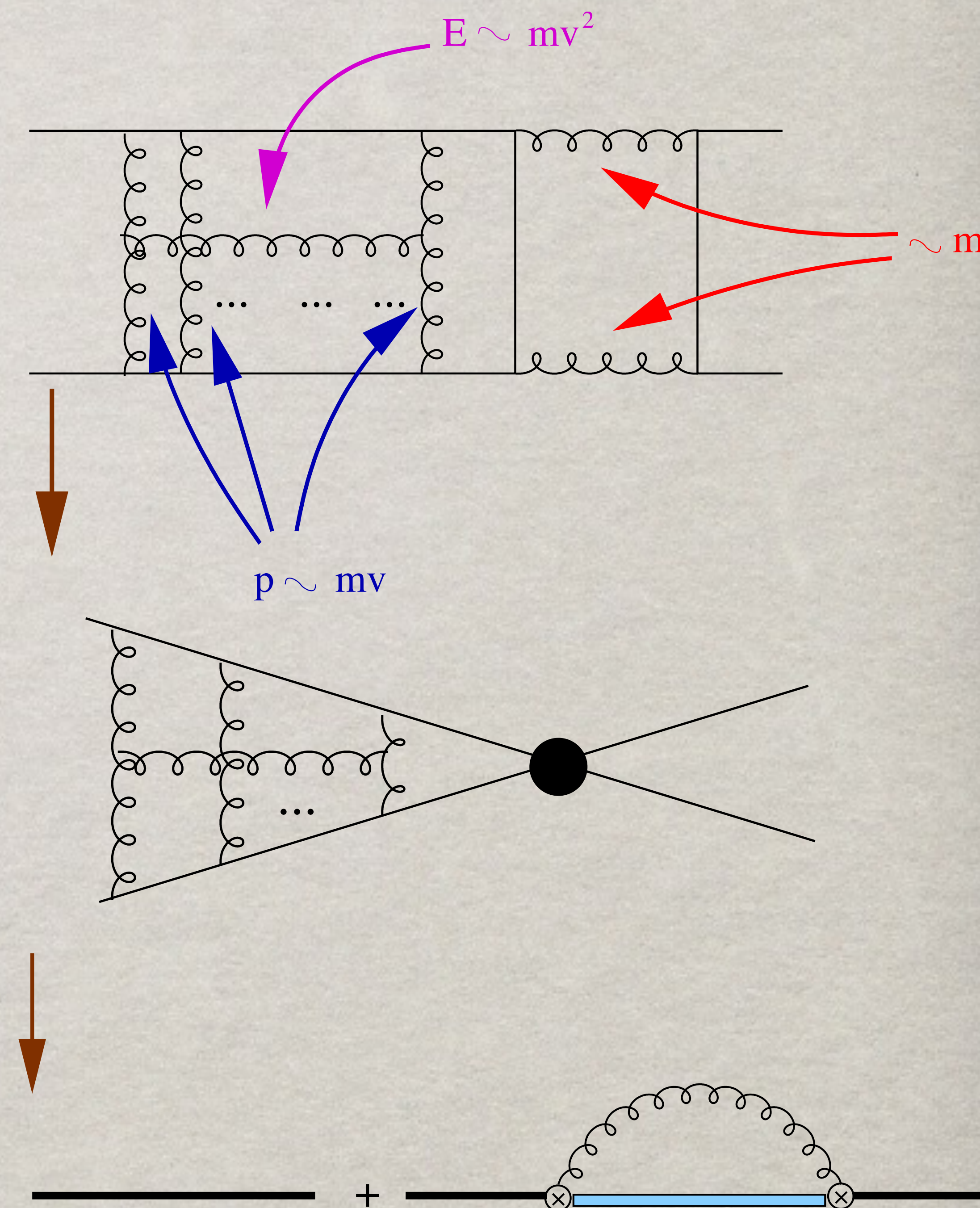

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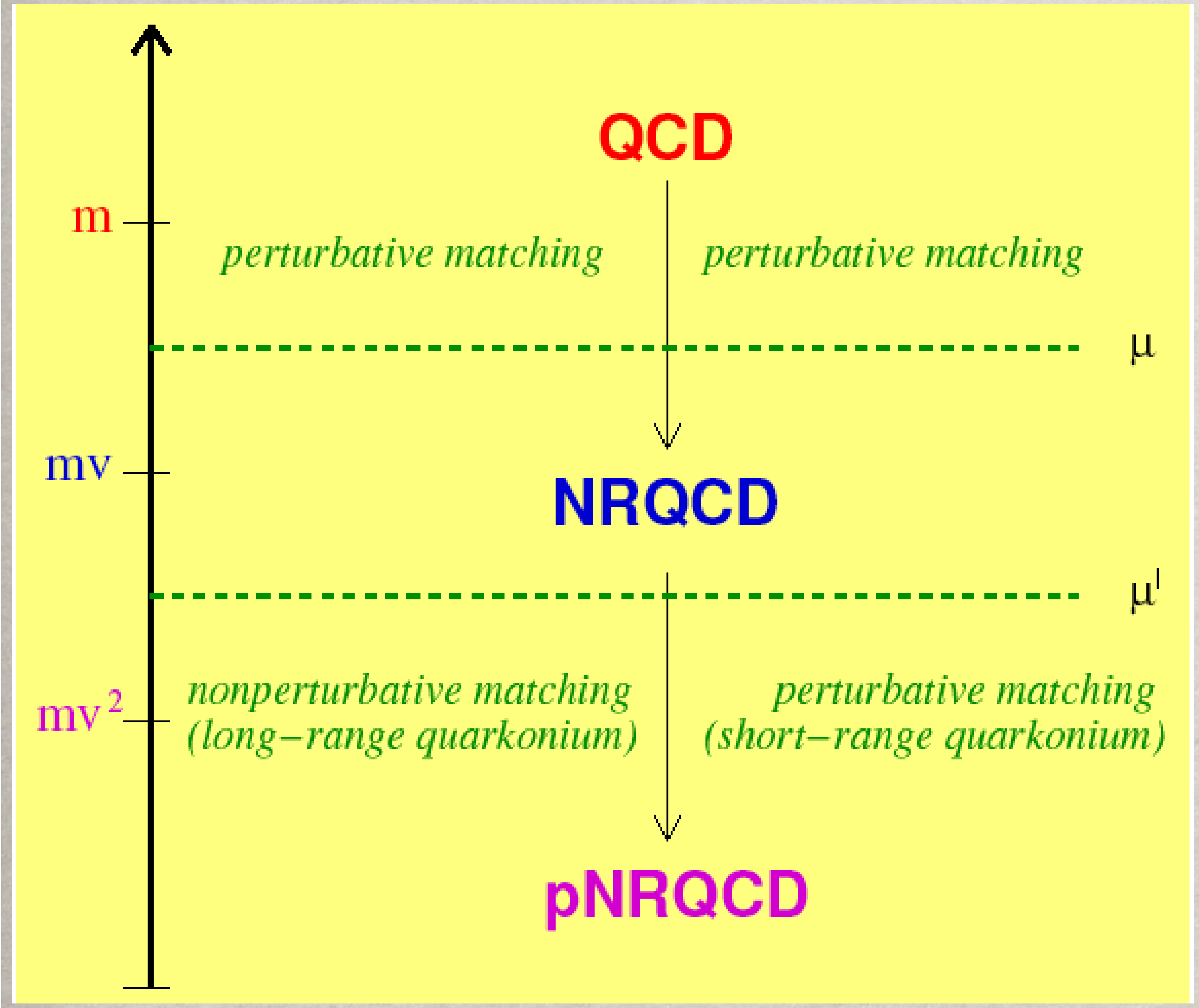

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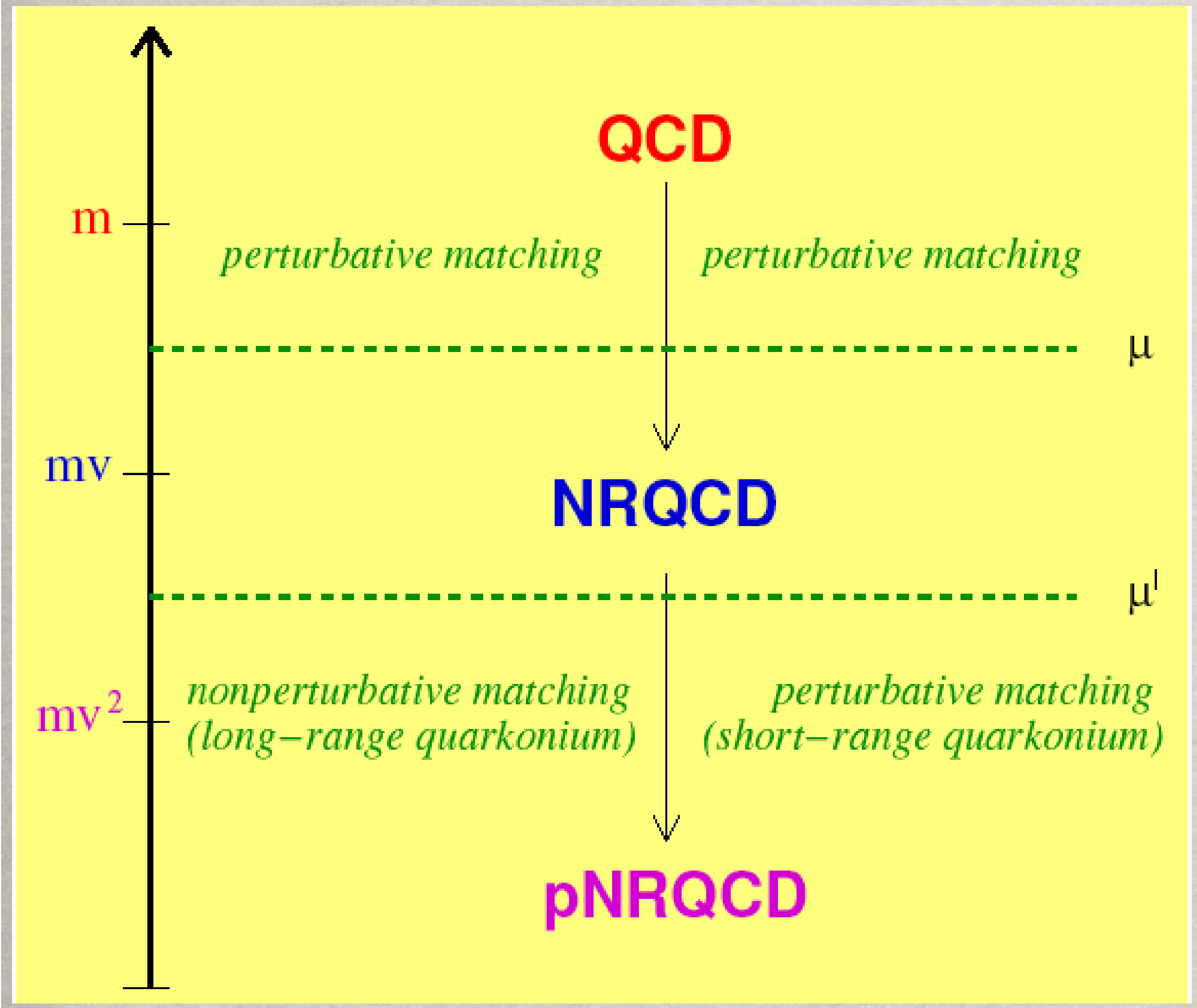
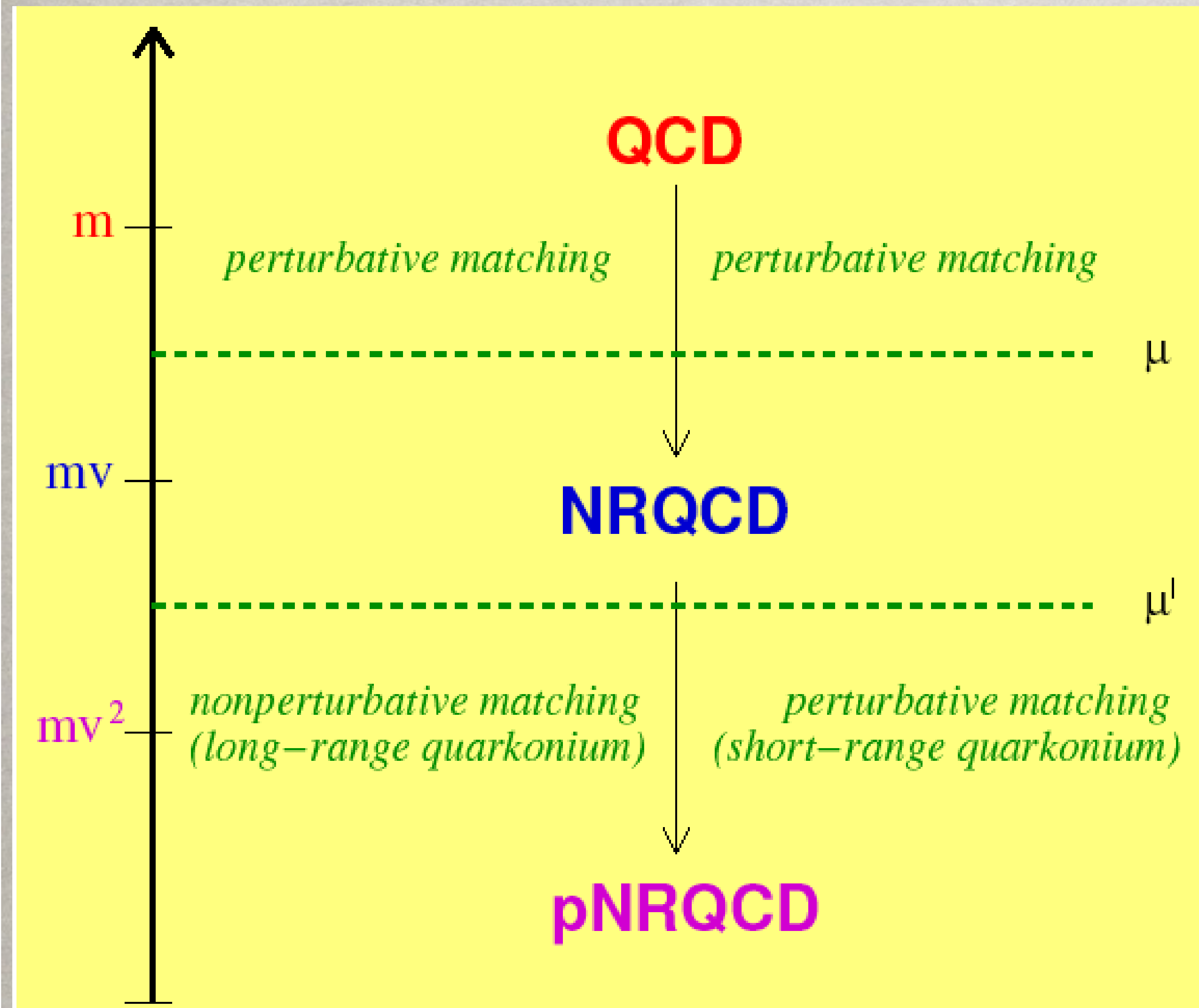
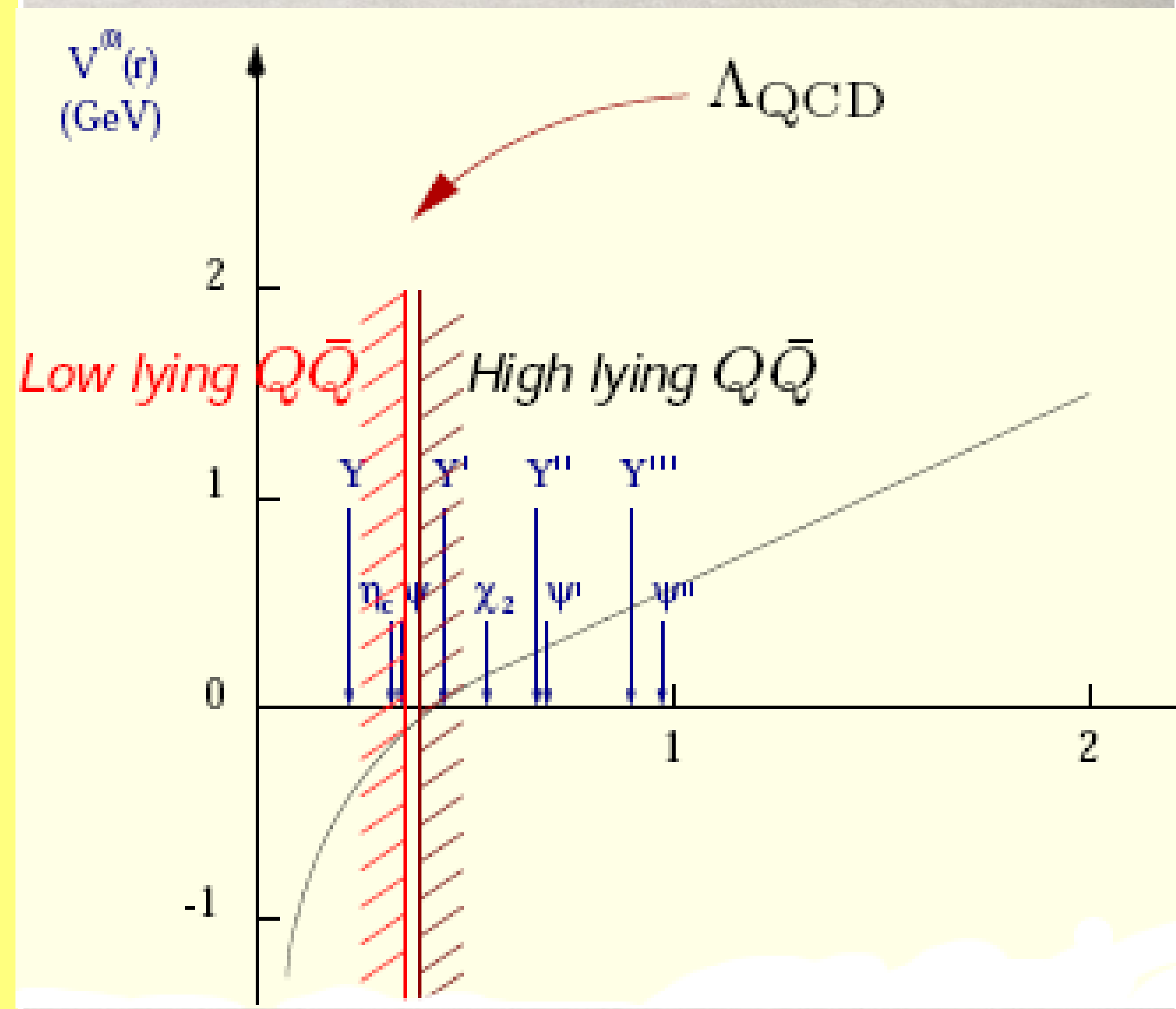


In QCD another scale is relevant $\Lambda_{\rm QCD}$

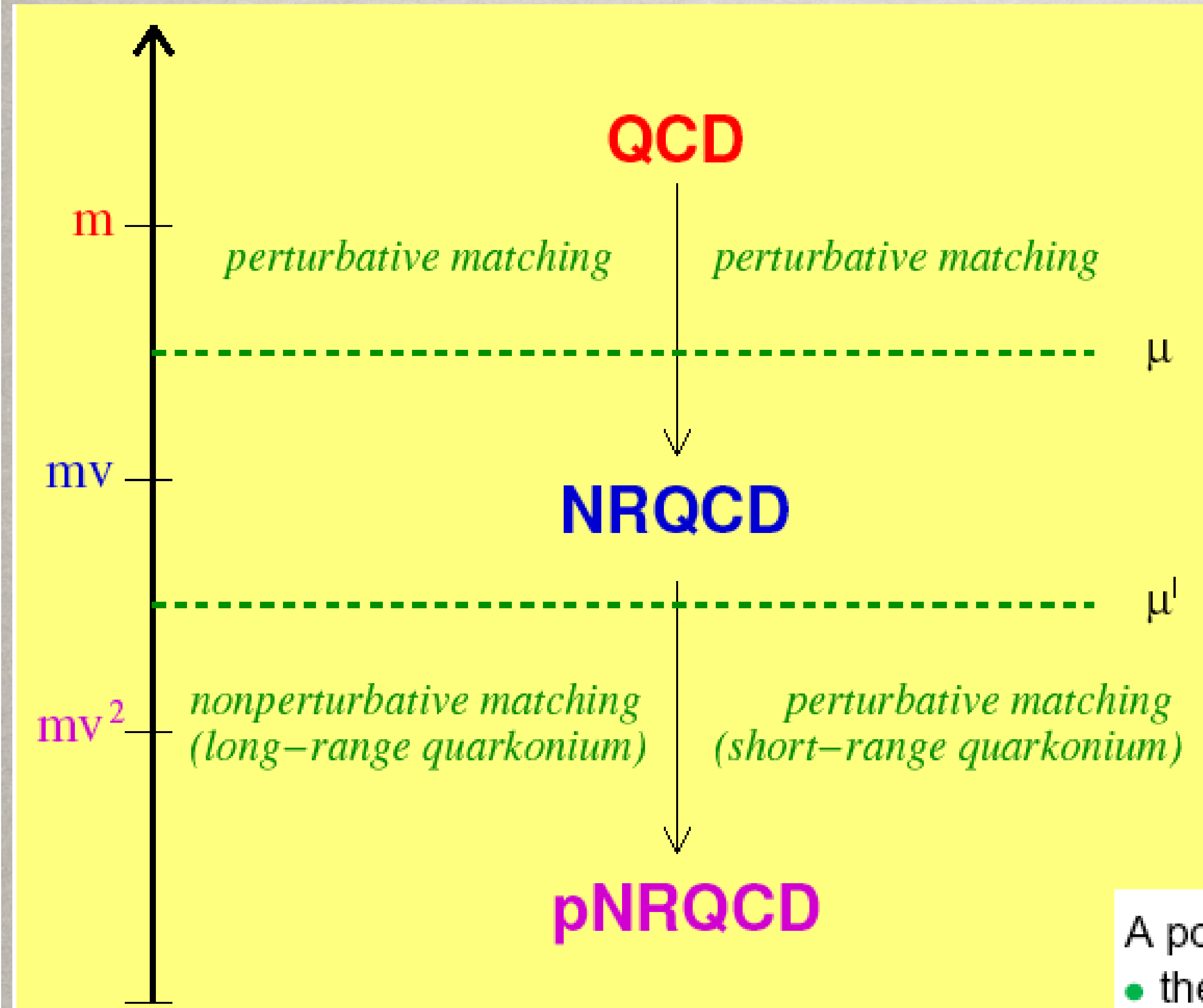
Quarkonium with NR EFT: pNRQCD

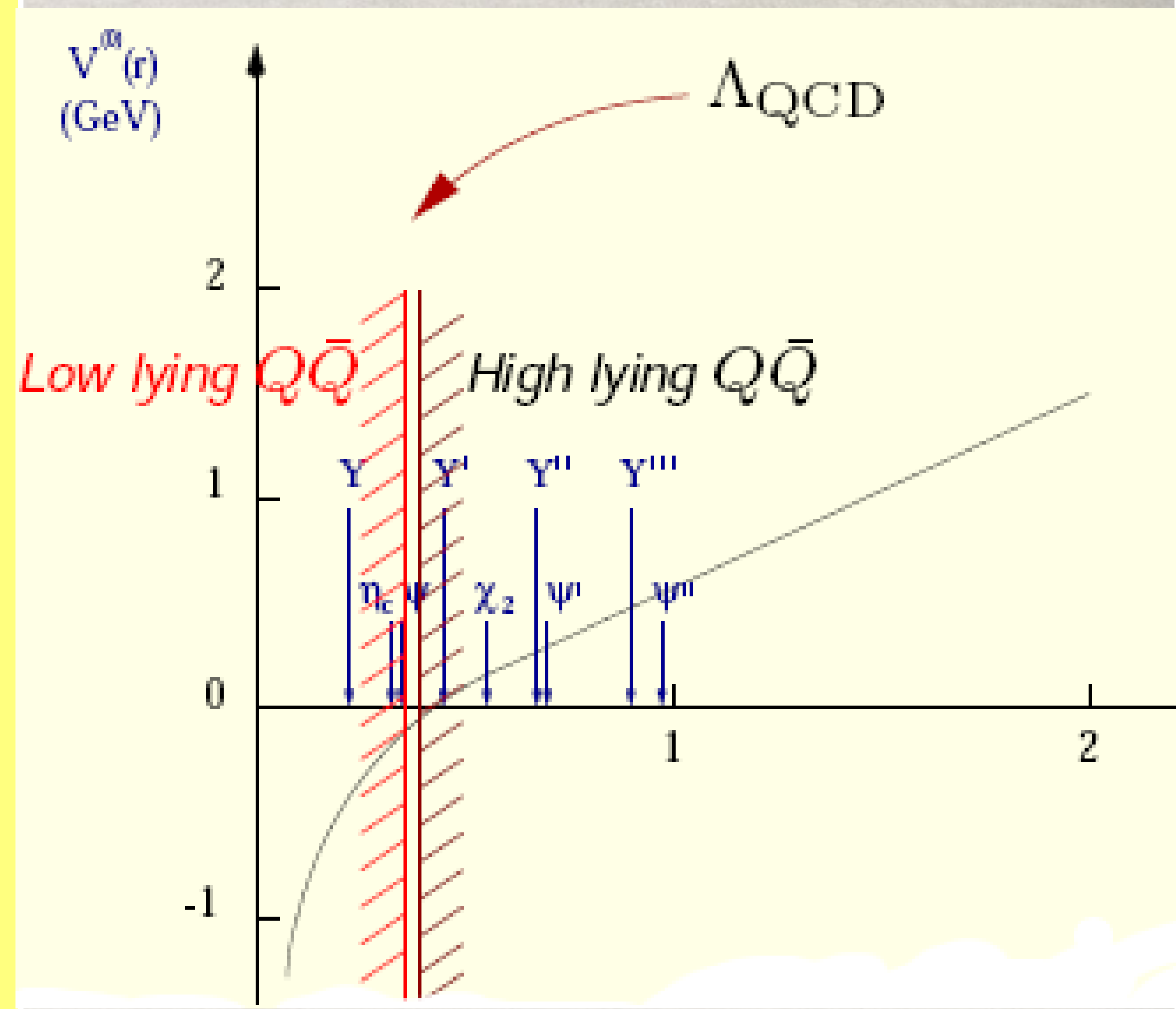




In QCD another scale is relevant Λ_{QCD}

Quarkonium with NR EFT: pNRQCD





A potential picture arises at the level of pNRQCD:

- the potential is perturbative if $mv \gg \Lambda_{QCD}$
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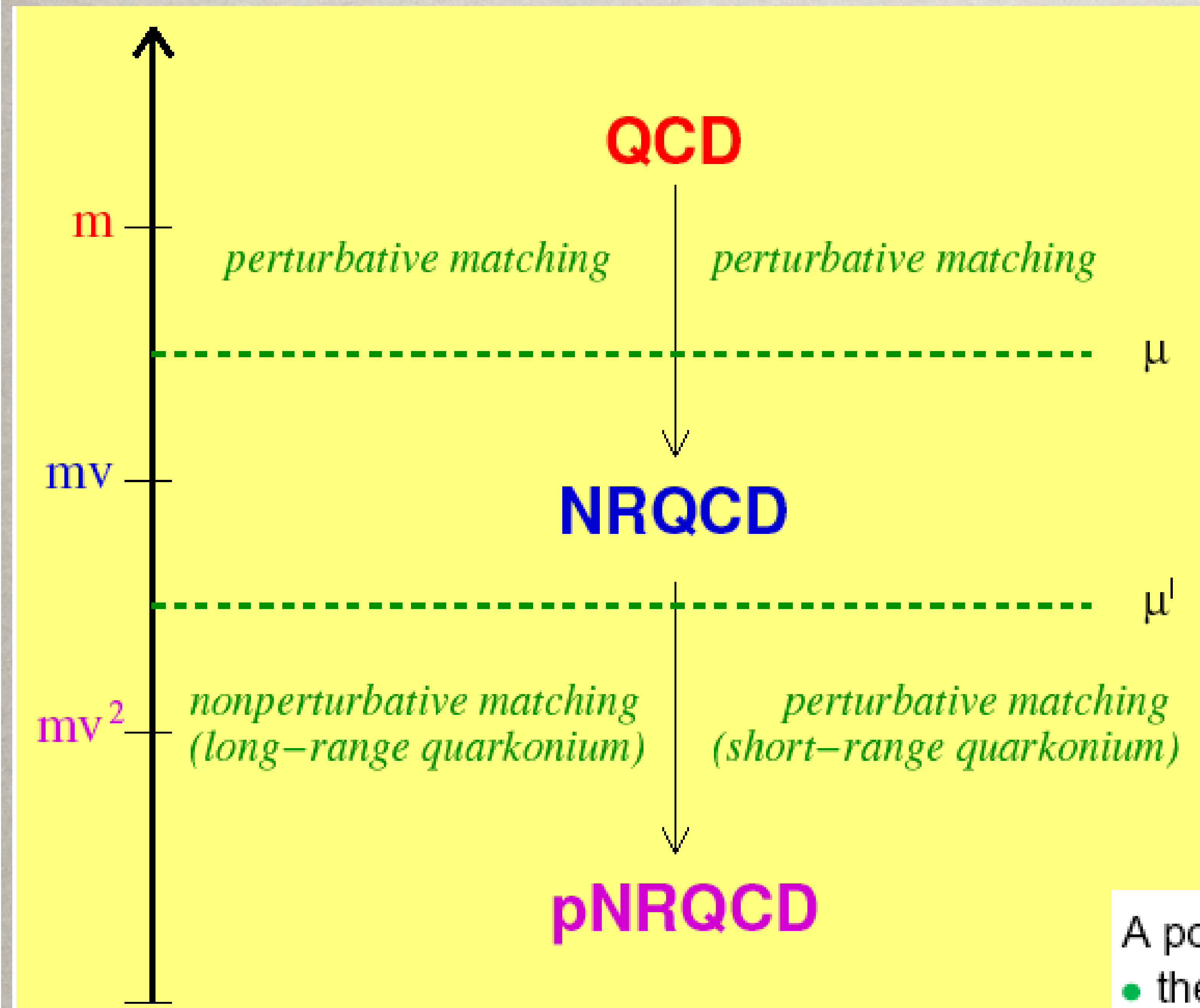

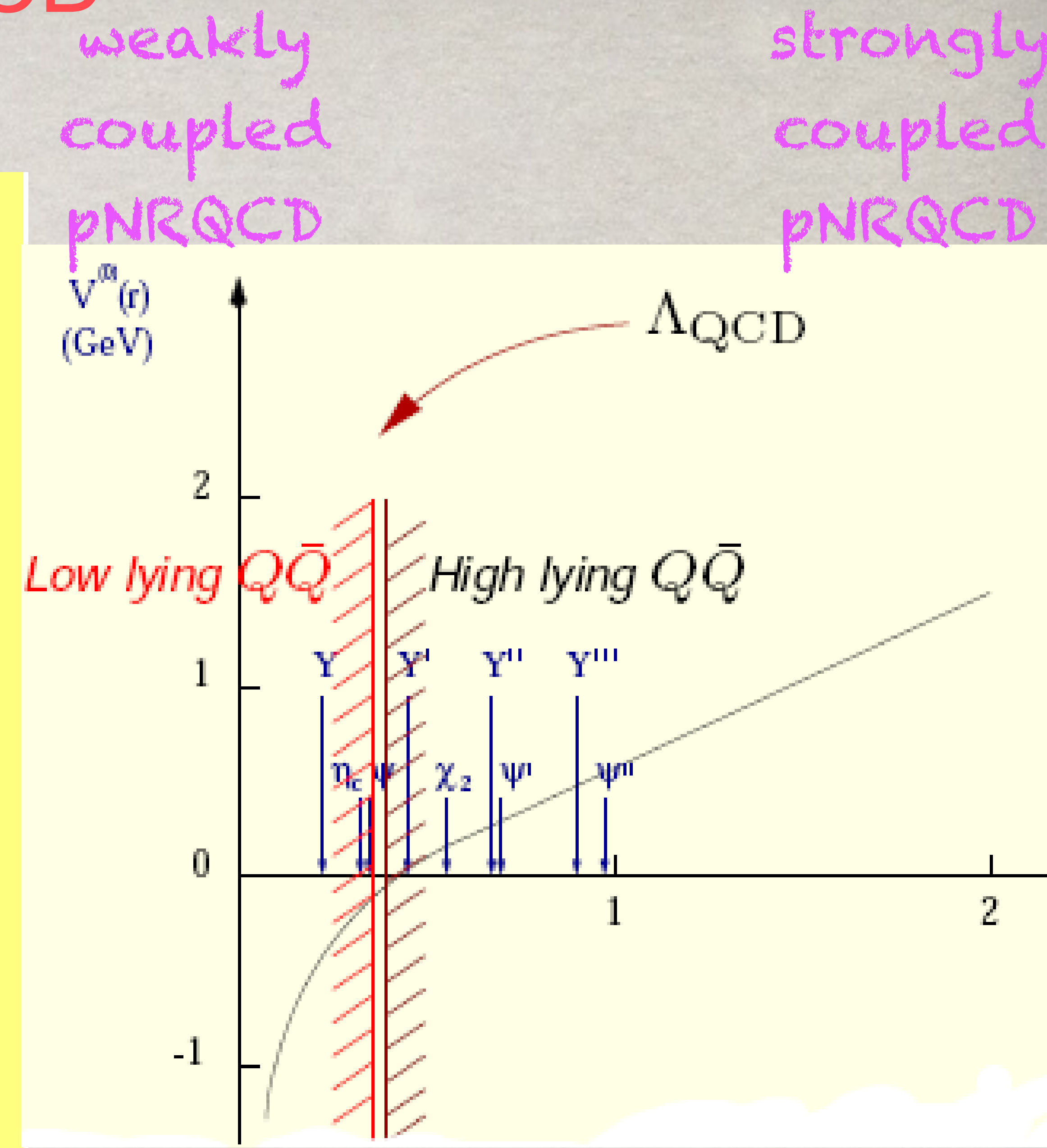

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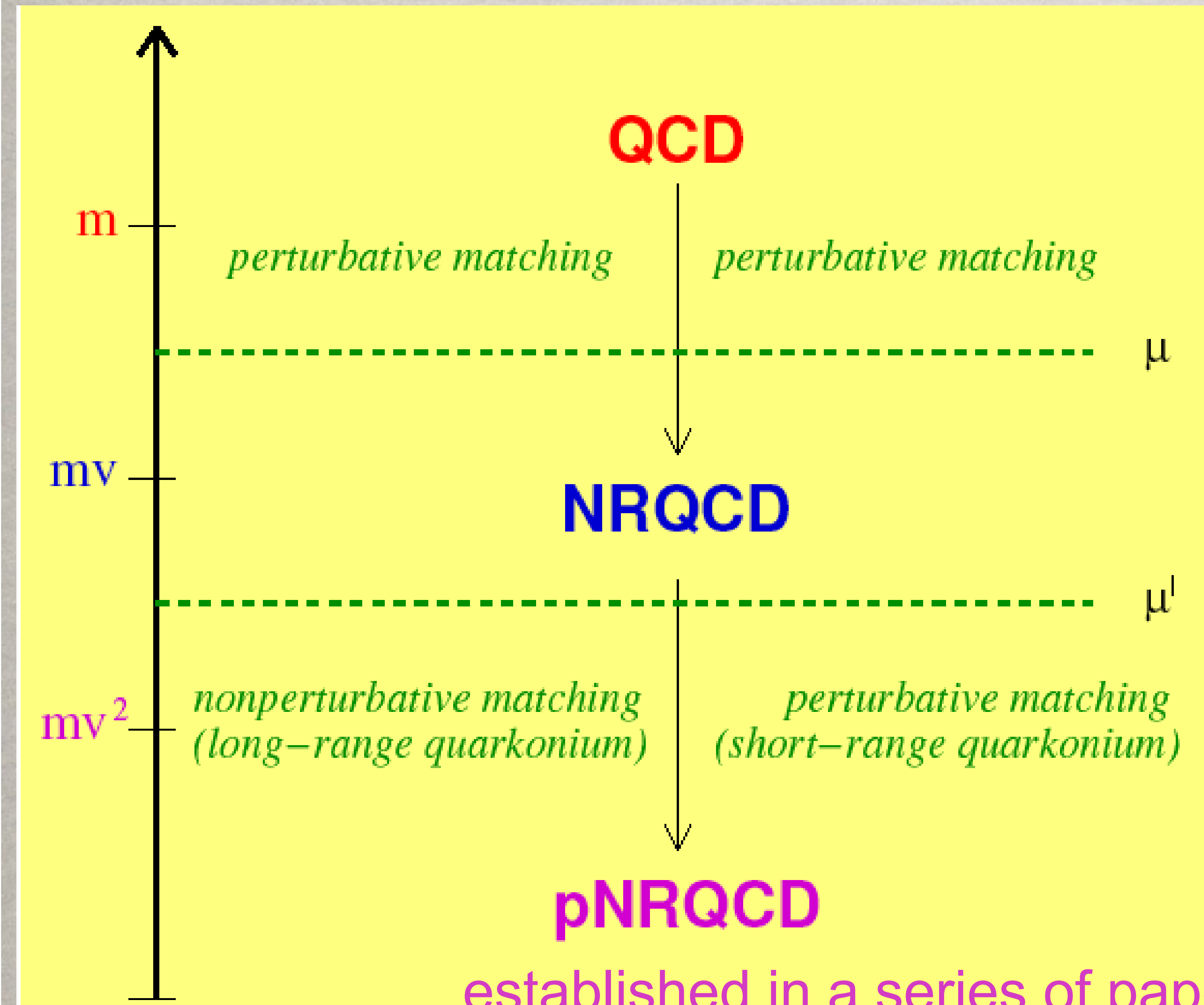
Quarkonium with EFT



Caswell, Lepage 86,
Lepage, Thacker 88
Bodwin, Braaten, Lepage 95......

Pineda, Soto 97, N.B. et al, 99,00,
Luke Manohar 97, Luke Savage 98,
Beneke Smirnov 98, Labelle 98
Labelle 98, Grinstein Rothstein 98
Kniehl, Penin 99, Griesshammer 00,
Manohar Stewart 00, Luke et al 00,
Hoang et al 01, 03->

established in a series of papers:
Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, Pineda, Soto 00--017
N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

Physics at the scale mv and mv^2 : pNRQCD
bound state formation

Physics at the scale mv and mv^2 : pNRQCD bound state formation

pNRQCD is today the theory used to address quarkonium bound states properties

pNRQCD and quarkonium Several cases for the physics at hand

Quarkonia states below and away from the strong decay threshold

The EFT has been constructed

*Work at calculating higher order perturbative corrections in v and alpha_s

*Resumming the log

*Calculating/extracting nonperturbatively the low energy quantities

*Extending the theory (electromagnetic effect, 3 bodies)

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The issue here is precision physics and the study of confinement

- Precise and systematic high order calculations allow the extraction of precise determinations of standard model parameters like the quark masses and alpha_s
- The eft has allowed to systematically factorize and to study the low energy nonperturbative contributions

Quarkonia states below in the quark gluon plasma at temperature T

pNRQCD at finite T has been constructed

Laine et al, 2007, Escobedo, Soto 2007 N. B. , Petreczky, Vairol. 2008
N. B. Escobedo, Ghiglieri, Vairo Soto, 2010-2014

The eft allows us to discover new, unexpected and important facts:

- The potential is neither the color singlet free energy nor the internal
- The quarkonium dissociation is a consequence of the apparence of a thermal decay width rather than being due to the color screening of the real part of the

We have now a coherent and systematical setup to calculate masses and width of quarkonium at finite T for small coupling: results to calculate study of non equilibrium evolution of quarkonium in a fireball using EFT for open quantum systems > R-AA calculation

N. B., Escobedo, Soto Vairo 2017

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N. B., Escobedo, Soto Vairo 2017

Quarkonia states at or above the strong decay threshold: X, Y, Z

pNRQCD for quark-antiquark and excited glue —>hybrids multiplets EFT of Born Oppenheimer

Berwein, N.B., Tarrus, Vairo 015

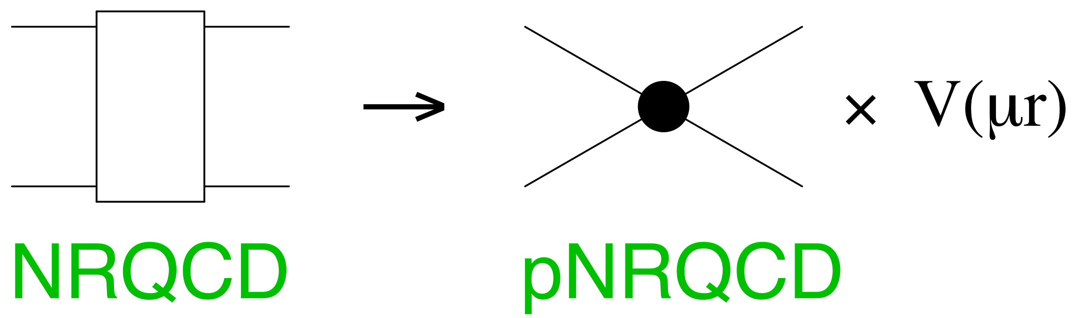
working now at including spin and extending to tetra quarks

Berwein, N.B., Lai, Segovia, Tarrus, Vairo 017

Quarkonium systems with small radius □ $r \ll \Lambda_{\mathrm{QCD}}^{-1}$

pNRQCD for quarkonia with small radius $r \ll \Lambda_{QCD}^{-1}$

Degrees of freedom that scale like mv are integrated out:



pNRQCD for quarkonia with small radius $r \ll \Lambda_{\rm QCD}^{-1}$

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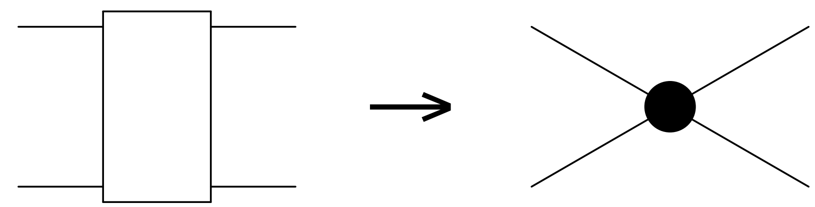

$\times$ V(μr)

NRQCD pNRQCD

- If $mv \gg \Lambda_{\rm QCD}$, the matching is perturbative

- Degrees of freedom: quarks and gluons

  Q-$\bar{Q}$ states, with energy $\sim \Lambda_{\rm QCD}, mv^2$ and momentum $\lesssim mv$
  $\Rightarrow$ (i) singlet S (ii) octet O

  Gluons with energy and momentum $\sim \Lambda_{\rm QCD}, mv^2$

- Definite power counting: $r \sim \dfrac{1}{mv}$ and $t, R \sim \dfrac{1}{mv^2}, \dfrac{1}{\Lambda_{\rm QCD}}$

  The gauge fields are multipole expanded:
  $A(R, r, t) = A(R, t) + \mathbf{r} \cdot \boldsymbol{\nabla} A(R, t) + \ldots$

  Non-analytic behaviour in $r \rightarrow$ matching coefficients V

weak pNRQCD $\quad r \ll \Lambda_{\rm QCD}^{-1}$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$

LO in r

S singlet field

O octet field

singlet propagator

octet propagator

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

weak pNRQCD $r \ll \Lambda_{\rm QCD}^{-1}$

Singlet static potential

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} + \mathrm{O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) \mathrm{O} \right\}$$

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- At leading order in r, the singlet S satisfies the QCD Schrödinger equation.

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Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

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LO in r

Octet static potential

- At leading order in r, the singlet S satisfies the QCD Schrödinger equation.
  - The (weak coupling) static potential is the Coulomb potential:

$$V_s(r) = -C_F \frac{\alpha_s}{r} + \ldots, \qquad V_o(r) = \frac{1}{2N} \frac{\alpha_s}{r} + \ldots, \qquad N = 3,\ C_F = \frac{4}{3}$$

S singlet field — singlet propagator

O octet field — octet propagator

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

weak pNRQCD $r \ll \Lambda_{\rm QCD}^{-1}$

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + {\rm Tr} \left\{ {\rm S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) {\rm S} + {\rm O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) {\rm O} \right\}$$

LO in r

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

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LO in r

$$+ V_A \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{S} + \mathrm{S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} \right\}$$
$$+ \frac{V_B}{2} \mathrm{Tr}\left\{ \mathrm{O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, \mathrm{O} + \mathrm{O}^\dagger \mathrm{O} \mathbf{r} \cdot g\mathbf{E} \right\}$$
$$+ \cdots$$

NLO in r

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-

weak pNRQCD $\quad r \ll \Lambda_{\rm QCD}^{-1}$

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu\, a} + {\rm Tr}\left\{ {\rm S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) {\rm S} \right. \\
& \left. + {\rm O}^\dagger \left(iD_0 - \frac{\mathbf{p}^2}{m} - V_o \right) {\rm O} \right\}
\end{aligned}
$$

LO in r

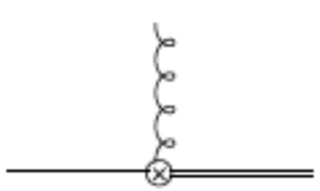
$$
\begin{aligned}
& + V_A {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, {\rm S} + {\rm S}^\dagger \mathbf{r} \cdot g\mathbf{E}\, {\rm O} \right\} \\
& + \frac{V_B}{2} {\rm Tr}\left\{ {\rm O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, {\rm O} + {\rm O}^\dagger {\rm O} \mathbf{r} \cdot g\mathbf{E} \right\} \\
& + \cdots
\end{aligned}
$$

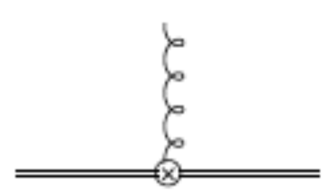
NLO in r

- Feynman rules:

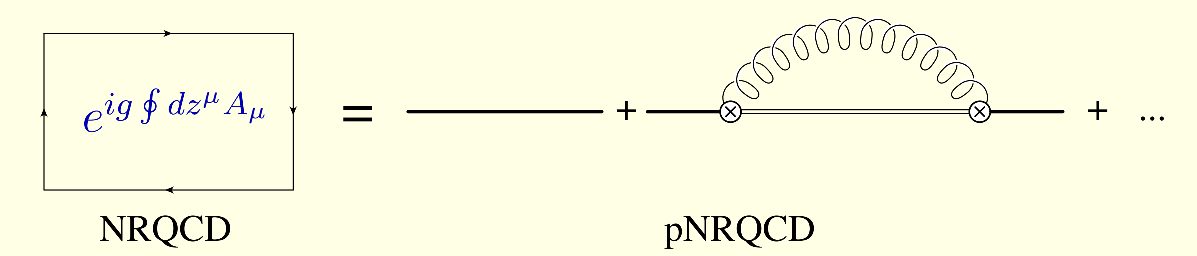
 $= \theta(t)\, e^{-itH_s}$

 $= \theta(t)\, e^{-itH_o} \left(e^{-i\int dt\, A^{\rm adj}} \right)$

 $= {\rm O}^\dagger \mathbf{r} \cdot g\mathbf{E}\, {\rm S}$

 $= {\rm O}^\dagger \{\mathbf{r} \cdot g\mathbf{E},\, {\rm O}\}$

QCD singlet static potential and singlet static energy



potential

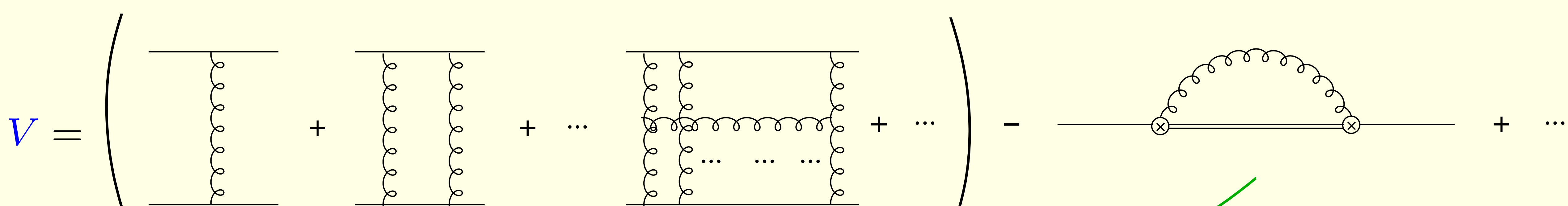
$$\lim_{T\to\infty} \frac{i}{T}\ln\langle \Box \rangle = V_s(r,\mu) - i\frac{g^2}{N_c}V_A^2\int_0^\infty dt\, e^{-it(V_o-V_s)}\langle \mathrm{Tr}(r\cdot E(t)\, r\cdot E(0))\rangle(\mu)+\dots$$

static energy

ultrasoft contribution

contributes from 3 loops

The potential is a Wilson coefficient of the EFT.
In general, it undergoes renormalization, develops scale dependence and satisfies renormalization group equations, which allow to resum large logarithms.



Quarkonium singlet static potential at N^4LO

$$
\begin{aligned}
V_s(r,\mu) \;\; &= \;\; -C_F \frac{\alpha_s(1/r)}{r} \Bigg[1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \left(\frac{\alpha_s(1/r)}{4\pi}\right)^2 \\
&+ \left(\frac{16\,\pi^2}{3} C_A^3 \,\ln r\mu + a_3\right) \left(\frac{\alpha_s(1/r)}{4\pi}\right)^3 \\
&+ \left(a_4^{L2} \ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\, C_A^3 \beta_0(-5 + 6\ln 2)\right) \ln r\mu + a_4\right) \left(\frac{\alpha_s(1/r)}{4\pi}\right)^4 \Bigg]
\end{aligned}
$$

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\end{aligned}
$$

a_1 Billoire 80

a_2 Schroeder 99, Peter 97

coeff $ln r\mu$ N.B. Pineda, Soto, Vairo 99

a_4^{L2}, a_4^L N.B., Garcia, Soto, Vairo 06

a_3 Anzai, Kiyo, Sumino 09, Smirnov, Smirnov, Steinhauser 09

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3LOOPS REDUCES TO 1LOOP IN THE EFT

a_4^{L2}, a_4^L N.B., Garcia, Soto

4LOOPS REDUCES TO 2LOOPS IN THE EFT

a_3 Anzai, Kiyo, Sumino 09, Smirnov, Smirnov, Steinhauser 09

Quarkonium singlet static potential at N^4LO

$$V_s(r,\mu) \;=\; -C_F \frac{\alpha_s(1/r)}{r}\Bigg[1 + a_1 \frac{\alpha_s(1/r)}{4\pi} + a_2 \left(\frac{\alpha_s(1/r)}{4\pi}\right)^2$$
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Two problems:

1)Bad convergence of the series due to large beta_0 terms

2) Large logs

Quarkonium singlet static potential at N^4LO

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for long it was believed that such series was not convergent problem for any phenomenological application

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\end{aligned}
$$

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The eft cures both:

1) Renormalon subtracted scheme

Beneke 98, Hoang, Lee 99, Pineda 01, N.B. Pineda Soto, Vairo 09

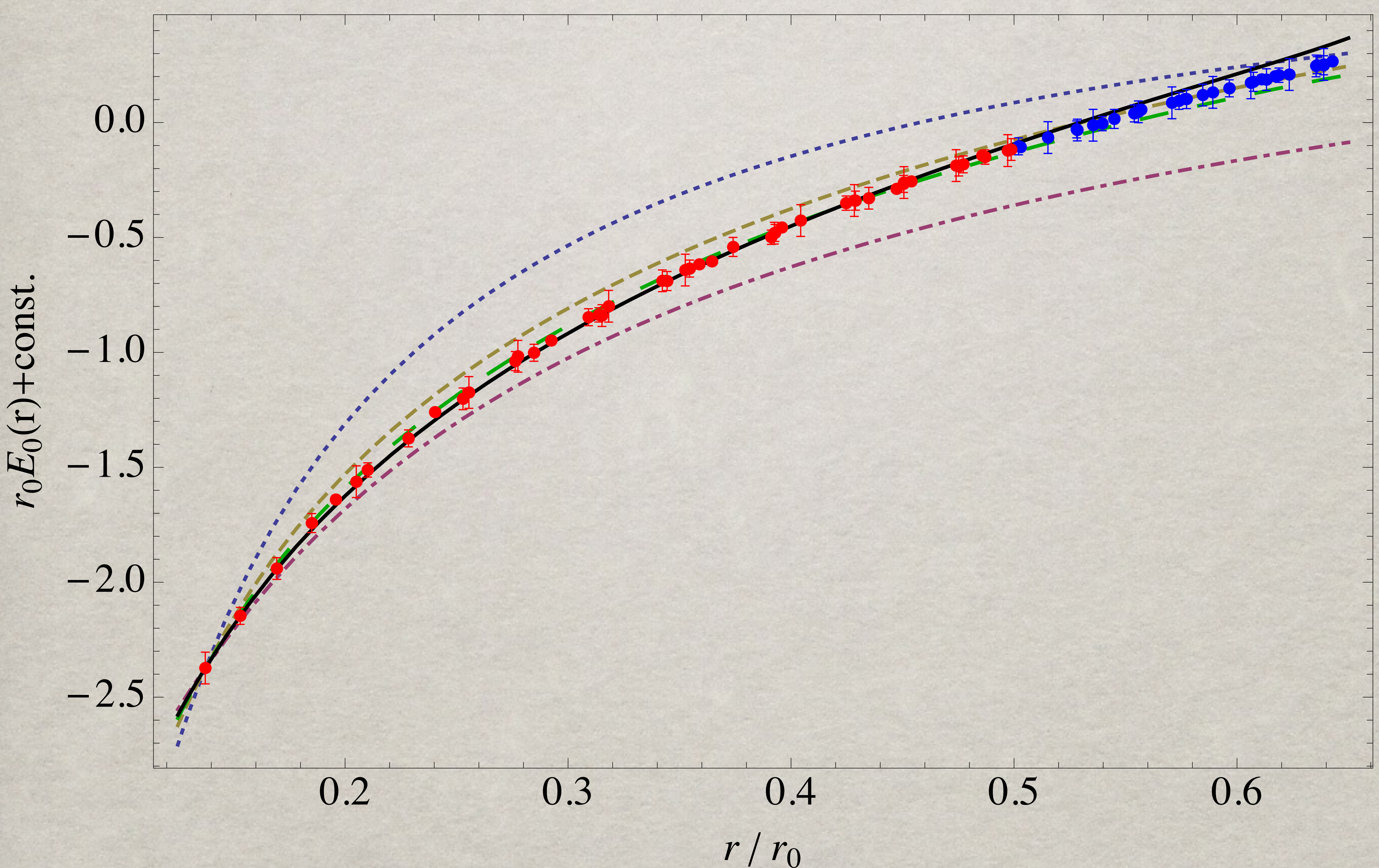
2) Renormalization group summation of the logs

up to N^3LL $(\alpha_s^{4+n} \ln^n \alpha_s)$

N. B Garcia, Soto Vairo 2007, 2009, Pineda, Soto

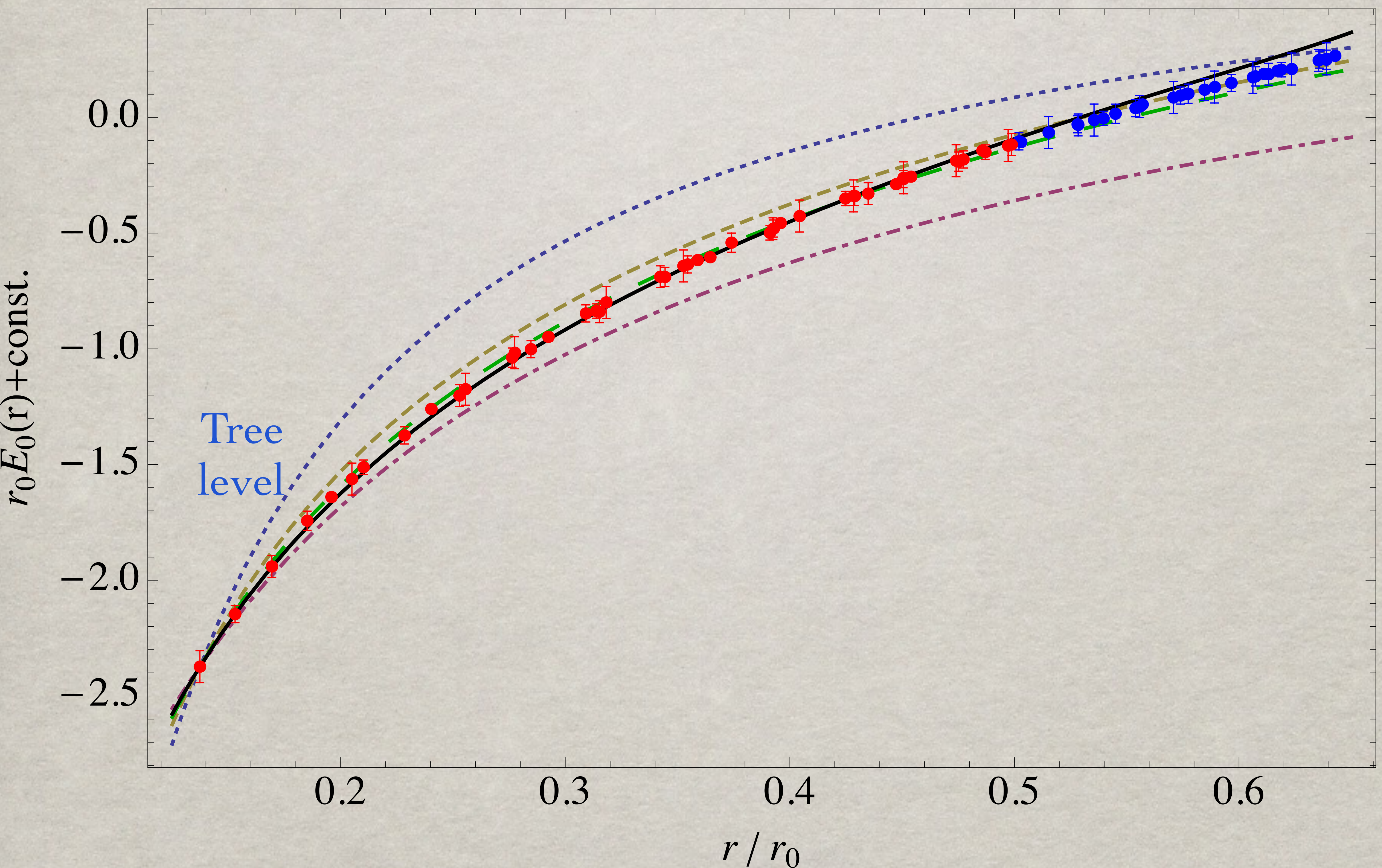
QQbar singlet static energy at N^3LI in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



QQbar singlet static energy at N^3Ll in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



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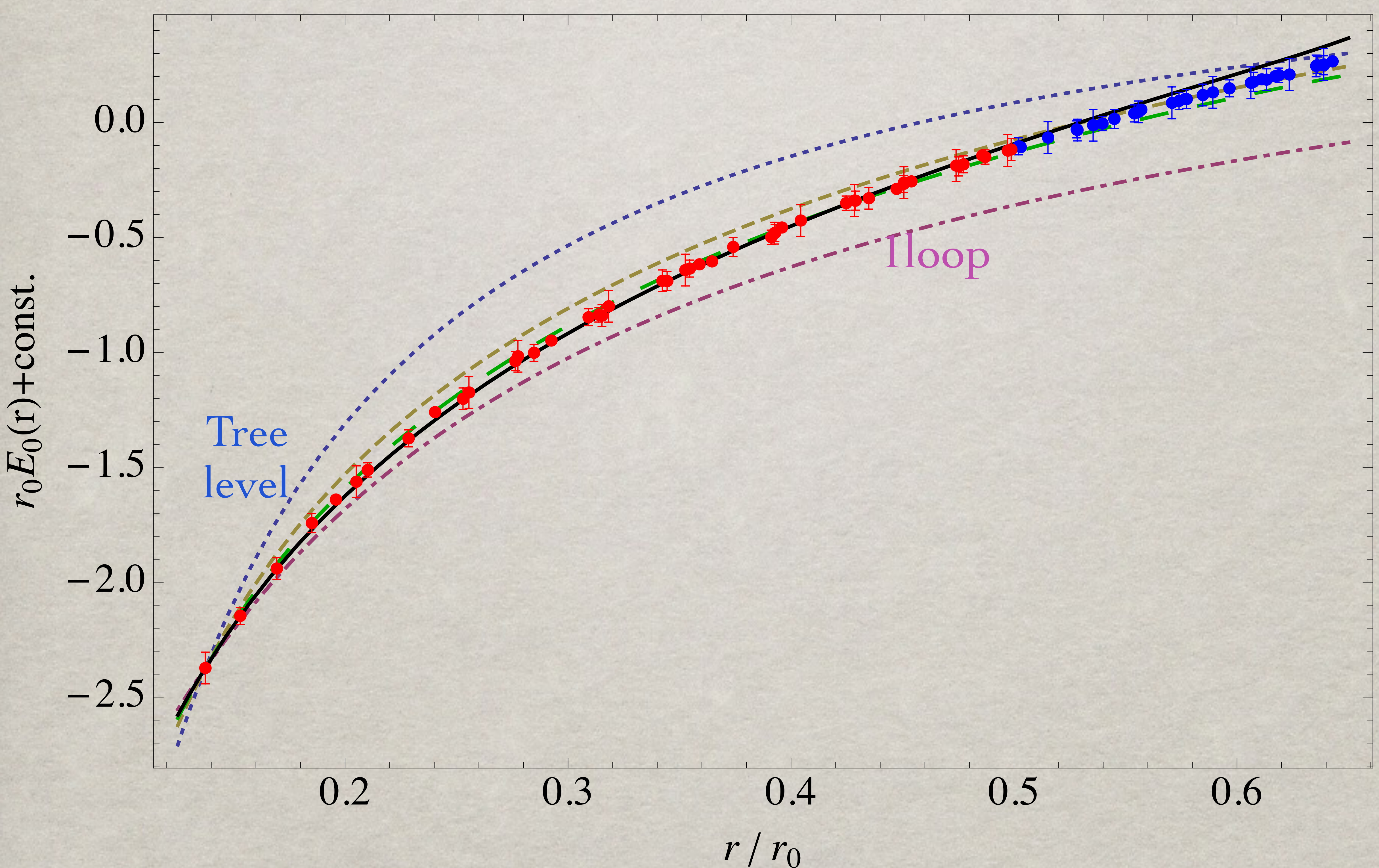
Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



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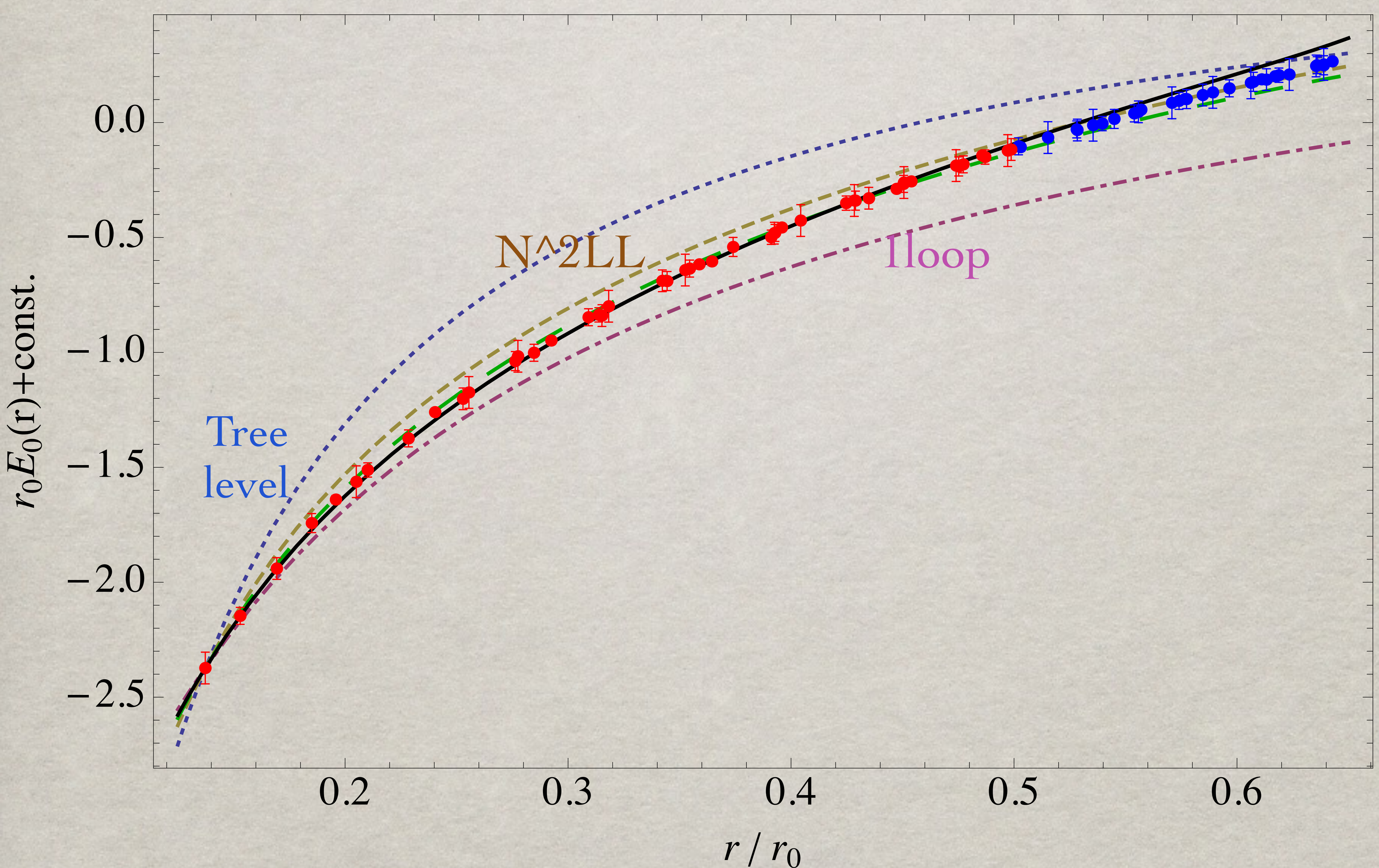
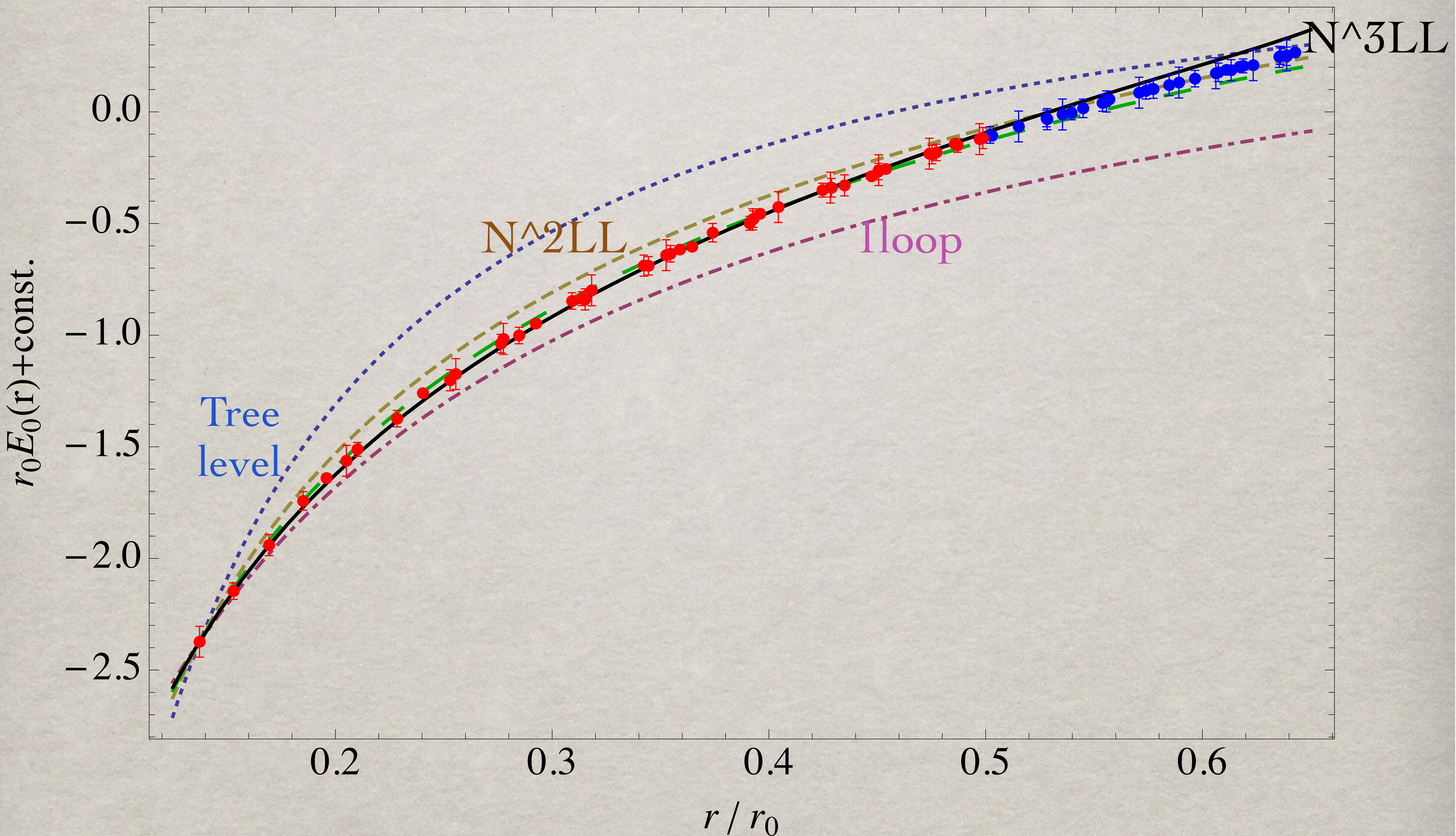


QQbar singlet static energy at N^3LI in comparison with unquenched (n_f=2+1) lattice data (red points,blue points)

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2012, 2014



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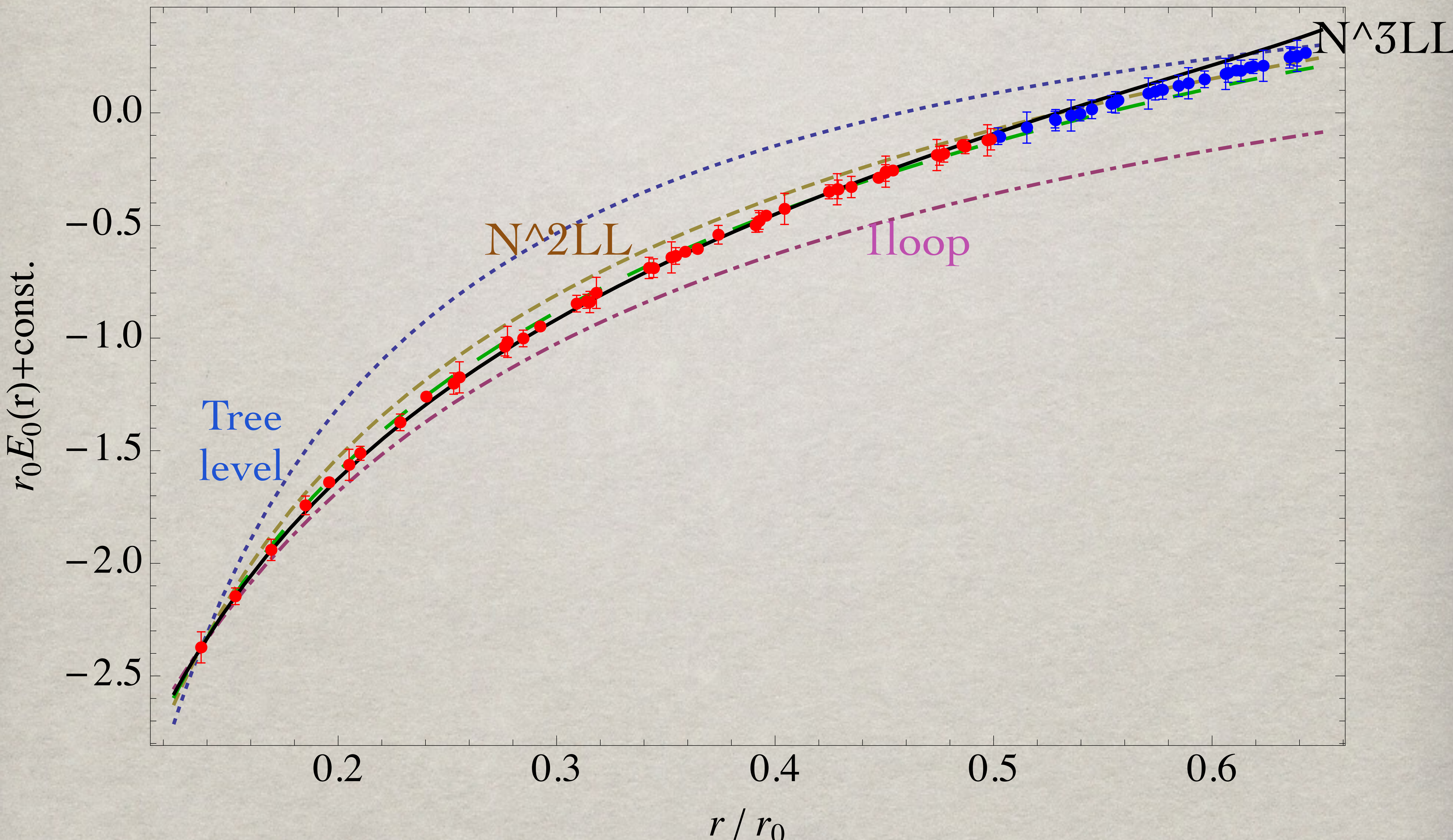

Good convergence to the lattice data

Lattice data less accurate in the unquenched case

α_s extraction

Bazanov, N. B., Garcia, Petreczky, Soto, Vairo , 2014

We obtain an extraction of alphas at N^3LO plus leading log resummation

$$\alpha_s(1.5\text{GeV}, n_f = 3) = 0.336^{+0.012}_{-0.008}$$

corresponding to

$$\alpha_s(M_z, n_f = 5) = 0.1166^{+0.0012}_{-0.0008}$$

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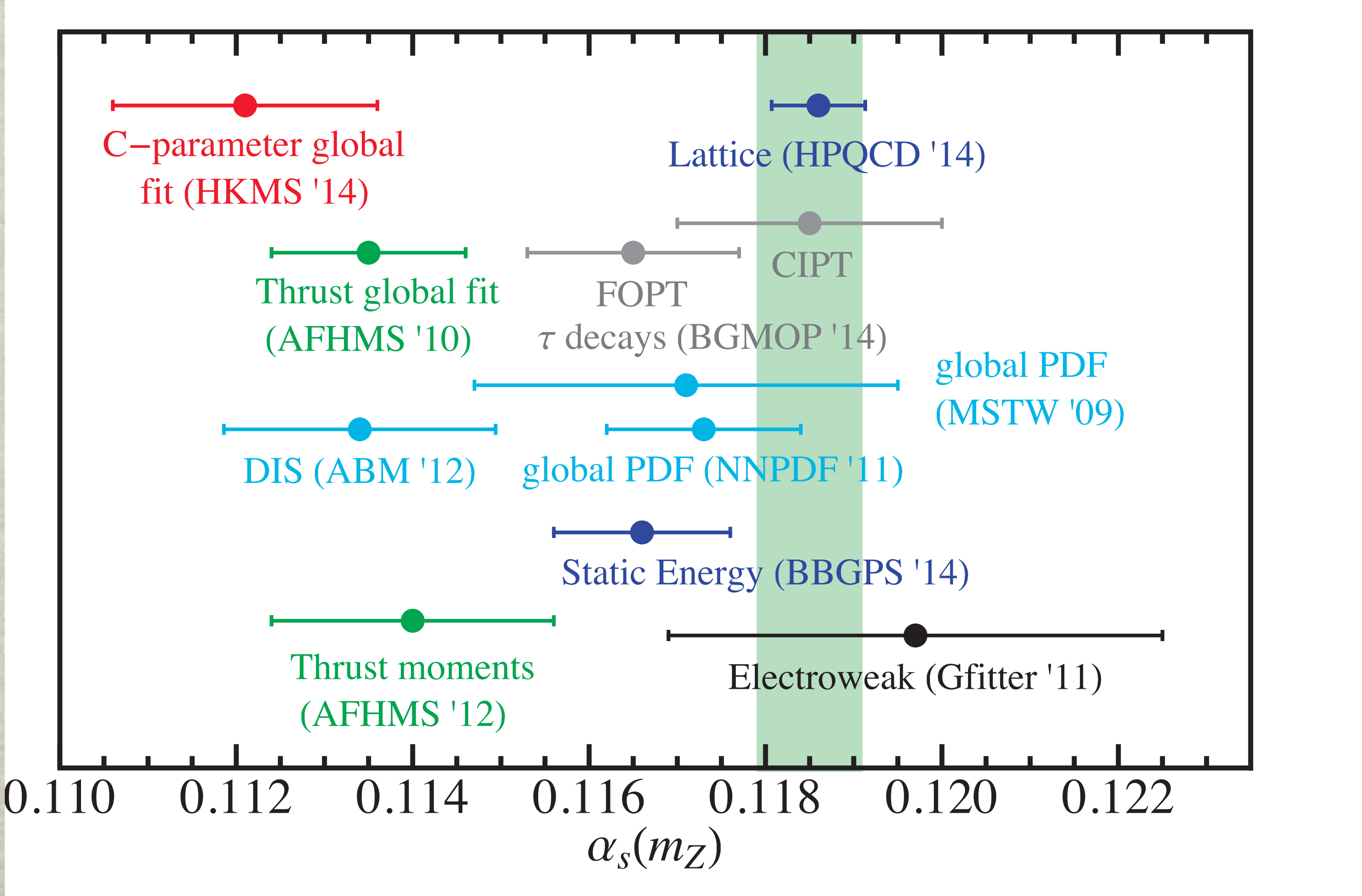

○ Hoang Kolodrubetz Mateu Stewart PRD 91 (2015) 094018

A precise knowledge of α_s is relevant for the physics of SM and BSM

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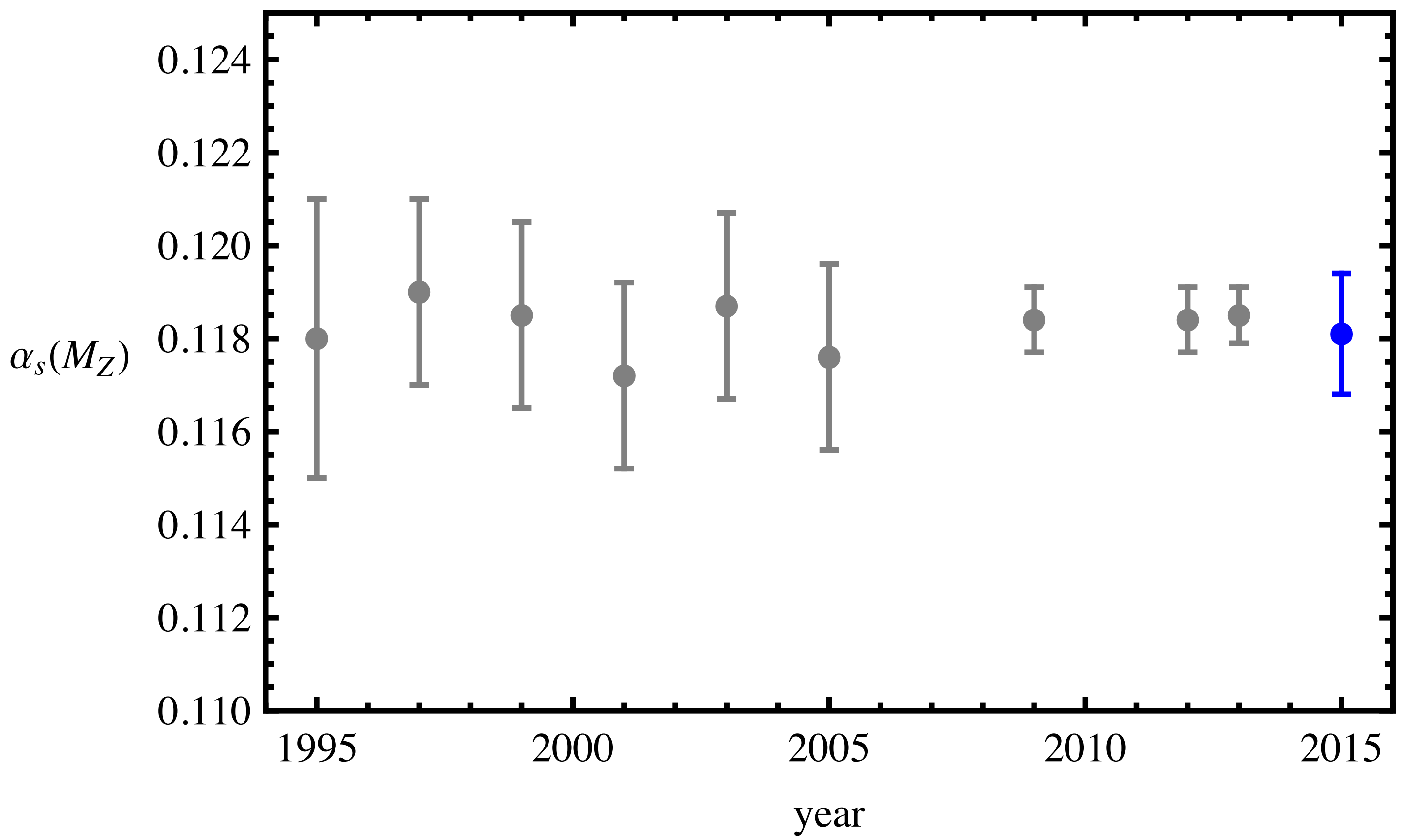
PDG average

In 2015, for the first time in over 20 years, the PDG uncertainty in α_s has increased!



The 2015 PDG average is $\alpha_\mathrm{s}(M_Z) = 0.1181 \pm 0.0013$.

- Bethke Dissertori Salam @ PDG2015 and ISMD2015

A precise knowledge of α_s is relevant for the physics of SM and BSM

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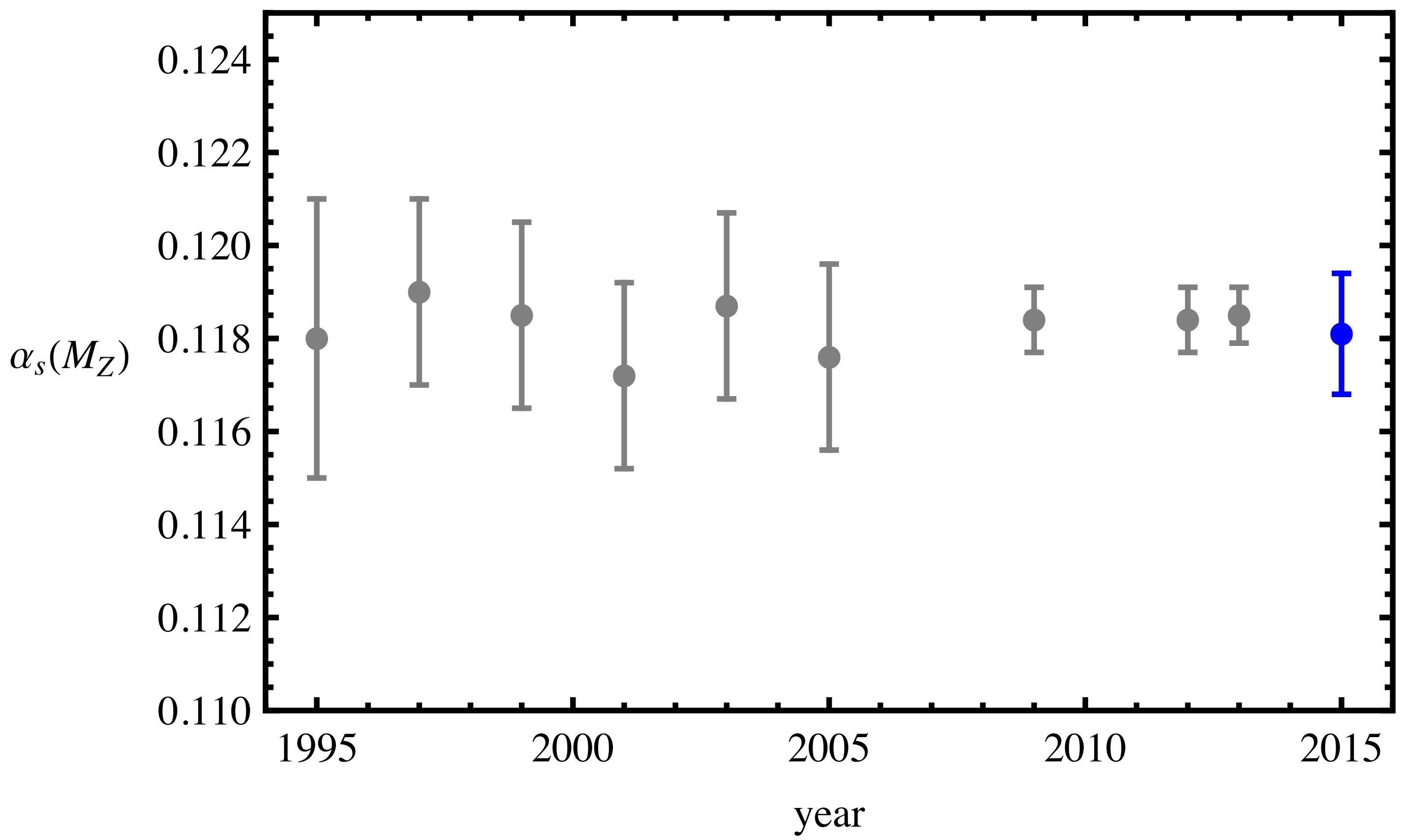

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○ Bethke Dissertori Salam @ PDG2015 and ISMD2015

we plan to reduce the error in our determination: new lattice data at smaller distance, new TUMQCD lattice collaboration N.B., Kronfeld, Petreczky, Vairo, Weber

Applications to Quarkonium physics: systems with small radius

for references see the QWG doc arXiv:1010.5827

- c and b masses at NNLO, N^{3}LO*, NNLL*;
- B_c mass at NNLO; Penin et al 04
- B_c^*, η_c, η_b masses at NLL; Kniehl et al 04
- Quarkonium 1P fine splittings at NLO;
- $\Upsilon(1S)$, η_b electromagnetic decays at NNLL;
- $\Upsilon(1S)$ and J/ψ radiative decays at NLO;
- $\Upsilon(1S) \to \gamma\eta_b$, $J/\psi \to \gamma\eta_c$ at NNLO;
- $t\bar{t}$ cross section at NNLL;
- QQq and QQQ baryons: potentials at NNLO, masses, hyperfine splitting, ... ; N. B. et al 010
- Thermal effects on quarkonium in medium: potential, masses (at $m\alpha_s^5$), widths, ...;

$$\mathcal{B}(J/\psi \to \gamma\eta_c(1S)) = (1.6 \pm 1.1)\%$$

$$\mathcal{B}(\Upsilon(1S) \to \gamma\eta_b(1S)) = (2.85 \pm 0.30) \times 10^{-4}$$

N. B. Yu Jia A. Vairo 2005

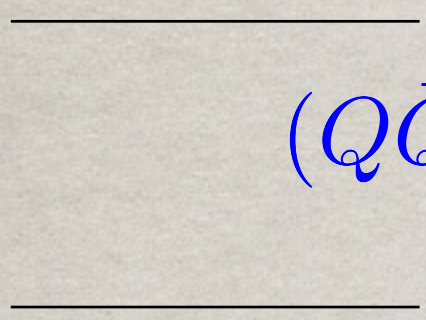
$$\Gamma(\eta_b(1S) \to \gamma\gamma) = 0.54 \pm 0.15 \text{ keV} .$$

$$\Gamma(\eta_b(1S) \to \text{ LH}) = 7\text{-}16 \text{ MeV}$$

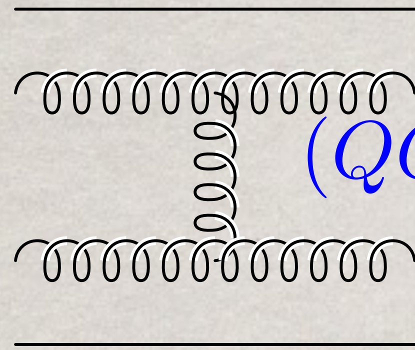
Y. Kiyo, A. Pineda, A. Signer 2010

Quarkonium systems with large radius □ $r \sim \Lambda_{QCD}^{-1}$

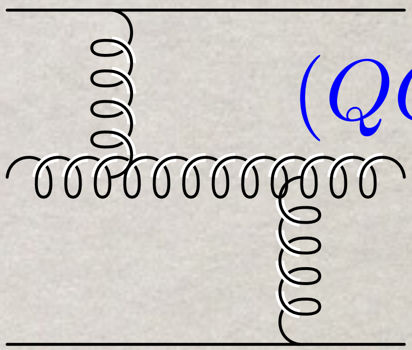
— Hitting the scale $\Lambda_{\rm QCD}$ $\quad r \sim \Lambda_{QCD}^{-1}$



$(Q\bar{Q})_1$



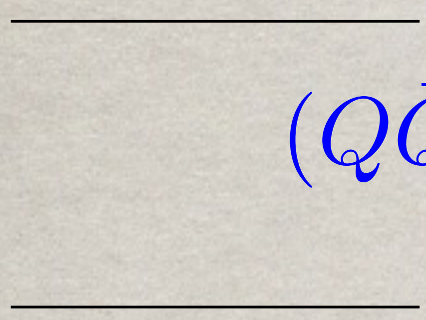
$(Q\bar{Q})_1$ + Glueball



$(Q\bar{Q})_8 G$

hybrid

Hitting the scale $\Lambda_{\rm QCD}$ $\quad r \sim \Lambda_{QCD}^{-1}$



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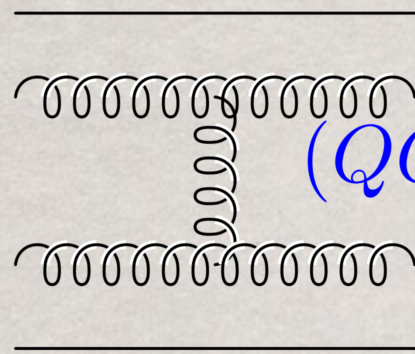

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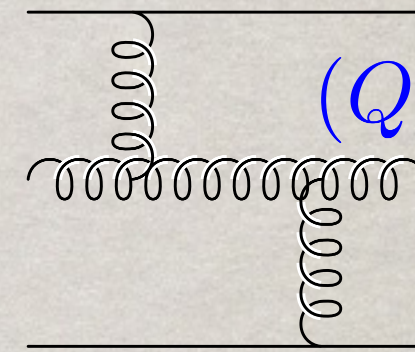

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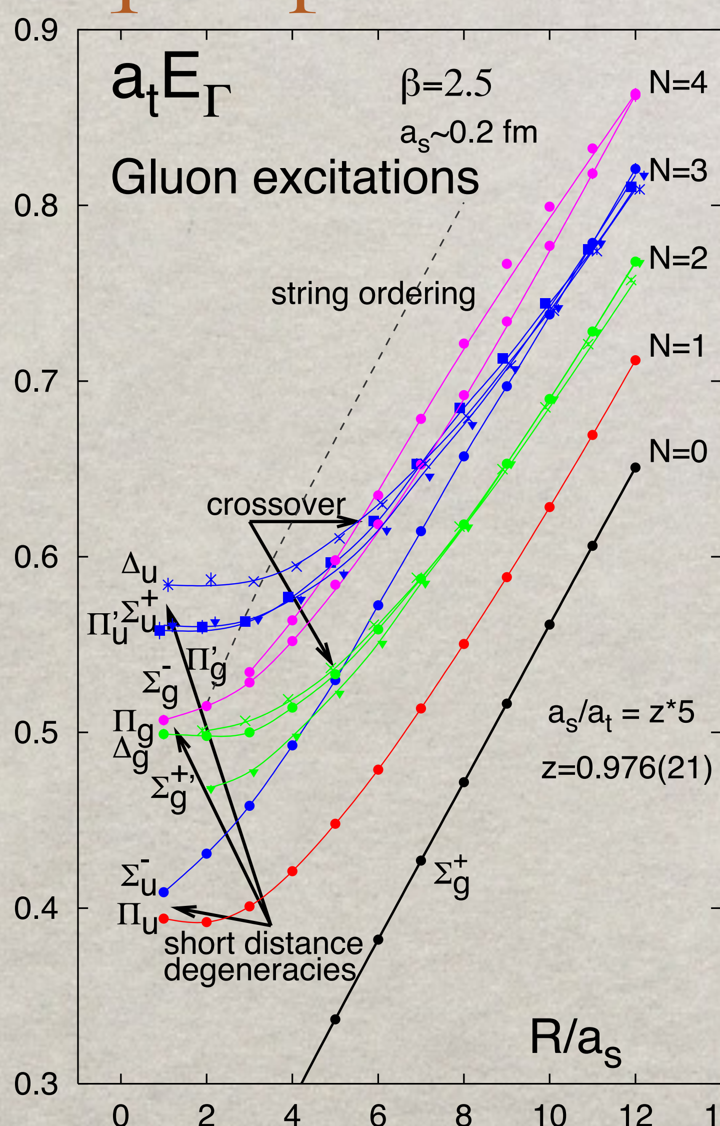
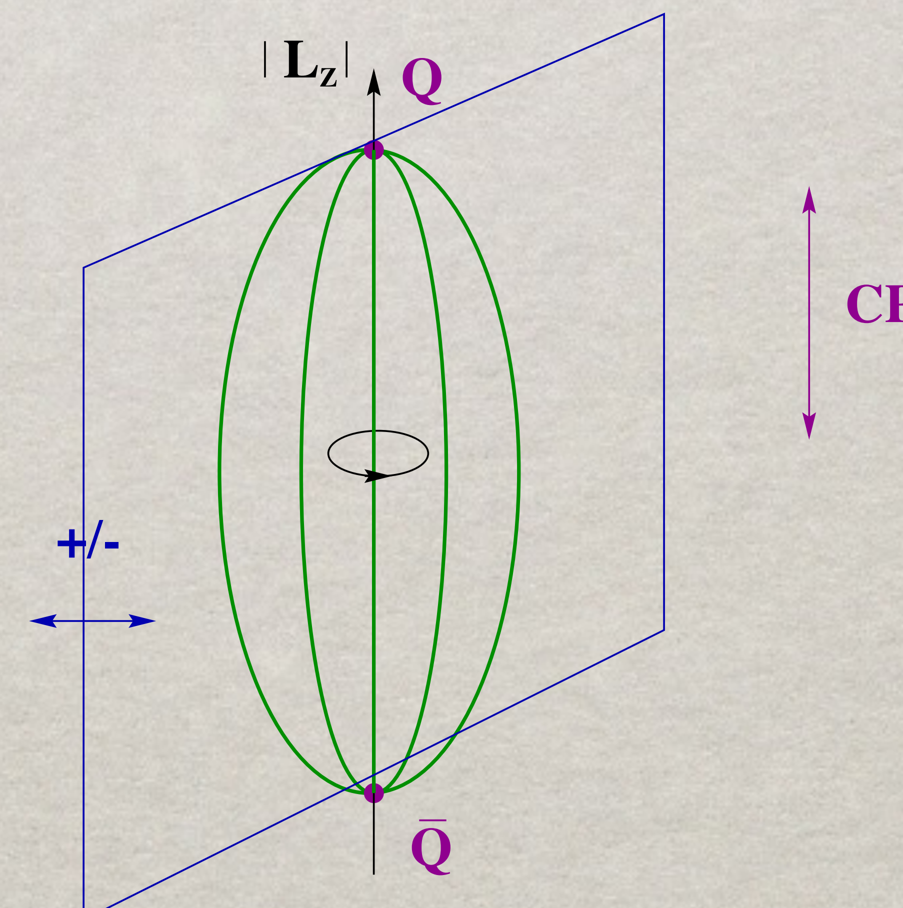
hybrid

Static qcd spectrum

Lattice





Symmetries of a diatomic molecule + C.C.

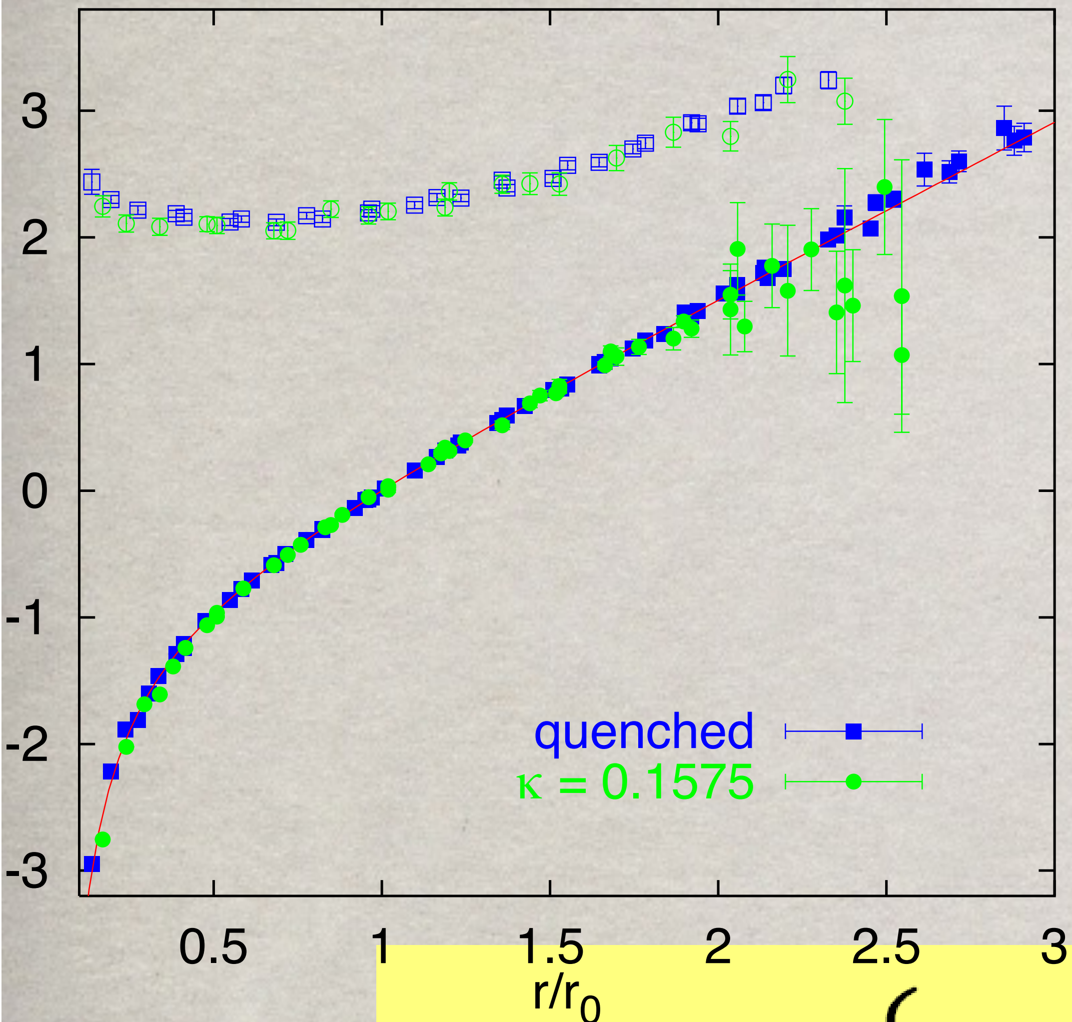
a) $|L_z| = 0, 1, 2, \ldots$
$\quad = \Sigma, \Pi, \Delta \ldots$

b) CP (u/g)

c) Reflection (+/−)
(for Σ only)

Juge Kuti Mornigstar 98-06 Static NRQCD

strongly coupled pNRQCD $r \sim \Lambda_{QCD}^{-1}$ $mv \sim \Lambda_{QCD}$



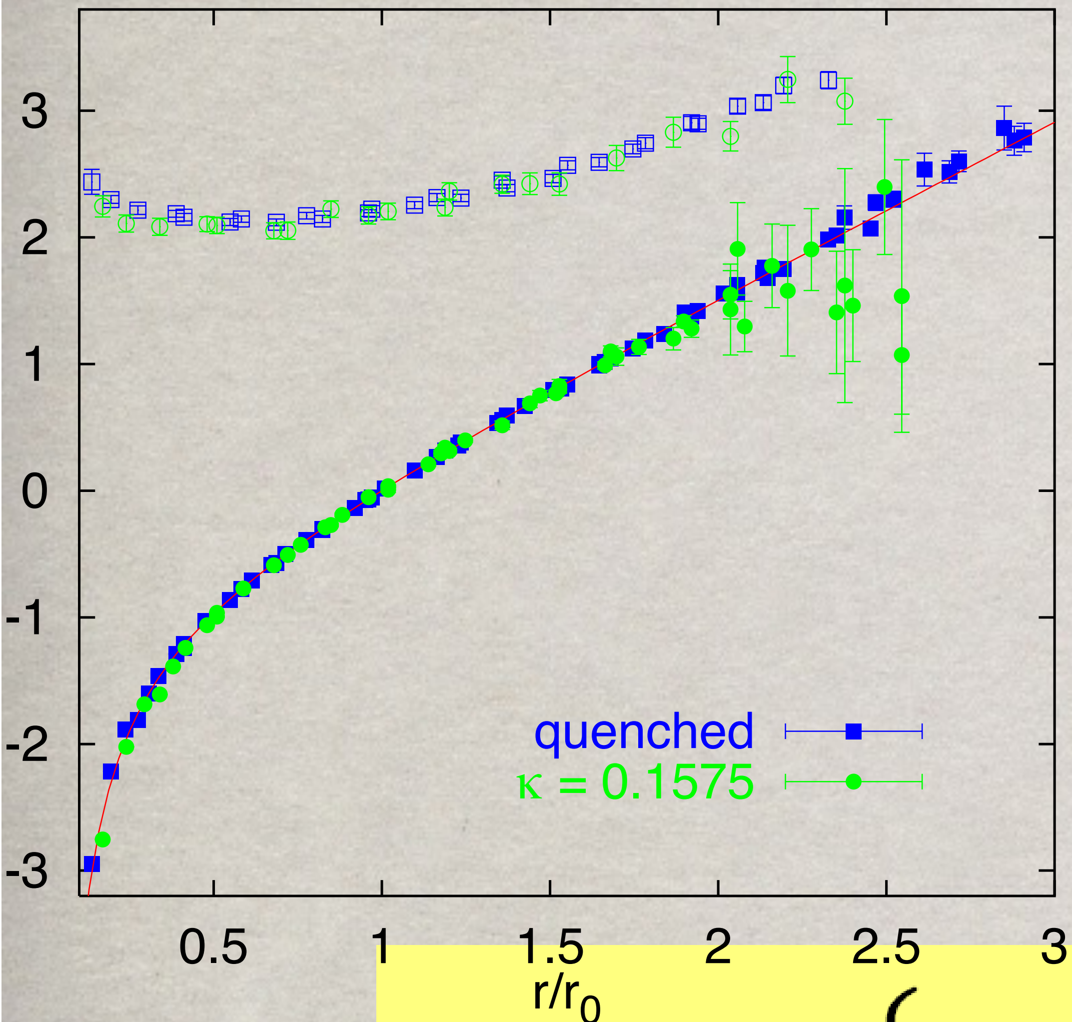
- integrate out all scales above mv^2
- gluonic excitations develop a gap Λ_{QCD} and are integrated out

⇒ The singlet quarkonium field S of energy mv^2 is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

$$\mathcal{L} = \mathrm{Tr}\left\{ \mathrm{S}^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathrm{S} \right\}$$

Brambilla Pineda Soto Vairo 00

strongly coupled pNRQCD $r \sim \Lambda_{QCD}^{-1}$ $mv \sim \Lambda_{QCD}$



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- A potential description emerges from the EFT Brambilla Pineda Soto Vairo 00
- The potentials $V = \mathrm{Re}V + ImV$ from QCD in the matching: get spectra and decays
- V to be calculated on the lattice or in QCD vacuum models

The matching condition is: $\langle H | \mathcal{H} | H \rangle = \langle nljs | \frac{\mathbf{p}^2}{m} + \sum_n \frac{V_s^{(n)}}{m^n} | nljs \rangle$

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$$H_{\rm NRQCD} = H^{(0)} + \frac{1}{m_Q} H^{(1,0)} + \frac{1}{m_{\bar{Q}}} H^{(0,1)} + \dots ,$$

$$H^{(0)} = \int d^3x \frac{1}{2} (\boldsymbol{E}^a \cdot \boldsymbol{E}^a + \boldsymbol{B}^a \cdot \boldsymbol{B}^a) - \sum_{j=1}^{n_f} \int d^3\boldsymbol{x}\, \bar{q}_j\, i\boldsymbol{D} \cdot \boldsymbol{\gamma}\, q_j ,$$

$$H^{(1,0)} = -\frac{1}{2} \int d^3x\, \psi^\dagger \left(\boldsymbol{D}^2 + g c_F\, \boldsymbol{\sigma} \cdot \boldsymbol{B}\right) \psi ,$$

$$H^{(0,1)} = \frac{1}{2} \int d^3x\, \chi^\dagger \left(\boldsymbol{D}^2 + g c_F\, \boldsymbol{\sigma} \cdot \boldsymbol{B}\right) \chi ,$$

$$\mathcal{H}^{(0)} |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = E_n^{(0)}(\mathbf{x}_1, \mathbf{x}_2) |\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

$$|\underline{n}; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)} = \psi^\dagger(\mathbf{x}_1)\chi(\mathbf{x}_2) |n; \mathbf{x}_1, \mathbf{x}_2\rangle^{(0)}$$

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and from this we obtain the

Quarkonium singlet static potential

$$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

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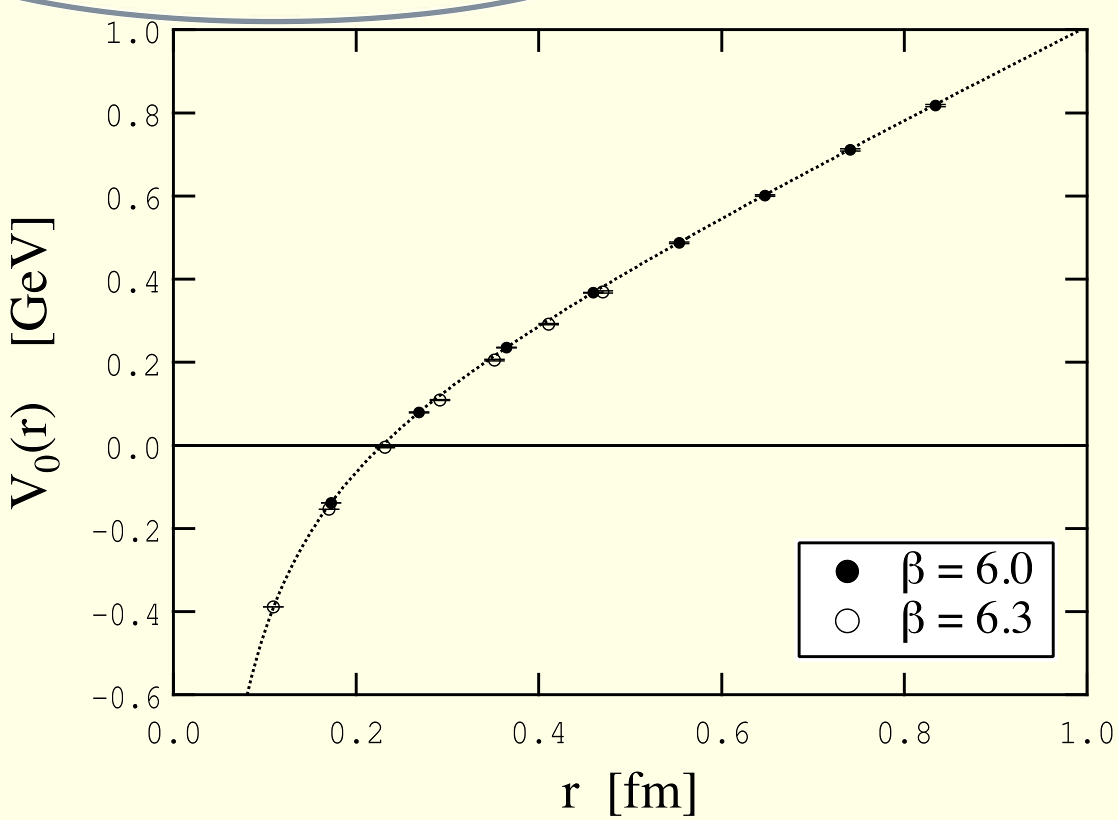
Quarkonium singlet static potential

$$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

$$V_s^{(0)} = \lim_{T\to\infty}\frac{i}{T}\ln\langle W(r\times T)\rangle = \lim_{T\to\infty}\frac{i}{T}\ln\langle \square \rangle$$

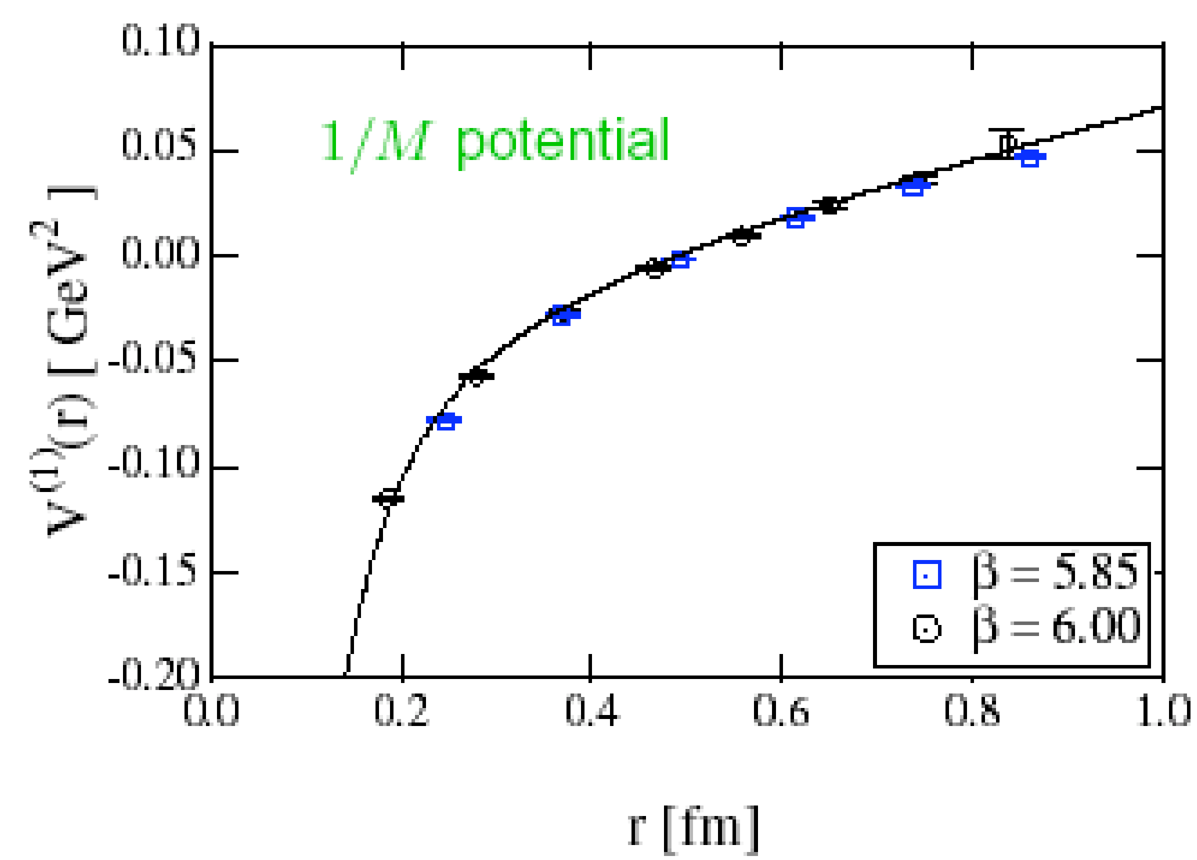
$$W = \langle \exp\{ig\oint A^\mu dx_\mu\}\rangle$$



○ Koma Koma NPB 769(07)79

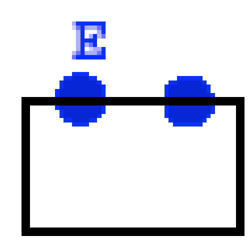
Quarkonium singlet static potential

Potentials are given in a factorized form as product of NRQCD matching coefficients and low energy terms. These are gauge invariant wilson loop with electric and magnetic insertions



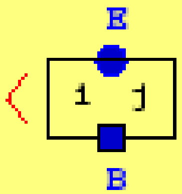
∘ Koma Koma Wittig PoS LAT2007(07)111

$$\frac{V_s^{(1)}}{m} = -\frac{1}{2m}\int_0^\infty dt\, t \left\langle \quad \right\rangle$$

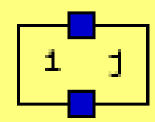


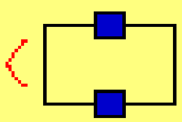
Brambilla et al 00

QCD Spin dependent potentials

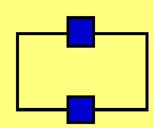
$V^{(2)}_{\rm SD} = \frac{1}{r}\Big(c_F \epsilon^{kij} \frac{2r^k}{r} i \int_0^\infty dt\, t \,\langle$ 

$\rangle - \frac{1}{2} V_s^{(0)\prime} \Big) (\mathbf{S}_1 + \mathbf{S}_2)\cdot \mathbf{L}$

$- {c_F}^2 \hat{r}_i \hat{r}_j i \int_0^\infty dt \Big(\langle$ 

$\rangle - \frac{\delta_{ij}}{3} \langle$  $\rangle \Big)$

$\times \Big(\mathbf{S}_1 \cdot \mathbf{S}_2 - 3(\mathbf{S}_1 \cdot \hat{\mathbf{r}})(\mathbf{S}_2 \cdot \hat{\mathbf{r}}) \Big)$

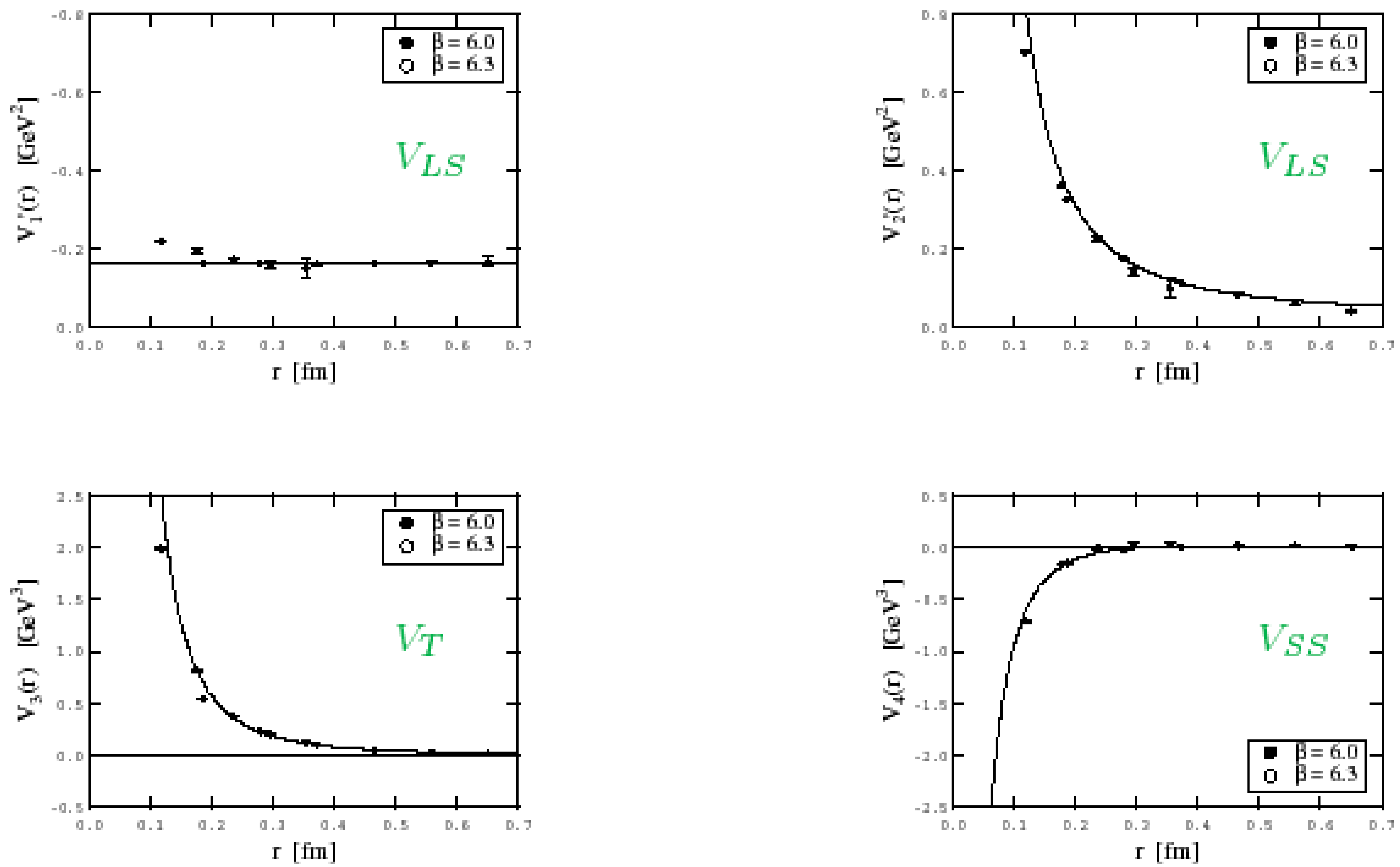
$+ \Big(\frac{2}{3} {c_F}^2 i \int_0^\infty dt \,\langle$  $\rangle - 4(d_2 + C_F d_4)\delta^{(3)}(\mathbf{r}) \Big) \mathbf{S}_1 \cdot \mathbf{S}_2$

Eichten Feinberg 81, Gromes 84, Chen et al. 95 Brambilla Vairo 99 Pineda, Vairo 00

-factorization: the NRQCD matching coefficients encode the physics at the large scale m, the potentials are given in terms of low energy nonperturbative Wilson loops

power counting; QM divergences absorbed NRQCD matching coefficients

Spin dependent potentials



Koma Koma Wittig 05, Koma Koma 06

Terrific advance in the data precision with Lüscher multivel algorithm!

Such data can distinguish different models for the dynamics of low energy QCD e.g. effective string model

N. B., Martinez, vairo 2014

For states close or above the strong decay threshold the situation is much more complicated.

there is no mass gap between quarkonium and the creation of a heavy-light mesons couple

$$m_{Q\bar{q}} + m_{\bar{Q}q} = 2m + 2\Lambda_{QCD}$$

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Many phenomenological models exist

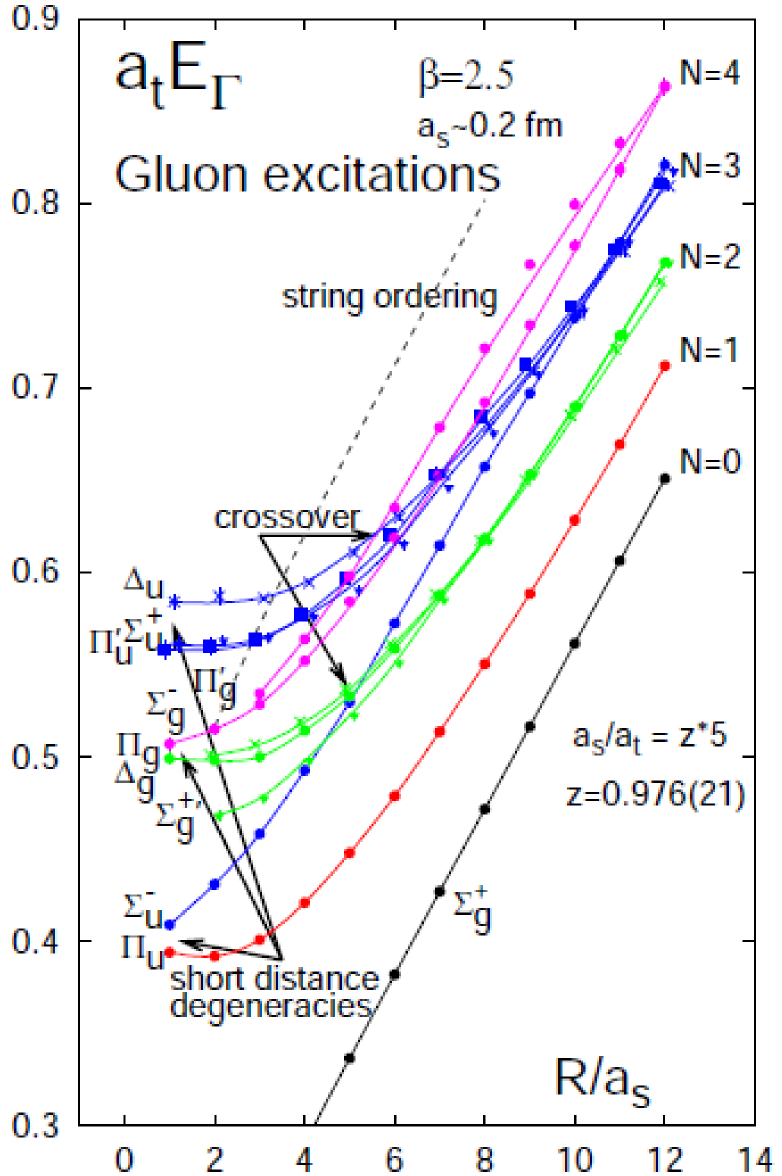
Heavy-quark heavy antiquark plus glue

Heavy-quark heavy antiquark plus glue

Define the symmetries of the system and the system static energies in NRQCD

Static Lattice energies

Juge Kuti Morningstar 2003

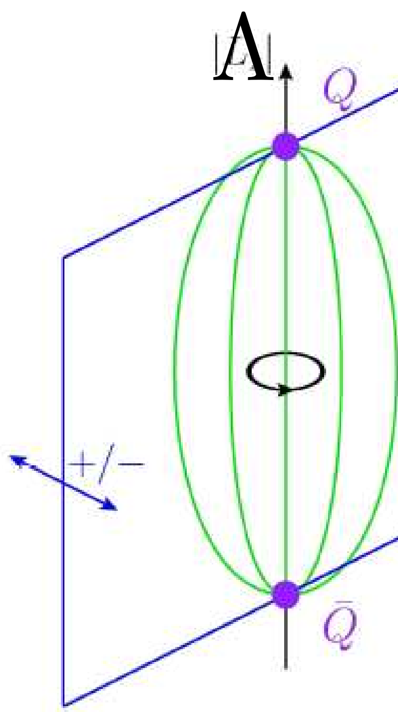


Symmetries

Static states classified by symmetry group $D_{\infty h}$

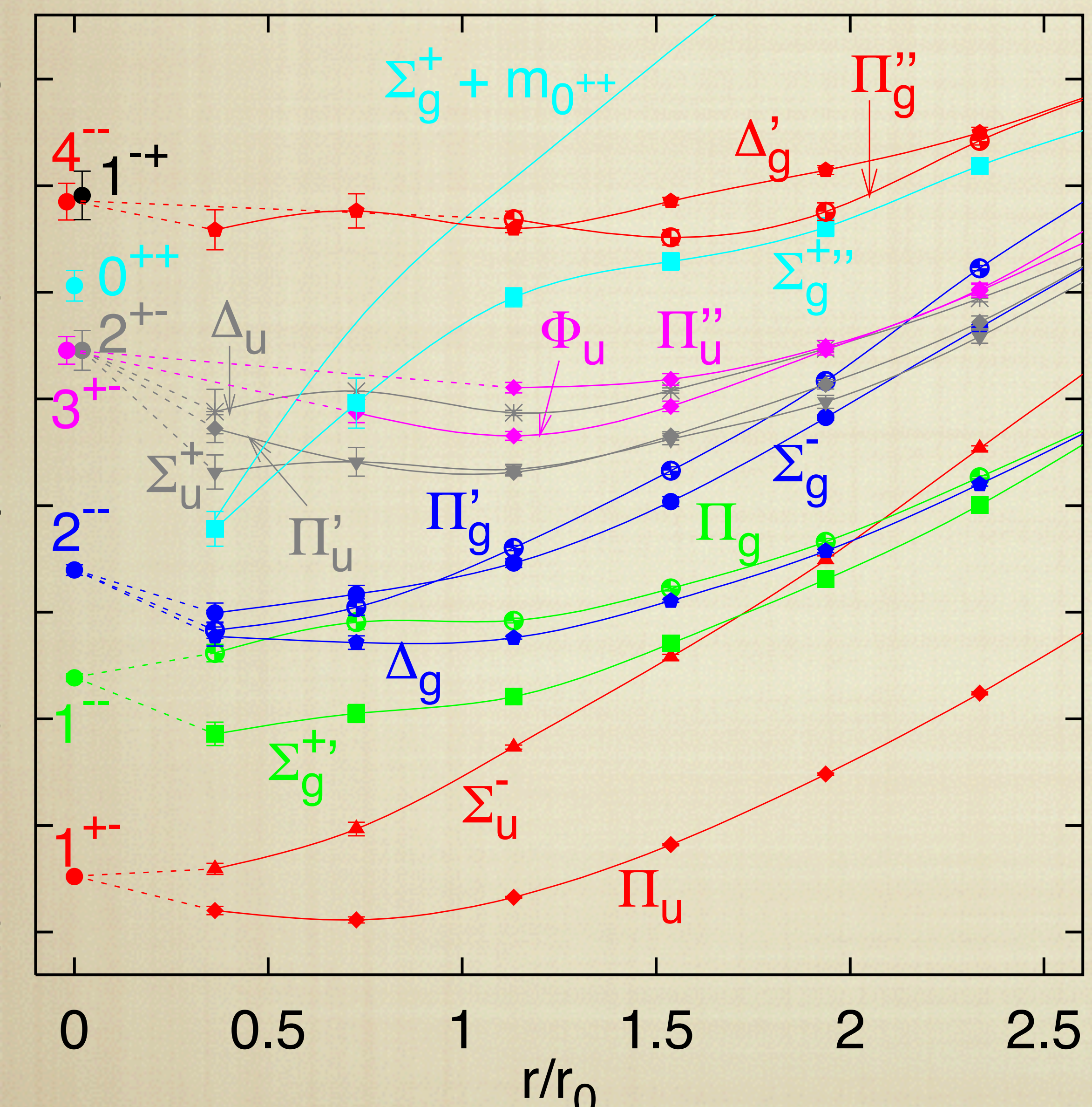
Representations labeled Λ^σ_η

- Λ rotational quantum number $|\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2\dots$ corresponds to $\Lambda = \Sigma, \Pi, \Delta\dots$
- η eigenvalue of CP: $g \hat{=} +1$ (gerade), $u \hat{=} -1$ (ungerade)
- σ eigenvalue of reflections
- σ label only displayed on Σ states (others are degenerate)



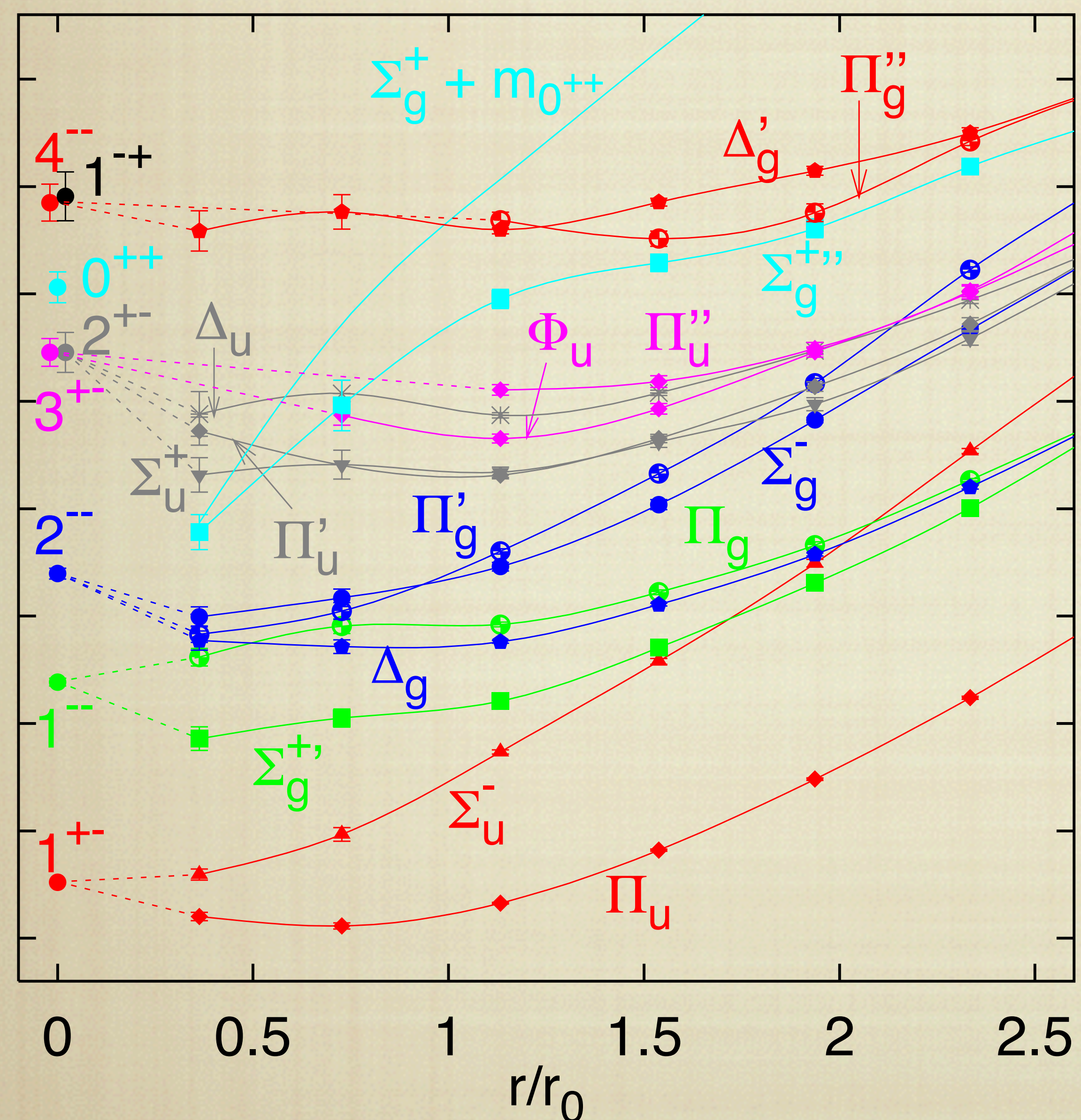
- The static energies correspond to the irreducible representations of $D_{\infty h}$
- In general it can be more than one state for each irreducible representation of $D_{\infty h}$, usually denoted by primes, e.g. Π_u, Π'_u, Π''_u...

pNRQCD gives the multiplets at short distance:gluelumps



pNRQCD gives the multiplets at short distance:gluelumps

In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.



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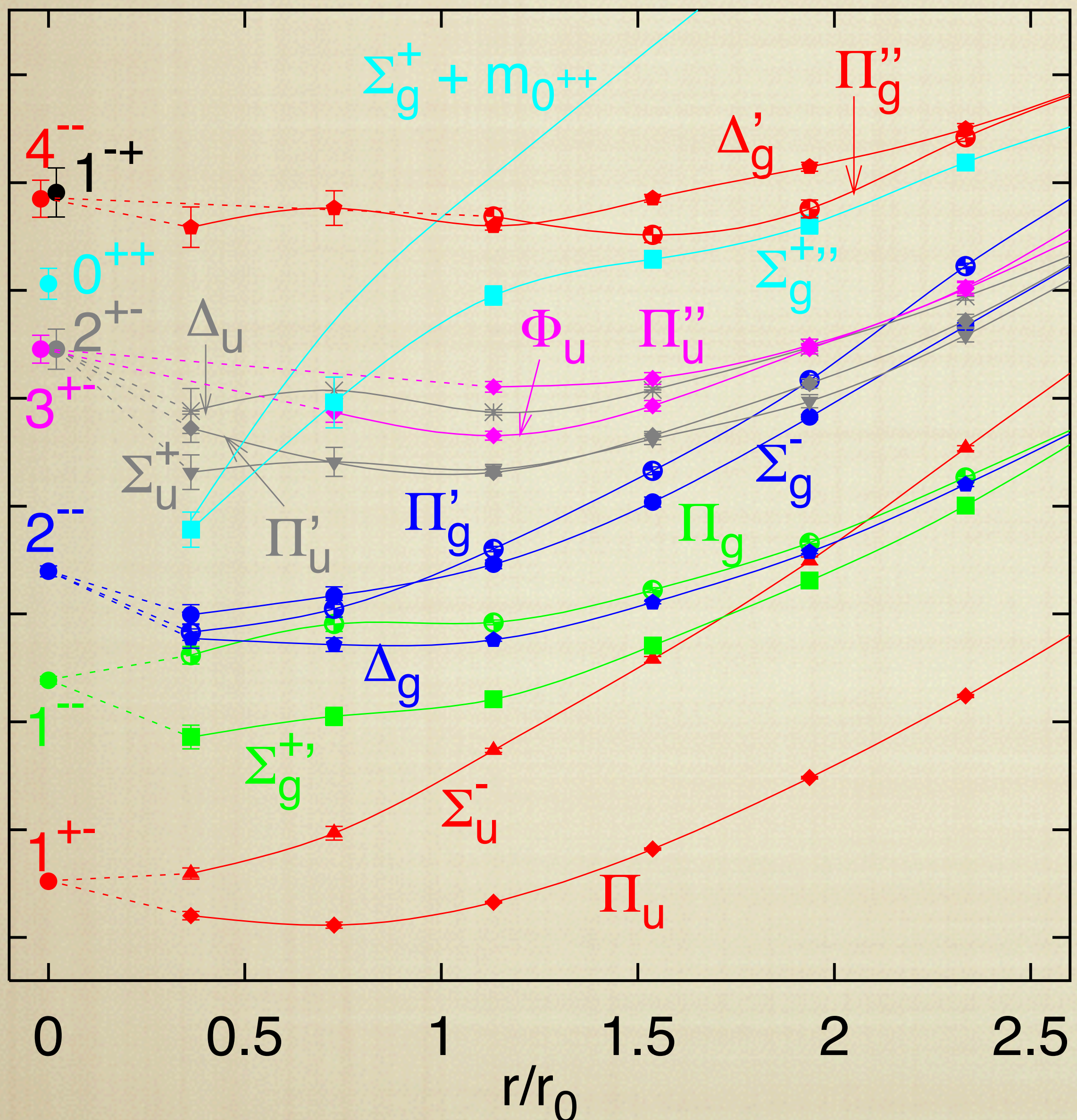
In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several Λ_η^σ representations contained in one J^{PC} representation:
- Static energies in these multiplets have same $r \to 0$ limit.



Gluonic excitation operators up to dim 3

Λ_η^σ	K^{PC}	H^a
Σ_u^-	1^{+-}	$\mathbf{r}\cdot\mathbf{B}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{E})$
Π_u	1^{+-}	$\mathbf{r}\times\mathbf{B}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{E})$
$\Sigma_g^{+\prime}$	1^{--}	$\mathbf{r}\cdot\mathbf{E}$, $\mathbf{r}\cdot(\mathbf{D}\times\mathbf{B})$
Π_g	1^{--}	$\mathbf{r}\times\mathbf{E}$, $\mathbf{r}\times(\mathbf{D}\times\mathbf{B})$
Σ_g^-	2^{--}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{B})$
Π_g'	2^{--}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{B}+\mathbf{D}(\mathbf{r}\cdot\mathbf{B}))$
Δ_g	2^{--}	$(\mathbf{r}\times\mathbf{D})^i(\mathbf{r}\times\mathbf{B})^j+(\mathbf{r}\times\mathbf{D})^j(\mathbf{r}\times\mathbf{B})^i$
Σ_u^+	2^{+-}	$(\mathbf{r}\cdot\mathbf{D})(\mathbf{r}\cdot\mathbf{E})$
Π_u'	2^{+-}	$\mathbf{r}\times((\mathbf{r}\cdot\mathbf{D})\mathbf{E}+\mathbf{D}(\mathbf{r}\cdot\mathbf{E}))$
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Brambilla Pineda Soto Vairo 00

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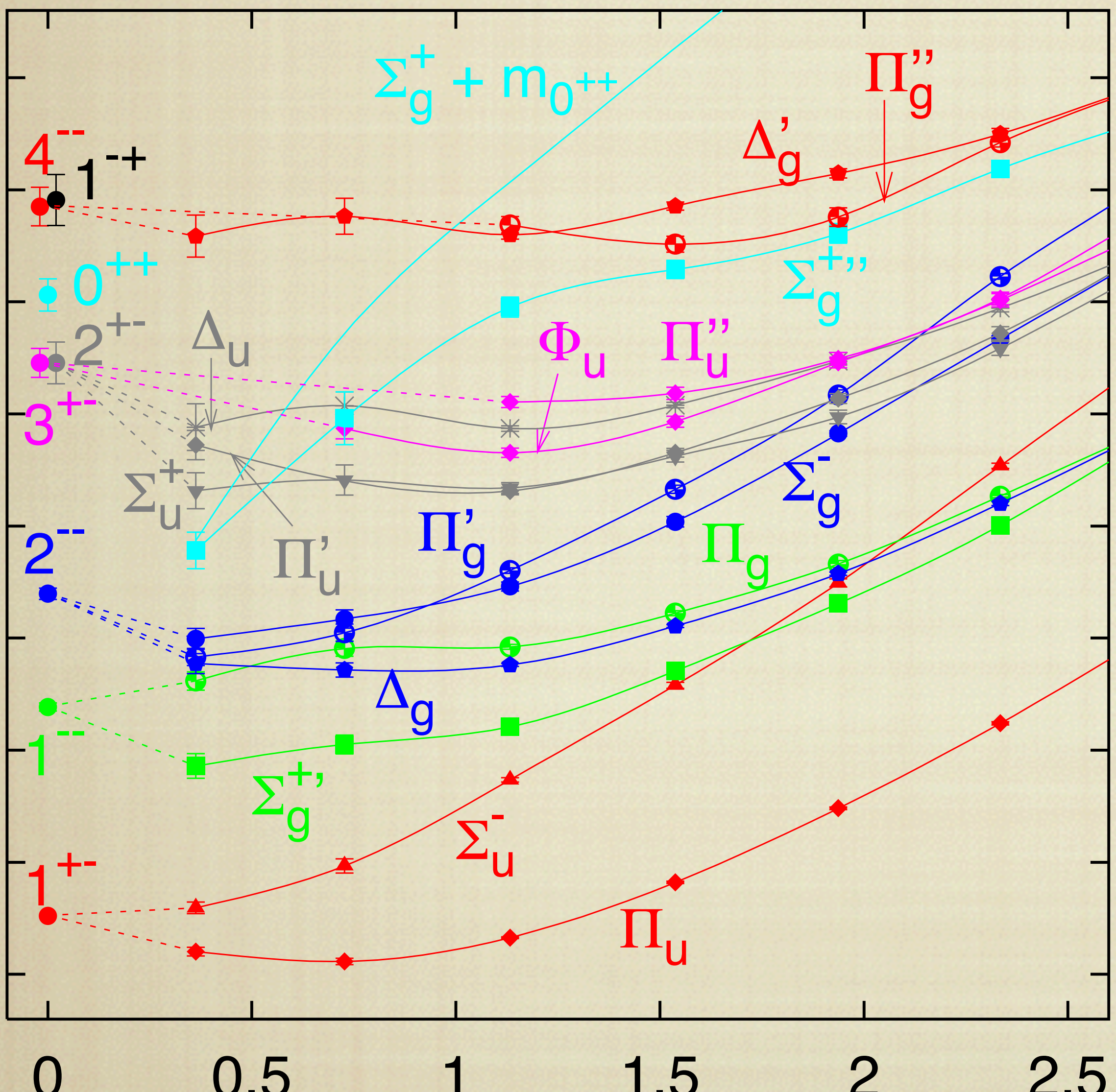

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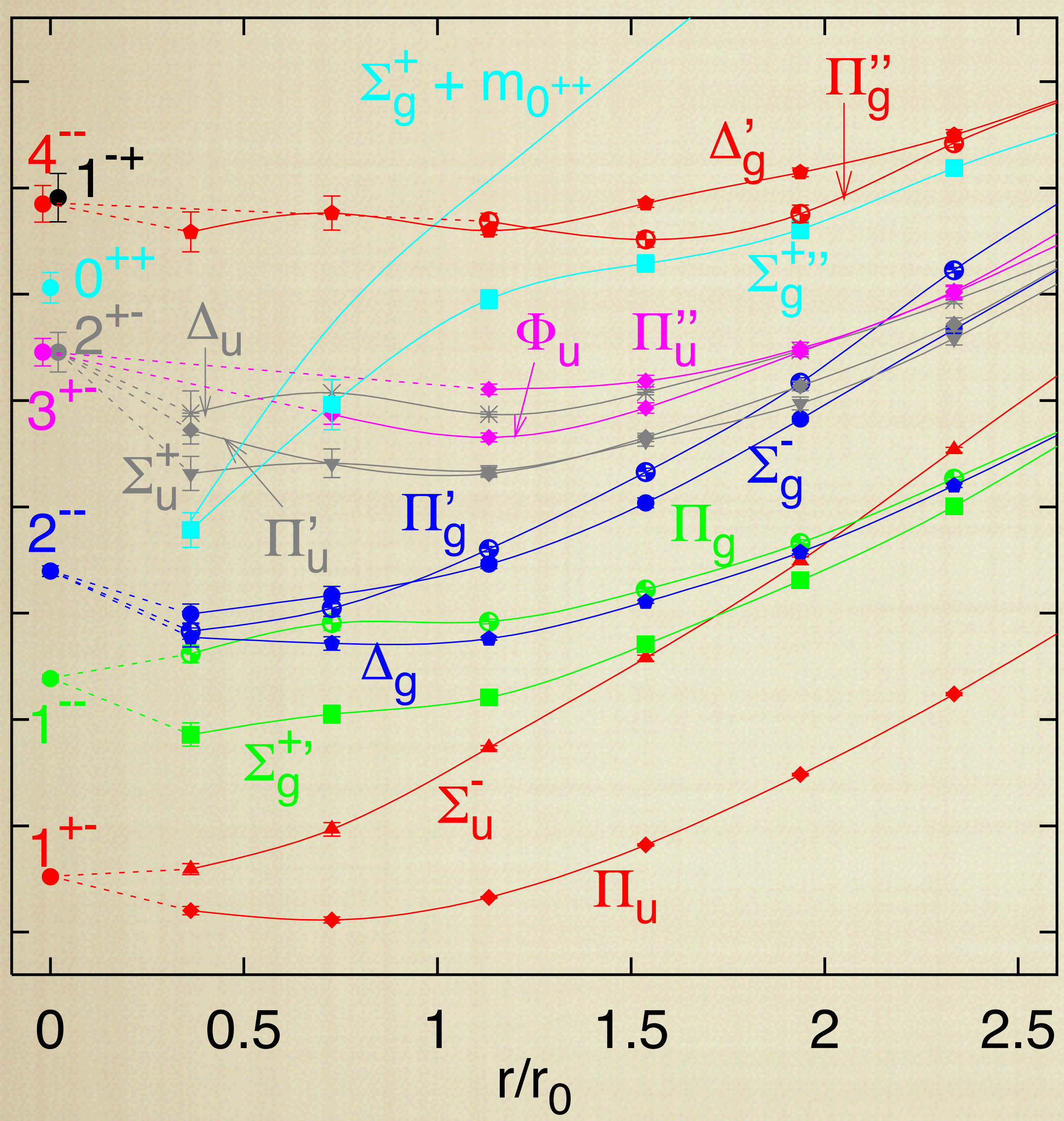
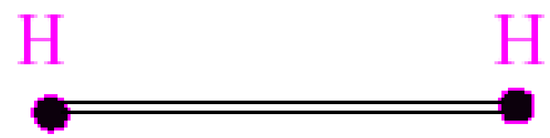
Brambilla Pineda Soto Vairo 00

The gluelump multiplets Σ_u^-, Π_u; $\Sigma_g^{+\prime}$, Π_g; Σ_g^-, Π'_g, Δ_g; Σ_u^+, Π'_u, Δ_u are degenerate.

Match to pNRQCD: one can determine the form of the potential

In the short-range hybrids become gluelumps, i.e., quark-antiquark octets, O^a, in the presence of a gluonic field, H^a: $H(R,r,t) = H^a(R,t)O^a(R,r,t)$.





$$= e^{-iTE_H}$$

$$E_H = V_o + \frac{i}{T}\ln\langle H^a(\frac{T}{2})\phi^{\rm adj}_{ab}H^b(-\frac{T}{2})\rangle$$

$$\langle H^a(\frac{T}{2})\phi^{\rm adj}_{ab}H^b(-\frac{T}{2})\rangle^{\rm np} \sim h\, e^{-iT\Lambda_H}$$

$$E_H(r) = V_o(r) + \Lambda_H \quad + b_{\Lambda_H} r^2$$

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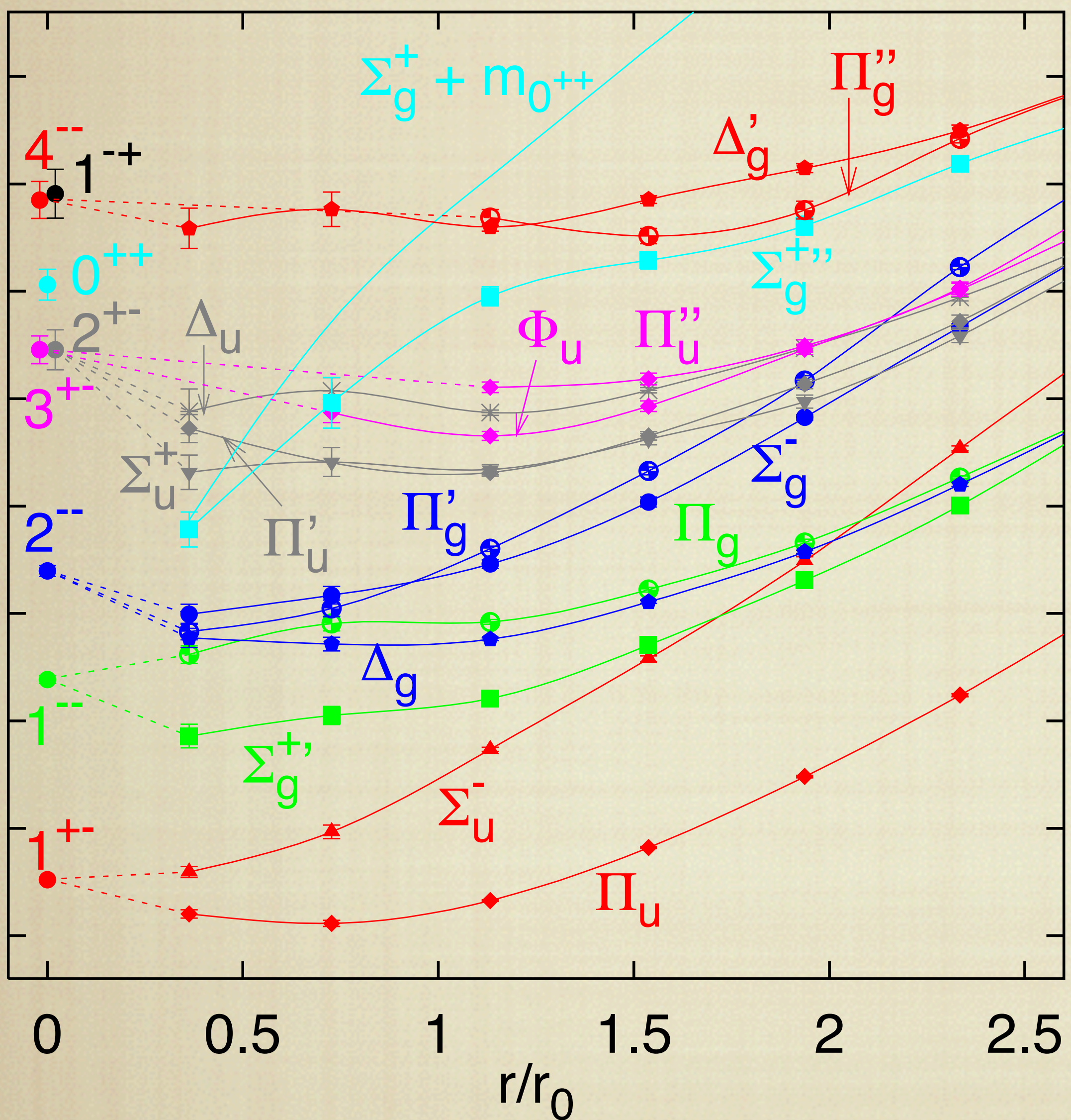

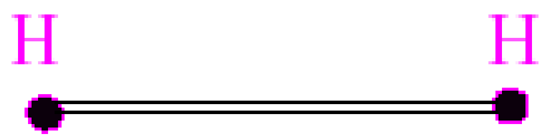

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octet potential

gluelump mass

correction softly breaking the symm

Match to pNRQCD: one can determine the form of the potential

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$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

Λ_H

- It is a non-perturbative quantity.
- It depends on the particular operator H^a, however it is the same for operators corresponding to different projections of the same gluonic operators.
- The gluelump masses have been determined in the lattice. Foster *et all* 1999; Bali, Pineda 2004; Marsh Lewis 2014
- At the subtraction scale $\nu_f = 1$ GeV: $\Lambda^{RS}_{1^{+-}} = 0.87(15)$ GeV.

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- We are going to fix it through a fit to the static energies lattice data.
- Breaks the degeneracy of the potentials.

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Octet potential at two loops; renormalon subtraction realised among pole mass, octet potential and gluelump mass, use RS scheme

Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

Coupled radial Schrödinger equations

Projection vectors in matrix elements allow for two different solutions (coupled or uncoupled) for the Σ_u^- and Π_u radial wave functions:

1st solution

$$\left[-\frac{1}{2\mu r^2}\,\partial_r r^2 \partial_r + \frac{1}{2\mu r^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)}\end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma \\ \psi_\Pi\end{pmatrix} = \mathcal{E}\begin{pmatrix}\psi_\Sigma \\ \psi_\Pi\end{pmatrix}$$

2nd solution

$$\left[-\frac{1}{2\mu r^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{2\mu r^2} + E_\Pi^{(0)}\right]\psi_\Pi = \mathcal{E}\,\psi_\Pi$$

- energy eigenvalue $\mathcal{E}$ gives hybrid mass: $m_H = m_Q + m_{\bar{Q}} + \mathcal{E}$
- $l(l+1)$ is the eigenvalue of angular momentum $\boldsymbol{L}^2 = \left(\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{L}_g\right)^2$
- the two solutions correspond to **opposite parity** states: $(-1)^l$ and $(-1)^{l+1}$
- corresponding eigenvalues under charge conjugation: $(-1)^{l+s}$ and $(-1)^{l+s+1}$
- Schrödinger equations can be solved numerically

For $l = 0$ the off-diagonal terms vanish, so the equations for $\psi_\Sigma^{(N)}$ and $\psi_{-\Pi}^{(N)}$ decouple. There exists only one parity state, and its radial wave function is given by a Schrödinger equation with the $E_\Sigma^{(0)}$ potential and an angular part $2/mr^2$.

Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019

$$E_H(r) = V_O(r) + \Lambda_H + b_H r^2$$

The Lambda -doubling effect breaks the degeneracy between opposite parity spin-symmetry multiplets and lowers the mass of the multiplets that get mixed contributions of different static energies.

1st solution

$$\left[-\frac{1}{2\mu r^2}\,\partial_r r^2 \partial_r + \frac{1}{2\mu r^2}\begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E_\Sigma^{(0)} & 0 \\ 0 & E_\Pi^{(0)}\end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma \\ \psi_\Pi\end{pmatrix} = \mathcal{E}\begin{pmatrix}\psi_\Sigma \\ \psi_\Pi\end{pmatrix}$$

2nd solution

$$\left[-\frac{1}{2\mu r^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{2\mu r^2} + E_\Pi^{(0)}\right]\psi_\Pi = \mathcal{E}\,\psi_\Pi$$

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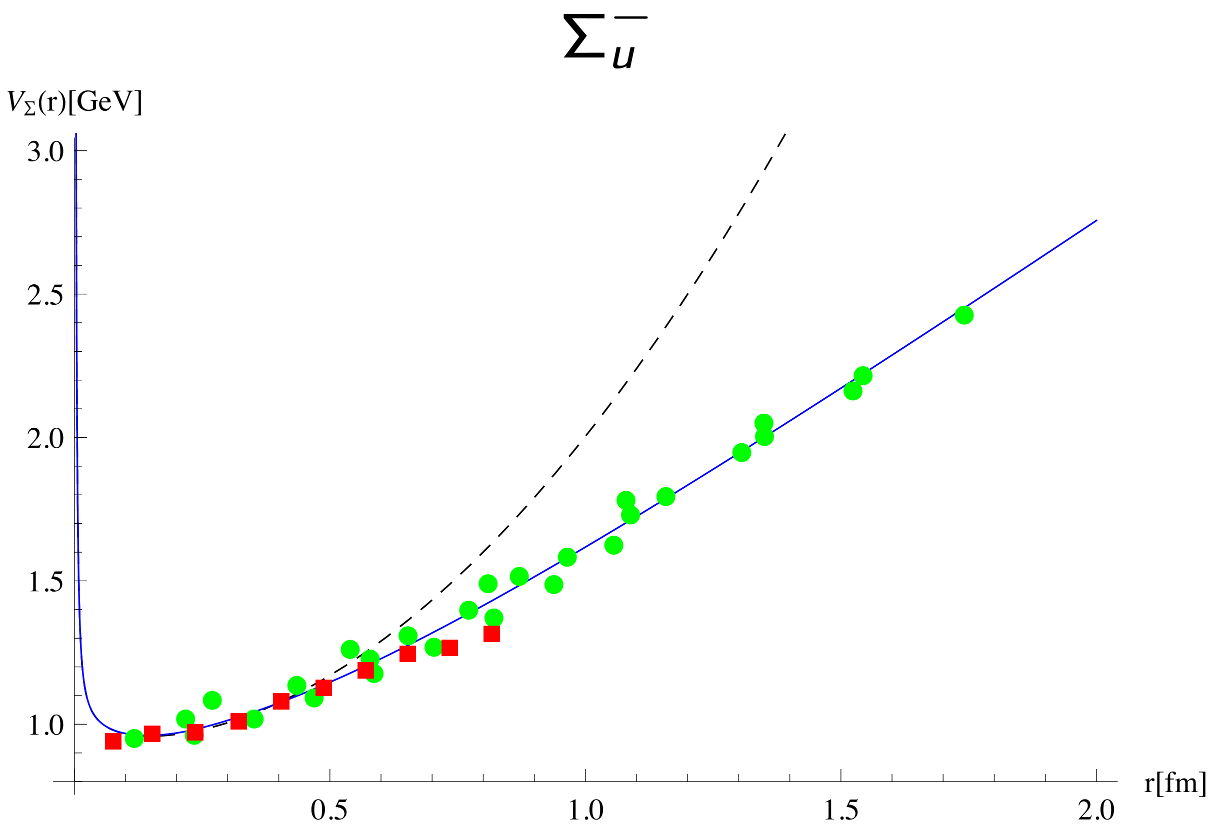
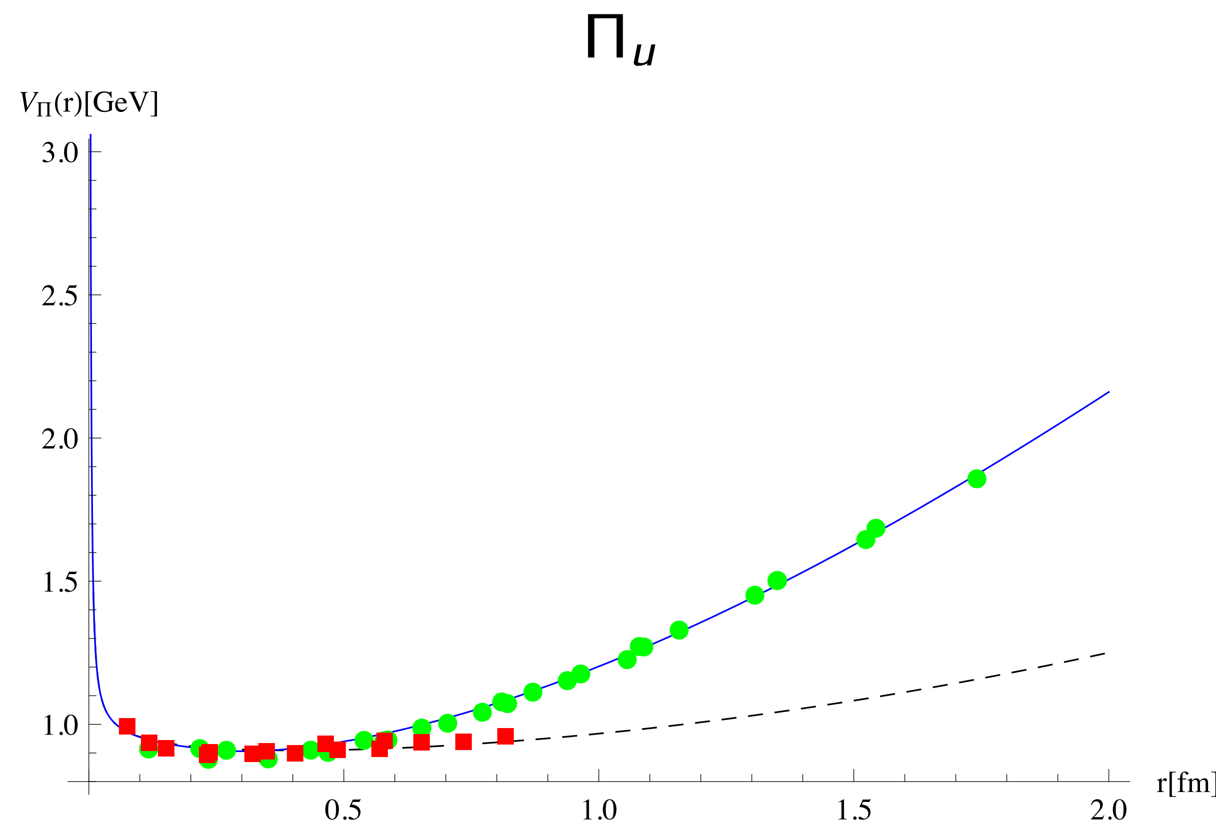




Lattice data: Bali, Pineda 2004; Juge, Kuti, Morningstar 2003, dashed line $V^{(0.5)}$, solid line $V^{(0.25)}$

$V^{(0.25)}$

- $r \leq 0.25$ fm: pNRQCD potential.
  - Lattice data fitted for the $r = 0 - 0.25$ fm range with the same energy offsets as in $V^{(0.5)}$.

$$b_\Sigma^{(0.25)} = 1.246\,\mathrm{GeV/fm}^2, \quad b_\Pi^{(0.25)} = 0.000\,\mathrm{GeV/fm}^2\,.$$

- $r > 0.25$ fm: phenomenological potential.
  - $\mathcal{V}'(r) = \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4$.
  - Same energy offsets as in $V^{(0.25)}$.
  - *Constraint:* Continuity up to first derivatives.

Berwein, N.B. , Tarrus, Vairo arXiv:1510.04299

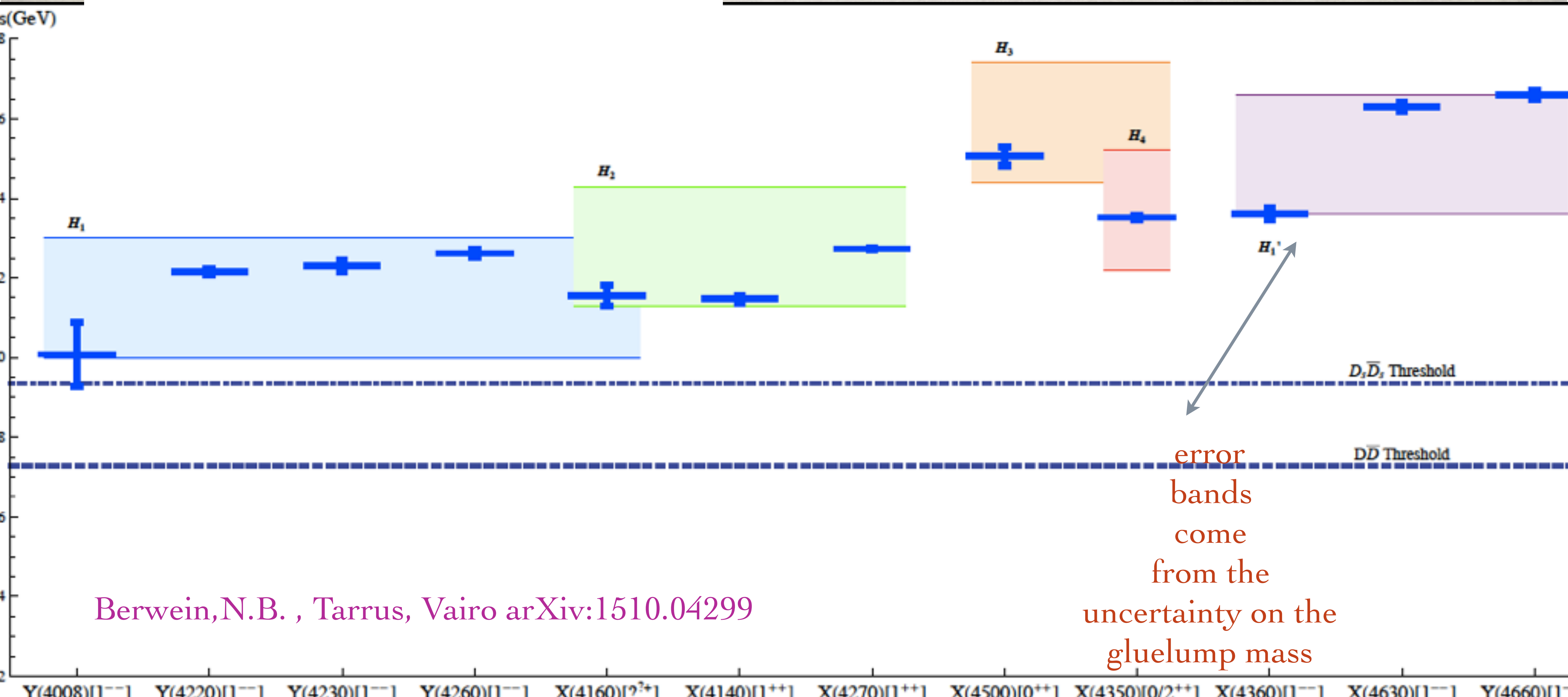
Identification with experimental states

Charmonium states

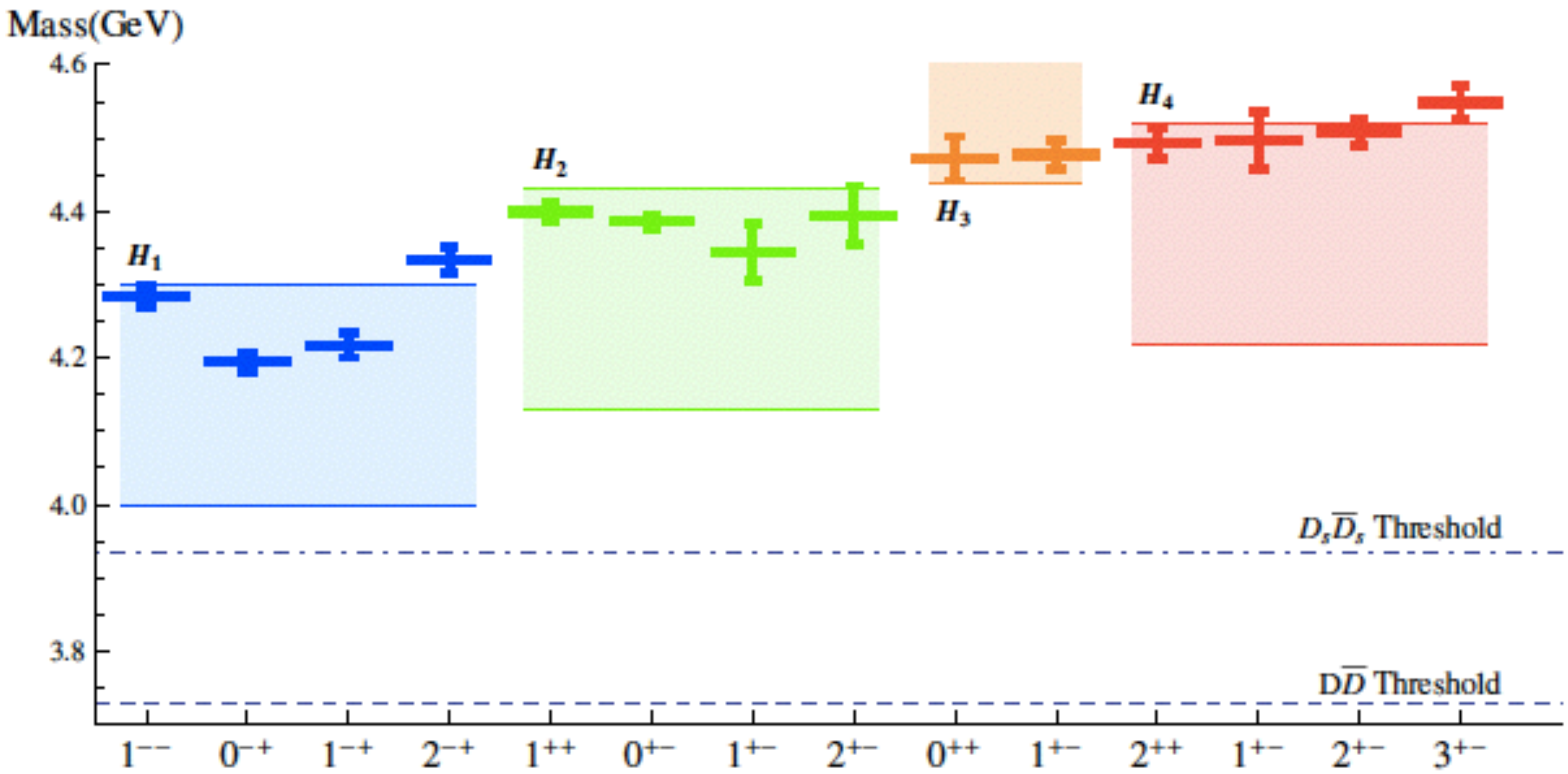
- Spin symmetry multiplets

H_1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u
H_6	$\{3^{--}, (2,3,4)^{-+}\}$	Σ_u^-, Π_u
H_7	$\{3^{++}, (2,3,4)^{+-}\}$	Π_u

we observe the Lambda-doubling pattern of molecular physics, multiplets that receive mixed contributions from Sigma_u and Pi_u have lower masses then those that remain pure Pi_u states



Charmonium hybrid states vs direct lattice data



- Berwein Brambilla Tarrus Vairo PRD 92 (2015) 114019
  lattice data from Liu et al JHEP 1207 (2012) 126

in the paper

The Born-Oppenheimer approximation in effective field theory language

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Gastão Krein†
Instituto de Física Teórica, Universidade Estadual Paulista Rua Dr. Bento Teobaldo Ferraz, 271 - Bloco II, 01140-070 São Paulo, SP, Brazil

Jaume Tarrús Castellà‡ and Antonio Vairo§
Physik-Department, Technische Universität München, James-Franck-Str. 1, 85748 Garching, Germany

we considered more general eigenstates of the octet sector the pNRQCD hamiltonian

$$|\kappa\rangle = O^{a\dagger}(\boldsymbol{r}, \boldsymbol{R})\, G^a_{i\kappa}(\boldsymbol{R})|\mathrm{US}\rangle\,,$$

$$\kappa = \{J^{PC}, f\}\,,$$

light flavour

project on $\int d^3r\, d^3R \sum_{i\kappa} |\kappa\rangle\, \Psi_{i\kappa}(t, \boldsymbol{r}, \boldsymbol{R}).$

obtain

$$L_{BO} = \int d^3R\, d^3r \sum_{\kappa} \Psi^\dagger_{i\kappa}(t, \boldsymbol{r}, \boldsymbol{R})\Big[(i\partial_t - h_o - \Lambda_\kappa)\,\delta^{ij} - \sum_\lambda P^i_{\kappa\lambda} b_{\kappa\lambda} r^2 P^j_{\kappa\lambda} + \cdots\Big]\Psi_{j\kappa}(t, \boldsymbol{r}, \boldsymbol{R})\,,$$

gives origin to a coupled Schroedinger equation

$$i\partial_t \Psi_{\kappa\lambda}(t, \boldsymbol{r}, \boldsymbol{R}) = \left[\left(-\frac{\nabla^2_r}{M} + V_o(r) + \Lambda_\kappa + b_{\kappa\lambda} r^2\right)\delta_{\lambda\lambda'} - \sum_{\lambda'} C_{\kappa\lambda\lambda'}\right]\Psi_{\kappa\lambda'}(t, \boldsymbol{r}, \boldsymbol{R})\,.$$

that can describe “tetraquarks” —> needs lattice calculations of tetraquarks static energies

coefficients C in calculation for any J and f by M. Berwein, N. Brambilla, A. Vairo 2017

Conclusions

Quarkonium is a golden system to study strong interactions

Nonrelativistic Effective Field Theories provide a systematic tool to investigate a wide range of heavy quarkonium observables in the realm of QCD

Allow us to make calculations with unprecented precision, where high order perturbative calculations are possible and to systematically factorize short from long range contributions where observables are sentitive to the nonperturbative dynamics of QCD

Allow us to give the appropriate definition and define a calculational scheme for quantities of huge phenomenological interest like the qqbar static energies, the qqbar potential at finite T

Allow us to obtain an open quantum systems description, to compute the out-of-equilibrium evolution of the subsystem and its non-trivial interaction with the environment (production, dissociation and recombination of quarkonium).

In the EFT framework heavy quark bound states become a unique laboratory for the study of strong interaction from the high energy to the low energy scales

Further applications and Outlook

The EFT described, pNRQCD,   has applications to another broad range of of problems in particle physics:

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Studies of thermal production of heavy Majorana neutrinos in early universe for Leptogenesis

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applications to atomic and molecular physics

http://itp.phy.pku.edu.cn/conference/qwg2017/

Multi-Scale Problems Using Effective Field Theories (INT-18-1b)
May 7 - June 1, 2018
E. Braaten, N. Brambilla, T. Schäfer, A. Vairo

INSTITUTE FOR NUCLEAR THEORY

INTERFACE OF EFFECTIVE FIELD THEORIES AND LATTICE GAUGE THEORY
15 October - 9 November 2018
Nora Brambilla, Andreas Kronfeld, Peter Petreczky, Antonio Vairo

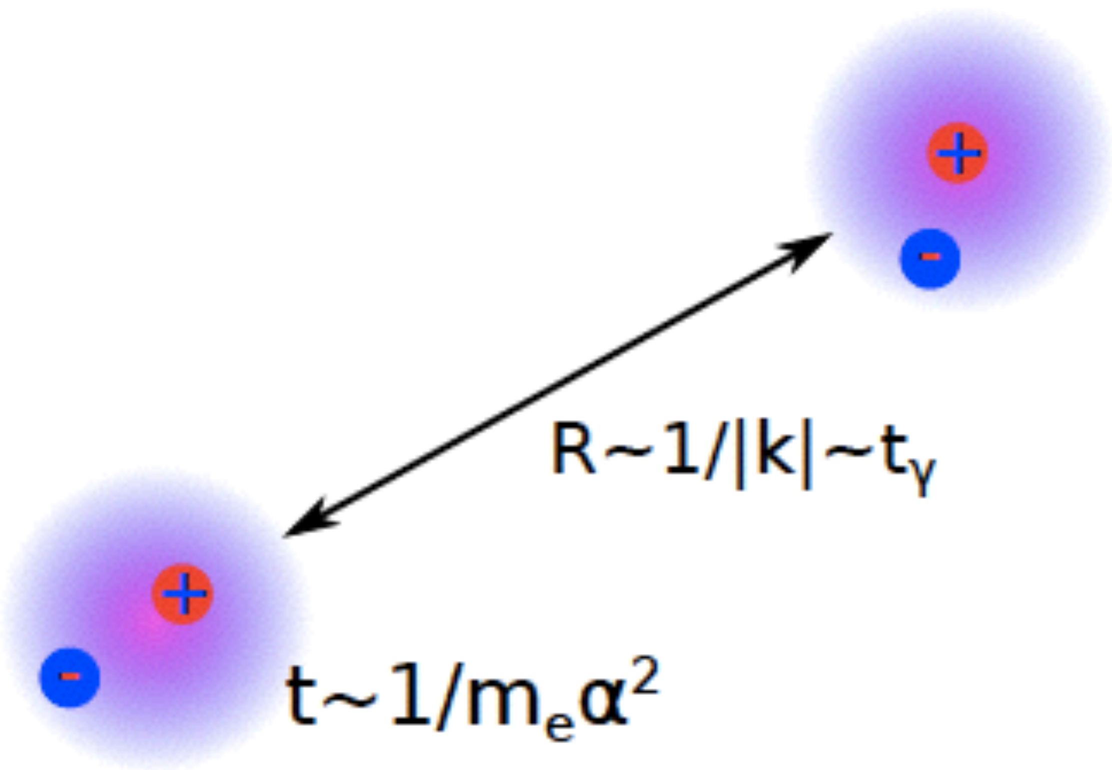
MIAPP Munich Institute for Astro- and Particle Physics

JOINT FGZ-PH Summer School on Methods of Effective Field Theory & Lattice Field Theory
26 June - 7 July 2017
Garching Forschungszentrum bei München,

More Scales

Quarkonium-Quarkonium/Quarkonium on nuclei
EFT of Van der Waals Interaction

- Consider van der Waals interactions between two hydrogen atoms in the ground state at the distance R



- For $t_\gamma \ll t$ (i.e. $|\mathbf{k}| \gg m_e\alpha^2$) the photon exchange is instantaneous.
- For $t_\gamma \gg t$ (i.e. $|\mathbf{k}| \ll m_e\alpha^2$) the photon exchange requires finite time (retardation effects).

Relevant scales

- momentum transfer $|\mathbf{k}| \sim 1/R$
- binding energy $m_e\alpha^2 \sim 1/t$

Possible scale hierarchies

- $|\mathbf{k}| >> m_e\alpha^2$: short range interaction with $W(R) \sim 1/R^6$ (London force)

  [London, 1930]
- $|\mathbf{k}| << m_e\alpha^2$: long range interaction with $W(R) \sim 1/R^7$ (Casimir-Polder force)

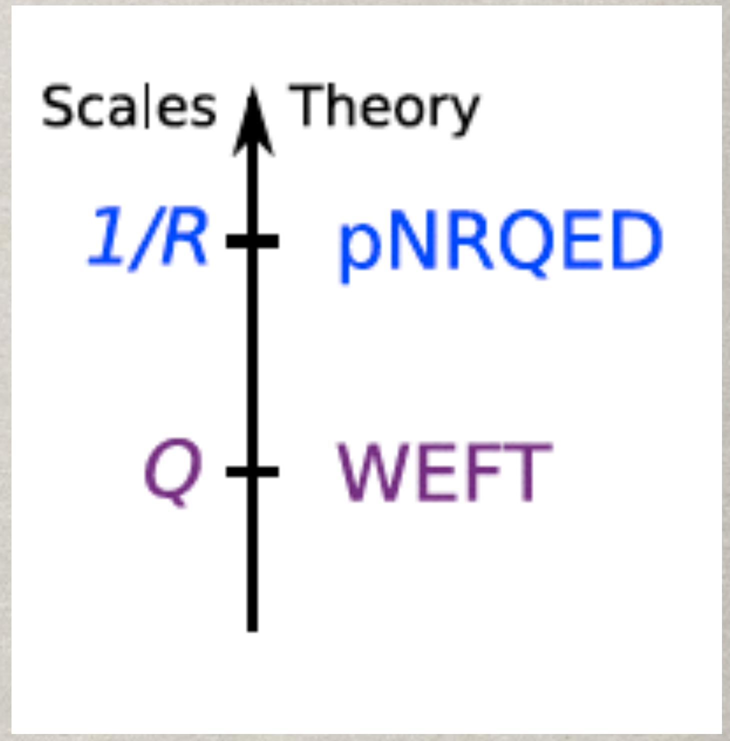
  [Casimir & Polder, 1948]
- $|\mathbf{k}| \sim m_e\alpha^2$: intermediate range

- Low-energy EFT of van der Waals forces at distances $|\mathbf{R}| \equiv |\mathbf{X}_1 - \mathbf{X}_2| \gg a_0$

$$L_{\text{WEFT}} = \int d^3X \sum_n S_n^\dagger(t, X)\Big[i\partial_0 - E_n + \frac{\boldsymbol{\nabla}_X^2}{2m_p} + 2\pi\alpha_n^{ij}\boldsymbol{E}_i\boldsymbol{E}_j + 2\pi\beta_n^{ij}\boldsymbol{B}_i\boldsymbol{B}_j - \frac{\langle n|\boldsymbol{\mu}|n\rangle\cdot e\boldsymbol{B}}{2m_e} + \dots\Big]S_n(t, X) - \int d^3X_1\, d^3X_2 \sum_{n_i, n_j} S_{n_i}^\dagger(t, X_1) S_{n_i}(t, X_1) W_{n_i, n_j}(X_1 - X_2)\, S_{n_j}^\dagger(t, X_2) S_{n_j}(t, X_2).$$

- S_{n_i} are U(1) fields describing H-atoms with quantum numbers n
- $\mathbf{E}$ and $\mathbf{B}$ are electric and magnetic fields
- α_n^{ij} and β_n^{ij} are static electric and magnetic polarizabilities

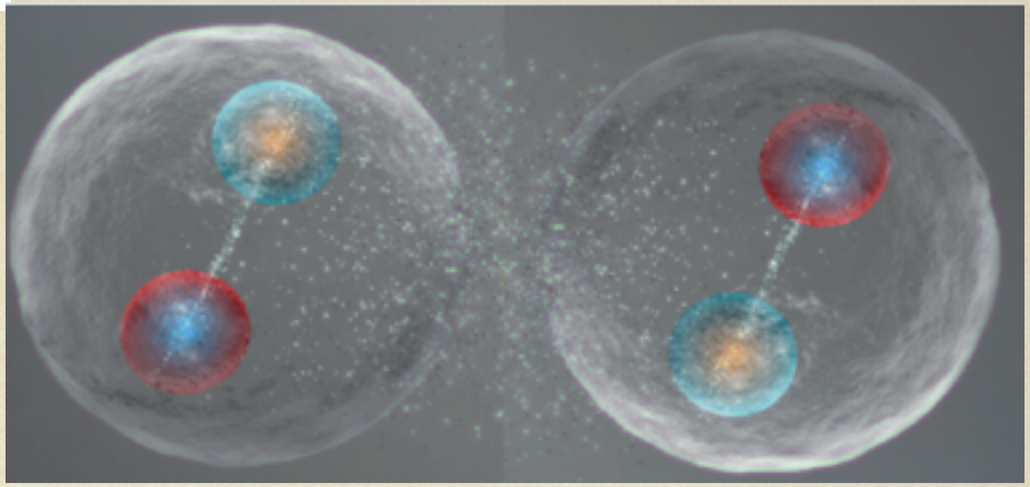
- $W_{n_i, n_j}(\mathbf{R})$ are the van der Waals potentials
- $1/|\mathbf{R}|$ corresponds to the typical momentum transfer $|\mathbf{k}|$
- Short distances: $m_e\alpha^2/|\mathbf{k}| \ll 1$
- Long distances: $|\mathbf{k}|/m_e\alpha^2 \ll 1$
- Intermediate distances: $|\mathbf{k}| \sim /m_e\alpha^2$



We have obtained the van der Waals potential also in the intermediate distance region (limits for short and large distance reproduce London and Casimir Polder) arXiv:1704.03476

Chromopolarizability & color van der Waals forces

$$\eta_b - \eta_b$$



Interactions between color neutral objects:

Via creation of instantaneous color dipole moments & gluon transitions in virtual color-octet intermediate state

— Polarizability—

— Chromopolarizability of 1S bottomonium; use pNRQC (potential Nonrelativistic QCD)

N. Brambilla, GK, J. Tarrús-Castellà, A. Vairo, PRD 93, 054002 (2016)

— Chromopolarizability of 1S bottomonium;

use pNRQC (potential Nonrelativistic QCD)

— van der Waals force between two bottomonia;

use QCD trace anomaly to match pNRQC to a chiral EFT

QCD ⟶ NRQCD ⟶ pNRQCD

pNRQCD ↓ gWEFT ⟶ χEFT ⟶ WEFT

Polarizability (gWEFT) van der Waals (WEFT)

E

- $\sim mv^2$ $pNRQCD$
- $\sim \Lambda_{QCD}$ $gWEFT$
- $\sim m_\pi \sim k$ χEFT
- $\sim k^2/m_\phi$ $WEFT$

m : bottom mass, v : relative velocity

$$m \gg mv \gg mv^2 \gg \Lambda_{\rm QCD}$$

m_ϕ : mass bottomonium, $r_{\phi\phi} \sim 1/m_\pi$: relative distance

$$\mathbf{k}^2_{\phi\phi}/m_\phi = m_\pi^2/m_\phi \ll m_\pi$$

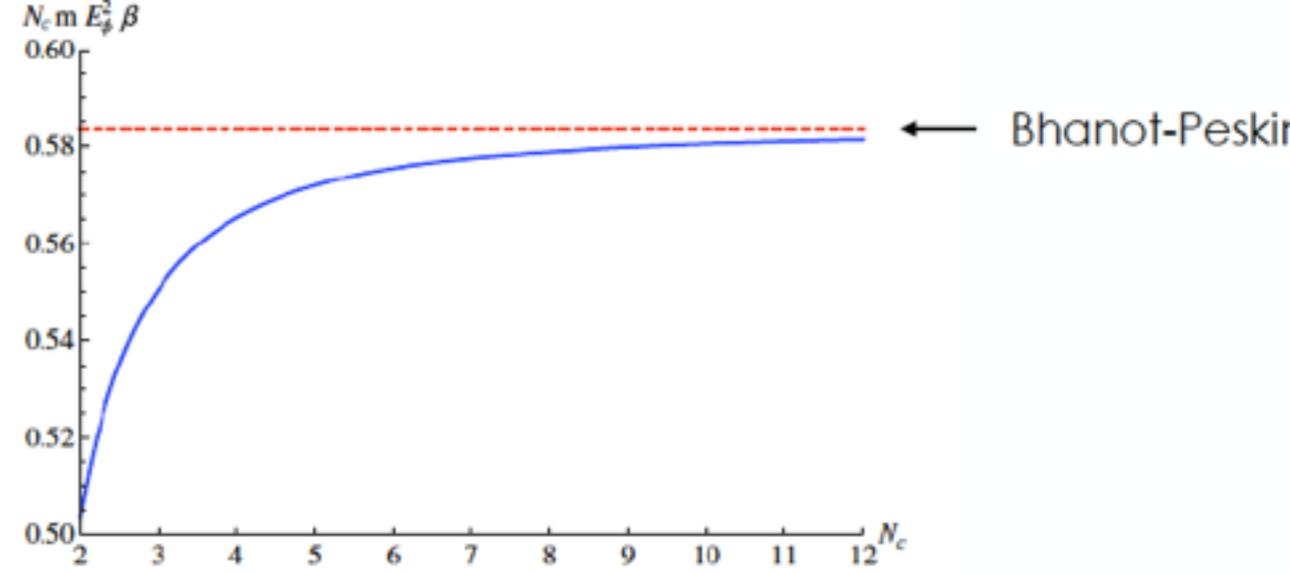


FIG. 3. The dependence of the polarizability on the number of colors. The dashed line at the constant value 7/12 corresponds to the large-N_c limit computed in Ref. [13].

Chromopolarizability

pNRQCD ⟶ gWEFT

$$L_{\rm gWEFT} = \int d^3\boldsymbol{R} \left\{ \phi^\dagger(t,\boldsymbol{R}) \left[i\partial_0 + E_\phi - \frac{\boldsymbol{\nabla}_R^2}{4m} + \frac{1}{2}\beta g^2 \boldsymbol{E}_a^2 + \cdots \right] \phi(t,\boldsymbol{R}) \right\} + \mathcal{L}_{\rm light}$$

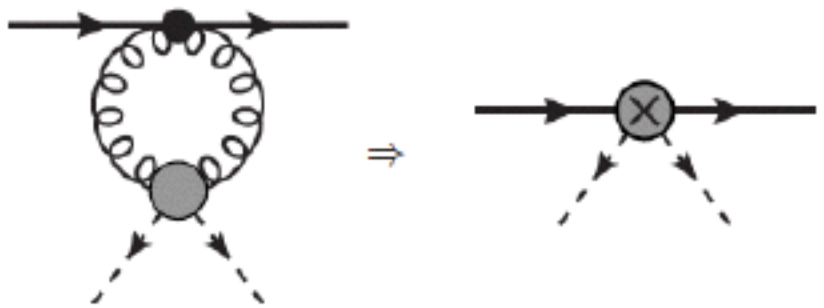
Chromopolarizability

$$\beta = -\frac{2V_A^2 T_F}{3N_c} \langle \phi | \boldsymbol{r} \frac{1}{E_\phi - h_o} \boldsymbol{r} | \phi \rangle$$

$$= -\frac{2V_A^2 T_F}{3N_c} \sum_l \int \frac{d^3p}{(2\pi)^3} |\langle \phi | \boldsymbol{r} | \boldsymbol{p}\, l \rangle|^2 \frac{1}{E_\phi - \frac{p^2}{m}}$$

van der Waals force

gWEFT ⟶ χEFT



QCD trace anomaly

$$g^2\langle\pi^+(p_1)\pi^-(p_2)|\boldsymbol{E}_a^2|0\rangle = \frac{8\pi^2}{3b}\left((p_1+p_2)^2\kappa_1 + m_\pi^2\kappa_2\right)$$

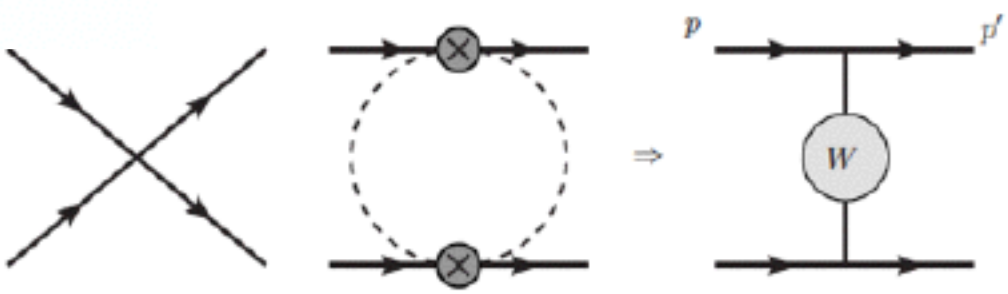
$$\kappa_1 = 1 - 9\kappa/4,\ \kappa_2 = 1 - 9\kappa/2 \qquad b = \frac{11}{3}N_c - \frac{2}{3}N_f$$

$$\kappa = 0.186 \pm 0.003 \pm 0.006 \longleftarrow \psi' \to J/\psi\pi^+\pi^-$$

– integrate out the pion

χEFT ⟶ WEFT

$$r_{\phi\phi} \sim 1/m_\pi \qquad \mathbf{k}_{\phi\phi}^2/m_\phi = m_\pi^2/m_\phi \ll m_\pi$$



— vdW potential

$$W(r) = \frac{1}{2\pi^2 r}\int_{2m_\pi}^{\infty} d\mu\,\mu\,e^{-\mu r}\,\mathrm{Im}\left[\widetilde{W}(\epsilon - i\mu)\right]$$

$$= -\frac{3\pi\beta^2 m_\pi^2}{8b^2 r^5}\left[\left(4\,(\kappa_2+3)^2\,(m_\pi r)^3 + \left(3\kappa_1^2 + 43\kappa_2^2 + 14\kappa_1\kappa_2\right) m_\pi r\right) K_1(2m_\pi r)\right.$$

$$\left. +2\left(2\,(\kappa_2+3)\,(\kappa_1+5\kappa_2)\,(m_\pi r)^2 + \left(3\kappa_1^2 + 43\kappa_2^2 + 14\kappa_1\kappa_2\right)\right) K_2(2m_\pi r)\right]$$

↓ asymptotic

$$W(r) = -\frac{3(3+\kappa_2)^2\pi^{3/2}\beta^2}{4b^2}\frac{m_\pi^{9/2}}{r^{5/2}}e^{-2m_\pi r}$$

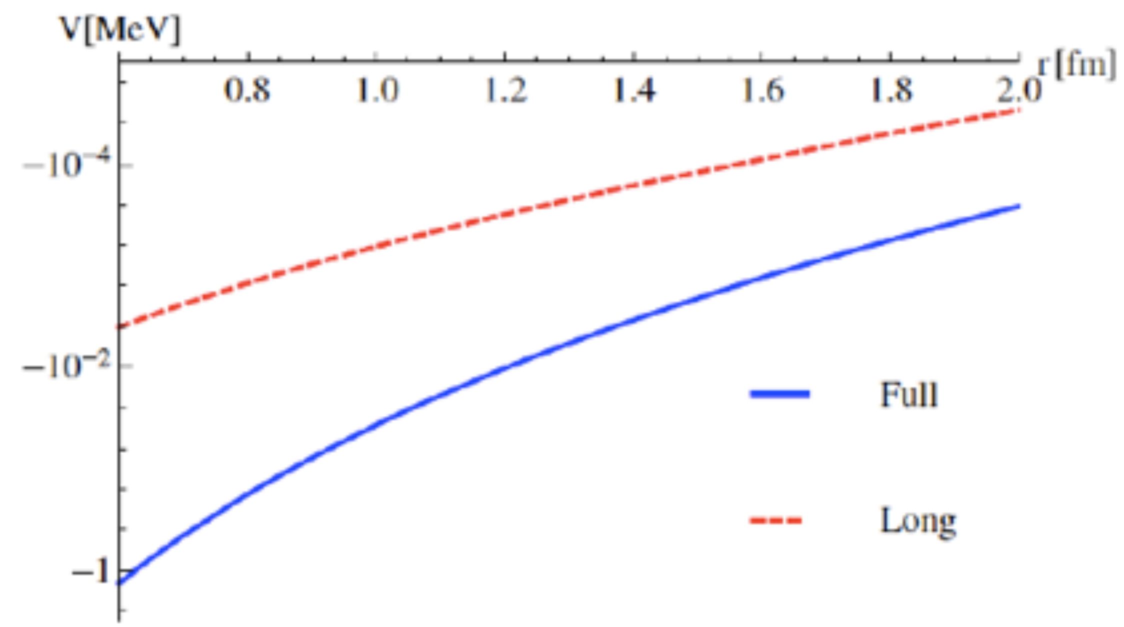


FIG. 9. Comparison of the van der Waals potential (40) (blue line) with its long-range expansion (41) (red line) for $\beta = 0.92\ \mathrm{GeV}^{-3}$ and other parameters like in Fig. 8.

Are there $\eta_b\,\eta_b$ bound-states?

It is likely, but depends somewhat on the medium- and short-range pieces