# Few-Body in EFT

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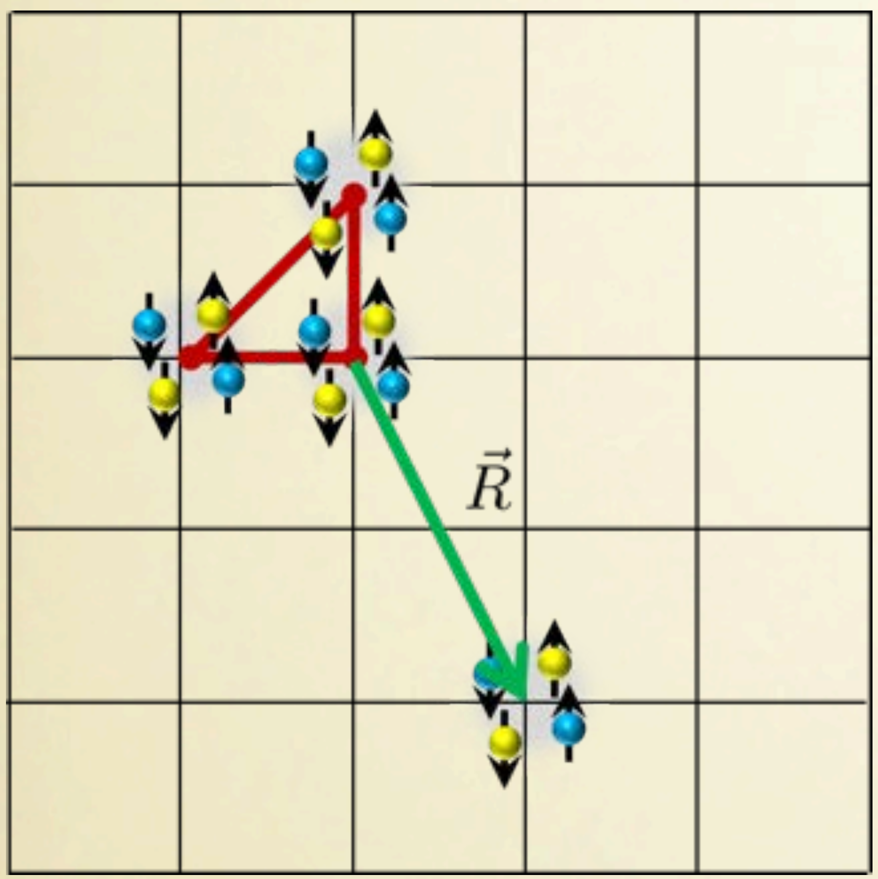
# Outline

- N-d scattering
- Field-redefinition
- p-p fusion

# A little detour on the lattice

- Consider: $a(b,\gamma)c$ ; $a(b,c)d$
- Need effective "cluster" Hamiltonian -- acts in cluster coordinates, spins,etc.
- Calculate reaction with cluster Hamiltonian. Many possibilities --- traditional methods, continuum EFT, lattice method

# Adiabatic Projection Method




Initial state $|\vec{R}\rangle$

Evolved state $|\vec{R}\rangle_\tau = e^{-\tau H}|\vec{R}\rangle$

$${}_\tau\langle\vec{R}'|H|\vec{R}\rangle_\tau$$

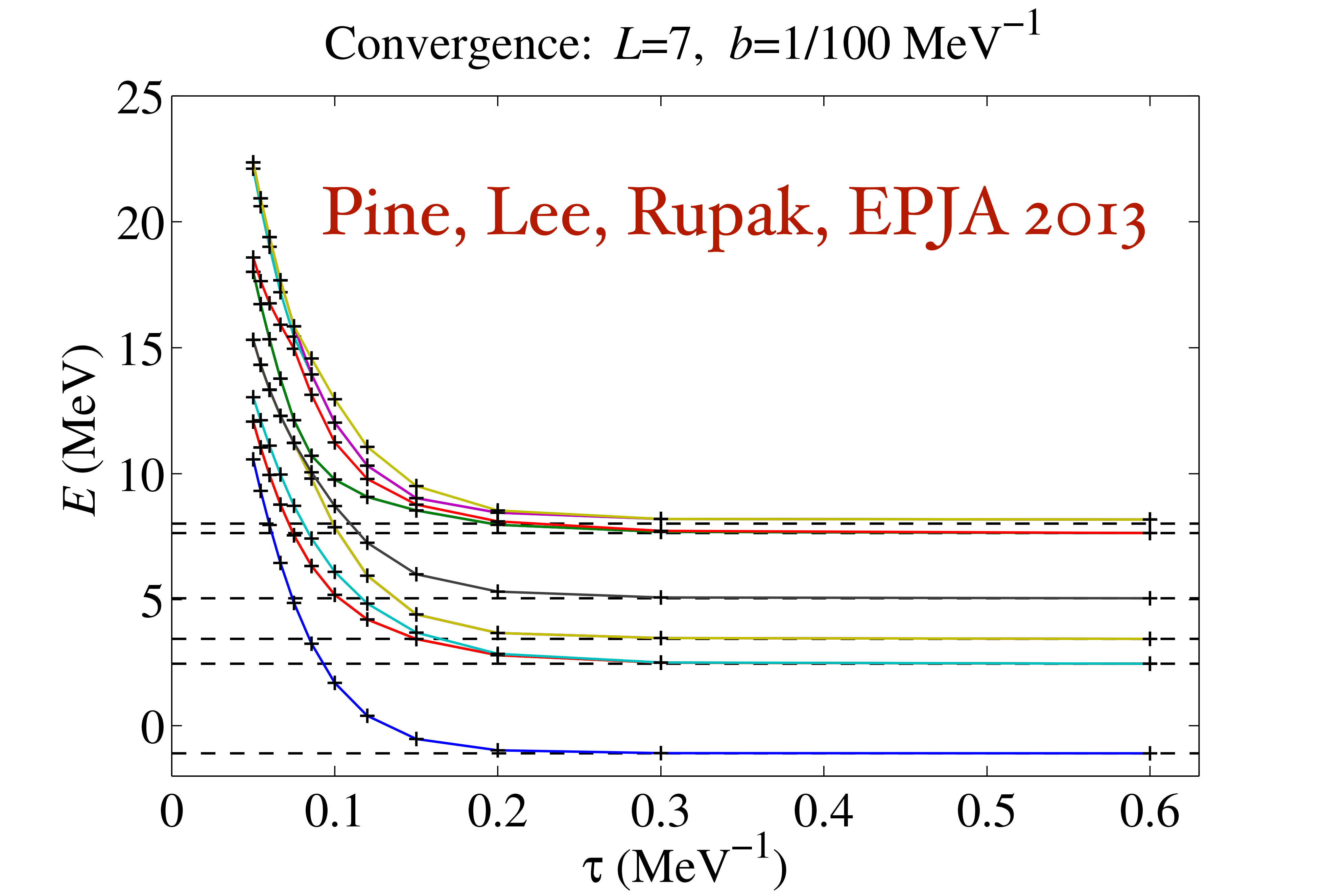
Energy measurements in cluster basis.
Divide by the norm matrix as these are not orthogonal basis $[N_\tau]_{\vec{R},\vec{R}'} =_\tau \langle\vec{R}|\vec{R}'\rangle_\tau$

Microscopic Hamiltonian $L^{3(A-1)}$

Cluster Hamiltonian $L^3$ ← smaller matrices in practice!!

-- acts on the cluster CM and spins

# Neutron-Deuteron System

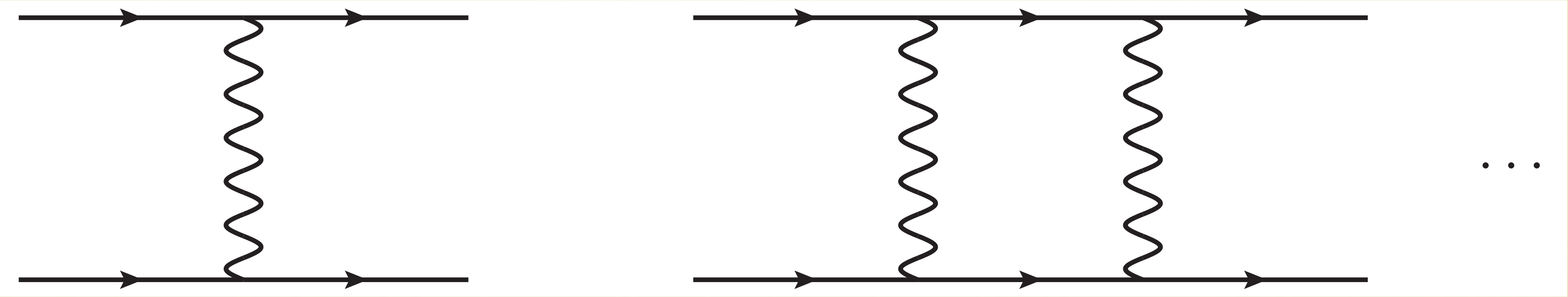



- grouping R found efficient

$\sim 30 \times 30$

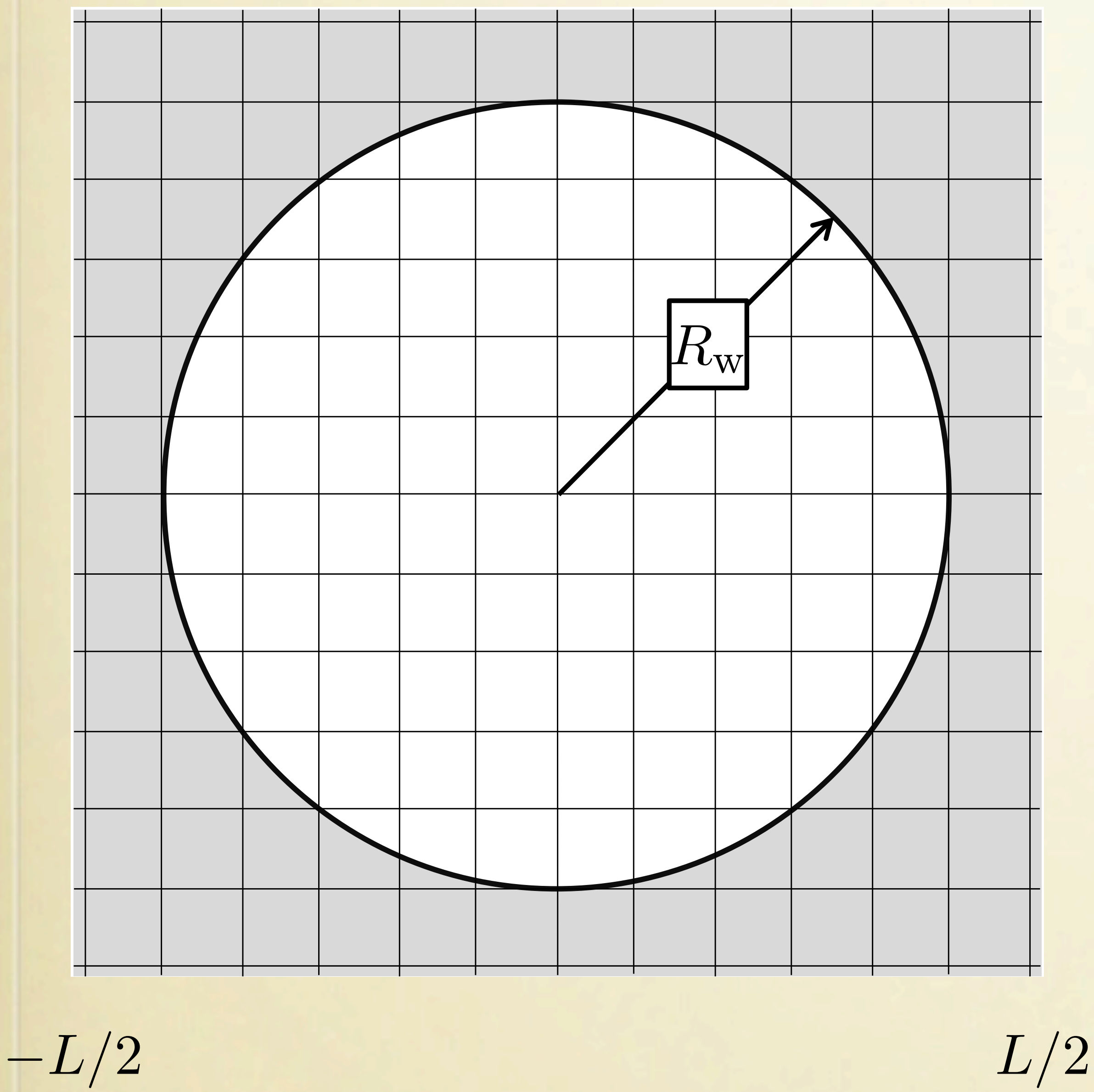
Something still missing ...

long range Coulomb




$\mathcal{O}\left(\frac{\alpha}{p^2}\right)$ $\mathcal{O}\left(\frac{\alpha}{p^2}\frac{\alpha\mu}{p}\right)$

# Spherical-wall method




$$\psi_{\text{short}}(r) \propto j_0(kr)\cot\delta_s - n_0(kr),$$
$$\psi_{\text{Coulomb}}(r) \propto F_0(kr)\cot\delta_{sc} + G_0(kr)$$

Adjust from free theory:

$$j_0(k_0 R_w) = 0$$

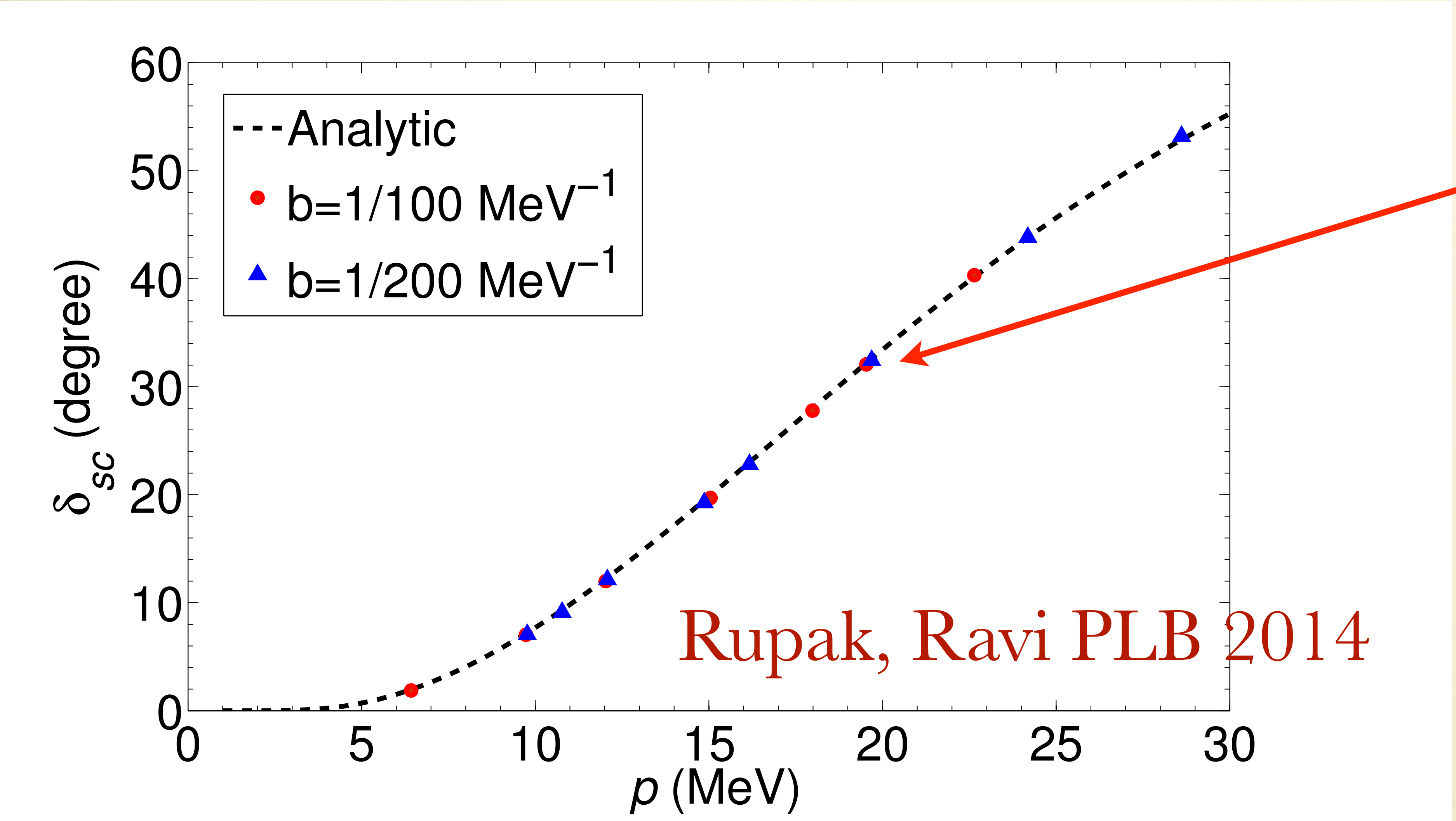
IR scale setting

Hard spherical wall boundary conditions, Borasoy et al. 2007

Carlson et al. 1984

Even older ?

# p-p Coulomb Subtracted Phase Shift



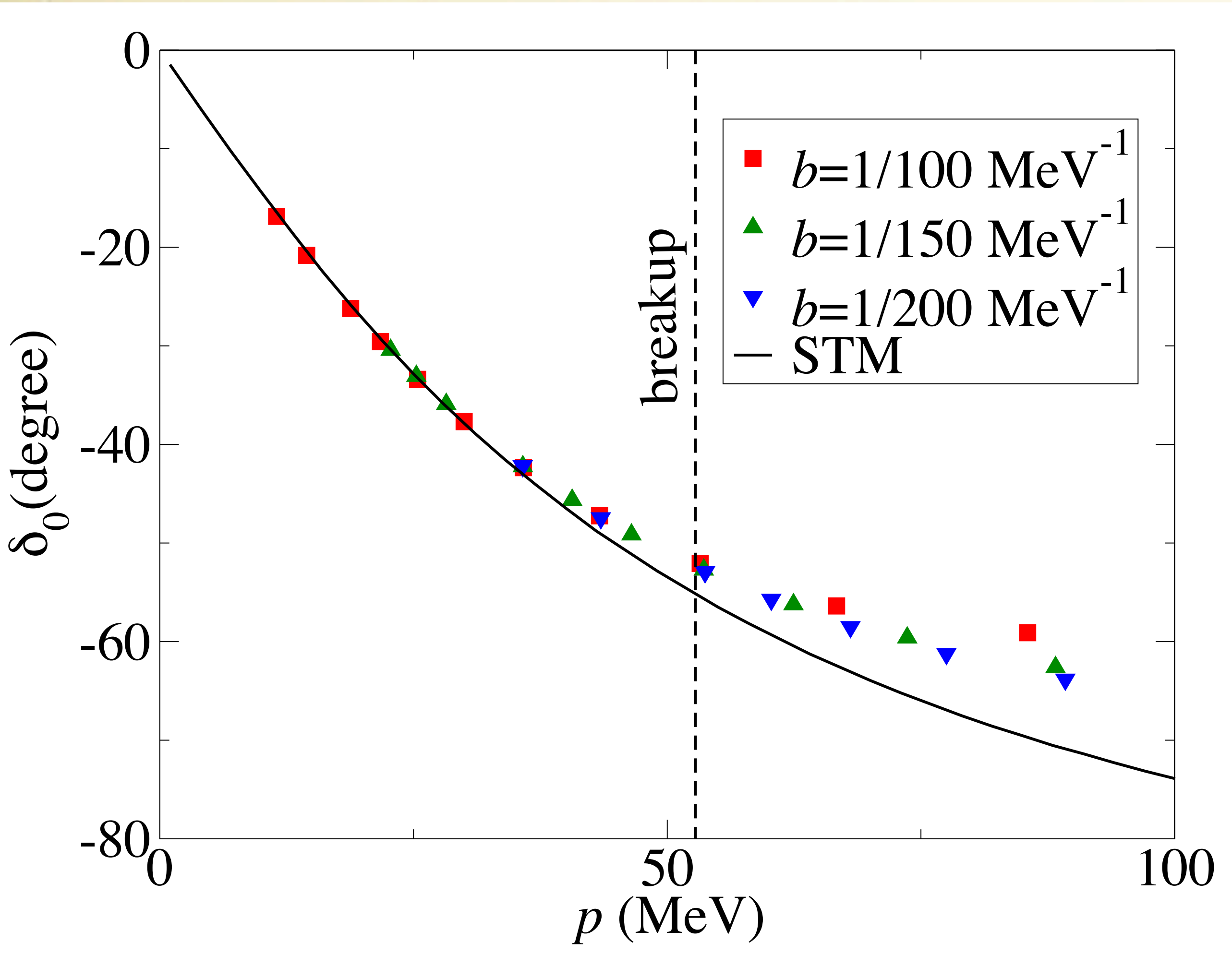

3% error in fits

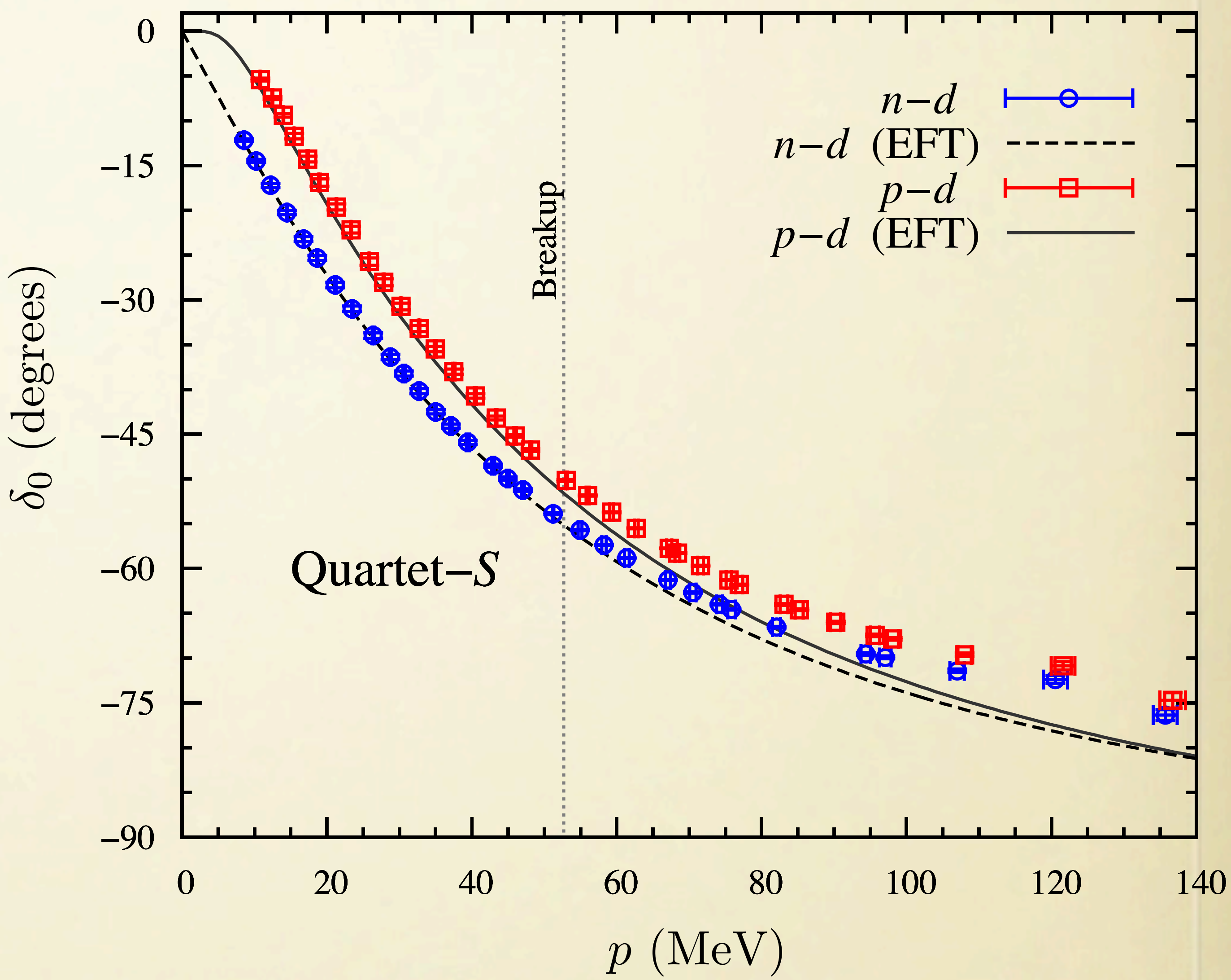
$$T = T_c + T_{sc}$$

$$T_c \approx \frac{2\pi}{\mu} \frac{e^{2i\sigma} - 1}{2ip}$$

$$T \approx \frac{2\pi}{\mu} \frac{e^{2i(\sigma+\delta_{sc})} - 1}{2ip}$$

# Improvement

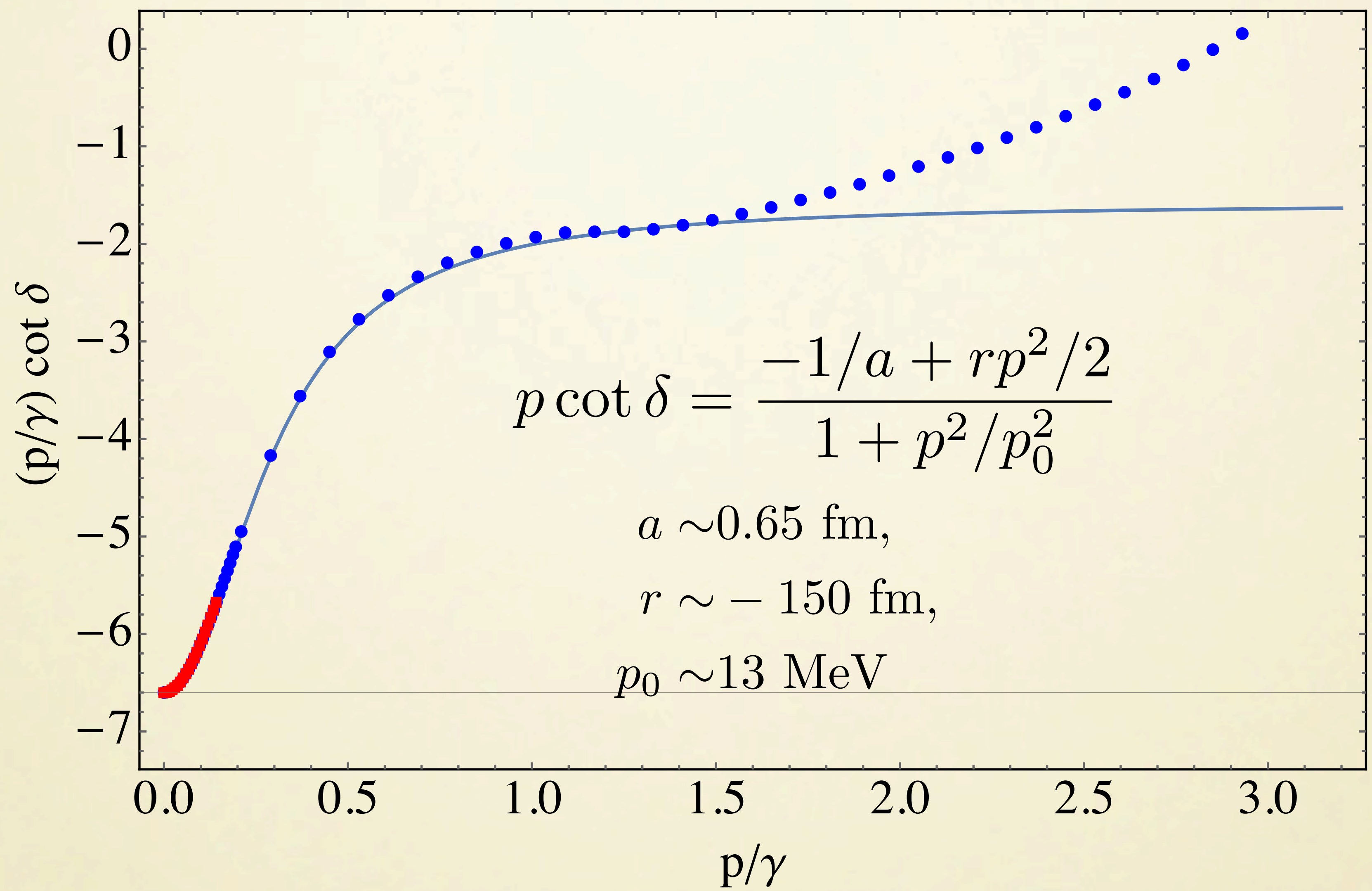



Pine, Lee, Rupak (2013)

Elhatisari, Lee, Meißner, Rupak (2016)

# N-D DOUBLET CHANNEL



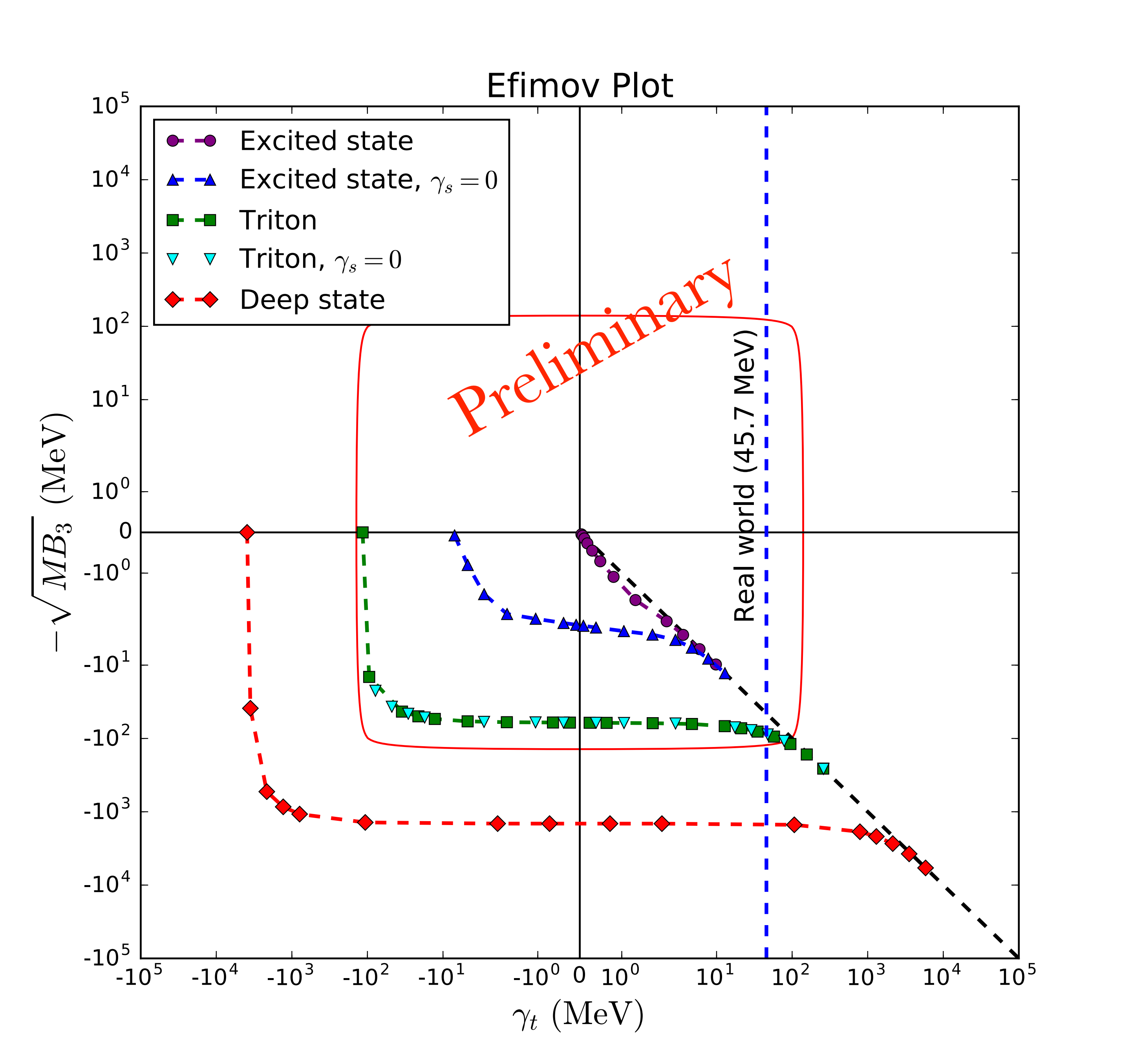

ERE form van Oers & Seagrave (1967)

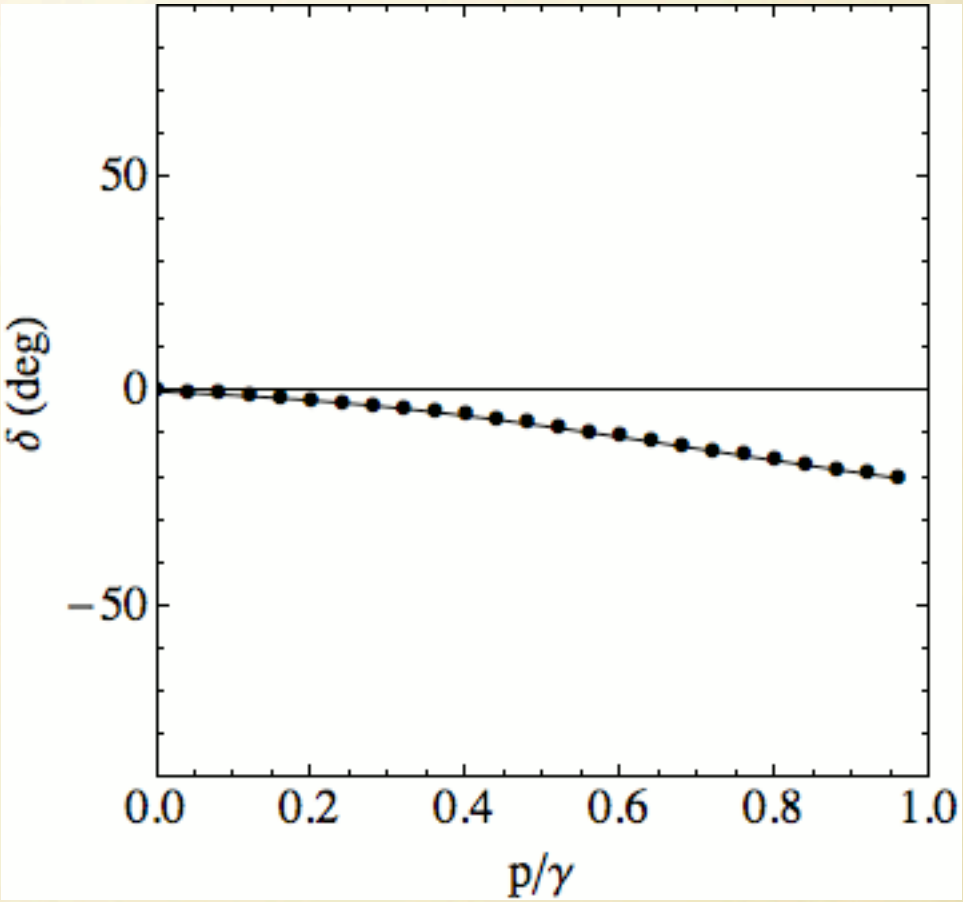
-what EFT for modified ERE

Virtual state at 0.5 MeV Girard & Fuda (1979)

- Efimov physics

# Efimov plot



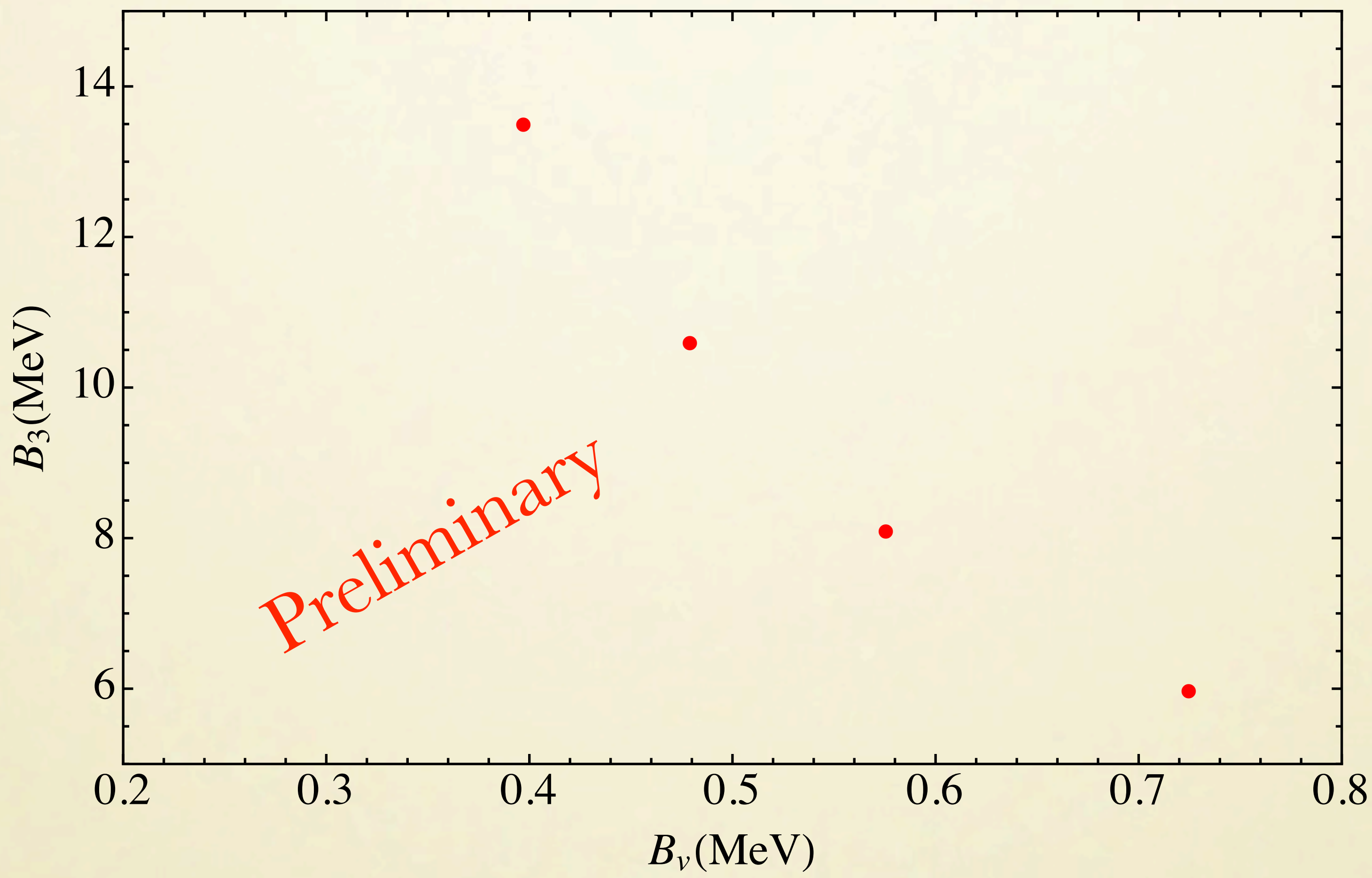




Higa, Rupak, Vaghani, van Kolck

Shallow virtual to bound state

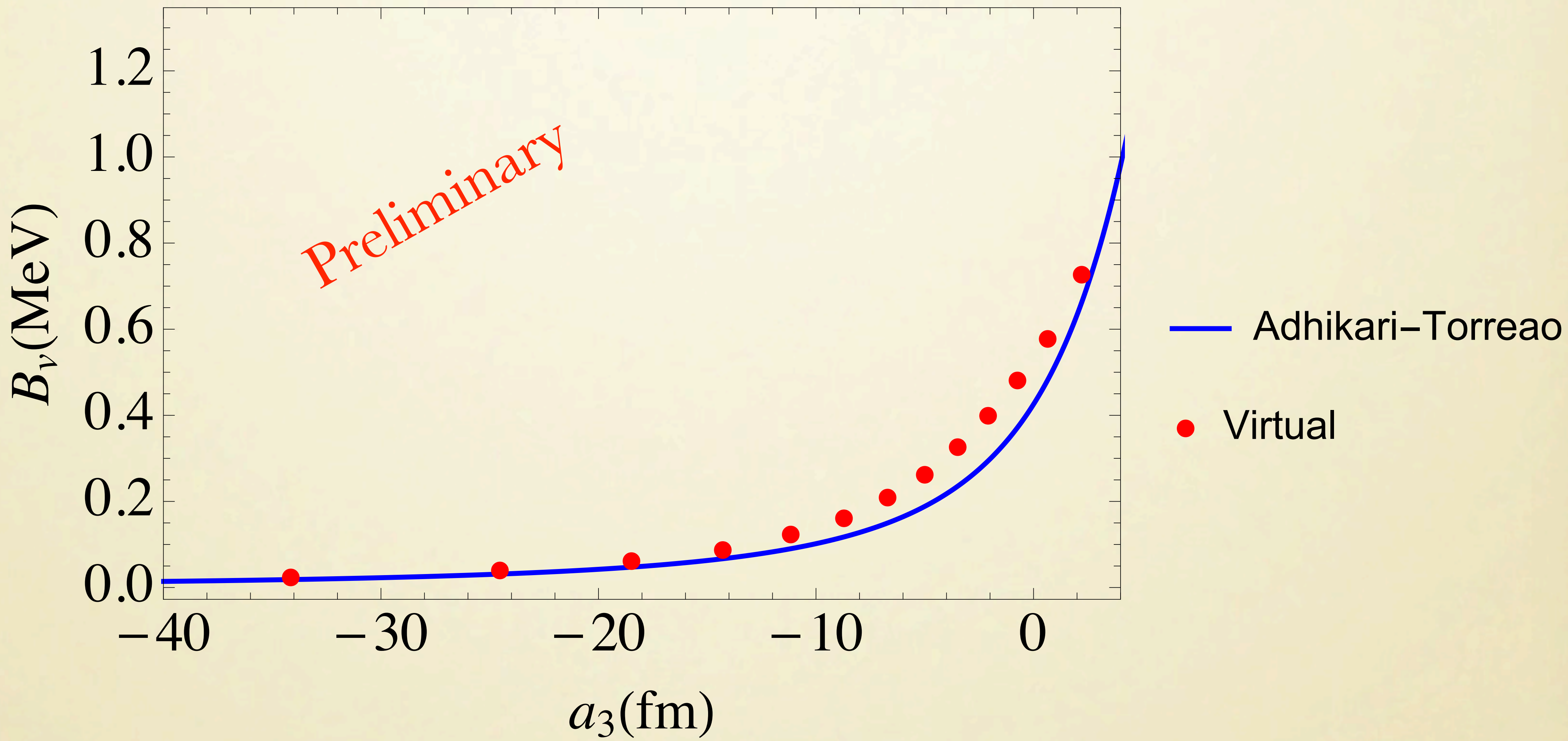
lattice QCD with B field, even with heavy pions?

# Phillips-Girard-Fuda




3-body correlation

# Adhikari-Torreao



# Field Redefinition

$$\mathcal{L} = N^\dagger(i\overrightarrow{\partial}_0 + \frac{\overrightarrow{\nabla}^2}{2M})N + d_t^\dagger(-i\overrightarrow{D}_t)d_t + d_s^\dagger(-i\overrightarrow{D}_t)d_s + t^\dagger(i\overrightarrow{D}_\Omega)t$$

$$-g_t(d_t^{i\,\dagger} N P_i N + h.c) - g_s(d_s^{a\,\dagger} N \bar{P}_a N + h.c)$$

$$-w_t(t^\dagger \sigma^i N d_t^i + h.c.) - w_s(t^\dagger \tau^a N d_s^a + h.c.)$$

Start here and write in terms of only nucleon fields

$$-i\overrightarrow{D}_t = (-i\overrightarrow{\partial}_0 - \frac{\overrightarrow{\nabla}^2}{4M} + \Delta_t),$$

$$-i\overrightarrow{D}_s = (-i\overrightarrow{\partial}_0 - \frac{\overrightarrow{\nabla}^2}{4M} + \Delta_s),$$

$$i\overrightarrow{D}_\Omega = (i\overrightarrow{\partial}_0 + \frac{\overrightarrow{\nabla}^2}{6M} + \Delta_\Omega)\ .$$

alternatives possible

# Integrate out fields

$$\frac{\partial \mathcal{L}}{\partial t^\dagger} = 0$$

$$\Rightarrow t = (i \overrightarrow{D}_\Omega)^{-1}(w_t \sigma^i N d_t^i + w_s \tau^a N d_s^a)$$

$$t^\dagger (i \overrightarrow{D}_\Omega) t$$

You see that it generates dimer-nucleon interaction ... more to come

$$\frac{\partial \mathcal{L}}{\partial d_t^{i\dagger}} = 0$$

$$\Rightarrow d_t^j = (-i \overrightarrow{\mathbb{D}}_t^{j\ i})^{-1}[g_t N P_i N + w_t w_s N^\dagger \sigma^i \tau^a (i \overrightarrow{D}_\Omega)^{-1} N d_s^a]$$

$$-i \overrightarrow{\mathbb{D}}_t^{i\ j} = -i \overrightarrow{D}_t \, \delta_{ij} - w_t^2 N^\dagger \sigma^i \sigma^j (i \overrightarrow{D}_\Omega)^{-1} N$$

Now things get interesting

# Finally integrate the last dimer field and write

$$\mathcal{L} = N^\dagger(i\,\overrightarrow{\partial}_0 + \frac{\overrightarrow{\nabla}^2}{2M})N - \frac{g_t^2}{2}[(NP_iN)^\dagger(-i\,\overrightarrow{\mathbb{D}}_t^{i\ j})^{-1}NP_jN + h.c.]$$

$$- \frac{1}{2}[g_s(N\bar{P}_aN)^\dagger + g_t w_t w_s (NP_kN)^\dagger(-i\,\overrightarrow{\mathbb{D}}_t^{k\ l})^{-1}N^\dagger(i\,\overrightarrow{D}_\Omega)^{-1}\sigma^l\tau^aN](-i\,\overrightarrow{\mathfrak{D}}^{a\ b})^{-1}$$

$$\times\,[g_sN\bar{P}_bN + g_t w_t w_s N^\dagger(i\,\overrightarrow{D}_\Omega)^{-1}\sigma^i\tau^bN(-i\,\overrightarrow{\mathbb{D}}_t^{i\ j})^{-1}NP_jN] + h.c.\ ,$$

$$(-i\,\overrightarrow{\mathfrak{D}}^{a\ b}) = [-i\,\overrightarrow{D}_s\,\delta^{a\ b} - w_s^2N^\dagger\tau^a\tau^b(i\,\overrightarrow{D}_\Omega)^{-1}N$$

$$- w_t^2w_s^2N^\dagger(i\,\overrightarrow{D}_\Omega)^{-1}\sigma^i\tau^aN(-i\,\overrightarrow{\mathbb{D}}_t^{i\ j})^{-1}N^\dagger\sigma^j\tau^b(i\,\overrightarrow{D}_\Omega)^{-1}N]$$

-- generates higher-body terms

-- need to remove time-derivatives

## Keep upto 3-body contact interactions

$$(-i\,\overrightarrow{\mathbb{D}}_t^{\,i\,j})^{-1} = (-i\,\overrightarrow{D}_t)^{-1}\delta^{i\,j} + \frac{w_t^2}{\Delta_t^2} N^\dagger\sigma^i\sigma^j(i\,\overrightarrow{D}_\Omega)^{-1}N + \frac{w_t^2}{\Delta_t^3\Delta_\Omega}(i\partial_0 + \frac{\overrightarrow{\nabla}^2}{4M})N^\dagger\sigma^i\sigma^j N\ ,$$

$$(-i\,\overrightarrow{\mathfrak{D}}^{\,a\,b})^{-1} = (-i\,\overrightarrow{D}_s)^{-1}\delta^{a\,b} + \frac{w_s^2}{\Delta_s^2} N^\dagger\tau^a\tau^b(i\,\overrightarrow{D}_\Omega)^{-1}N + \frac{w_s^2}{\Delta_s^3\Delta_\Omega}(i\partial_0 + \frac{\overrightarrow{\nabla}^2}{4M})N^\dagger\tau^a\tau^b N \equiv (-i\,\overrightarrow{\mathbb{D}}_s^{\,a\,b})^{-1}\ ,$$

## Almost home, and write

$$\mathcal{L} = N^\dagger(i\,\overrightarrow{\partial}_0 + \frac{\overrightarrow{\nabla}^2}{2M})N - \frac{g_t^2}{2}[(NP_iN)^\dagger(-i\,\overrightarrow{\mathbb{D}}_t^{\,i\,j})^{-1}NP_jN + h.c.]$$
$$- \frac{g_s^2}{2}[(N\bar{P}_aN)^\dagger(-i\,\overrightarrow{\mathbb{D}}_s^{\,a\,b})^{-1}N\bar{P}_bN + h.c.]$$
$$- g_tg_sw_tw_s[(N\bar{P}_aN)^\dagger(-i\,\overrightarrow{D}_s)^{-1}N^\dagger(i\,\overrightarrow{D}_\Omega)^{-1}\sigma^i\tau^aN(-i\,\overrightarrow{D}_t)^{-1}NP_iN + h.c.]$$
$$\equiv \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \ldots$$

# Field Redefinition

To remove time-derivative from two-body try

$$N \to N + a_1 P_i^\dagger N^\dagger (N P_i N) + b_1 \bar{P}_a^\dagger N^\dagger (N \bar{P}_a N)$$

Bedaque, Grießhammer (2000)
Bedaque, Rupak, Grießhammer, Hammer (2003)

After a good amount of elbow grease

$$\mathcal{L}_2 = -\frac{g_t^2}{\Delta_t}(N P_i N)^\dagger N P_i N - \frac{g_s^2}{\Delta_s}(N \bar{P}_a N)^\dagger N \bar{P}_a N$$
$$- \frac{g_t^2}{8M\Delta_t^2}[(N P_i N)^\dagger (N \overleftrightarrow{\nabla}^2 P_i N) + h.c.] - \frac{g_s^2}{8M\Delta_s^2}[(N P_a N)^\dagger (N \overleftrightarrow{\nabla}^2 P_a N) + h.c.]$$

using $a_1 = g_t^2/\Delta_t^2$

$b_1 = g_s^2/\Delta_s^2$

Removing time-derivatives from three-body contact interaction requires, another field redefinition. However, the leading momentum independent term is simple

$$-\left[\frac{g_t^2 w_t^2}{\Delta_t^2\Delta_\Omega}+\frac{g_t^4}{6\Delta_t^3}\right](NP_iN)^\dagger N^\dagger\sigma^i\sigma^j N(NP_jN)-\left[\frac{g_s^2 w_s^2}{\Delta_s^2\Delta_\Omega}+\frac{g_s^4}{6\Delta_s^3}\right](N\bar{P}_aN)^\dagger N^\dagger\tau^a\tau^b N(N\bar{P}_bN)$$

$$-\left[\frac{g_t g_s w_t w_s}{\Delta_t\Delta_s\Delta_\Omega}-\frac{g_t^2 g_s^2}{4\Delta_t^2\Delta_s}-\frac{g_t^2 g_s^2}{4\Delta_t\Delta_s^2}\right][(N\bar{P}_aN)^\dagger N^\dagger\tau^a\sigma^i N(NP_iN)+h.c.]$$

Bedaque, Rupak, Grießhammer, Hammer (2003)

Need to pull out the SU(4) symmetric piece

# P-P FUSION

$$|NN(s;\vec{k},p)\rangle = \frac{p}{\sqrt{4\pi}}\frac{1}{(2\pi)^3}\int d\Omega_{\hat{p}}\left[N(\vec{k}/2+\vec{p})\mathcal{P}^{(s)}N(\vec{k}/2-\vec{p})\right]^\dagger|0\rangle$$

Projector

cm, relative momentum

$$\langle NN(s';\vec{k}',p')|NN(s;\vec{k},p)\rangle = \delta^{(3)}(\vec{k}-\vec{k}')\delta(p-p')\delta ss'$$

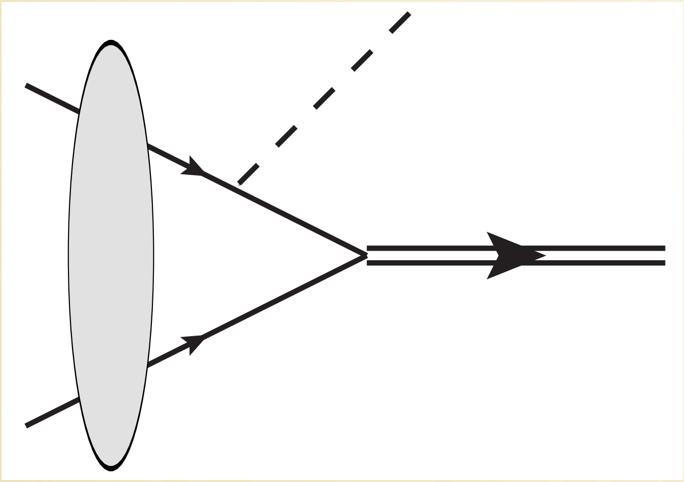
with projectors

$$\sum_{\text{Ave. pol}} \text{Tr}\left[\mathcal{P}^{(s)}\mathcal{P}^{(s')}{}^\dagger\right] = \frac{1}{2}\delta_{ss'}$$

Chen, Rupak, Savage (1999)

$$\langle NN(s';\vec{k}',p')|N_a^* N_b^* \mathcal{O}_{ab;cd} N_c N_d|NN(s;\vec{k},p)\rangle = \frac{1}{2}\int d\cos\theta \mathcal{P}_{ab}^{(s')} \mathcal{O}_{ab;cd}\left[\mathcal{P}_{ab}^{(s')}{}^{\dagger}\right]_{cd}$$

Chen, Rupak, Savage (1999)
Fleming, Mehen, Stewart (2000)



$$\sim \sqrt{Z_d} h g_A (\vec{\epsilon}_d^{\,*} \times \vec{\epsilon}^{\,*})_x \hat{p}_y (-2\mu)^2 \int \frac{d^3q}{(2\pi)^3} \frac{\psi_{\vec{p}}^{(+)}(\vec{q})}{q^2+\gamma^2}$$

project the appropriate p-wave channels

s-wave comparison $|\langle d;x|A_y^-|pp\rangle| = g_A C_\eta \sqrt{\frac{32\pi}{\gamma^3}}\Lambda(p)\delta_{xy}$

indices contracted

# Thank you