Regulator dependence and isospin breaking

Sebastian König

EMMI RRTF Workshop:
"The systematic treatment of the Coulomb interaction in few-body systems"

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The Ohio State University

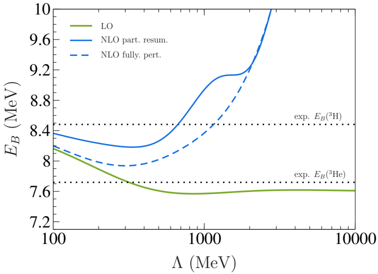
NSF

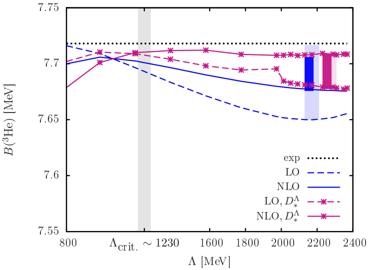
NUCLEI
Nuclear Computational Low-Energy Initiative

Motivation: counterterm controversy

Should there be a new counterterm at NLO!

Yes! Vanasse *et al.*, PRC **89** (2014) 064003 No! Kirscher+Gazit, PLB **755** (2016) 253





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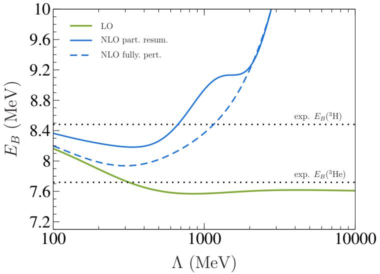

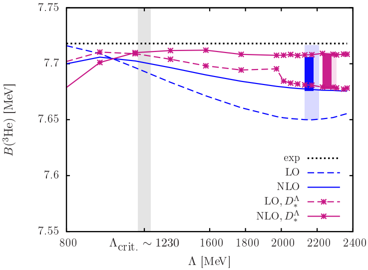

Vanasse *et al.*

- momentum-space formalism
- sharp cutoff (non-local)
- can take Λ arbitrarily large

Kirscher and Gazit

- configuration-space (R)RGM
- Gaussian regulators (local)
- limited cutoff range

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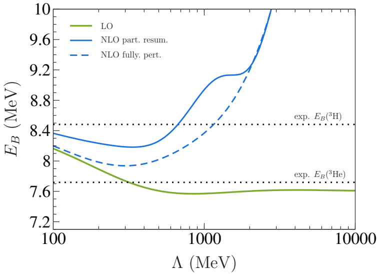

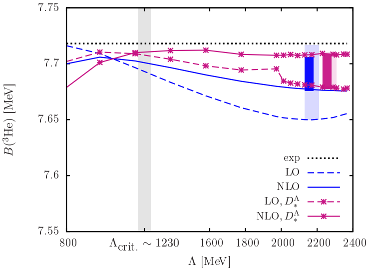

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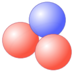
power counting $\leftrightarrow$ regulators?

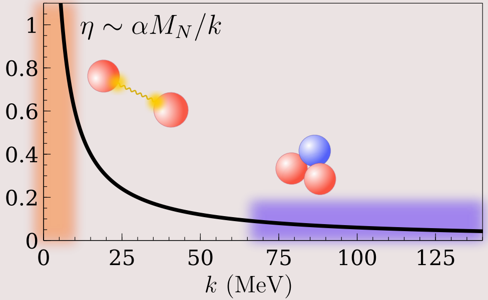
Coulomb corrections for nuclei



How much Coulomb should we really iterate?

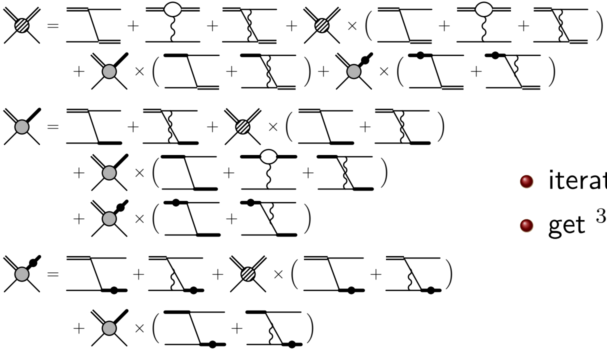
Coulomb regimes

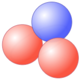
trinucleon binding momentum ~ 80 MeV . . .



$\hookrightarrow$ **should Coulomb not be a small perturbative correction?**

Nonperturbative vs. perturbative and helium





- iterate $\mathcal{O}(\alpha)$ diagrams...
- get $^3\mathrm{He}$ pole directly

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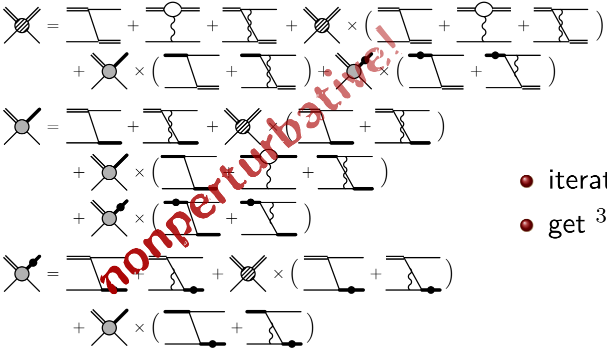

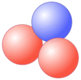

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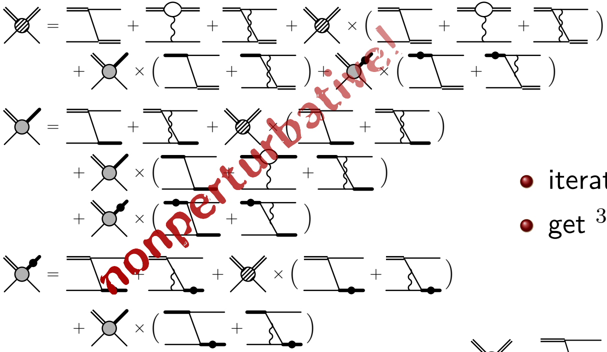

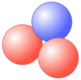

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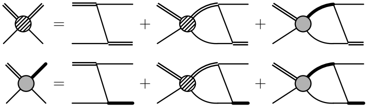
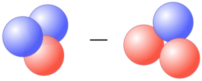
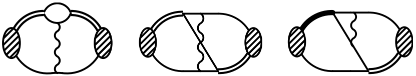


- use trinucleon wavefunctions
- fully perturbative in α!

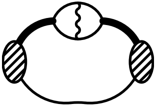
$\Delta E :$ 

$$\Delta E = \langle \psi | V_C | \psi \rangle$$



SK, Grießhammer, Hammer, J. Phys. G **42** 045101 (2015)

Coulomb bubble divergence



- an additional diagram is logarithmically divergent. . .
- . . . but this divergence comes from the photon-bubble subdiagram!

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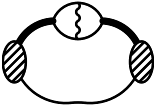

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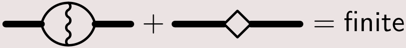
Strategy

SK, Grießhammer, Hammer, van Kolck, JPG **43** 055106 (2016), 1508.05085 [nucl-th]

1. isolate divergence: 
2. **take the leading-order $^1S_0$ in the unitarity limit!**

$$a_{^1S_0} = -23.7 \approx \infty \rightsquigarrow 1/a_{^1S_0} \approx 0$$

3. include divergent diagram together with finite $a_{^1S_0}$



A new expansion

Take the leading-order singlet channel in the unitarity limit!

$a_t = -23.4 \approx \infty$!

$\sigma_t = \sigma_t^{(0)} + \sigma_t^{(1)}$

$$\boxed{\sigma_t^{(0)} - 2\Lambda/\pi = 0}$$

$$\mathrm{i}\Delta_t^{(0)}(p_0, \mathbf{p}) \frac{-\mathrm{i}}{\sigma_t + y_t^2 I_0(p_0, \mathbf{p})} \to \frac{-\mathrm{i}}{\sqrt{\frac{\mathbf{p}^2}{4} - M_N p_0 - \mathrm{i}\varepsilon}}$$

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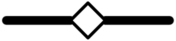
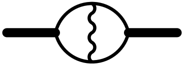
NLO corrections

- scattering length: $\sigma_t^{(1)} \sim -1/a_t$
- effective range: $c_t^{(1)} = \dfrac{M_N r_{0t}}{2}$



$\sim \; -\mathrm{i}\sigma^{(1)}_{t(,pp)}$

$\sim \; -\mathrm{i}c_t^{(1)}$



$$\mathrm{i}\Delta_t^{(1)}(p_0,\mathbf{p}) = \mathrm{i}\Delta_t^{(0)}(p_0,\mathbf{p}) \times \left[-\mathrm{i}\sigma_t^{(1)} - \mathrm{i}c_t^{(1)}\left(p_0 - \frac{\mathbf{p}^2}{4M_N}\right)\right] \times \mathrm{i}\Delta_t^{(0)}(p_0,\mathbf{p})$$

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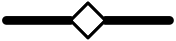

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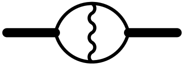

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- new ${}^1S_0$ LO is isospin-symmetric and parameter-free
- **allows matching between perturbative and non-perturbative Coulomb regimes**

Divergence dissection

Vanasse, Egolf, Kerin, SK, Springer, PRC **89** 064003 (2014)

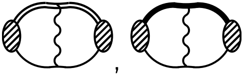
Look at structure of p-d three-body force:

$$H_{0,1}^{(\alpha)}(\Lambda) = h_I^{(\alpha)}(\Lambda) + h_\kappa^{(\alpha)}(\Lambda)$$

$$h_I^{(\alpha)}(\Lambda) = -\frac{3\pi(1+s_0^2)}{16}\left\{\boxed{\frac{1}{12}(r_{p\text{–}p} - r_t)\Lambda\left[1 - \cdots\right]}\right.$$

$+$ various terms $\sim \log\Lambda$, all $\propto (r_{p\text{–}p} - r_t)$ or $\propto (\gamma_{p\text{–}p} - \gamma_t)\Big\} / \sin^2(\cdots)$

$$h_\kappa^{(\alpha)} = -\frac{\sqrt{3}\kappa\pi(1+s_0^2)}{48}\Big\{\text{various terms} \sim \log\Lambda$$



that do not vanish in the isospin limit $+ \Psi(\Lambda)\Big\} / \sin^2(\cdots)$

- pieces associated with $r_{p\text{–}p} \neq r_t$ and $\alpha\rho_d$, αr_t
- these have been relagated to a higher order in the new scheme!

SK, Grießhammer, Hammer, van Kolck, JPG **43** 055106 (2016), 1508.05085 [nucl-th]

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Regulator dependence: overview

power counting $\leftrightarrow$ regulators?

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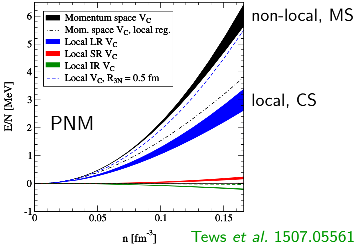
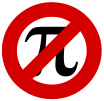
similar issues in chiral EFT!



Tews *et al.* 1507.05561 [nucl-th]



pionless EFT can study this question in a clean scenario without other complications!

- chiral EFT with Weinberg counting does not absorb all cutoff dependence in counterterms
- pionless EFT formulated with RG invariance
- different schemes can still give different runnings for LECs

Pionless two- and three-body regularization

Momentum-space overview

- typically used in pionless EFT with dibaryons:
  dim. reg. (PDS) in two-body sector, cutoff in three-body equation

$$\Delta(p) \sim \frac{1}{\underbrace{-1/a}_{=\sigma-\mu} + \sqrt{\frac{\mathbf{p}^2}{4} - M_N p_0 - \mathrm{i}\varepsilon}} \ , \ T(E;k,p) = K(E;k,p) + \int_0^{\Lambda} \mathrm{d}q^2 K(E;k,q) D(E;q) T(E;q,p)$$

- as long as everything converges, this should not matter

  Bedaque *et al.*, NPA **676** 357 (2000)

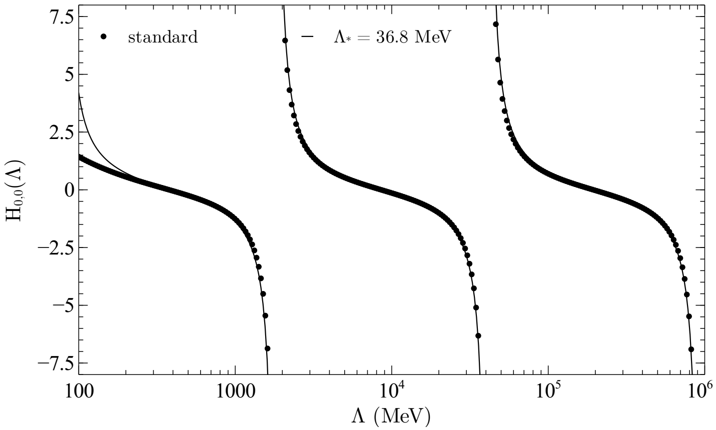
- **to check this:** use cutoff regulator for two-body system,
  keep finite-Λ corrections

  Vanasse, Coulomb RRTF Part I (Jan. 2016)

- Faddeev calculations use Gaussian regulator functions for two- and three-body sector

  Platter *et al.*, PRA **70** 052101 (2004), PLB **607** 254 (2005), ...

Three-body force with two-body cutoff regulator



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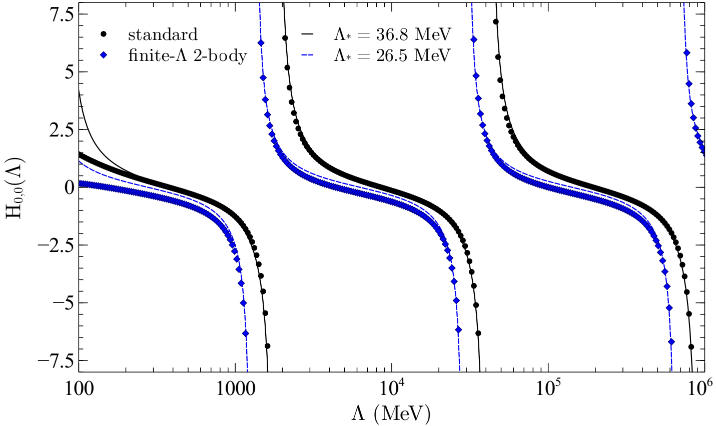
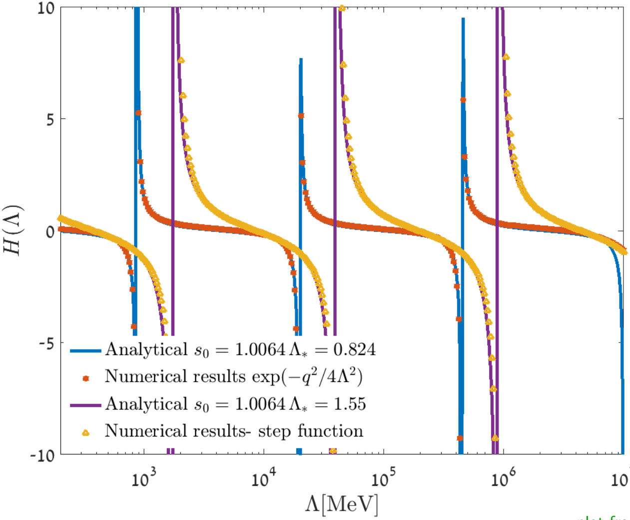


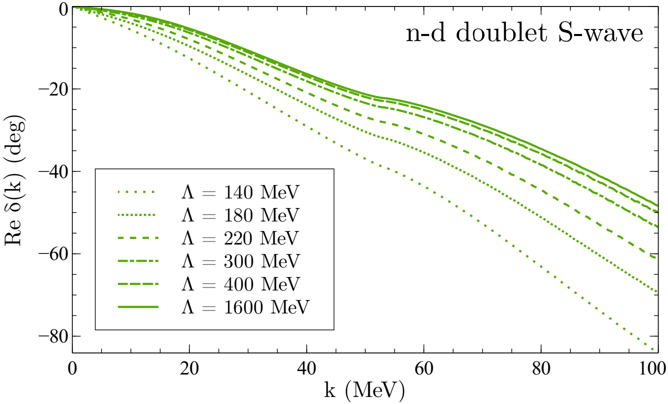
$$\Delta(p_0,\mathbf{p}) \sim \frac{1}{-\gamma + \sqrt{\frac{\mathbf{p}^2}{4} - M_N p_0 - \mathrm{i}\varepsilon} - \frac{2}{\pi}\sqrt{\frac{\mathbf{p}^2}{4} - M_N p_0 - \mathrm{i}\varepsilon}\,\arctan\left(\sqrt{\frac{\mathbf{p}^2}{4} - M_N p_0 - \mathrm{i}\varepsilon}/\Lambda\right)}$$

Three-body force with Gaussian regulators



plot from H. Deleon, Jan. 2016

nd doublet phase shift with two-body cutoff regulator



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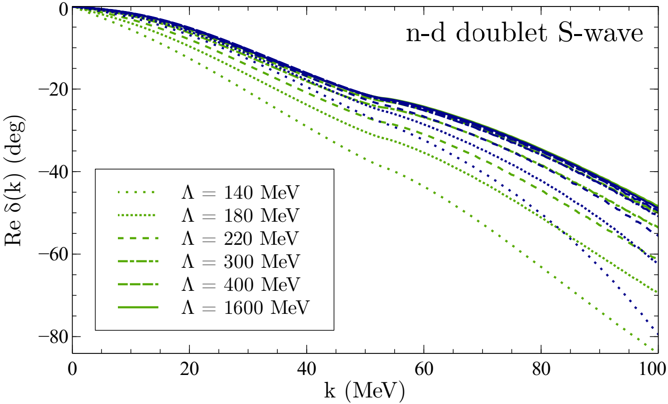

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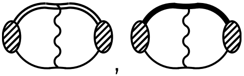

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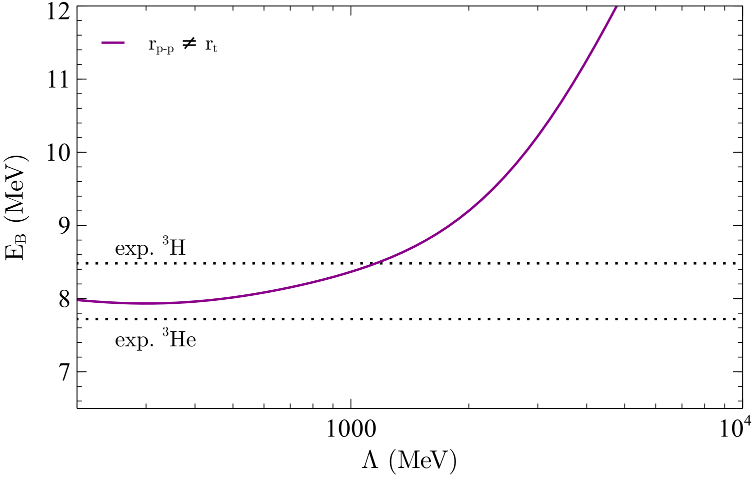

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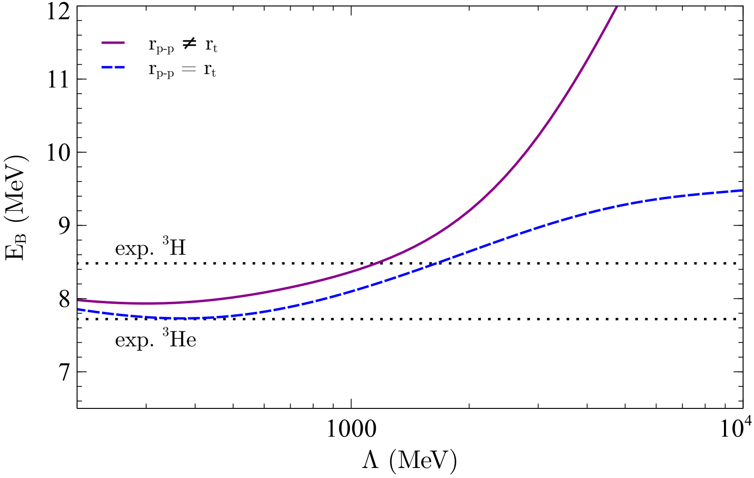

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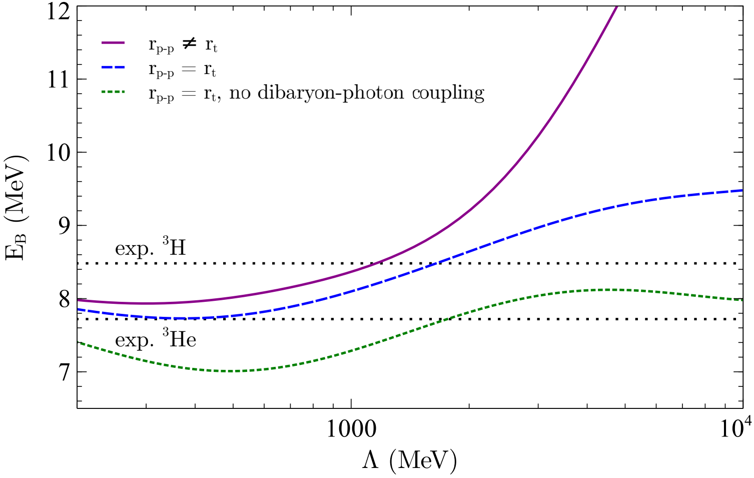

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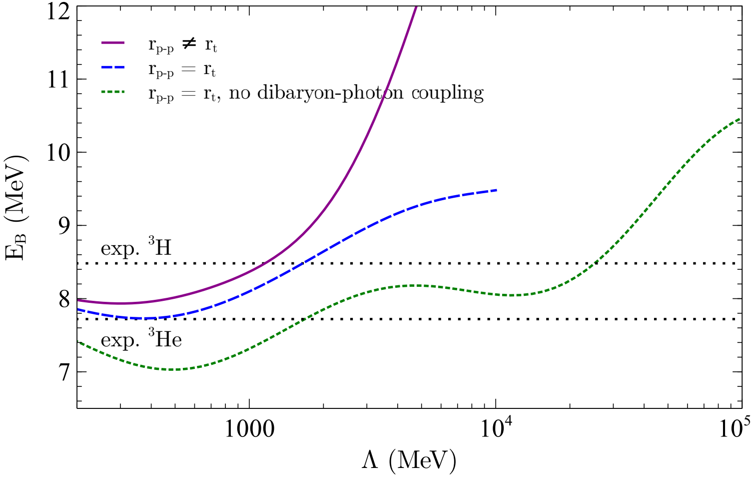

NLO divergence from isospin breaking (no Coulomb)

- fit three-nucleon force with isospin-symmetric $^1S_0$ dibaryon
- use this in calculation with separate n-n channel