Bound State Calculation in Three-Body Systems with Short Range Interactions

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Ingredients of $\mathrm{EFT}_{\not{\pi}}$

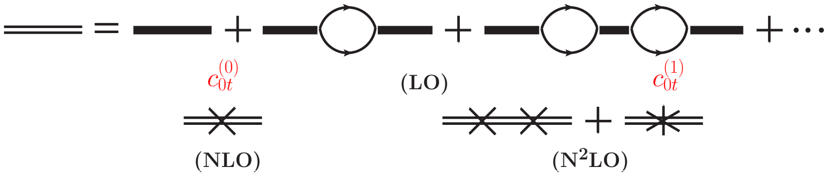
- For momenta $p < m_\pi$ pions can be integrated out as degrees of freedom and only nucleons and external currents are left.
- For any effective (field) theory write down all terms with degrees of freedom that respect symmetries.
- Develop a power counting to organize terms by their relative importance.
- Calculate respective observables up to a given order in the power counting.

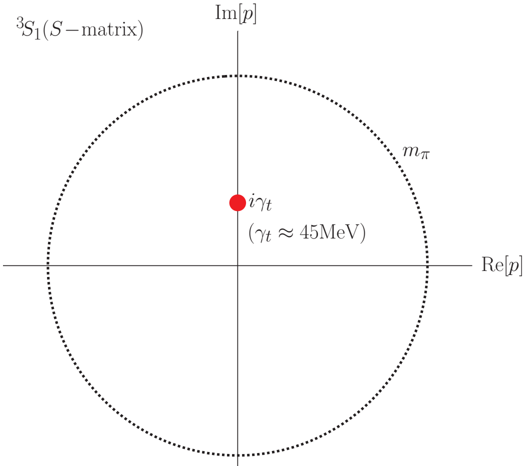
The two-body Lagrangian to $\mathrm{N^2LO}$ in $\mathrm{EFT}_{\not\pi}$ is

$$\mathcal{L}_2 = \hat{N}^\dagger\left(i\partial_0+\frac{\vec{\nabla}^2}{2M_N}\right)\hat{N}+\hat{t}_i^\dagger\left(\Delta_t-\sum_{n=0}^{1}c_{nt}\left(i\partial_0+\frac{\vec{\nabla}^2}{4M_N}+\frac{\gamma_t^2}{M_N}\right)^{n+1}\right)\hat{t}_i$$
$$+\hat{s}_a^\dagger\left(\Delta_s-\sum_{n=0}^{1}c_{ns}\left(i\partial_0+\frac{\vec{\nabla}^2}{4M_N}+\frac{\gamma_s^2}{M_N}\right)^{n+1}\right)\hat{s}_a$$
$$+y_t\left[\hat{t}_i^\dagger\hat{N}^TP_i\hat{N}+\mathrm{H.c.}\right]+y_s\left[\hat{s}_a^\dagger\hat{N}^T\bar{P}_a\hat{N}+\mathrm{H.c.}\right].$$

- c_{0t}, c_{0s}-range corrections

The LO dressed deuteron propagator is given by a bubble sum





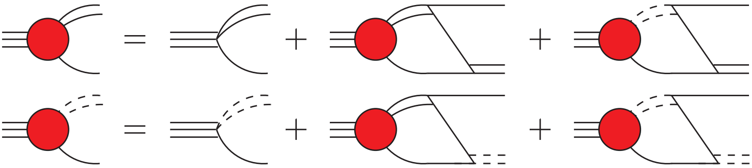
(Z-parametrization) At LO coefficients are fit to reproduce the deuteron pole and at NLO to reproduce the residue about the deuteron pole (Phillips,Rupak, and Savage (2000)).

Doublet S-wave and Bound state

The three-body Lagrangian is

$$\mathcal{L}_3 = \hat{\psi}^\dagger \left[\Omega - h_2(\Lambda)\left(i\partial_0 + \frac{\vec{\nabla}^2}{6M_N} + \frac{\gamma_t^2}{M_N}\right)\right]\hat{\psi} + \sum_{n=0}^{\infty}\left[\omega_{t0}^{(n)}\hat{\psi}^\dagger\sigma_i\hat{N}\hat{t}_i - \omega_{s0}^{(n)}\hat{\psi}^\dagger\tau_a\hat{N}\hat{s}_a\right] + \mathrm{H.c.}.$$

where ψ is an auxiliary triton field. The LO triton vertex function $\mathcal{G}_0(E,p)$ is given by following coupled integral equations (Hagen, Hammer, and Platter (2013))



$$\mathcal{G}_0(E,p) = \widetilde{\mathbf{B}}_0 + \mathbf{K}_0^{\ell=0}(q,p,E) \otimes \mathcal{G}_0(E,q),$$

The LO kernel in cluster-configuration (c.c) space is

$$\mathbf{K}_0^{\ell}(q,p,E) = -\frac{2\pi}{qp}Q_0\left(\frac{q^2+p^2-M_NE-i\epsilon}{qp}\right)\begin{pmatrix}1 & -3\\ -3 & 1\end{pmatrix}$$
$$\times\begin{pmatrix}D_t(E,\vec{\mathbf{q}}) & 0\\ 0 & D_s(E,\vec{\mathbf{q}})\end{pmatrix}.$$

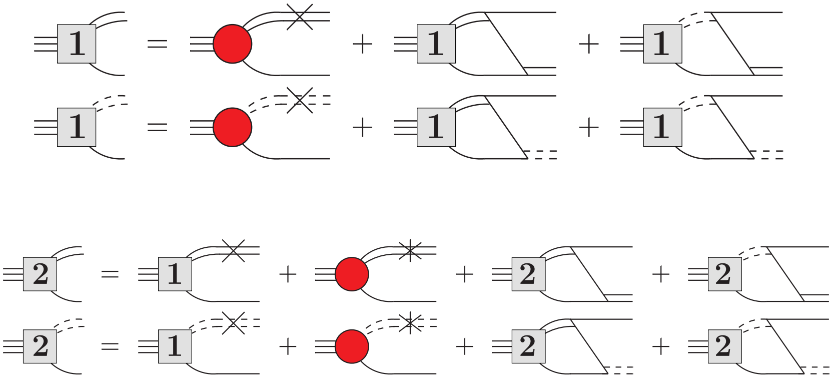
The LO triton vertex function and the inhomogeneous term $\widetilde{\mathbf{B}}_0$ are c.c. space vectors given by

$$\boldsymbol{\mathcal{G}}_0(E,p) = \left(\begin{array}{c}\mathcal{G}_{0,\psi\to Nt}(E,p)\\ \mathcal{G}_{0,\psi\to Ns}(E,p)\end{array}\right), \widetilde{\mathbf{B}}_0 = \left(\begin{array}{c}1\\ 1\end{array}\right).$$

The $\otimes$ operator is given by

$$A(q)\otimes B(q) = \frac{1}{2\pi^2}\int_0^{\Lambda} dq q^2 A(q)B(q).$$

The NLO ($\mathcal{G}_1(E,p)$) and NNLO ($\mathcal{G}_2(E,p)$) triton vertex functions are



The NLO triton vertex function is

$$\mathcal{G}_1(E,p)=\mathcal{G}_0(E,p)\circ\mathbf{R}_1\left(E-\frac{\vec{\mathbf{p}}^2}{2M_N},\vec{\mathbf{p}}\right)+\mathbf{K}_0^{\ell=0}(q,p,E)\otimes\mathcal{G}_1(E,q),$$

and the NNLO triton vertex function

$$\mathcal{G}_2(E,p)=\left[\mathcal{G}_1(E,p)-\mathbf{c}_1\circ\mathcal{G}_0(E,p)\right]\circ\mathbf{R}_1\left(E-\frac{\vec{\mathbf{p}}^2}{2M_N},\vec{\mathbf{p}}\right)$$
$$+\mathbf{K}_0^{\ell=0}(q,p,E)\otimes\mathcal{G}_2(E,q),$$

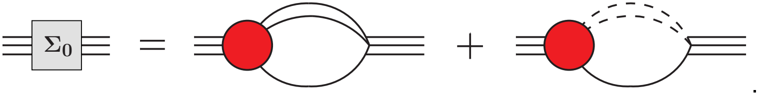
where

$$\mathbf{R}_1(p_0,\vec{\mathbf{p}})=\begin{pmatrix}\frac{Z_t-1}{2\gamma_t}\left(\gamma_t+\sqrt{\frac{1}{4}\vec{\mathbf{p}}^2-M_Np_0-i\epsilon}\right)\\ \frac{Z_s-1}{2\gamma_s}\left(\gamma_s+\sqrt{\frac{1}{4}\vec{\mathbf{p}}^2-M_Np_0-i\epsilon}\right)\end{pmatrix},$$

and

$$\mathbf{c}_1=\begin{pmatrix}Z_t-1\\ Z_s-1\end{pmatrix}.$$

Defining



The dressed triton propagator is given by the sum of diagrams

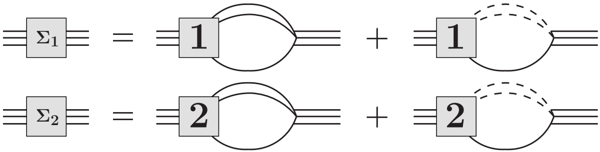
which yields

$$i\Delta_3(E) = \frac{i}{\Omega} - \frac{i}{\Omega} H_{\mathrm{LO}} \Sigma_0(E) \frac{i}{\Omega} + \cdots$$
$$= \frac{i}{\Omega} \frac{1}{1 - H_{\mathrm{LO}} \Sigma_0(E)},$$

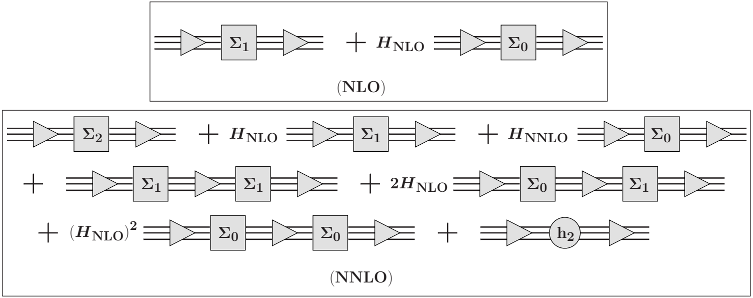
where

$$H_{\mathrm{LO}} = -\frac{3\omega_t^2}{\pi\Omega} = -\frac{3\omega_s^2}{\pi\Omega} = \frac{3\omega_t \omega_s}{\pi\Omega}.$$

Defining the functions



The NNLO triton propagator is



Calculating three-body forces and wavefunction renormalization of triton

- **Method 1:** Fix triton pole position at each order (Fixes three-body forces if binding energy fit to). Calculate residue about triton pole at each order to get triton wavefunction renormalization.
- **Method 2:** Note that in general triton pole and wavefunction renormalization given by perturbative expansion

$$1 - H\Sigma(E) =$$
$$1 - (H_0 + H_1 + \cdots)(\Sigma_0(B_0 + B_1 + \cdots) + \Sigma_1(B_0 + B_1 + \cdots) + \cdots) = 0$$

$$Z_\psi = \frac{1}{\Sigma'(E)} = \frac{1}{\Sigma_0'(E) + \Sigma_1'(E) + \cdots} = \frac{1}{\Sigma_0'(E)} - \frac{\Sigma_1'(E)}{\Sigma_0'(E)} + \cdots$$

Properly Renormalized Vertex Function

Ensuring that triton propagator has pole at triton binding energy gives conditions

$$H_{\mathrm{LO}} = \frac{1}{\Sigma_0(B)} \quad , \quad H_{\mathrm{LO}}\Sigma_1(B) + H_{\mathrm{NLO}}\Sigma_0(B) = 0,$$

$$H_{\mathrm{LO}}\Sigma_2(B) + H_{\mathrm{NLO}}\Sigma_1(B) + \left(H_{\mathrm{NNLO}} + \frac{4}{3}(M_N B + \gamma_t^2)\widehat{H}_2\right)\Sigma_0(B) = 0.$$

Triton wavefunction renormalization is residue about pole leads to triton vertex functions

$$\boldsymbol{\Gamma}_0(p) = \sqrt{Z_\psi^{\mathrm{LO}}}\mathcal{G}_0(B,p) \quad , \quad \sqrt{Z_\psi^{\mathrm{LO}}} = \sqrt{\frac{\pi}{\Sigma_0'(B)}}$$

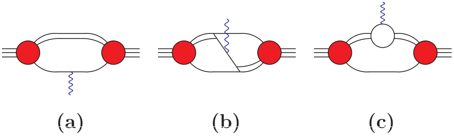
$$\boldsymbol{\Gamma}_1(p) = \sqrt{Z_\psi^{\mathrm{LO}}}\left[\mathcal{G}_1(B,p) - \frac{1}{2}\frac{\Sigma_1'}{\Sigma_0'}\mathcal{G}_0(B,p)\right].$$

$$\boldsymbol{\Gamma}_2(p) = \sqrt{Z_\psi^{\mathrm{LO}}}\left[\mathcal{G}_2(B,p) - \frac{1}{2}\frac{\Sigma_1'}{\Sigma_0'}\mathcal{G}_1(B,p) + \left\{\frac{3}{8}\left(\frac{\Sigma_1'}{\Sigma_0'}\right)^2 - \frac{1}{2}\frac{\Sigma_2'}{\Sigma_0'} - \frac{2}{3}M_N\widehat{H}_2\frac{\Sigma_0^2}{\Sigma_0'}\right\}\mathcal{G}_0(B,p)\right].$$

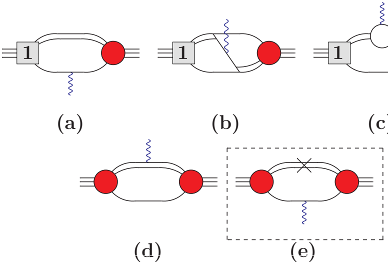
Triton Charge Form Factor

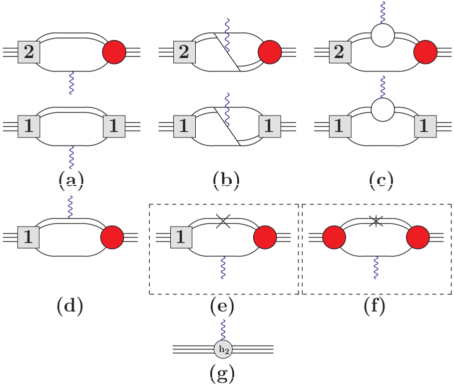
Charge form factor of triton at LO given by three diagrams

$$\hat{N}^{\dagger}\left[i\partial_0 + ie\left(\frac{1+\tau_3}{2}\right)\hat{A}_0\right]\hat{N}$$



NLO and NNLO triton charge form factor





LO triton charge form factor given by

$$Z_{\psi}^{\mathrm{LO}}\sum_{j=a,b,c}\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4p}{(2\pi)^4}\mathcal{G}_0^T(E,\vec{\mathbf{P}},p_0,\vec{\mathbf{p}})\chi_j(E,\vec{\mathbf{K}},\vec{\mathbf{P}},p_0,k_0,\vec{\mathbf{p}},\vec{\mathbf{k}})$$
$$\times\,\mathcal{G}_0(E,\vec{\mathbf{K}},k_0,\vec{\mathbf{k}}),$$

where $\mathcal{G}_0(E,\vec{\mathbf{K}},k_0,\vec{\mathbf{k}})$ is LO triton vertex function in a frame boosted by momentum $\vec{\mathbf{K}}$

$$\mathcal{G}_0(E,\vec{\mathbf{K}},k_0,\vec{\mathbf{k}})=\widetilde{\mathbf{B}}_0$$
$$+\left[\mathbf{R}_0\left(q,k,\frac{2}{3}B_0+k_0-\frac{\vec{\mathbf{K}}\cdot\vec{\mathbf{k}}}{3M_N}+\frac{\vec{\mathbf{k}}^2}{2M_N}\right)\mathbf{D}^{(0)}\left(B_0-\frac{\vec{\mathbf{q}}^2}{2M_N},\vec{\mathbf{q}}\right)\right]$$
$$\otimes\,\mathcal{G}_0(B_0,\vec{\mathbf{q}}).$$

In the Breit frame we have $\vec{\mathbf{Q}}=\vec{\mathbf{P}}-\vec{\mathbf{K}}$.

Focusing on diagram (a) we find

$$\left(\chi_a^{ji}(E,\vec{\mathbf{K}},\vec{\mathbf{P}},p_0,k_0,\vec{\mathbf{p}},\vec{\mathbf{k}})\right)^{\mu\alpha}_{\nu\beta} = ie(2\pi)^4\delta\left(k_0-p_0\right)\boldsymbol{\delta}^{(3)}\left(\vec{\mathbf{k}}-\vec{\mathbf{p}}-\frac{2}{3}\vec{\mathbf{Q}}\right)$$
$$\times\, i\mathbf{D}^{(0)}\!\left(\frac{2}{3}E+k_0,\vec{\mathbf{k}}+\frac{2}{3}\vec{\mathbf{K}}\right)\frac{i}{\frac{1}{3}E-k_0-\frac{(\vec{\mathbf{k}}-\frac{1}{3}\vec{\mathbf{K}})^2}{2M_N}+i\epsilon}$$
$$\times\,\frac{i}{\frac{1}{3}E-k_0-\frac{(\vec{\mathbf{k}}-\frac{2}{3}\vec{\mathbf{Q}}-\frac{1}{3}\vec{\mathbf{P}})^2}{2M_N}+i\epsilon}\left(\frac{1+\tau_3}{2}\right)^{\mu}_{\nu}\delta^{\alpha}_{\beta}\delta^{ij}.$$

Projecting in the doublet S-wave channel gives

$$\chi_a(E,\vec{\mathbf{K}},\vec{\mathbf{P}},p_0,k_0,\vec{\mathbf{p}},\vec{\mathbf{k}}) = ie(2\pi)^4\delta\left(k_0-p_0\right)\boldsymbol{\delta}^{(3)}\left(\vec{\mathbf{k}}-\vec{\mathbf{p}}-\frac{2}{3}\vec{\mathbf{Q}}\right)$$
$$\times\, i\mathbf{D}^{(0)}\!\left(\frac{2}{3}E+k_0,\vec{\mathbf{k}}+\frac{2}{3}\vec{\mathbf{K}}\right)\frac{i}{\frac{1}{3}E-k_0-\frac{(\vec{\mathbf{k}}-\frac{1}{3}\vec{\mathbf{K}})^2}{2M_N}+i\epsilon}$$
$$\times\,\frac{i}{\frac{1}{3}E-k_0-\frac{(\vec{\mathbf{k}}-\frac{2}{3}\vec{\mathbf{Q}}-\frac{1}{3}\vec{\mathbf{P}})^2}{2M_N}+i\epsilon}\begin{pmatrix}0 & 0\\ 0 & 2/3\end{pmatrix}.$$

Triton charge form factor

LO charge form-factor contribution from diagram (a) is

$$F_0^{(a)}(Q^2) = Z_\psi^{\rm LO}\left\{\widetilde{\boldsymbol{\mathcal{G}}}_0^T(p)\otimes\boldsymbol{\mathcal{A}}_0(p,k,Q)\otimes\widetilde{\boldsymbol{\mathcal{G}}}_0(k)\right.$$
$$\left.+2\widetilde{\boldsymbol{\mathcal{G}}}_0^T(p)\otimes\boldsymbol{\mathcal{A}}_0(p,Q)+\mathcal{A}_0(Q)\right\},$$

and NLO contribution is

$$F_1^{(a)}(Q^2) = Z_\psi^{\rm LO}\left\{\widetilde{\boldsymbol{\mathcal{G}}}_0^T(p)\otimes\boldsymbol{\mathcal{A}}_1(p,k,Q)\otimes\widetilde{\boldsymbol{\mathcal{G}}}_0(k)\right.$$
$$+\,2\widetilde{\boldsymbol{\mathcal{G}}}_1^T(p)\otimes\boldsymbol{\mathcal{A}}_0(p,k,Q)\otimes\widetilde{\boldsymbol{\mathcal{G}}}_0(k)$$
$$\left.+2\widetilde{\boldsymbol{\mathcal{G}}}_0^T(p)\otimes\boldsymbol{\mathcal{A}}_1(p,Q)+2\widetilde{\boldsymbol{\mathcal{G}}}_1^T(p)\otimes\boldsymbol{\mathcal{A}}_0(p,Q)+\mathcal{A}_1(Q)\right\},$$

where

$$\widetilde{\boldsymbol{\mathcal{G}}}_n(p) = \mathbf{D}^{(0)}\!\left(B_0-\frac{\vec{\mathbf{p}}^2}{2M_N},\vec{\mathbf{p}}\right)\boldsymbol{\mathcal{G}}_n(B_0,p).$$

Triton charge form factor

The vector term is

$$\boldsymbol{\mathcal{A}}_n(p,Q) = -\frac{M_N}{2\pi}\Big|_0^1 \int_0^\Lambda dq q^2 \int_{-1}^1 dx \frac{1}{qQx} \frac{1}{p\sqrt{q^2 - \frac{2}{3}qQx + \frac{1}{9}Q^2}}$$

$$\times\ Q_0\left(\frac{p^2+q^2+\frac{1}{9}Q^2+(y-\frac{2}{3})qQx - M_N B_0}{p\sqrt{q^2-\frac{2}{3}qQx+\frac{1}{9}Q^2}}\right)$$

$$\times\ D_s^{(n)}\!\left(B_0 - \frac{q^2}{2M_N} - \frac{Q^2}{12M_N} + \left(\frac{1}{2}-y\right)\frac{qQx}{M_N}, \vec{\mathbf{q}}\right)\begin{pmatrix} 2 \\ -2/3 \end{pmatrix},$$

and scalar term is

$$\mathcal{A}_n(Q) = \frac{M_N}{4\pi^2}\Big|_0^1 \int_0^\Lambda dq q^2 \int_{-1}^1 dx \frac{1}{qQx}\frac{2}{3}$$

$$\times\ D_s^{(n)}\!\left(B_0 - \frac{q^2}{2M_N} - \frac{Q^2}{12M_N} + \left(\frac{1}{2}-y\right)\frac{qQx}{M_N}, \vec{\mathbf{q}}\right).$$

LO triton charge form factor

$$F_0(Q^2) = F_0^{(a)}(Q^2) + F_0^{(b)}(Q^2) + F_0^{(c)}(Q^2),$$

NLO triton charge form factor

$$F_1(Q^2) = \left(F_1^{(a)}(Q^2) + F_1^{(b)}(Q^2) + F_1^{(c)}(Q^2) + F_1^{(d)}(Q^2)\right) - \frac{\Sigma_1'}{\Sigma_0'} F_0(Q^2),$$

and NNLO triton charge form factor

$$\begin{aligned} F_2(Q^2) = & \left(F_2^{(a)}(Q^2) + F_2^{(b)}(Q^2) + F_2^{(c)}(Q^2) + F_2^{(d)}(Q^2)\right) \\ & - \frac{\Sigma_1'}{\Sigma_0'} \left(F_1^{(a)}(Q^2) + F_1^{(b)}(Q^2) + F_1^{(c)}(Q^2) + F_1^{(d)}(Q^2)\right) \\ & + \left(\left(\frac{\Sigma_1'}{\Sigma_0'}\right)^2 - \frac{\Sigma_2'}{\Sigma_0'} - \frac{4}{3} M_N \widehat{H}_2 \frac{\Sigma_0^2}{\Sigma_0'}\right) F_0(Q^2) + \frac{4}{3} M_N \widehat{H}_2 \frac{\Sigma_0^2}{\Sigma_0'} \\ & - \frac{\left\langle r_p^2 \right\rangle}{6} Q^2 F_0(Q^2) - \frac{\left\langle r_n^2 \right\rangle}{6} Q^2 F_n(Q^2). \end{aligned}$$

How to get charge radius

The triton charge form factor expanded in powers of Q^2 yields

$$F(Q^2) = 1 - \frac{\left\langle r_{^3H}^2 \right\rangle}{6} Q^2 + \cdots$$

- **Method 1:** Calculate charge form factor for various low values of Q^2. Fit a line as function of Q^2 to the resulting data. The slope of this line is related to the charge radius.
- **Method 2:** Expand all diagrams as functions of Q^2 and take only Q^2 pieces. Then calculate this and obtain the charge radius. Has advantage of allowing more integrals to be done analytically. Therefore is more numerically stable and allows higher cutoffs to be calculated.

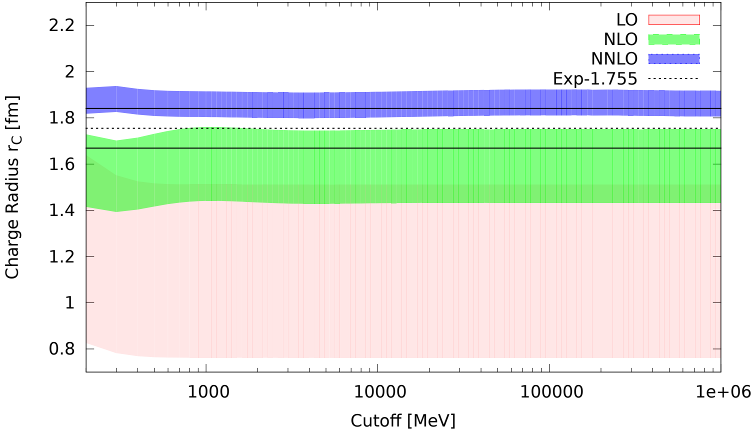
The triton charge radius squared to NNLO is given by

$$\left\langle r_{^3\mathrm{H}}^2\right\rangle_2 = \left\langle r_{^3\mathrm{H}}^2\right\rangle_{\widetilde{2}} + \left\langle r_p^2\right\rangle + 2\left\langle r_n^2\right\rangle .$$

Taking the square root and expanding perturbatively gives

$$r_c = \sqrt{\left\langle r_{^3\mathrm{H}}^2\right\rangle_0}\left(\underbrace{1}_{\mathrm{LO}} + \underbrace{\frac{1}{2}\frac{\left\langle r_{^3\mathrm{H}}^2\right\rangle_1}{\left\langle r_{^3\mathrm{H}}^2\right\rangle_0}}_{\mathrm{NLO}} + \underbrace{\frac{1}{2}\frac{\left\langle r_{^3\mathrm{H}}^2\right\rangle_2}{\left\langle r_{^3\mathrm{H}}^2\right\rangle_0} - \frac{1}{8}\left(\frac{\left\langle r_{^3\mathrm{H}}^2\right\rangle_1}{\left\langle r_{^3\mathrm{H}}^2\right\rangle_0}\right)^2}_{\mathrm{N^2LO}} + \cdots\right).$$

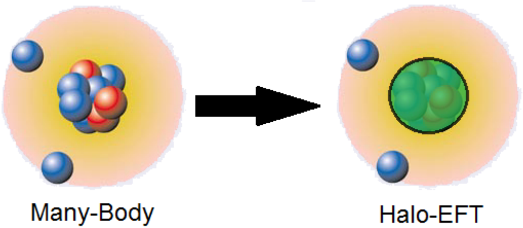
LO $\mathrm{EFT}_{\not\pi}$ prediction via wavefunctions $r_C = 2.1 \pm .6 fm$ (Platter and Hammer (2005))



and now for something completely different...?

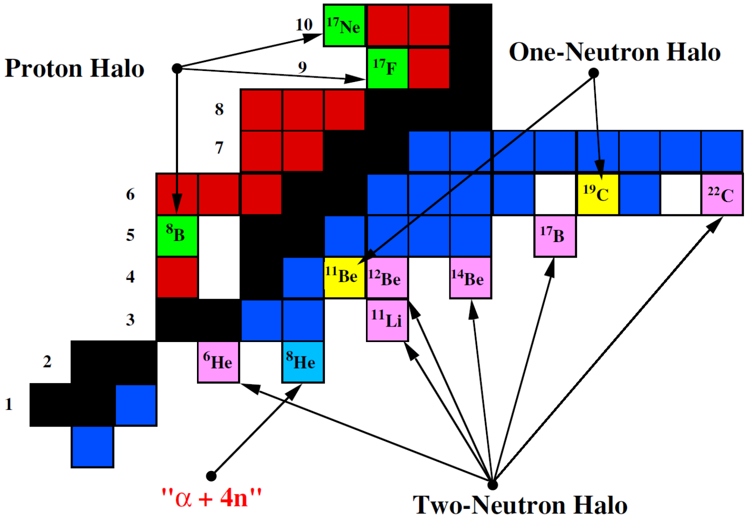
Halo-Nuclei

- For halo-nuclei $R_{halo} > R_{core}$, can expand in powers of R_{core}/R_{halo}.



- If a probe has De Broglie wavelength λ, and $\lambda > R_{core}$ the structure of the core cannot be resolved and it can be treated as a fundamental degree of freedom.
- Breakdown scales of halo-EFT set by $E_{\star}$ (first excited state energy of core) and m_{π}

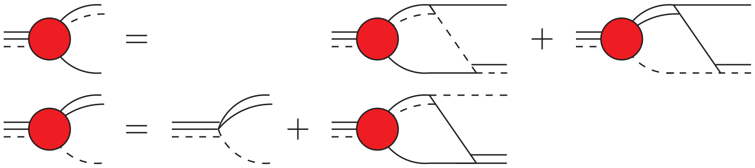
Halo-Nuclei



http://www.nupecc.org/report97/report97.pdf

Halo-Nuclei

- LO halo-nuclei vertex function given by (Hagen, Hammer, and Platter (2013))



- S-wave interactions in both two and three-body sector
- Nearly identical to pionless EFT
- Differences from pionless EFT: core is spin-0, three-body force chosen differently, and parameters will have different values

Unitary equal mass limit

Calculation of LO halo-nuclei charge radius nearly identical to triton charge radius calculation. In Unitary limit and equal mass limit it is found

Authors	$mE_{3B}\left\langle r_c^2\right\rangle$
Vanasse	.224
Hagen et al.	.265

Using analytical techniques in (Braaten and Hammer (2006)) it can be shown that $mE_{3B}\left\langle r_c^2\right\rangle = (1+s_0^2)/9 \approx .224$ in the unitary and equal mass limit. Changing a single factor in the code of Hagen et al. they would also obtain .224.

Conclusions

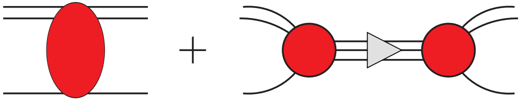
- Can now calculate bound state properties strictly perturbatively in $\mathrm{EFT}_{\not\pi}$.
- Further work needs to be done considering other bound state properties and including Coulomb interactions to probe properties of Helium-3.
- Various techniques can calculate bound state properties of the triton. These techniques should all be benchmarked against known analytical solutions in certain limits.
- Techniques should give $M_N E_{^3\mathrm{H}} \left\langle r^2_{^3\mathrm{H}} \right\rangle = .224...$ in unitary and equal mass limit.
- Techniques should produce Efimov spectrum in unitary limit.

Conclusions and Future directions

- Calculating the *nd* scattering amplitude to higher orders in $\mathrm{EFT}_{\not\pi}$ strictly perturbatively is made easier by new techniques.
- Calculating *nd* scattering to higher orders will allow investigation of polarization observables, in particular A_y.
- *nd* scattering to $\mathrm{N}^4\mathrm{LO}$ will require insertion of three-body *SD*-mixing terms, three-body *P*-wave corrections, and etc...
- Now that bound states can also be calculated perturbatively, one can consider calculations including external currents such as $\gamma + {}^3\mathrm{He} \rightarrow p + d$, $\gamma + {}^3\mathrm{H} \rightarrow n + d$,$\gamma + {}^3\mathrm{He} \rightarrow \gamma + {}^3\mathrm{He}$, $\gamma + {}^3\mathrm{H} \rightarrow \gamma + {}^3\mathrm{H}$, and ${}^3\mathrm{H} \rightarrow e^- + \bar{\nu}_e + {}^3\mathrm{He}$.
- Further work needs to be done on disagreement in halo-nuclei

LO three-body force

The LO doublet S-wave amplitude for nd scattering is given by the sum of diagrams



,

which gives

$$T_{LO} = t_{LO} + H_{LO}\frac{1}{1 - H_{LO}\Sigma_0(E)}\pi Z_{LO}\left(G_{0,Nt\to Nt}(E,k)\right)^2,$$

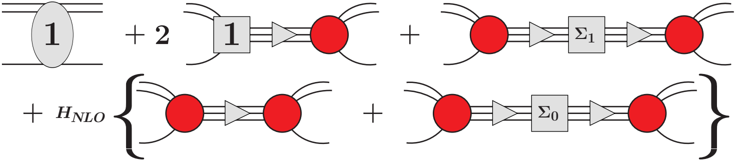
where

$$t_{LO} = Z_{LO}t^{\ell=0}_{0,Nt\to Nt}(k,k)$$

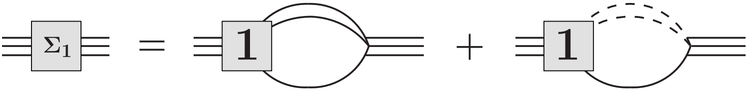
Fitting to the doublet S-wave nd scattering length a_{nd}, H_{LO} is given by

$$H_{LO} = \frac{x}{1 + x\Sigma_0(E)} \quad , \quad x = \frac{-\left(\frac{3\pi a_{nd}}{M_N} + T_{LO}\right)}{\pi Z_{LO}\left(G_{0,Nt\to Nt}(E,k)\right)^2}$$

The NLO doublet *S*-wave amplitude for *nd* scattering is given by the sum of diagrams



where



The $\mathrm{N^2LO}$ contribution can be calculated similarly but has many more contributions.

Integral equations for *nd* scattering amplitude

Projecting out in total angular momentum $\vec{\mathbf{J}} = \vec{\mathbf{L}} + \vec{\mathbf{S}}$ we obtain the set of coupled integral equations in cluster configuration space

$$
\begin{aligned}
\mathbf{t}^{(n)}{}_{\beta,\alpha}(k,p) &= \mathbf{K}^{(n)}{}_{\beta,\alpha}(k,p,E) \\
&+ \sum_{\gamma} \sum_{i=1}^{n-1} \mathbf{K}^{(n-i)}{}_{\beta,\gamma}(q,p,E) \otimes \left(\mathbf{R}^{(0)} \left(E - \frac{q^2}{2M_N}, q \right) \circ \mathbf{t}^{(i)}_{\gamma,\alpha}(k,q) \right) \\
&+ \sum_{i=1}^{n-1} \mathbf{R}^{(n-i)} \left(E - \frac{p^2}{2M_N}, p \right) \circ \mathbf{t}^{(i)}_{\beta,\alpha}(k,p) \\
&+ \mathbf{K}^{(0)}_{\beta,\beta}(q,p,E) \otimes \left(\mathbf{R}^{(0)} \left(E - \frac{q^2}{2M_N}, q \right) \circ \mathbf{t}^{(n)}_{\beta,\alpha}(k,q) \right)
\end{aligned}
$$

where $\alpha = J, L, S, \beta = J, L', S'$, and $\gamma = J, L'', S''$ and

$$A(q) \otimes B(q) = \frac{1}{2\pi^2} \int_0^{\Lambda} dq q^2 A(q) B(q),$$

$\circ$: Schur product in cluster-configuration space

Three-body breakup cross-section